Statics - TAM 210 & TAM 211

Lecture 21

March 5, 2018

Chap 7.2

Announcements

- Upcoming deadlines:
- Monday (3/5)
  - Mastering Engineering Tutorial 8
- Tuesday (3/6)
  - PL HW 6
- Quiz 4 (3/7-9)
  - Sign up at CBTF
  - Up thru and including Lecture 19 (Frames & Machines). Note that quiz and lecture material always builds on earlier fundamental concepts.
- Quiz 5 (3/14-16)
- No class Friday March 9, enjoy EOH!
- No Prof. H-W office hours on Friday March 9
- If building is picketed, no class Wednesday March 7. Will have online video

Chapter 7: Internal Forces

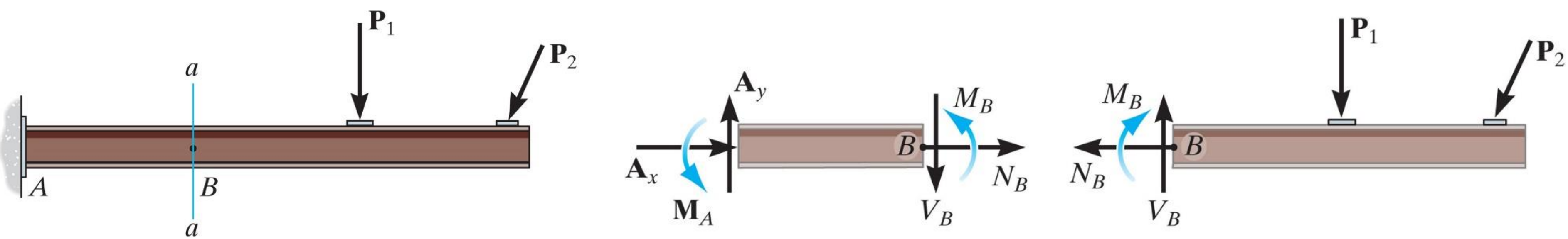
Goals and Objectives

- Determine the internal loadings in members using the method of sections
- Generalize this procedure and formulate equations that describe the internal shear and bending moment throughout a member
- Be able to construct or identify shear and bending moment diagrams for beams when distributed loads, concentrated forces, and/or concentrated couple moments are applied

Recap: Internal loadings in structural members

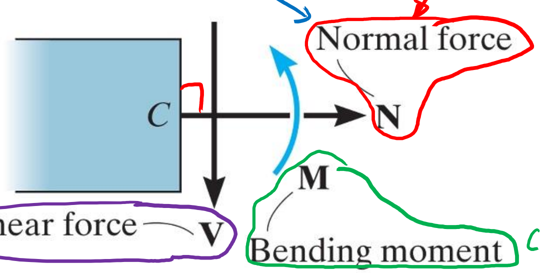
Structural Design: need to know the loading acting within the member in order to be sure the material can resist this loading

Cutting members at internal points reveal **internal forces and moments.** ⇒ use Method of Sections



tendency to prevent translation

⊥ to cut or xsection



Counter couple moment

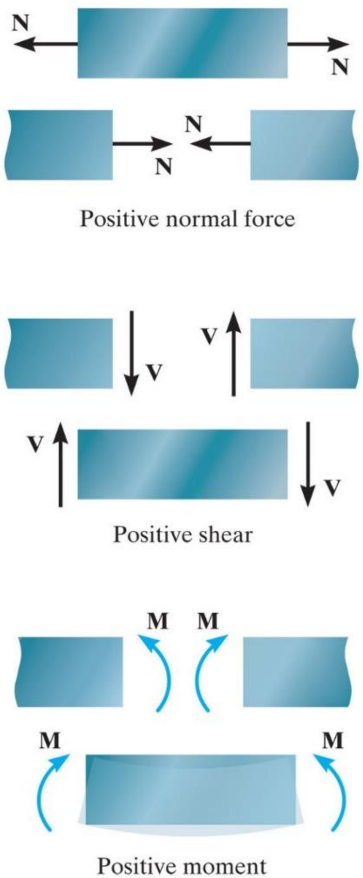
|| to cut or tangent to xsection

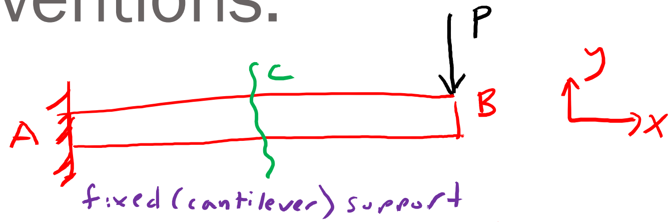
tendency to prevent bending

N, V, M ← key labels to learn

2D

Recap: Sign conventions:

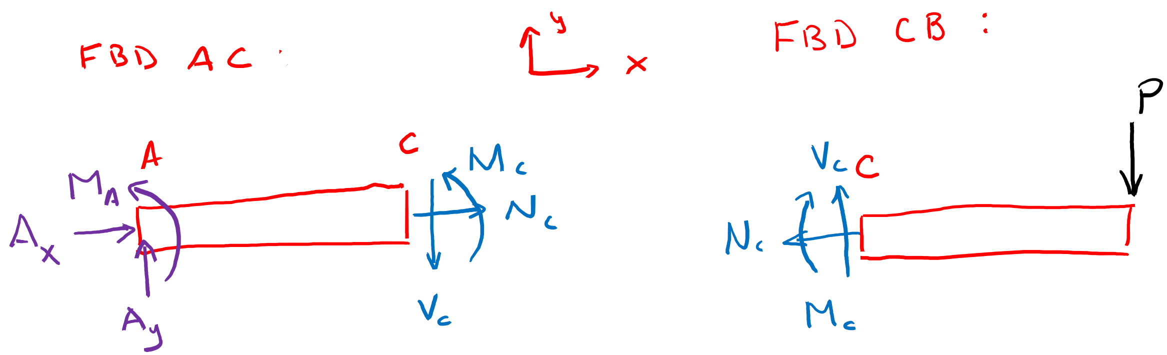




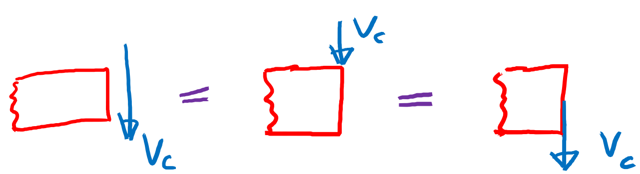
If beam AB is cut at C, draw FBDs of sections AC, CB illustrating assumptions of N, V, M drawn in positive directions.

FBD AC:

FBD CB:



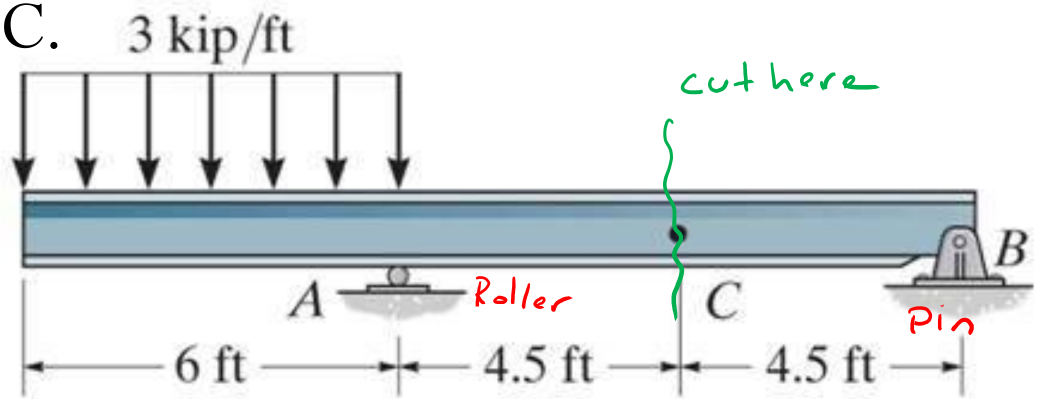
Note: although draw V off the side of the cut section, V is actually applied at the cut.



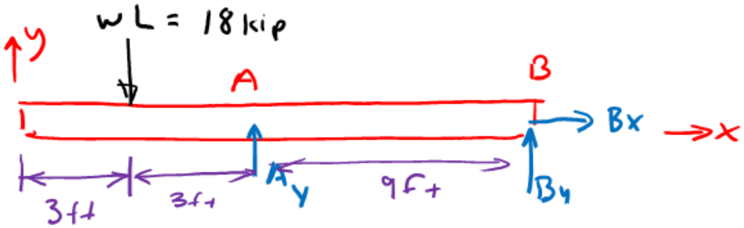
Recap: Procedure for analysis:

1. Find support reactions (free-body diagram of entire structure)
2. Pass an imaginary section through the member
3. Draw a free-body diagram of the segment that has the least number of loads on it
4. Apply the equations of equilibrium

Find the internal forces at point C.



FBD of entire beam



3 unknowns (A_y, B_x, B_y)

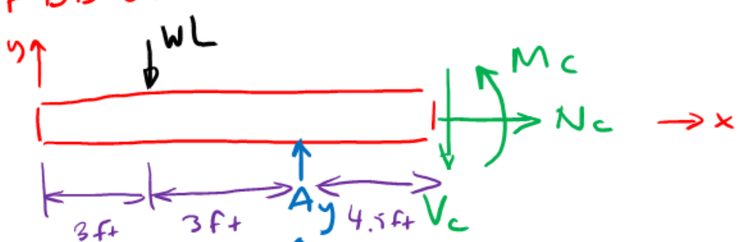
use 3 EoE to solve for A_y, B_x, B_y.

ΣF_x: $\boxed{B_x = 0}$, ΣF_y: $A_y + B_y - wL = 0$ $\rightarrow$ $\boxed{B_y = -6 \text{ kip}}$

$+\circlearrowleft \Sigma M_B$: $(12\,ft)\,wL - (9\,ft)\,A_y = 0$ $\rightarrow$ $\boxed{A_y = 24 \text{ kip}}$

B $\downarrow B_y$

FBD of left section:



known after solve above

3 unknowns (N_c, V_c, M_c); assuming know A_y

use EoE:

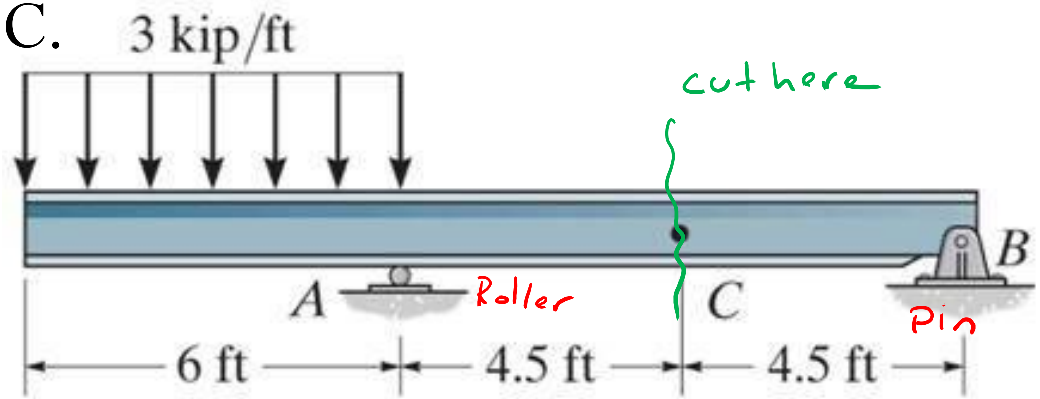
ΣF_x: $\boxed{N_c = 0}$

ΣF_y: $A_y - wL - V_c = 0$ $\Rightarrow$ $\boxed{V_c = 6 \text{ kip}}$

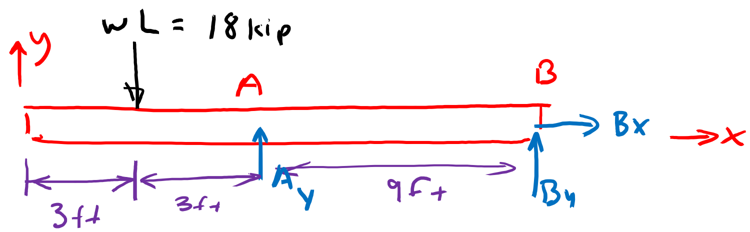
$+\circlearrowleft \Sigma M_c$: $M_c - (4.5\,ft)\,A_y + (7.5\,ft)\,wL = 0$

$\Rightarrow$ $\boxed{M_c = -27 \text{ kip}\cdot\text{ft}}$

Find the internal forces at point C.



FBD of entire beam

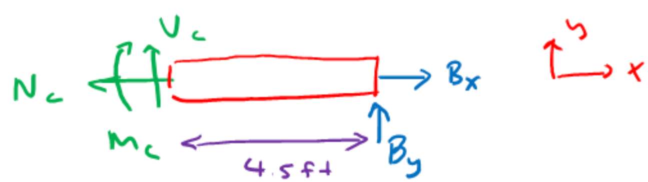


3 unknowns (A_y, B_x, B_y)

use 3 EoE to solve for A_y, B_x, B_y. $\boxed{A_y = 24 \text{ kip}}$ $\boxed{B_x = 0}$ $\boxed{B_y = -6 \text{ kip}}$

Alternatively, could examine right section:

FBD of right section



3 unknowns (N_c, V_c, M_c) assuming know B_x, B_y

use EoE:

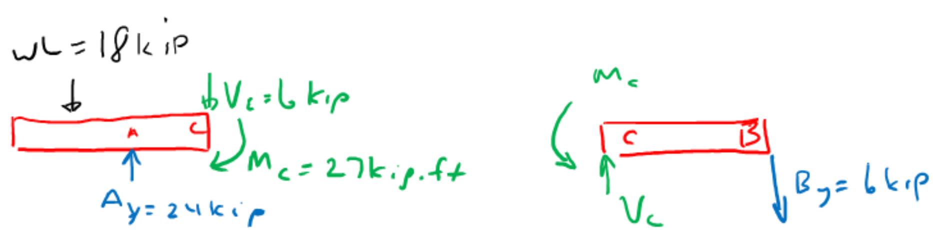
$\Sigma F_x: \; B_x - N_c = 0 \Rightarrow \boxed{N_c = 0}$

$\Sigma F_y: \; B_y + V_c = 0 \rightarrow \boxed{V_c = 6 \text{ kip}}$

$+\circlearrowleft \Sigma M_c: \; -M_c + (4.5\text{ft})B_y = 0$

$\rightarrow \boxed{M_c = -27 \text{ kip}\cdot\text{ft}}$

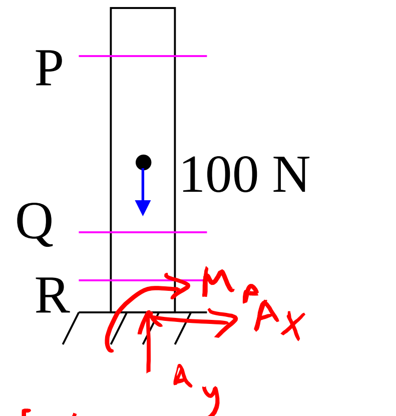
$\therefore$ Actual Forces & Moments:



Note changes in directions of arrows for B_y & M_c from original FBDs due to negative values in solutions.

Note that column is massless.

1. A column is loaded with a vertical 100 N force. At which sections are the internal loads the same?



A) P, Q, and R

B) P and Q

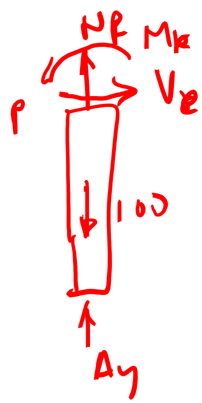
C) Q and R

D) None of the above.

Entire: $A_y = 100N$

$A_x = 0$

$M_A = 0$



$V_P = 0$

$\Sigma F_y: N_P + \cancel{A_y}^{100} - 100 = 0$

$\boxed{N_P = 0}$

$\Sigma M_P: M_P = 0$

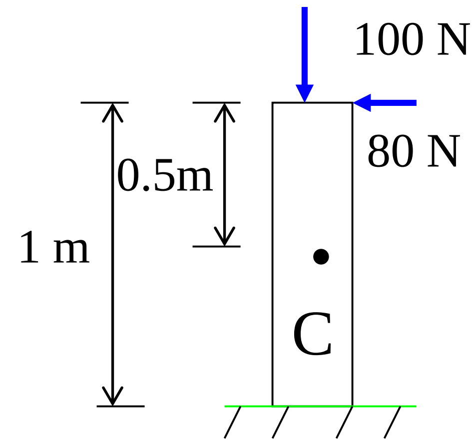
$V_Q = 0$

$\Sigma F_y: A_y + N_Q = 0$

$\boxed{N_Q = -100N}$

$\Sigma M_Q: M_Q = 0$

2. Determine the magnitude of the internal loads (normal, shear, and bending moment) at point C.



N V M

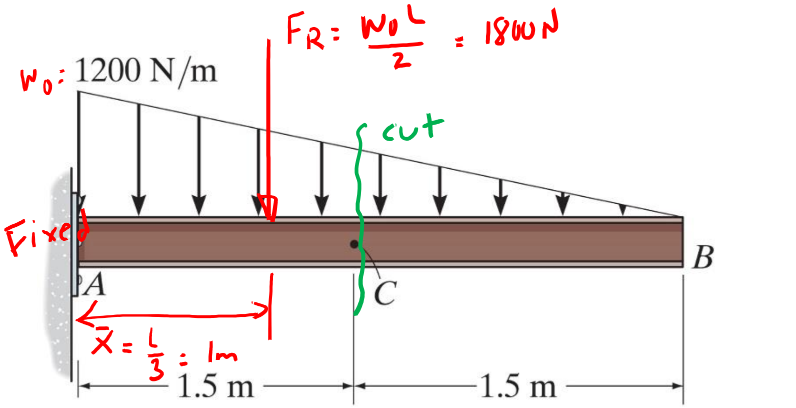
A) (100 N, 80 N, 80 N m)

B) (100 N, 80 N, 40 N m)

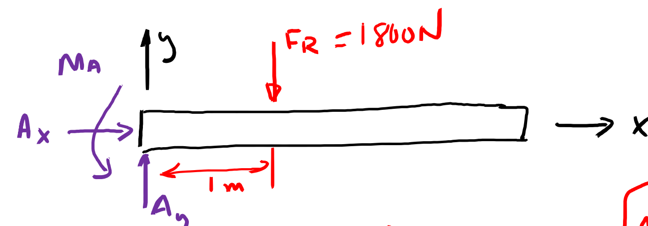
C) (80 N, 100 N, 40 N m)

D) (80 N, 100 N, 0 N m)

Find the internal forces and moments at C



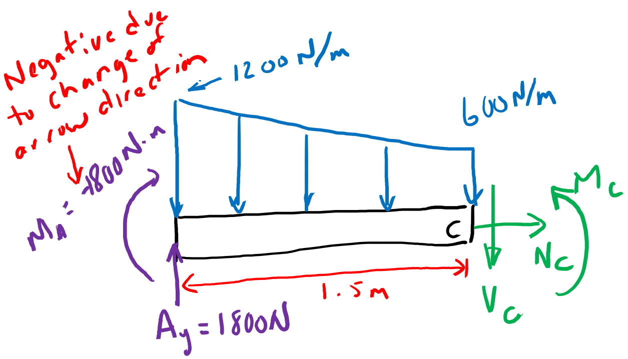
FBD of entire beam:

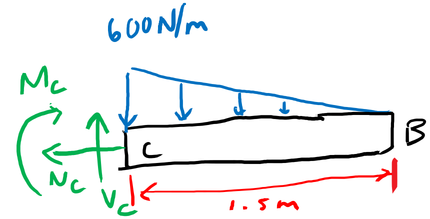


$\Sigma F_x : \boxed{A_x = 0}$, $\Sigma F_y : A_y - F_R = 0 \Rightarrow \boxed{A_y = F_R = 1800N}$ $A_y \uparrow$

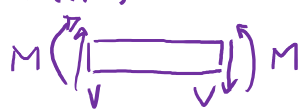
$\circlearrowleft + \Sigma M_A : M_A - (1m)(1800N) = 0 \Rightarrow \boxed{M_A = 1800N}$ $M_A \circlearrowleft$

Let's look at FBDs of Left & Right sides when cut at C:





let's draw rxn force & moment arrows following positive conventions for shear & bending moments



3 unknowns

N_c, V_c, M_c

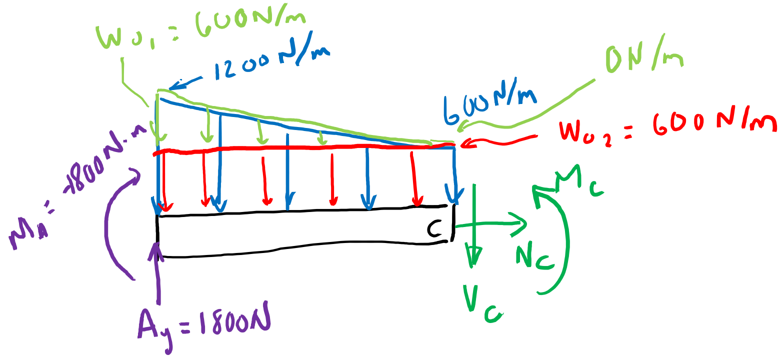
3 unknowns

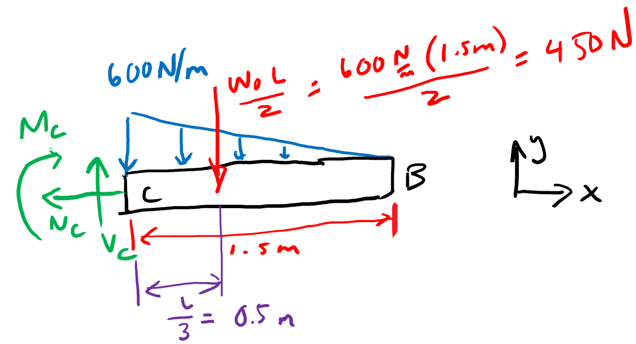
N_c, V_c, M_c

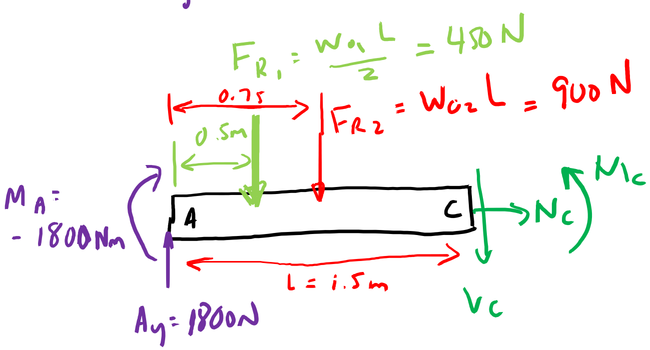
We can solve for unknown internal forces with either left or right side:

Left side:
Divide distributed load into F_{R_1} for triangle and F_{R_2} for rectangle

Right side:
simply find F_R for distributed load







$\Sigma F_x: \boxed{N_c = 0}$

$\Sigma F_y: A_y - F_{R_1} - F_{R_2} - V_c = 0$

$V_c = 1800N - 450N - 900N$

$\boxed{V_c = 450N}$ $V_c \downarrow$

$+\circlearrowleft \Sigma M_A: -M_A - (0.5m)F_{R_1} - (0.75m)F_{R_2} - (1.5m)V_c + M_c = 0$

$\boxed{M_c = -225 \text{ Nm}}$ Since negative $\Rightarrow$ assumed arrow direction on FBD is incorrect; should be $\circlearrowright M_c$

$\Sigma F_x: -N_c = 0 \rightarrow \boxed{N_c = 0}$

$\Sigma F_y: V_c - 450N \rightarrow \boxed{V_c = 450N}$ $V_c \uparrow$

$+\circlearrowleft \Sigma M_c: -M_c - (0.5m)(450N) = 0$

$\boxed{M_c = -225 \text{ N m}}$ $M_c \circlearrowright$

Note that choosing left FBD takes more steps, but get the same result.

What are internal forces along the length of the beam?

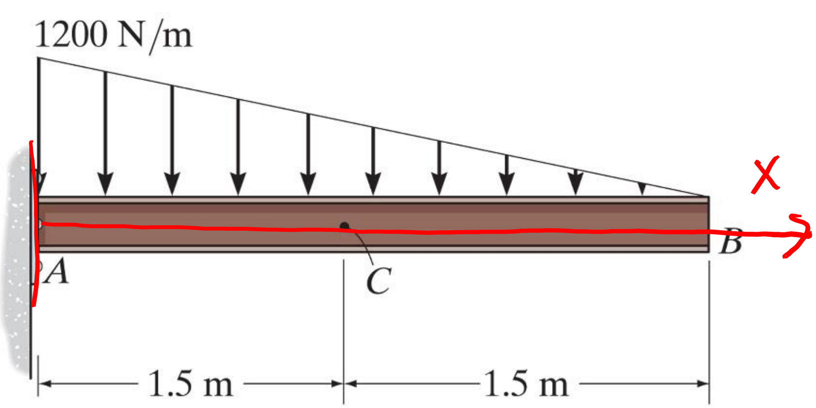
Shear Force and Bending Moment Diagrams

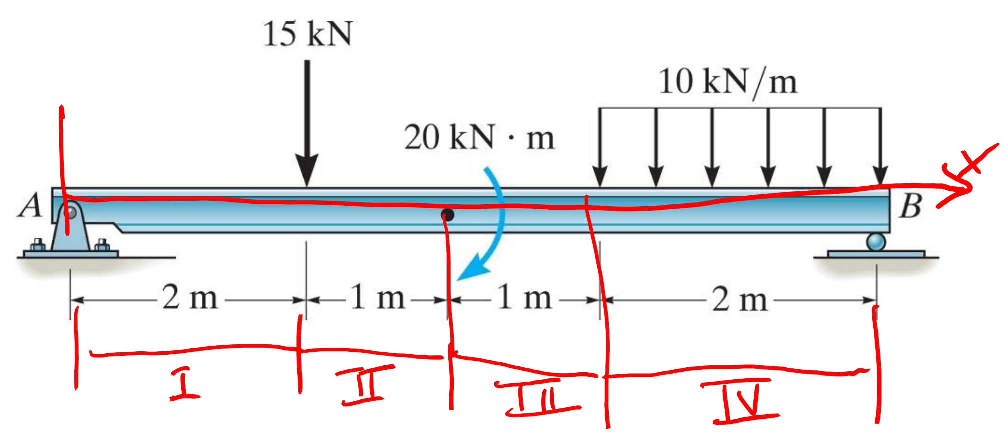
Goal: provide detailed knowledge of the variations of internal shear force and bending moments (V and M) throughout a beam when perpendicular distributed loads, concentrated forces, and/or concentrated couple moments are applied.

Normal forces (N) in such beams are zero, so we will not consider normal force diagrams.

Procedure

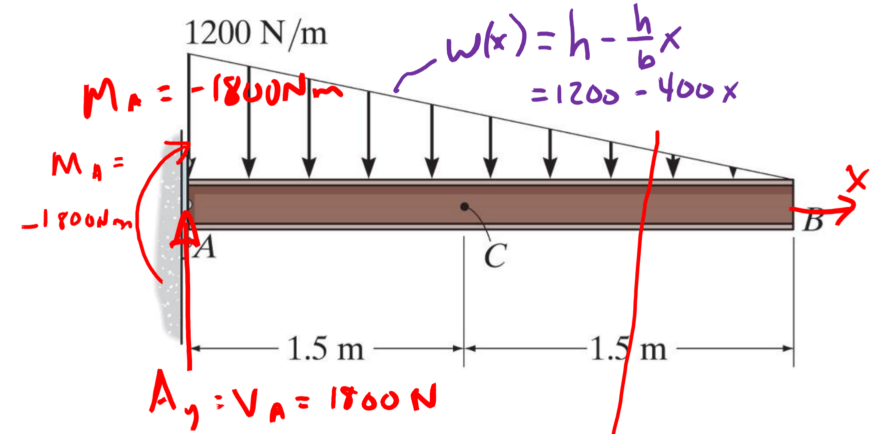
1. Find support reactions (free-body diagram of entire structure)
2. Specify coordinate x (start from left)
3. Divide the beam into sections according to loadings
4. Draw FBD of a section
5. Apply equations of equilibrium to derive V and M as functions of x: V(x), M(x)





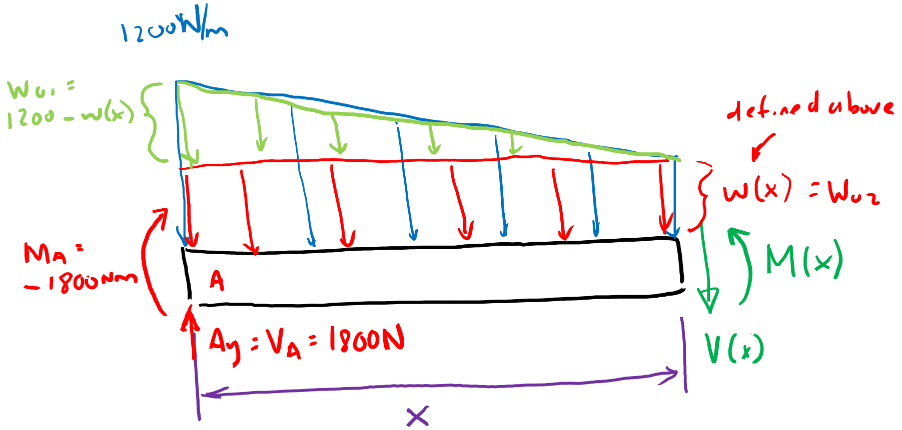
Draw the shear and bending moment diagrams for the beam.

From previous example, we know that the support reactions are: $A_x = 0$, $A_y = 1800\,N \uparrow$, $M_A = -1800\,Nm$



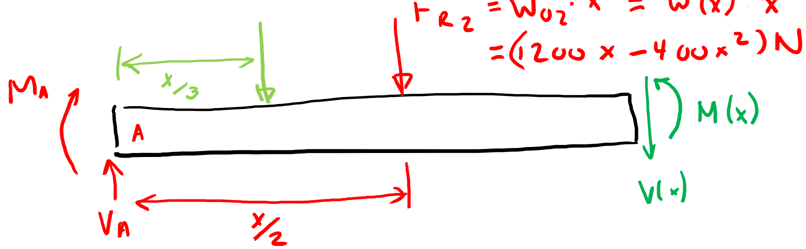
We are interested in finding $V(x)$ & $M(x)$ as these vary along the length of the beam.

So for any length x of the beam, we get the following generic FBD as a function of x.



$$F_{R_1} = W_{0_1} \frac{L}{2} = [1200 - w(x)]\frac{x}{2} = (200x^2)\,N$$

$$F_{R_2} = W_{0_2} \cdot x = w(x) \cdot x$$
$$= (1200x - 400x^2)\,N$$



$\Sigma F_x: A_y - F_{R1} - F_{R2} - V(x) = 0$

$$\boxed{V(x) = (200x^2 - 1200x + 1800)\ N}$$

Quadratic

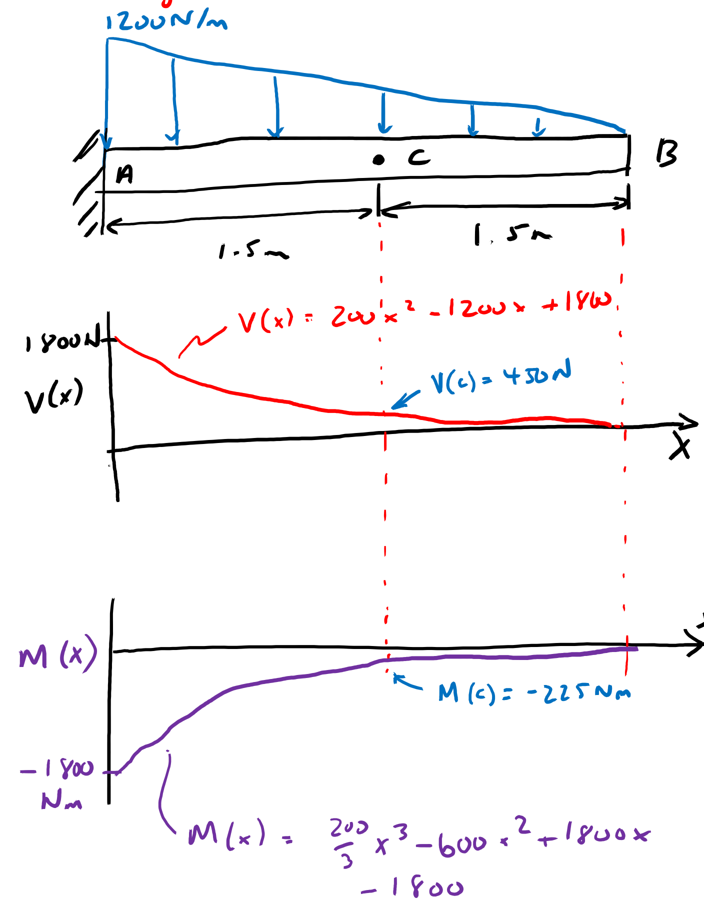
Boundary conditions:

$V(x=0) = 1800\ N = A_y$

$V(x=L=3m) = 0\ N$

cf. $V(@C = 1.5m) = 450\ N$ ✓ w/ previous example

Draw Shear Force V(x) & Bending Moment M(x) diagrams



$+\circlearrowleft \Sigma M_A: -M_A - \left(\frac{x}{3}\right)F_{R1} - \left(\frac{x}{2}\right)F_{R2} - x \cdot V(x) + M(x) = 0$

$$\boxed{M(x) = \left(\frac{200}{3}x^3 - 600x^2 + 1800x - 1800\right)\ Nm}$$

3rd Order Polynomial

B.C.

$M(0) = -1800\ Nm = M_A$

$M(L) = 0$

cf. $M(@C = 1.5m) = -225\ Nm$ ✓ w/ previous

Note that since the applied load is a single distributed load along the entire length of the beam, then V(x) and M(x) are continuous functions. We will see that V(x) and M(x) will be discontinuous functions when multiple loads are applied to a beam, and these discontinuities will happen at the transitions between loading regions.