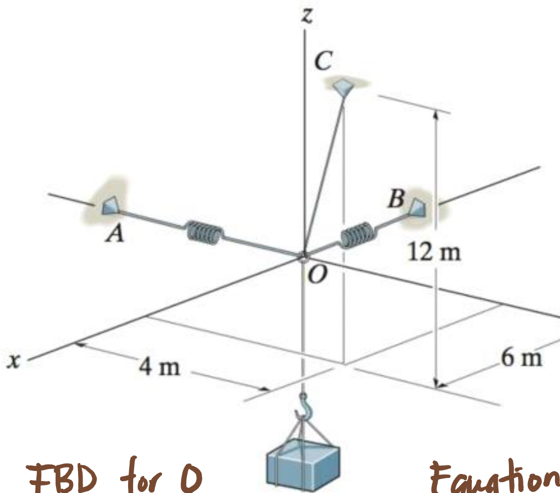
Goals and Objectives

- Example problems for particle(s) at equilibrium in 2D and 3D.

1

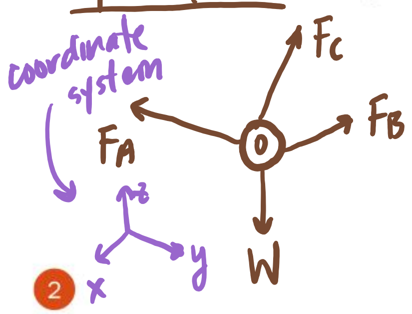
Example – Single Particle in 3D



Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of k = 360 N-m.

Plan: Use ring O to relate all the forces involved (known & unknown) to each other.

FBD for O



2

Equations of Equilibrium (EoE)

$$\Sigma F_x = F_{Ax} + F_{Bx} + F_{Cx} = 0$$
$$\Sigma F_y = F_{Ay} + F_{By} + F_{Cy} = 0$$
$$\Sigma F_z = F_{Az} + F_{Bz} + F_{Cz} - W = 0$$

• Find the Cartesian components of each force through the given figure:

$F_{spring} = ks$

$$\vec{F}_A = F_{Ax}\hat{i} + F_{Ay}\hat{j} + F_{Az}\hat{k} = 0\hat{i} - ks_A\hat{j} + 0\hat{k} = 0$$
$$\vec{F}_B = F_{Bx}\hat{i} + F_{By}\hat{j} + F_{Bz}\hat{k} = -ks_B + 0\hat{j} + 0\hat{k} = 0$$
$$\vec{F}_C = F_{Cx}\hat{i} + F_{Cy}\hat{j} + F_{Cz}\hat{k} = F_C\hat{u}_C = F_C\left(\frac{6}{14}\right)\hat{i} + F_C\left(\frac{4}{14}\right)\hat{j} + F_C\left(\frac{12}{14}\right)\hat{k}$$
$$\hat{u}_C = \frac{6\hat{i} + 4\hat{j} + 12\hat{k}}{\sqrt{6^2+4^2+12^2}} = \frac{6}{14}\hat{i} + \frac{4}{14}\hat{j} + \frac{12}{14}\hat{k}$$

EoE becomes

$$\Sigma F_x = 0 + (-ks_B) + F_C\left(\frac{6}{14}\right) = 0$$
$$\Sigma F_y = (-ks_A) + 0 + F_C\left(\frac{4}{14}\right) = 0$$
$$\Sigma F_z = 0 + 0 + F_C\left(\frac{12}{14}\right) - W = 0$$

Solve for s_A & s_B with $W = (20\,kg)(9.81\,\frac{m}{s^2})$

$F_C \approx 229$ N

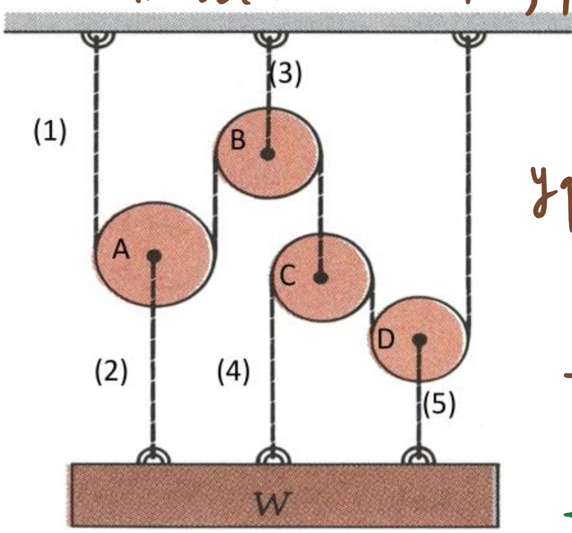
$s_A \approx 0.182$ m

$s_B \approx 0.273$ m

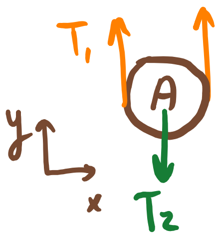
Example – System of Particles

The five ropes can each take 1500 N without breaking. How heavy can *W* be without breaking any?

Plan: See which cable will experience the most force by relating them to each other via FBD, pick 1 as reference for comparison. (e.g. T_1)



FBD for A

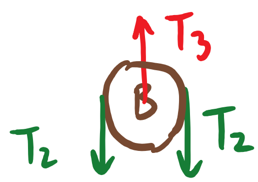


EoE

$$\Sigma F_y = 2T_1 - T_2 = 0$$

$$T_2 = 2T_1$$

FBD for B

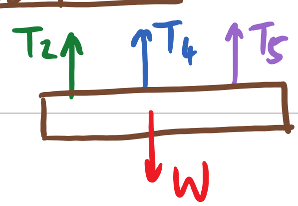


EoE

$$\Sigma F_y = T_3 - 2T_2 = 0$$

$$T_3 = 2T_2 = 4T_1$$

FBD for W



EoE

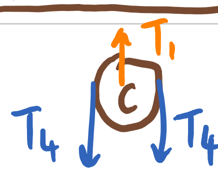
$$\Sigma F_y = T_2 + T_4 + T_5 - W = 0$$

$$\rightarrow W = T_2 + T_4 + T_5$$

$$= 2T_1 + \tfrac{1}{2}T_1 + T_1$$

$$= 3\tfrac{1}{2}T_1$$

FBD for C

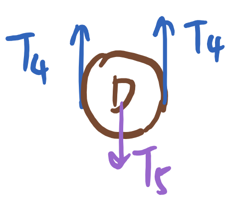


EoE

$$\Sigma F_y = T_1 - 2T_4 = 0$$

$$T_4 = \tfrac{1}{2}T_1$$

FBD for D



EoE

$$\Sigma F_y = 2T_4 - T_5 = 0$$

$$T_5 = 2T_4 = T_1$$

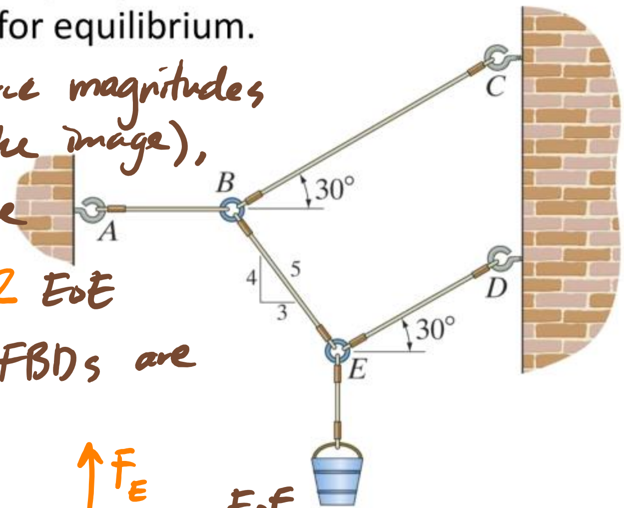
- Since $T_3\ (=4T_1) > T_2\ (=2T_1) > T_5\ (=T_1) > T_4\ (=\tfrac{1}{2}T_1)$,

T_3 experiences the highest load, so it is the limiting factor.

Let $T_3 = 1500\,N = 4T_1 = 4\dfrac{W}{(7/2)}$, the maximum $W = 1312.5\,N$

Example – System of Particles

The 30-kg bucket is supported at E by a system of five cords. Determine the force in each cord for equilibrium.



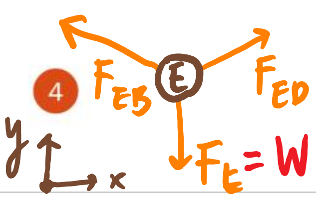
There are 5 cords with unknown force magnitudes (but unknown direction based on the image), 5 equations are needed to solve for them. Each FBD can produce 2 EoE ($\Sigma F_x = 0$ & $\Sigma F_y = 0$), at least 3 FBDs are necessary to solve the problem.

First use FBD for the bucket:

EoE

$\Sigma F_y = F_E - W = 0 \longrightarrow F_E = W$

Second, FBD for ring E:

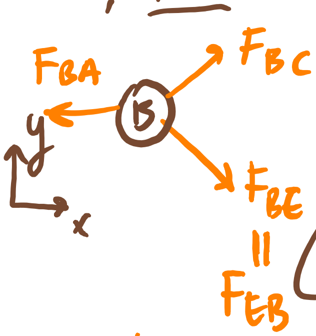


EoE

$$\begin{cases} \Sigma F_x = -F_{EB}\left(\frac{3}{5}\right) + F_{ED}\cos 30^\circ = 0 \\ \Sigma F_y = F_{EB}\left(\frac{4}{5}\right) + F_{ED}\sin 30^\circ - W = 0 \end{cases}$$

$\rightarrow$ Solve for F_{EB} and F_{ED}.

Third, FBD for B:



(it's the same

EoE

$$\begin{cases} \Sigma F_x = -F_{BA} + F_{BC}\cos 30^\circ + F_{BE}\left(\frac{3}{5}\right) = 0 \\ \Sigma F_y = F_{BC}\sin 30^\circ - F_{BE}\left(\frac{4}{5}\right) = 0. \end{cases}$$

$\rightarrow$ With F_{BE} ($= F_{EB}$) solved from the previous part, now solve for F_{BA} and F_{BC}.

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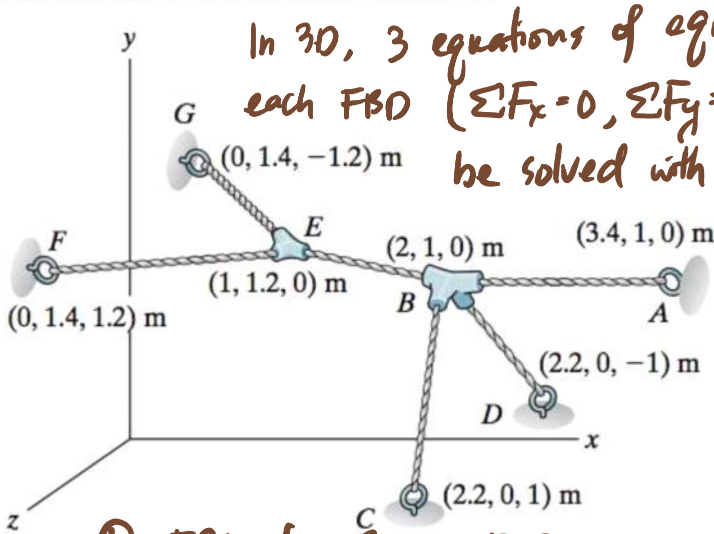
Example – System of Particles

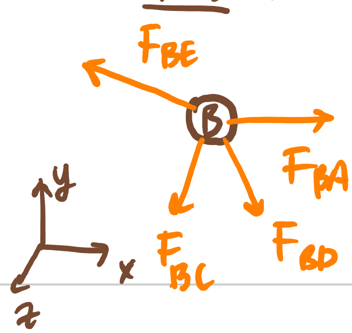
Determine the tension in each cable for the system below, given that the tension in cable *AB* is 10 N.



In 3D, 3 equations of equilibrium are available for each FBD ($\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$), 3 unknowns can be solved with each FBD. There are 5 unknowns here ($F_{BC}, F_{BD}, F_{BE}, F_{EF}, F_{EG}$), at least 2 FBD are needed to solve.

13

① FBD for B



EoE

$$\Sigma F_x = F_{BAx} + F_{BCx} + F_{BDx} + F_{BEx} = 0$$

$$\Sigma F_y = F_{BAy} + F_{BCy} + F_{BDy} + F_{BEy} = 0$$

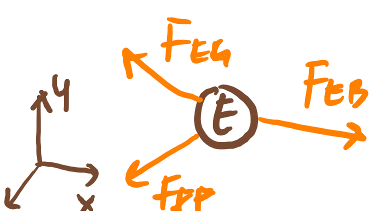
$$\Sigma F_z = F_{BAz} + F_{BCz} + F_{BDz} + F_{BEz} = 0$$

- Use position vector to derive force directions. For example: $\vec{F}_{BA} = F_{BA}\,\hat{u}_{BA} = F_{BA}\left(\frac{\vec{r}_{BA}}{r_{BA}}\right)$

$$\hat{u}_{BA} = \frac{1.4\hat{i} + 0\hat{j} + 0\hat{k}}{1.4} = \hat{i}, \text{ so } \vec{F}_{BA} = F_{BA}\,\hat{i}$$

→ Once F_{BC}, F_{BD}, and F_{BE} are solved from above, then

② FBD for E



EoE

There are only 2 unknowns (F_{EF} & F_{EG}) left to solve, so we only need 2 EoE, so

$$\Sigma F_x = F_{EBx} + F_{EFx} + F_{EGx} = 0$$

$$\Sigma F_x = F_{EBx} + F_{EFx} + F_{EGx} = 0$$

$$\Sigma F_y = F_{EBy} + F_{EFy} + F_{EGy} = 0 .$$