Chapter 5: Equilibrium of Rigid Bodies

Equilibrium of a Rigid Body

Static equilibrium:

$\sum \boldsymbol{F} = \boldsymbol{0}$ (zero forces = no translation)

$\sum(\boldsymbol{M}) = \boldsymbol{0}$ (zero moment = no rotation)

Maintained by reaction forces and moments

forces from supports / constraints are exactly enough to produce zero forces and moments

Assumption of rigid body

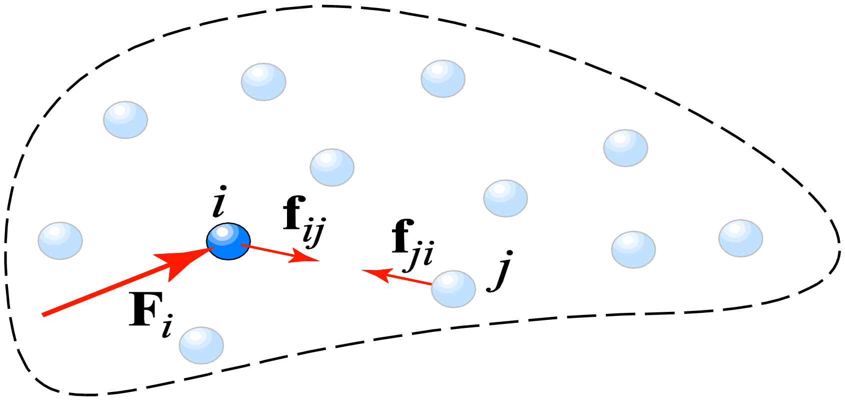
Shape and dimensions of body remain **unchanged** by application of forces.

More precisely:
All **deformations of bodies** are small enough to be ignored in analysis.

Equilibrium of a Rigid Body

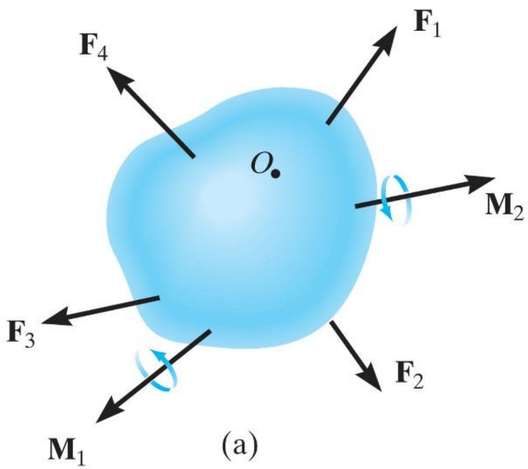
This subject is of central importance in statics. We regard a rigid body as a collection of particles.

$\boldsymbol{F}_i$ = resultant external force on particle i
$\boldsymbol{f}_{ij}$ = internal force on particle i by particle j
$\boldsymbol{f}_{ji}$ = internal force on particle j by particle i

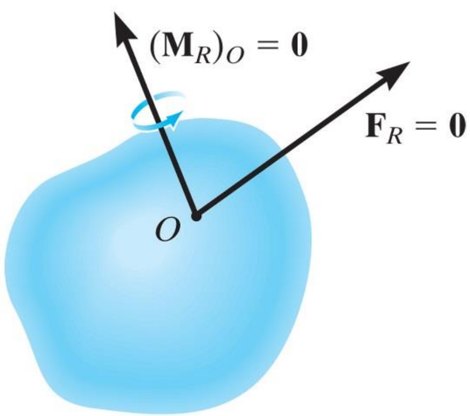


Note that $\boldsymbol{f}_{ji} = \boldsymbol{f}_{ij}$ by Newton's third law and therefore the internal forces will not appear in the equilibrium equations.

We can reduce the force and couple moment system acting on a body to an equivalent resultant force and a resultant couple moment at an arbitrary point O.



(a)



Process of solving rigid body equilibrium problems

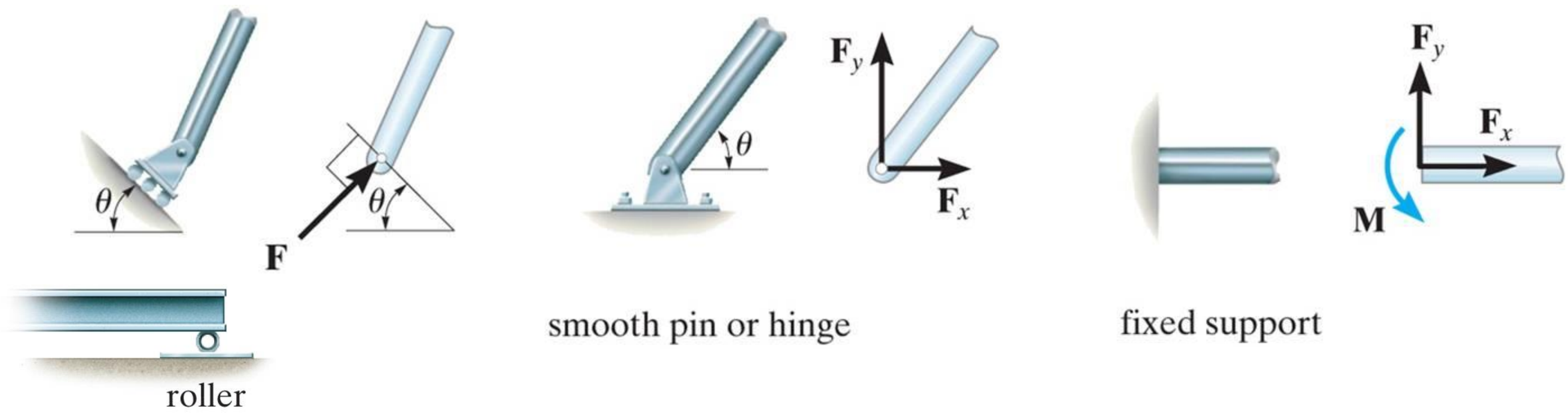
1. Create idealized model (modeling and assumptions)

2. Draw free body diagram showing ALL the external (applied loads and supports)

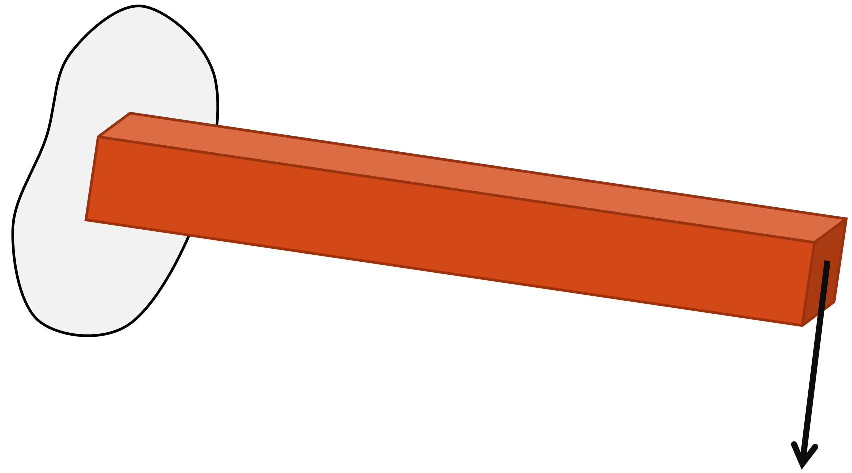
3. Apply equations of equilibrium

Equilibrium in two-dimensional bodies

Support reactions



- If a support prevents the translation of a body in a given direction, then a force is developed on the body on that direction
- If a rotation is prevented, a couple moment is exerted on the body



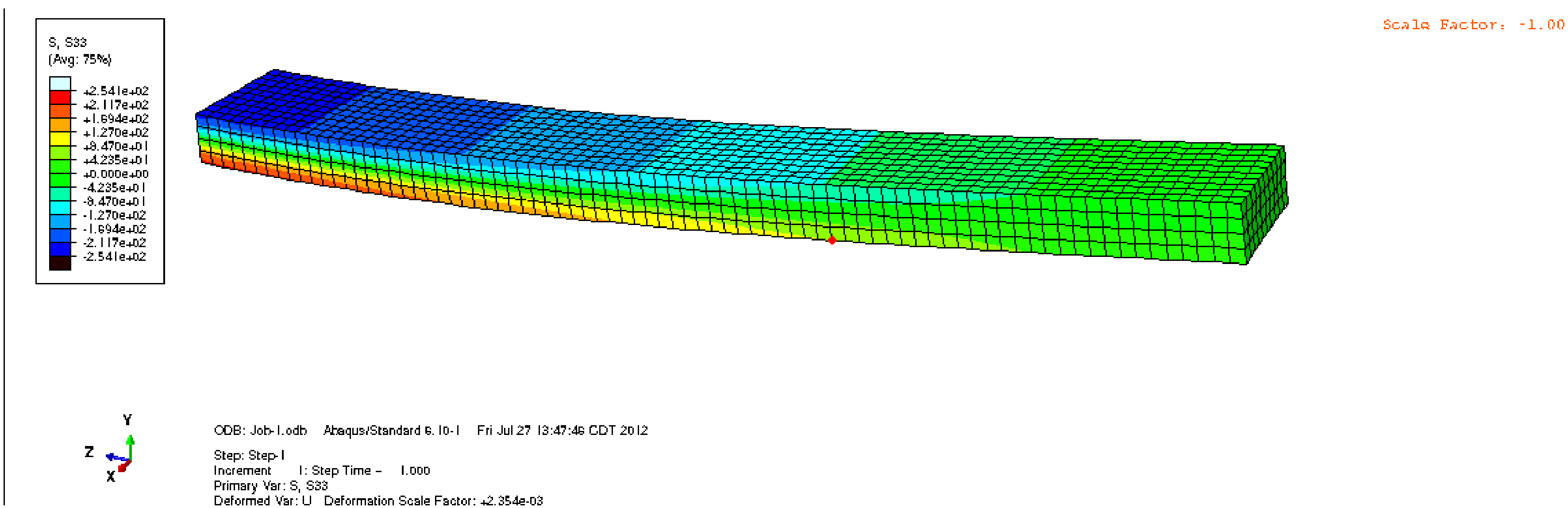

S, S33
(Avg: 75%)
+2.541e+02
+2.117e+02
+1.694e+02
+1.270e+02
+8.470e+01
+4.235e+01
+0.000e+00
-4.235e+01
-8.470e+01
-1.270e+02
-1.694e+02
-2.117e+02
-2.541e+02
Scale Factor: -1.00
Y
Z
X
ODB: Job-1.odb Abaqus/Standard 6.10-1 Fri Jul 27 13:47:49 CDT 2012
Step: Step-1
Increment 1: Step Time = 1.000
Primary Var: S, S33
Deformed Var: U Deformation Scale Factor: +2.354e-03

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

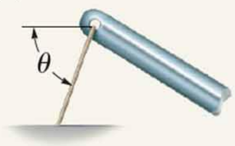
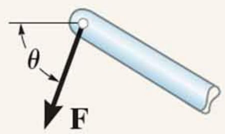
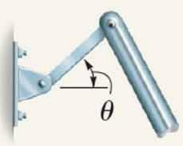
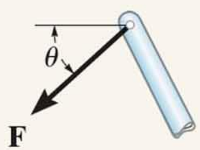
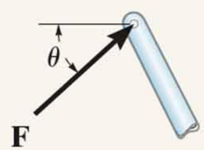
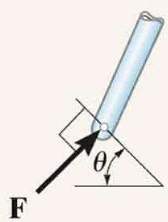
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

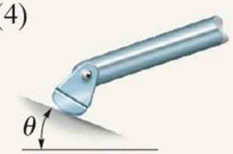
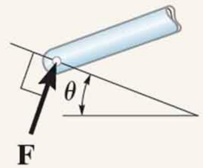
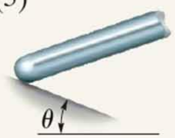
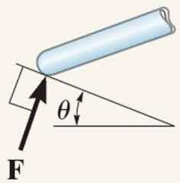
Types of Connection	Reaction	Number of Unknowns
(4)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(5)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

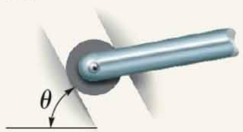
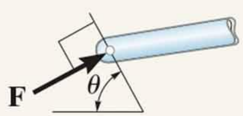
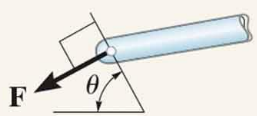
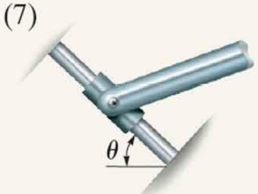
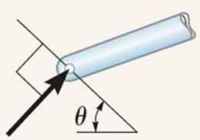
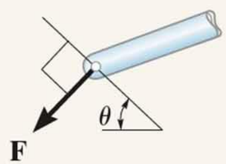
Types of Connection	Reaction	Number of Unknowns
(6)  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
(7)  member pin connected to collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

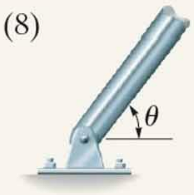
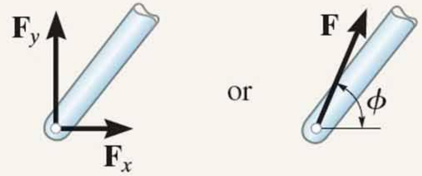
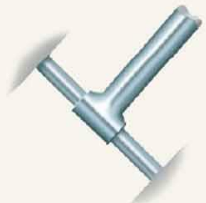
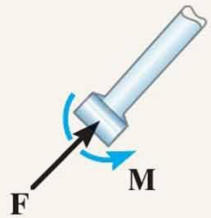
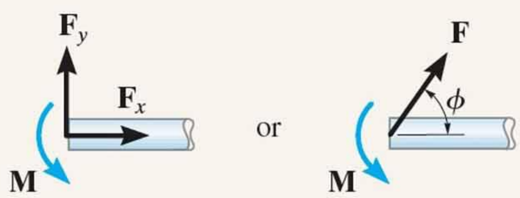
Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

Figure: 05_PH002

The rocker support for this bridge girder allows horizontal movement so the bridge is free to expand and contract due to a change in temperature. (4)

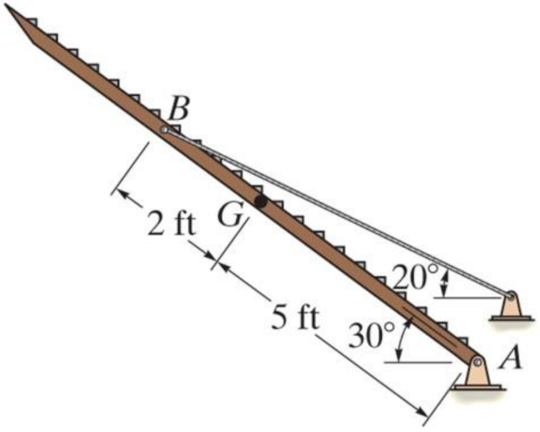
Figure: 05_PH001

The cable exerts a force on the bracket in the direction of the cable. (1)

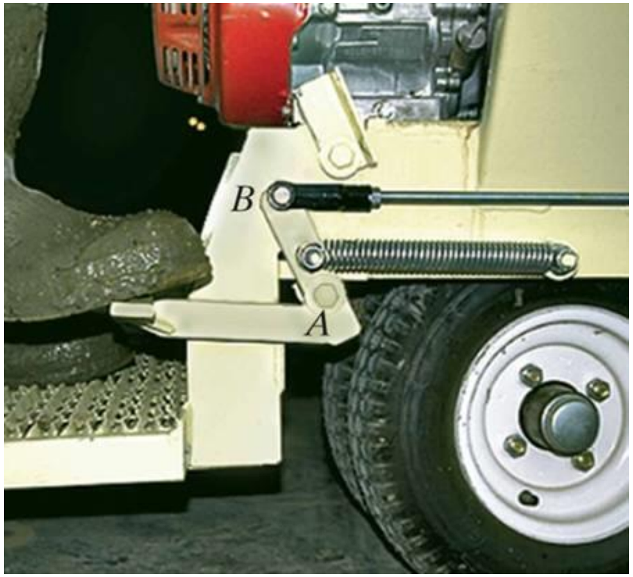
Figure: 05_PH004

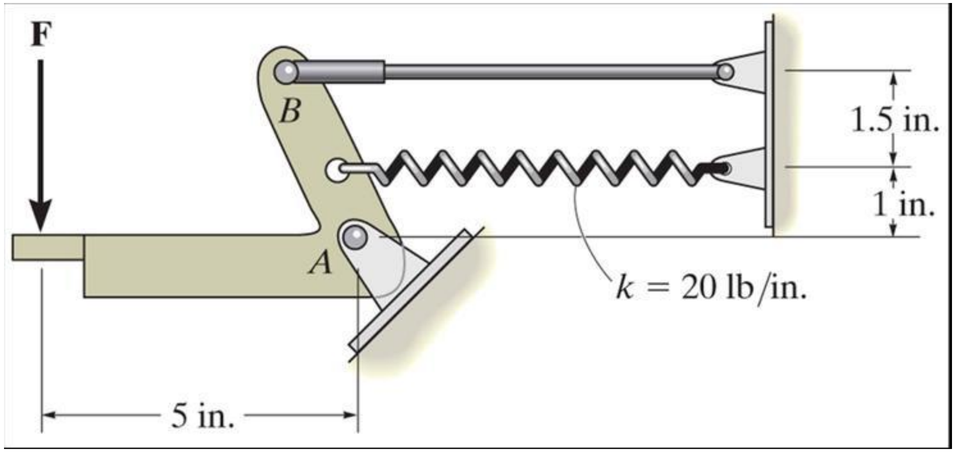
This utility building is pin supported at the top of the column. (8)

The uniform truck ramp has weight 400 lb and is pinned to the body of the truck at each side and held in the position shown by the two side cables. Determine the reaction forces at the pins and the tension in the cables.



The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at *B* is 20 lb. Determine the vertical force applied to the pedal.

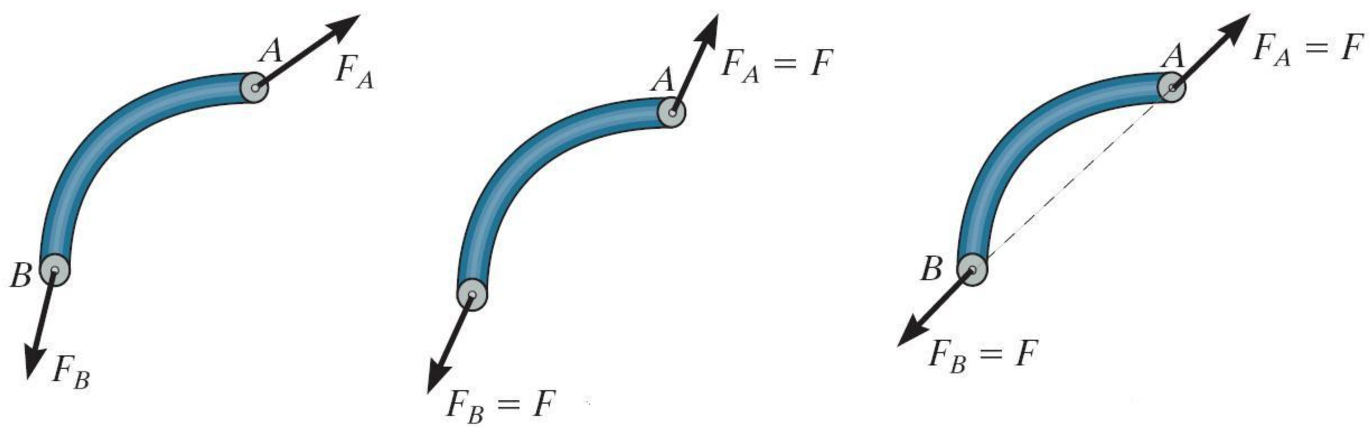




Two-force members

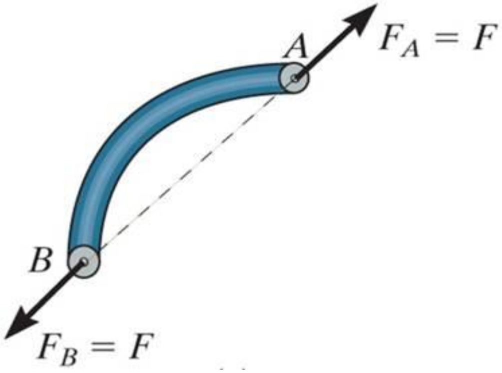
As the name implies, two-force members have forces applied at only two points.

If we apply the equations of equilibrium to such members, we can quickly determine that **the resultant forces at *A* and *B* must be equal in magnitude and act in the opposite directions along the line joining points *A* and *B*.**

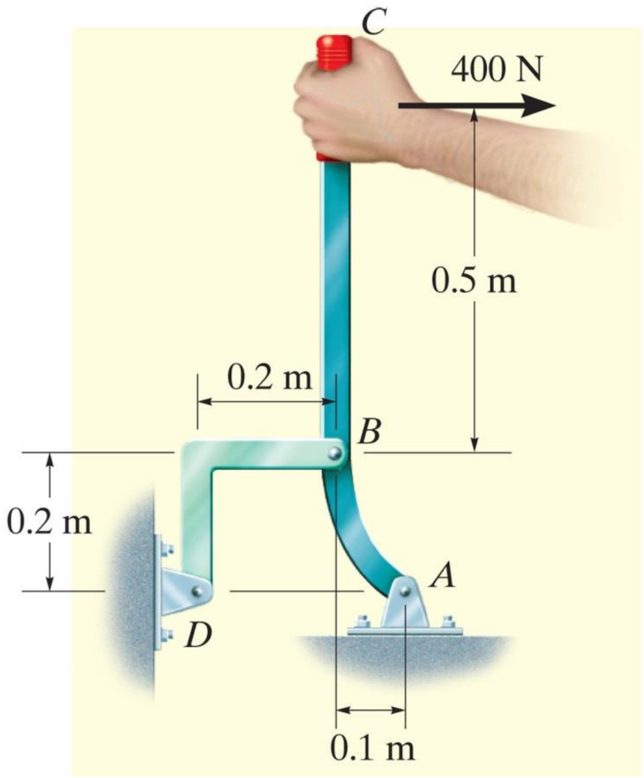


Examples of two-force members

In the cases above, members *AB* can be considered as two-force members, provided that their weight is neglected.



This fact **simplifies** the equilibrium analysis of some rigid bodies since the directions of the resultant forces at *A* and *B* are thus known (along the line joining points *A* and *B*).

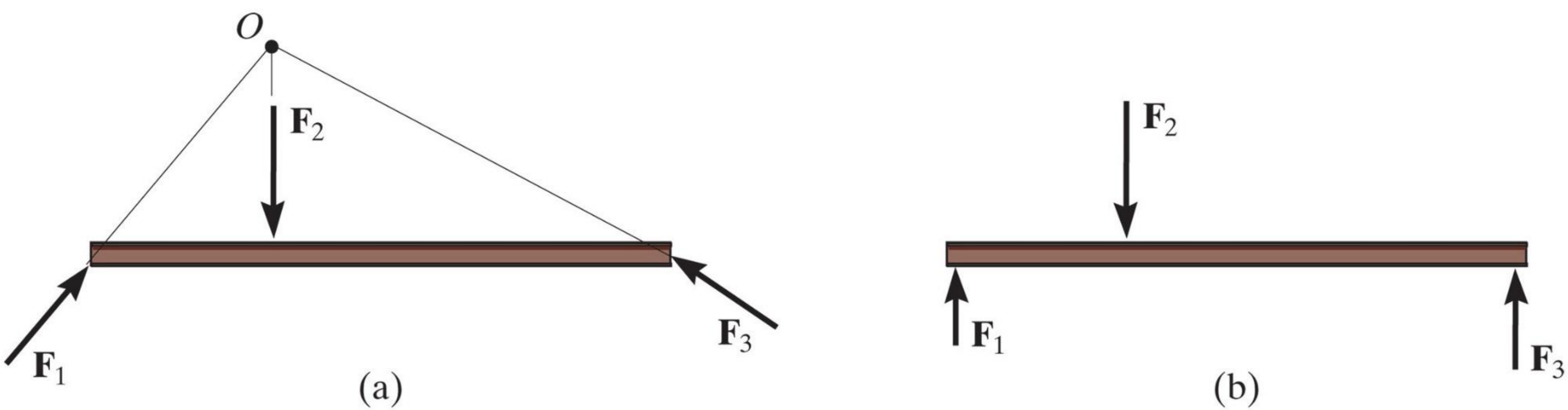


The lever *ABC* is pin supported at *A* and connected to a short link *BD*. If the weight of the members is negligible, determine the reaction forces at pins *D* and *A*.

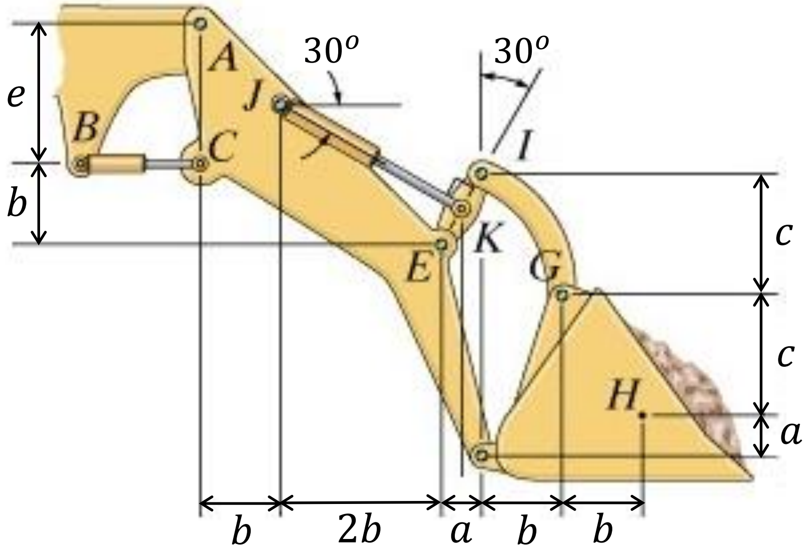
Three-force members

As the name implies, three-force members have forces applied at only three points.

Moment equilibrium can be satisfied only if the three forces are concurrent or parallel force system

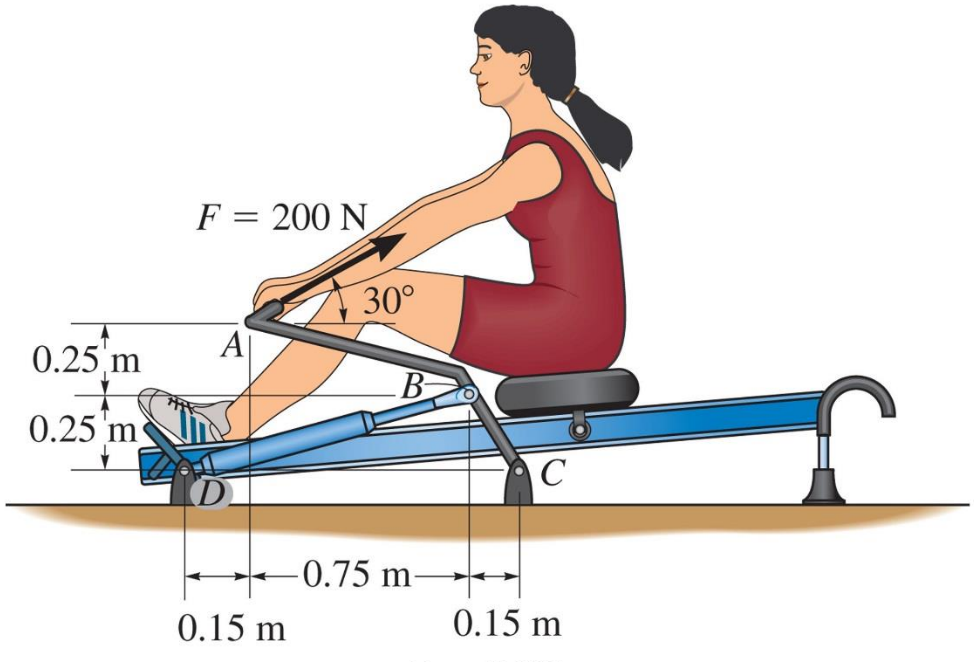


Three-force member



How many "two-force" members in this system?

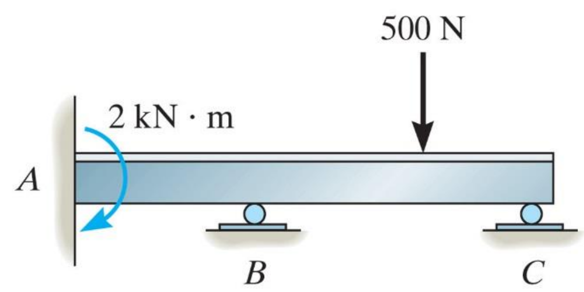
A) 0
B) 1
C) 2
D) 3
E) 4

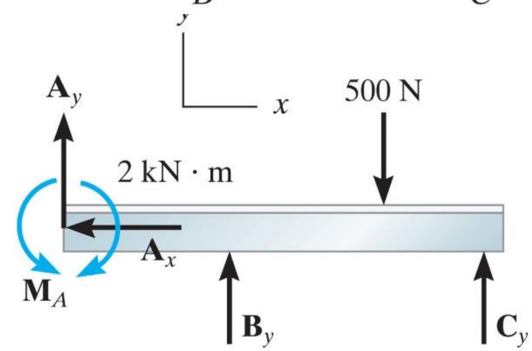


The woman exercises on the rowing machine. If she exerts a holding force of $F = 200$ lb on the handle ABC, determine the reaction force at pin C and the force developed along the hydraulic cylinder BD on the handle.

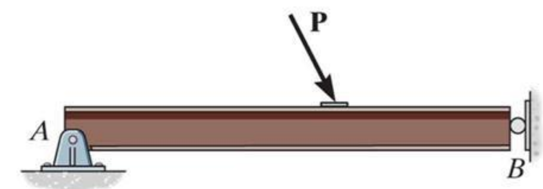
Constraints

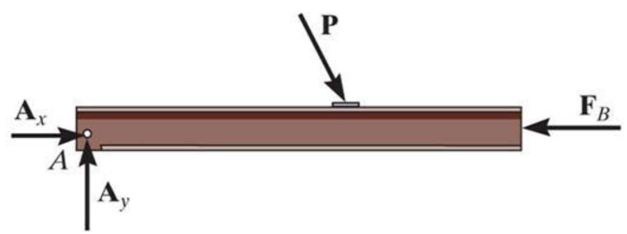
To ensure equilibrium of a rigid body, it is not only necessary to satisfy equations of equilibrium, but the body must also be properly constrained by its supports



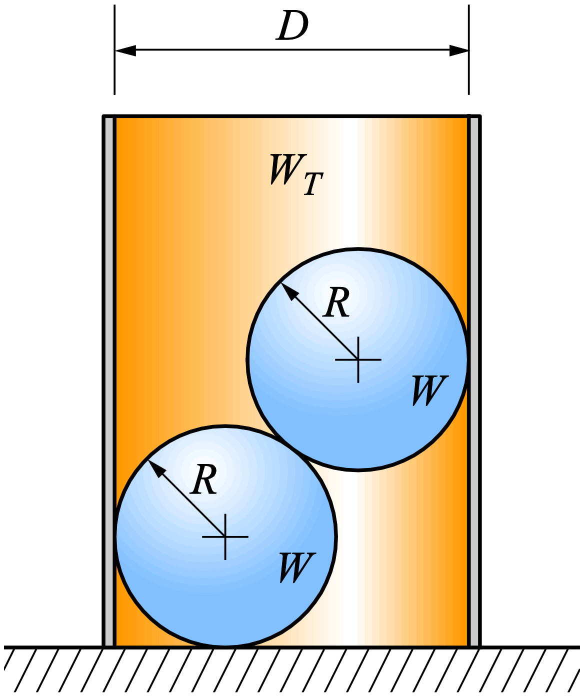


- **Redundant constraints**: the body has more supports than necessary to hold it in equilibrium; the problem is STATICALLY INDERTERMINATE and cannot be solved with statics alone





- **Improper constraints:** In some cases, there may be as many unknown reactions as there are equations of equilibrium. However, if the supports are not properly constrained, the body may become unstable for some loading cases.



Two marbles, which of radius R and weight W, are placed inside a hollow thin-walled tube of diameter D. Note that $D < 4R$, so that only one marble touches the floor. Find the minimum weight W_T of the tube such that it will not turn over. All surfaces are smooth.

Equilibrium in three-dimensional bodies

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

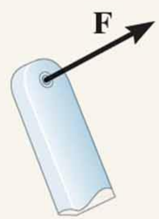
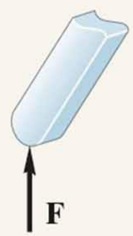
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

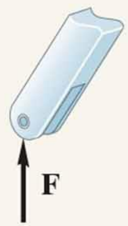
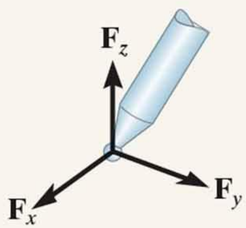
Types of Connection	Reaction	Number of Unknowns
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

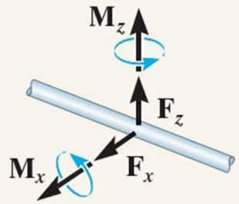
Types of Connection	Reaction	Number of Unknowns
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are *generally not applied* if the body is supported elsewhere. See the examples.

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

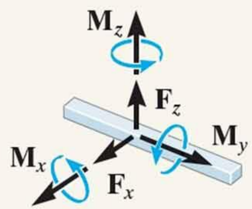
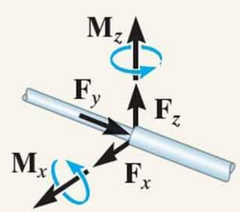
Types of Connection	Reaction	Number of Unknowns
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

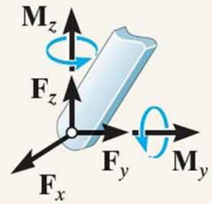
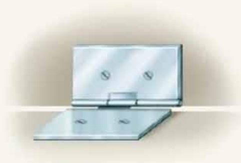
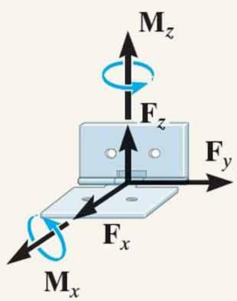
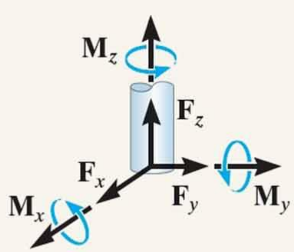
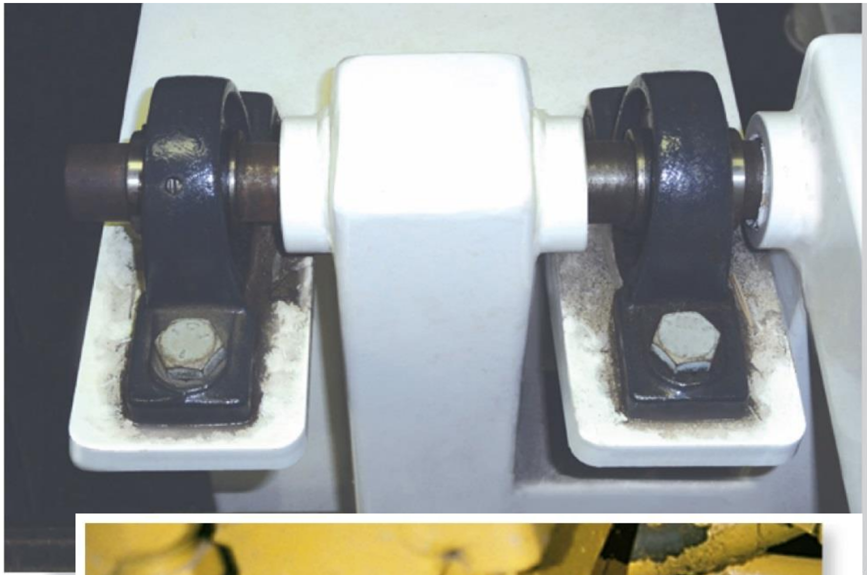
Types of Connection	Reaction	Number of Unknowns
(8)  single smooth pin	 	Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(9)  single hinge	 	Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(10)  fixed support	 	Six unknowns. The reactions are three force and three couple-moment components.

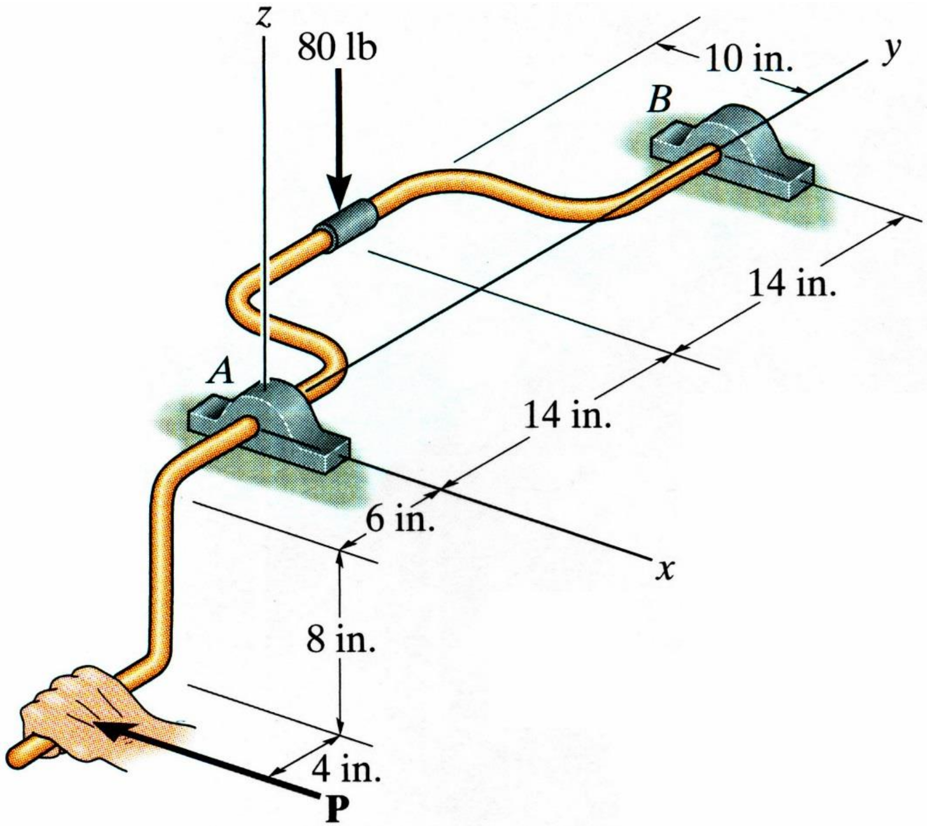
The journal bearings support the ends of the shaft. (5)



This ball-and-socket joint provides a connection for the housing of an earth grader to its frame. (4)

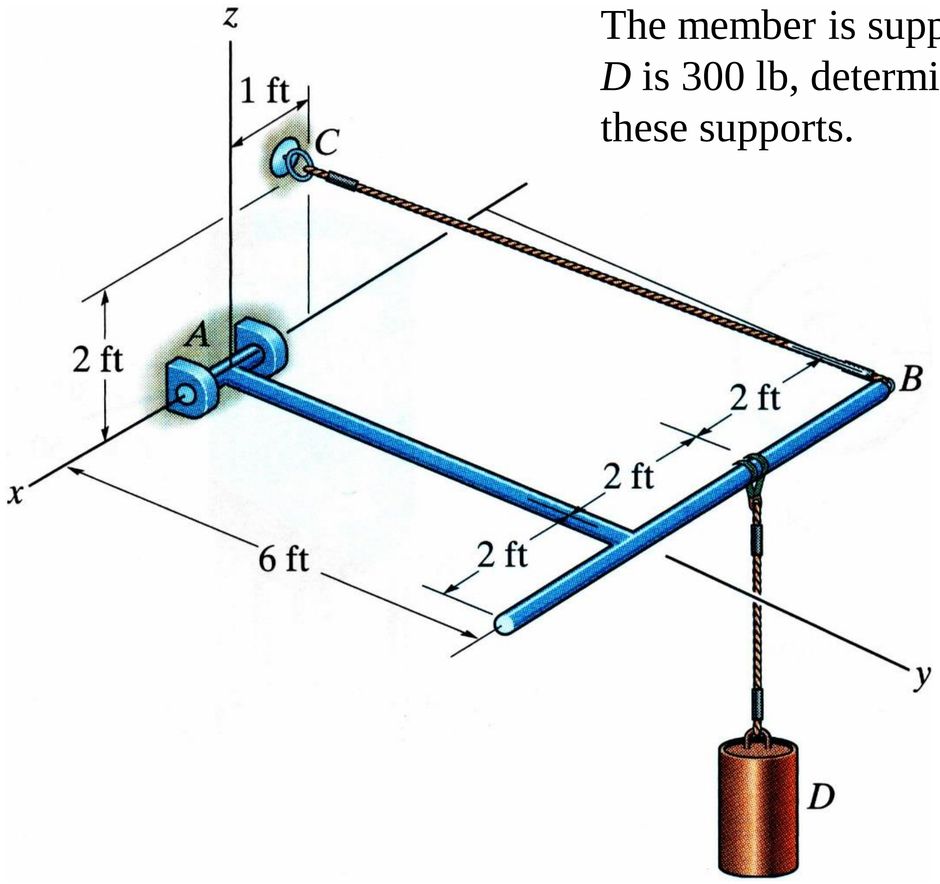
This pin is used to support the end of the strut used on a tractor. (8)

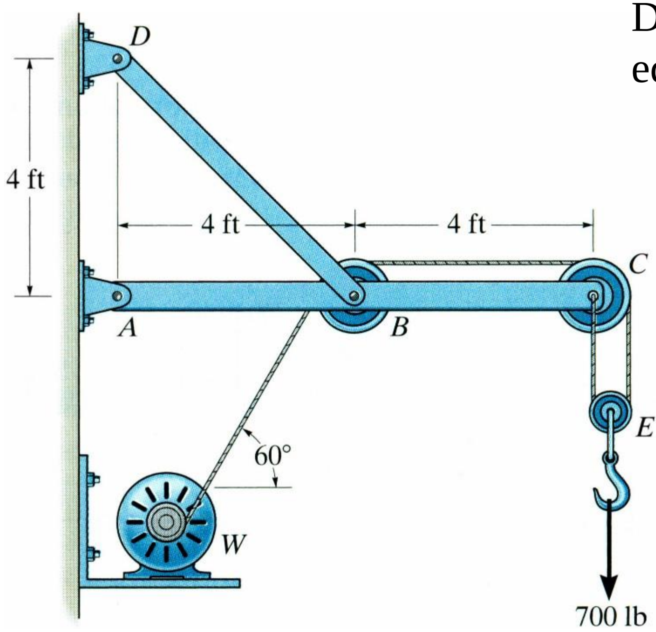
This thrust bearing is used to support the drive shaft on a machine. (7)



A vertical force of 80 lb acts on the crankshaft. Determine the horizontal equilibrium force *P* that must be applied to the handle and the *x*, *y*, *z* components of force at the smooth journal bearing *A* and the thrust bearing *B*. The bearings are properly aligned and exert only force reactions on the shaft.

The member is supported by a pin at *A* and cable *BC*. If the load at *D* is 300 lb, determine the *x*, *y*, *z* components of the reactions at these supports.





Draw a free-body diagram of the entire machine and set it into equilibrium. Neglect pulley sizes. Let *P* denote the 700 lb load.