

Announcements

- Quiz 2 re-try this week (Thu-Sat)
 - Same material, different problems
 - Come to my office hours for Quiz 2 questions
- No lecture Friday (2/15) 😊
 - Friday office hours will still meet

☐ Upcoming deadlines:

- Friday (2/15)
 - Written Assignment
- Tuesday (2/19)
 - PL HW

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Objective

- Free body diagram for 2D rigid body
- Types of constraints
- Equations of equilibrium for 2D rigid body

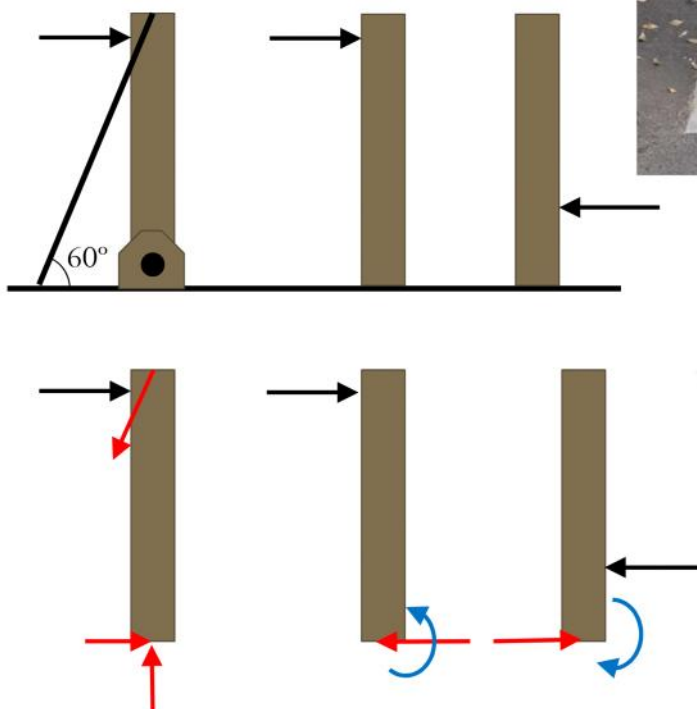
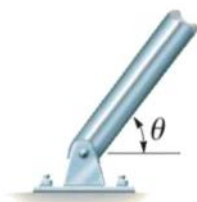
Equilibrium in two-dimensional bodies

Why different support?



Equilibrium in two-dimensional bodies

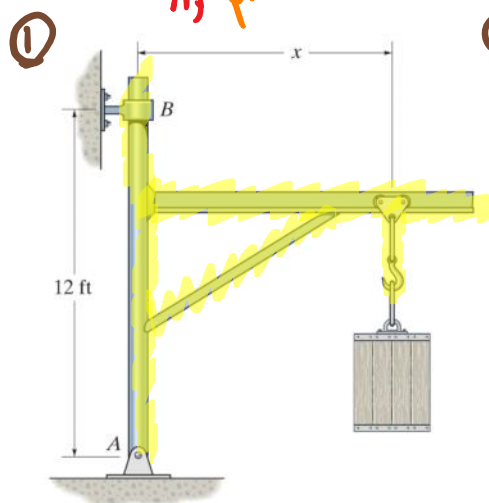
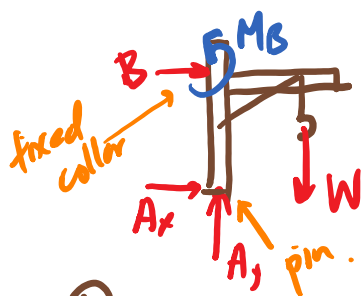
Active Forces vs. Support reaction components



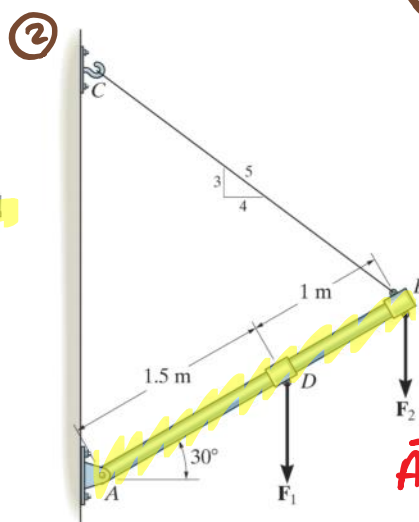
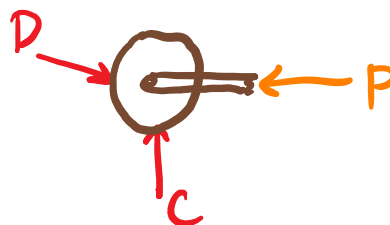
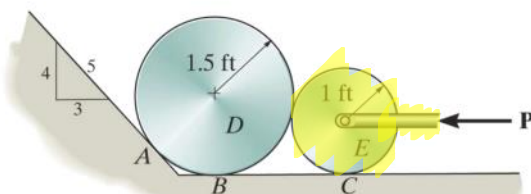
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Free Body Diagrams

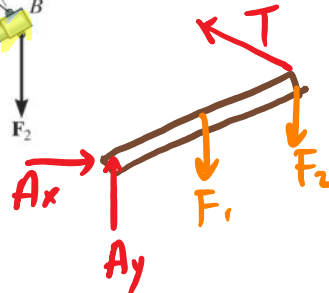
① FBD for frame AB



③ FBD for disk E



FBD for AB



Constraints

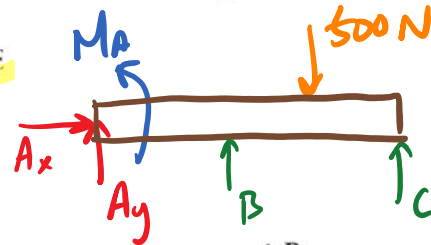
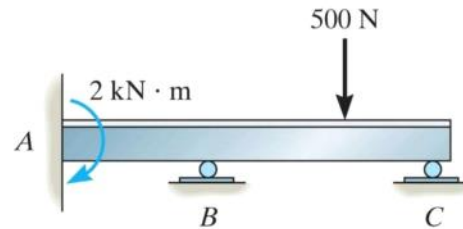
To ensure equilibrium of a rigid body, it is not only necessary to satisfy equations of equilibrium, but the body must also be properly constrained by its supports

- **Redundant constraints:** the body has more supports than necessary to hold it in equilibrium; the problem is **STATICALLY INDETERMINATE** and cannot be solved with statics alone

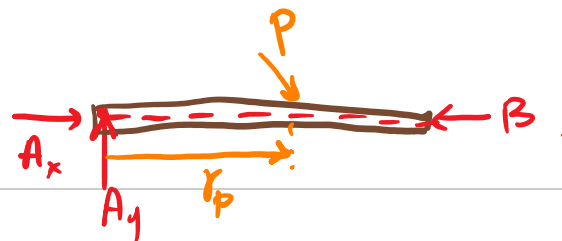
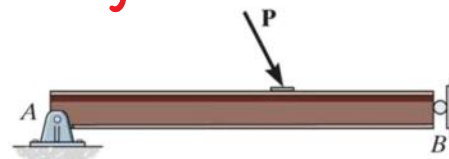
Equilibrium Equation:

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

- **Improper constraints:** In some cases, there may be as many unknown reactions as there are equations of equilibrium. However, if the supports are not properly constrained, the body may become unstable for some loading cases.



5 unknowns w/
4 equations

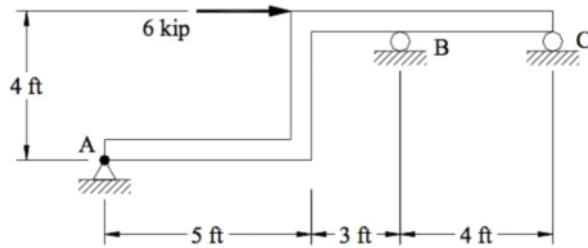


$\sum M_A = \vec{r}_P \times \vec{P} \neq 0$
 \rightarrow equilibrium cannot be
 achieve

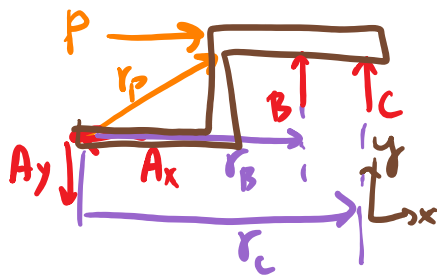
Constraints

Proper, redundant, or improper constraints

a)



• Properly constrained.



$$\Sigma F_x = 0 = P - A_x = 0 \quad \checkmark$$

$$\Sigma M_A = \vec{r}_P \times \vec{P} + \vec{r}_B \times \vec{B} + \vec{r}_C \times \vec{C} = 0 \quad \checkmark$$

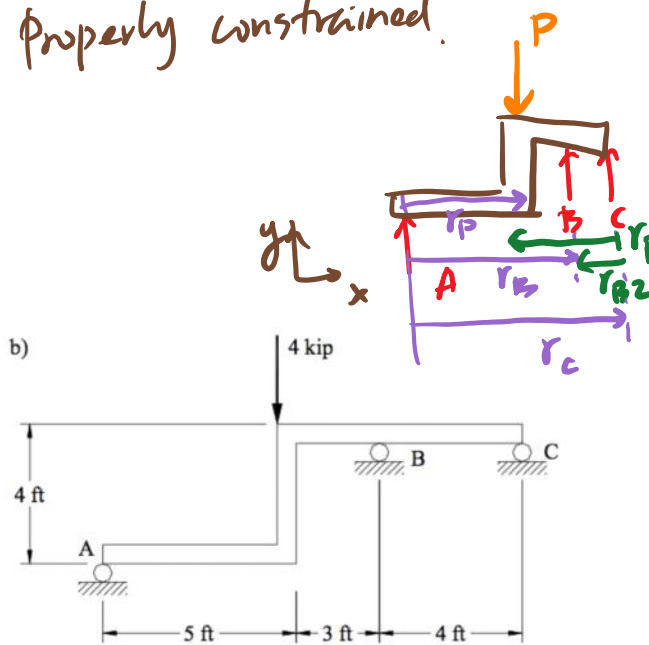
$(-\hat{k}) \quad (\hat{k}) \quad (\hat{k})$

$$\Sigma F_y = B + C - A = 0 \quad \checkmark$$

Constraints

Proper, redundant, or improper constraints

- properly constrained.



$$\Sigma F_y = -P + A + B + C = 0 \quad \checkmark$$

$$\Sigma M_A = \vec{r}_P \times \vec{P} + \vec{r}_B \times \vec{B} + \vec{r}_C \times \vec{C} = 0 \quad \checkmark$$

$(-\hat{k}) \quad (+\hat{k}) \quad (+\hat{k})$

$$\Sigma M_C = \vec{r}_{B2} \times \vec{B} + \vec{r}_A \times \vec{A} + \vec{r}_{P2} \times \vec{P}$$

Redundancy? No.

3 unknowns

3 eqns. of equil.

Equations of Equilibrium options:

- 1.) $\Sigma F_x = 0$ 1.) $\Sigma F_y = 0$
- 2.) $\Sigma F_y = 0$ 2.) $\Sigma M_A = 0$
- 3.) $\Sigma M_A = 0$ 3.) $\Sigma M_C = 0$.

- You may use combinations of M equations about different points if less than 2 F equations are used.

Constraints

Proper, redundant, or improper constraints

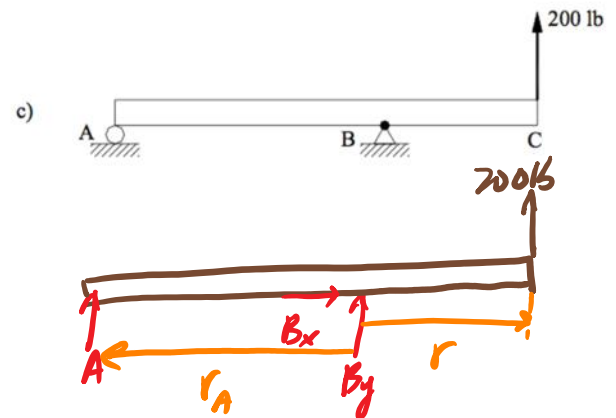
Properly constrained

$$\sum F_x = B_x = 0$$

$$\sum F_y = A + B_y - 200 \text{ lb} = 0$$

$$\sum M_B = r(200 \text{ lb}) - r_A A = 0$$

Redundant? No.

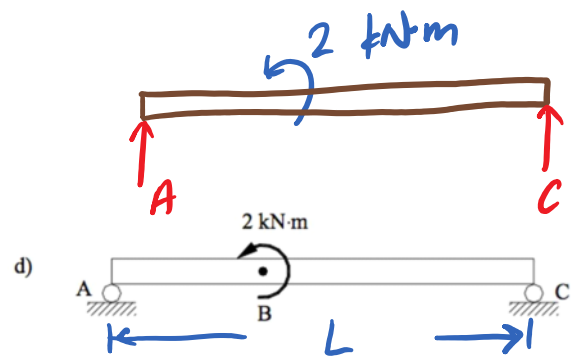


Constraints

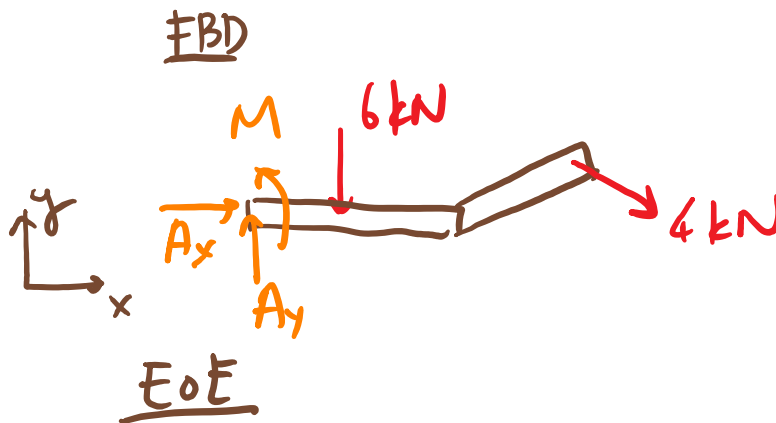
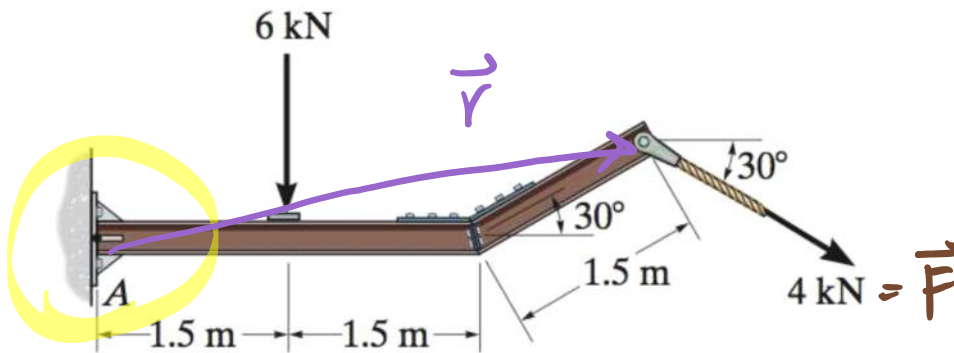
Proper, redundant, or improper constraints

- Improperly constrained.

$$\sum M_A = 2 \text{ kN}\cdot\text{m} + l \cdot C \neq 0$$



Determine the components of the support reactions at the fixed support A on the cantilevered beam.



$$\sum F_x = A_x + (4 \text{ kN}) \cos 30^\circ = 0$$

$$\sum F_y = A_y - 6 \text{ kN} - (4 \text{ kN}) \sin 30^\circ = 0$$

$$\sum M_A = M - (1.5 \text{ m})(6 \text{ kN}) + \vec{r} \times (\vec{F}) = 0$$