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Announcements

- Quiz 1 continues
- If you are just joining us – check out the course website for all the logistics you need to know:

  https://courses.engr.illinois.edu/tam210
- Complete CATME survey <u>before</u> Sunday (1/27)

❑ Upcoming deadlines:

- Friday (01/25 – TODAY!)
  - Written Assignment #1
- Tuesday (01/29)
  - PL HW

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Chapter 3: Equilibrium of a particle

Goals and Objectives

- Practice following general procedure for analysis.
- Introduce the concept of a free-body diagram for an object modeled as a particle.
- Solve particle equilibrium problems using the equations of equilibrium.

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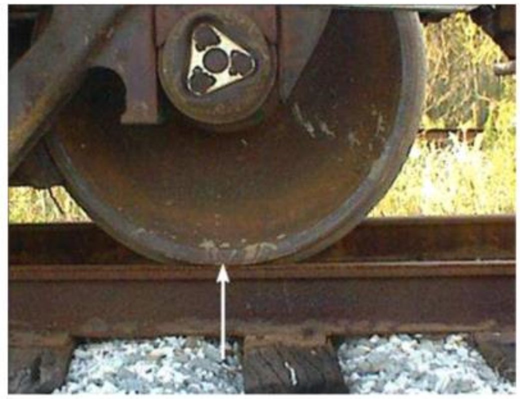
Fundamental concepts



Basic quantities:

- Length
- Time
- Volume
- Mass

Idealizations:

- Particle:

  Has mass but neglect size (no geometry)

- Rigid Body:

  A combination of particles at a fixed distance, no deformation

- Concentrated Force:

  Loading acting at a point

Understanding and applying these things allows for amazing achievements in engineering! (planes, robotics, etc)

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Applications

For a spool of given weight, how would you find the forces in cables AB and AC?

If designing a spreader bar (BC) like this one, you need to know the forces to make sure the rigging (A) doesn't fail.

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General procedure for analysis

1. Read the problem carefully; write it down carefully.

2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.

3. Apply principles needed.

4. Solve problem symbolically. Make sure equations are dimensionally homogeneous

5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).

6. See if answer is reasonable.

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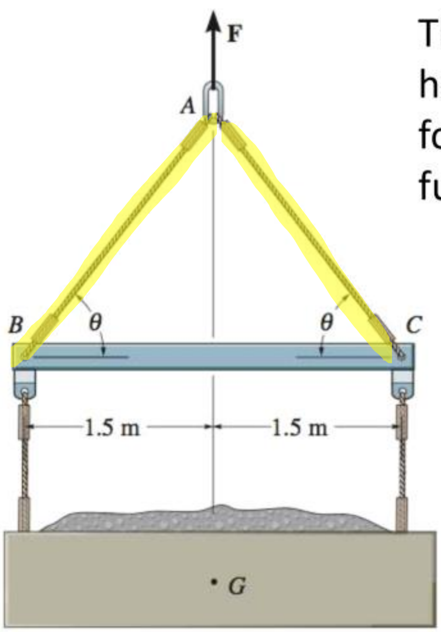
Free body diagram



The lift sling is used to hoist a container having a mass of 500 kg. Determine the force in each of the cables AB and AC as a function of θ.

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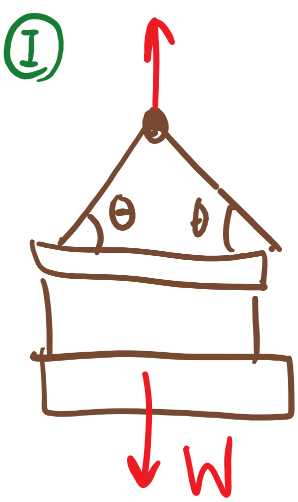
- Force in the cable
  = force exerted by the cable
- Bodies where cable exert force on: ring A and bar BC at B.
  → use these places to find force in cable AB.

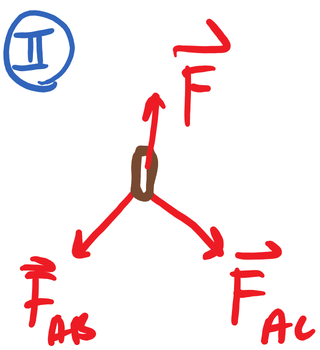
- Possible FBD for this system:

I



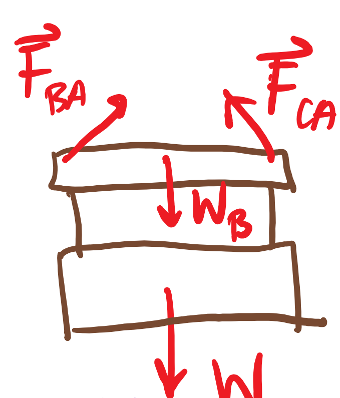
- Not useful for

II



- Useful FBD for solving $\vec{F}_{AB}$ & $\vec{F}_{AC}$, if $\vec{F}$ is a given parameter.

III



- Useful for finding $\vec{F}_{BA}$ & $\vec{F}_{CA}$ if W

- Not useful for this problem. Forces in cables AB & AC are internal, so they won't show up in the FBD.

T_{AB} & T_{AC}, ...

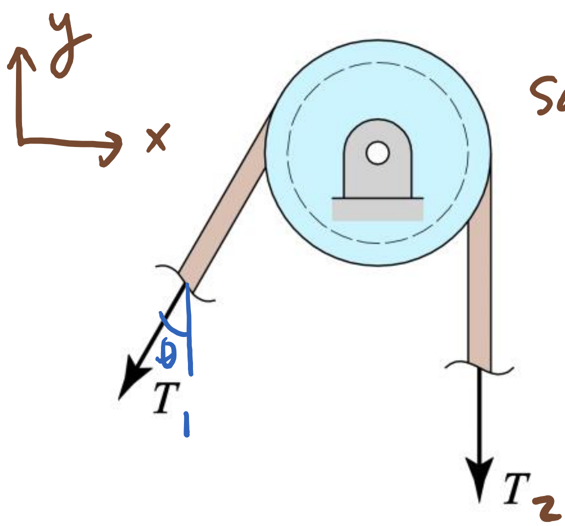
given parameter.

F_{BA} & F_{CA} ...

and W_B are the given parameters.

Idealizations

Pulleys are (usually) regarded as frictionless; then the tension in a rope or cord around the pulley is the same on either side.



Frictionless pulley

same magnitude $\Rightarrow T_1 = T_2$

→ changes direction

example

$\vec{T}_1 = -T_1 \sin\theta\,\hat{i} - T_1\cos\theta\,\hat{j}$

$\vec{T}_2 = -T_2\,\hat{j}$

→ $\vec{T}_1$ & $\vec{T}_2$ have different directions, but same magnitude.

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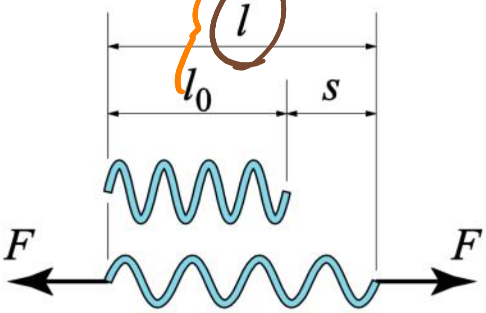
Idealizations

Springs are (usually) regarded as linearly elastic; then the tension is proportional to the *change* in length *s*.

difference



$$F = ks = k(l - l_0)$$

Linearly elastic spring

ex. $s = 3m$, $k = 5\frac{N}{m}$

$\rightarrow F_{spring} = ks = (5\frac{N}{m})(3m)$

$= 15N$

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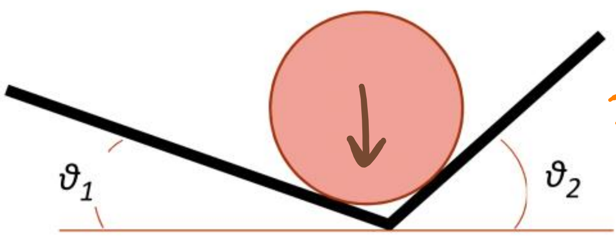
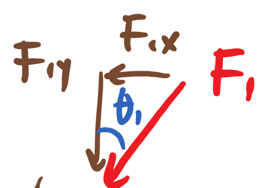
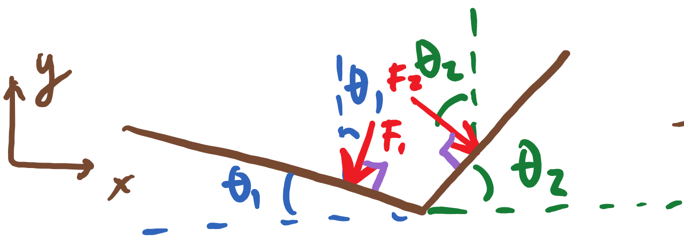
Idealizations



Contact force in smooth surface:

Force direction ⊥ to surface





example

$F_{1x} = -F_1 \sin\theta_1$

$F_{1y} = -F_1 \cos\theta_1$

$\vec{F}_2 = F_2 \sin\theta_2 \,\hat{i} - F_2 \cos\theta_2 \,\hat{j}$

$\vec{F}_1 = -F_1 \sin\theta_1 \,\hat{i} - F_1 \cos\theta_1 \,\hat{j}$

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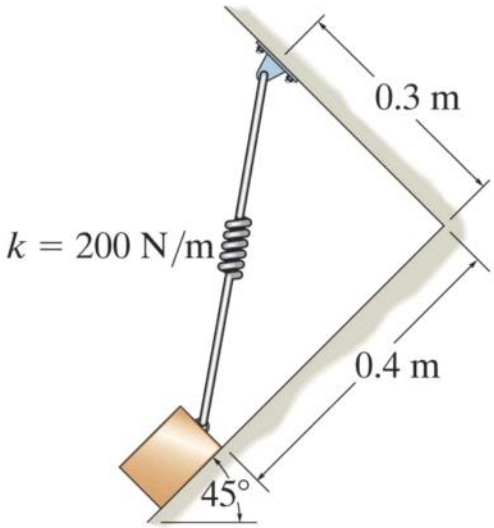
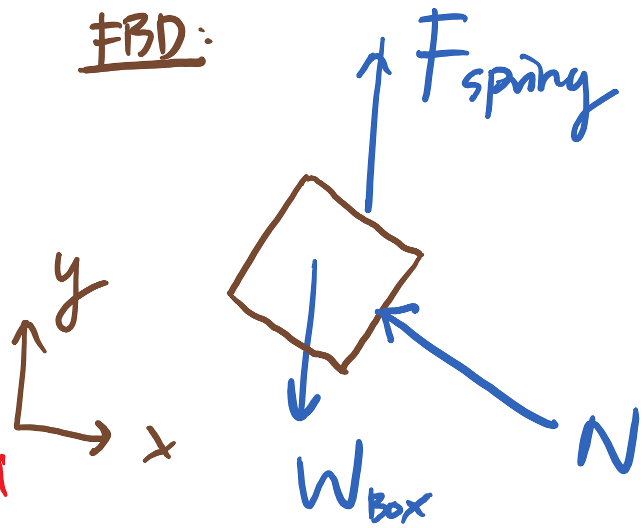
Free Body Diagram Example



Given: s ($l - l_0$ of spring)

Find: W_{Box}

FBD:



$F_{spring} = ks$

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Side note: We will ignore friction force (between box & ground) for now.

always include a coordinate system with FBD

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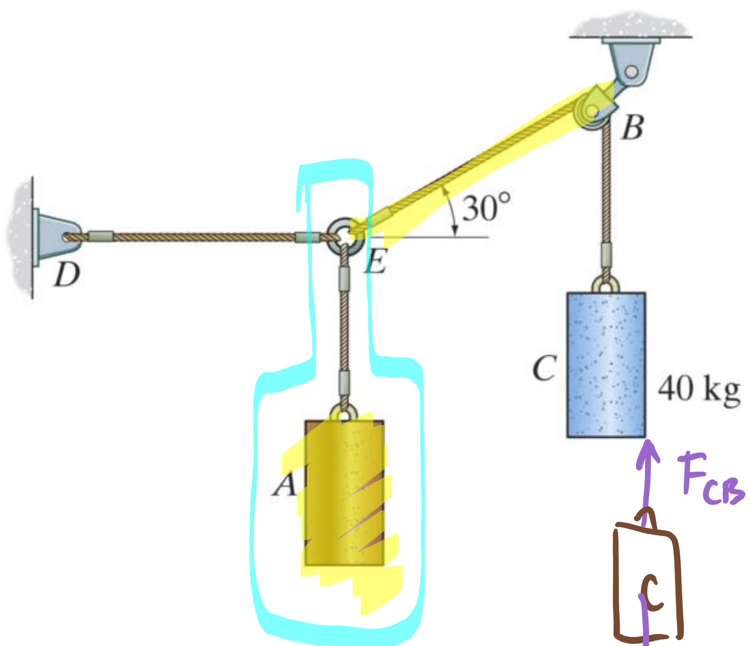
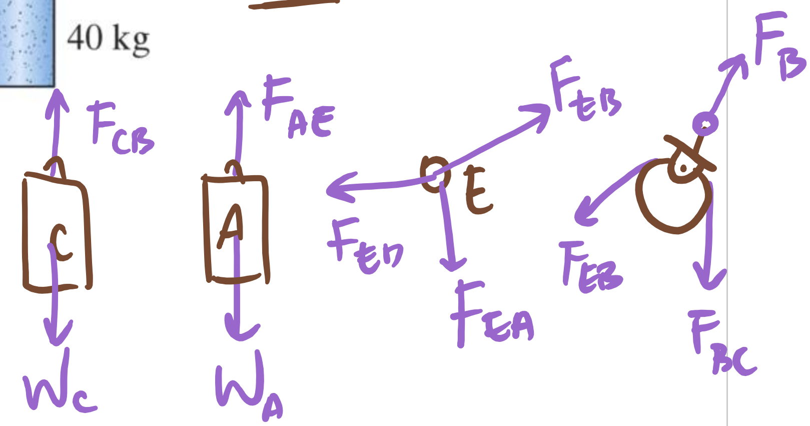
Free Body Diagram Example



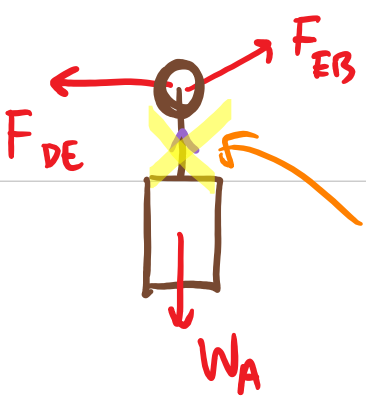
Given: W_A, W_C

Find: $\vec{F}_{EB}$

FBD



$F_{EB} = F_{BC}$



internal force should not be included in FBD

Equilibrium of a particle

According to Newton's first law of motion , a particle will be in **equilibrium** (that is, it will remain at rest or continue to move with constant velocity) if and only if

$F = ma$, if $a = 0$, the $F = 0$

in space: $\Sigma \vec{F} = 0$

In three dimensions, equilibrium requires:

Eqns of Equilibrium

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \qquad \left(\begin{array}{l} a_x = 0 \\ a_y = 0 \\ a_z = 0 \end{array} \right)$$

Coplanar forces: if all forces are acting in a single plane, such as the "xy" plane, then the equilibrium condition becomes

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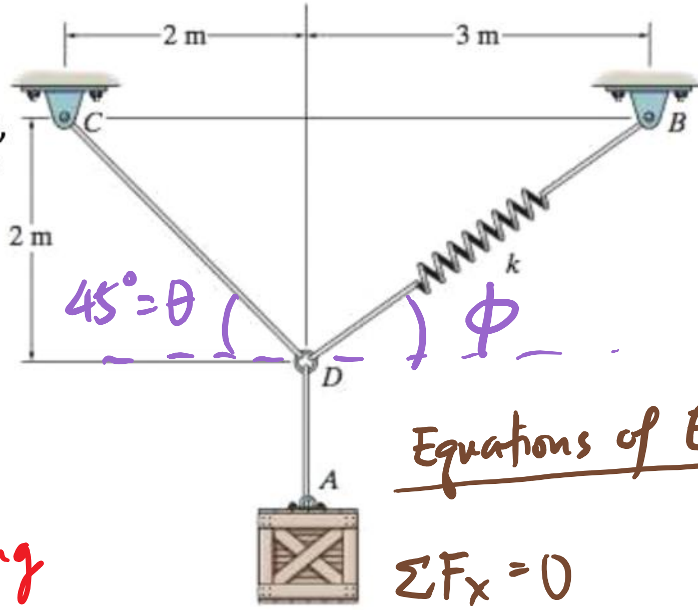
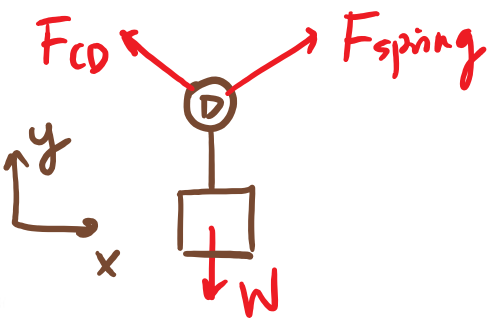
Example

If the spring *DB* has an unstretched length of 2 m, determine the stiffness of the spring to hold the 40-kg crate in the position shown.



FBD



Equations of Equilibrium

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\rightarrow \Sigma F_x = F_{CDx} + F_{spring,x}$

$\Sigma F_y = F_{CDy} + F_{spring,y} + W$

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$$F_{CD,x} = -F_{CD}\cos 45°$$

$$F_{spring,x} = ks\cos\phi$$

$$\Sigma F_x = -F_{CD}\cos 45° + ks\cos\phi$$