Announcements

- CBTF Quiz 7 this week!
- Last day of office hours and Piazza help: Wed, Dec. 13
- No discussion sections next week

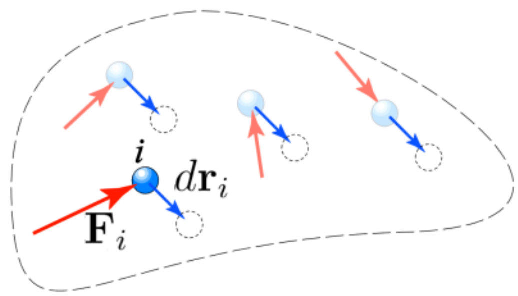
❑ Upcoming deadlines:

- Saturday (12/9)
  - ME HW27
- Tuesday (12/12)
  - PL HW26

Definition of Work

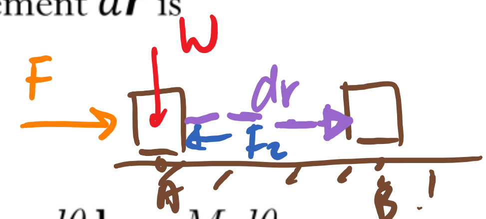
Work of a force

A force does work when it undergoes a displacement in the direction of the line of action.



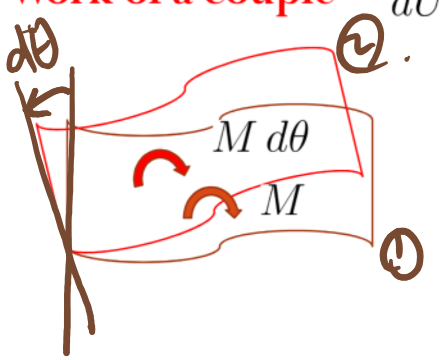
The work dU produced by the force $\boldsymbol{F}$ when it undergoes a differential displacement $d\boldsymbol{r}$ is given by

$$dU = \mathbf{F} \cdot d\mathbf{r}$$



Work of a couple

$$dU = M\mathbf{k} \cdot d\theta\,\mathbf{k} = M\,d\theta$$



dr // F = work (positive)

dr ⊥ W = no work.

dr // F_2 : negative work

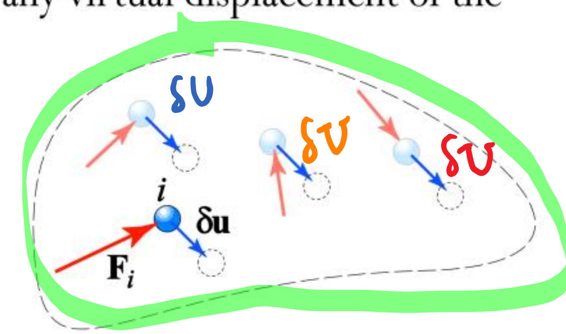
Virtual Displacements

A *virtual displacement* is a conceptually possible displacement *or* rotation of all *or* part of a system of particles. The movement is assumed to be possible, but actually does not exist. These "movements" are first order differential quantities: $\delta x, \delta y, \delta \theta$

Virtual Work

The principle of virtual work states that if a body is in equilibrium, then the algebraic sum of the virtual work done by all the forces and couple moments acting on the body is zero for any virtual displacement of the body. Thus,

$$\delta U = 0$$



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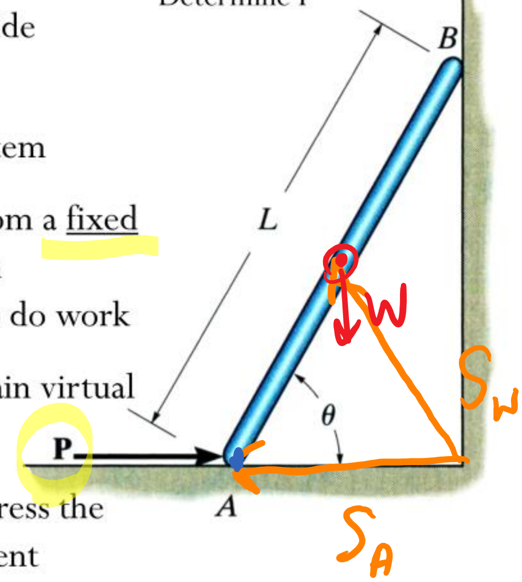
Procedure for Analysis

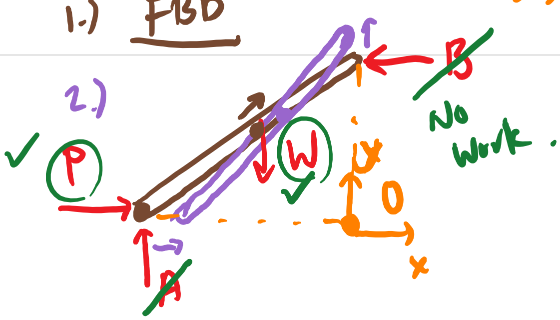
1. Draw FBD of the entire system and provide coordinate system
2. Sketch the "deflected position" of the system
3. Define position coordinates measured from a <u>fixed point</u> and select the parallel line of action component and remove forces that do no do work
4. <u>Differentiate</u> position coordinates to obtain virtual displacement
5. Write the virtual work equation and express the virtual work of each force/ couple moment
6. Factor out the comment virtual displacement term and solve

- Thin rod of weight *W*
- Smooth wall and floor
- Determine P



1.) FBD

2.)



No Work.

3.) $\vec{s}_A = -L\cos\theta\,\hat{i}$

$\vec{s}_W = -\frac{L}{2}\cos\theta\,\hat{i} + \frac{L}{2}\sin\theta\,\hat{j}$

$\cancel{\vec{s}_B} = L\sin\theta\,\hat{j}$

4). $\delta\left(\vec{s}_A = -L\cos\theta\,\hat{i}\right)$

$\delta\vec{s}_A = -L(-\sin\theta)\,\delta\theta\,\hat{i}$

$\delta\left(\vec{s}_W = -\frac{L}{2}\cos\theta\,\hat{i} + \frac{L}{2}\sin\theta\,\hat{j}\right)$

$\delta\vec{s}_W = -\frac{L}{2}(-\sin\theta)\,\delta\theta\,\hat{i} + \frac{L}{2}\cos\theta\,\delta\theta\,\hat{j}$

$\delta U = \vec{F}\cdot\delta\vec{r}$

$\vec{P} = P\hat{i}$

$\vec{W} = -W\hat{j}$

$\Rightarrow$ 5.) $\delta U = 0 = \vec{P}\cdot\delta\vec{s}_A + \vec{W}\cdot\delta\vec{s}_W$

$$= P(L\sin\theta\,\delta\theta) + (-W)\left(\frac{L}{2}\cos\theta\,\delta\theta\right)$$

$$= \underbrace{L\delta\theta\left(P\sin\theta - \frac{W}{2}\cos\theta\right) = 0}$$

$$\cancel{L=0}, \quad \cancel{\delta\theta=0}, \quad \underbrace{P\sin\theta - \frac{W}{2}\cos\theta} = 0 \Rightarrow \boxed{P = \frac{W\cos\theta}{2\sin\theta}}$$

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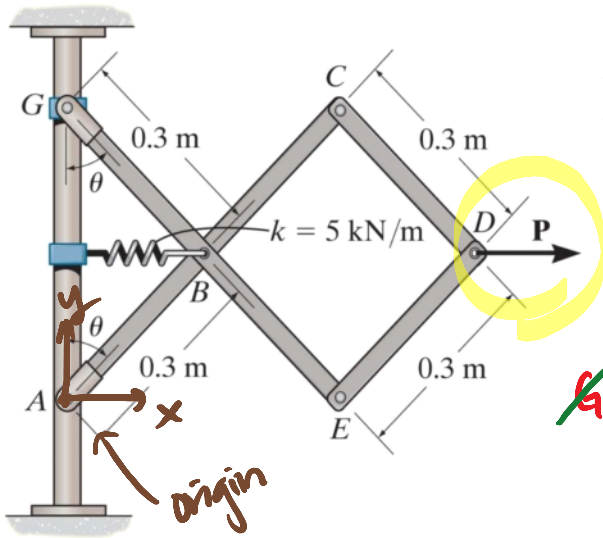
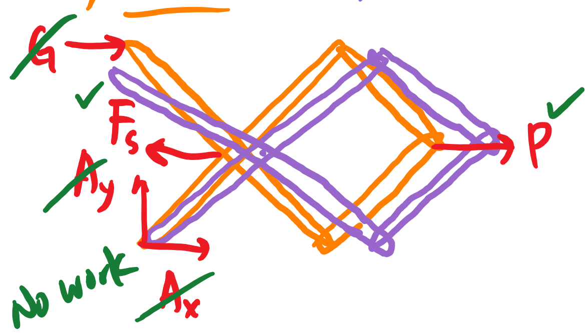


Determine the required force P needed to maintain equilibrium of the scissors linkage when the angle is 60 degrees. The spring is unstretched when the angle is 30 degrees.

1.) FBD 2)



3.) $\vec{S}_B = 0.3\sin\theta\,\hat{i} + 0.3\cos\theta\,\hat{j}$

$\vec{S}_D = 3(0.3\sin\theta)\,\hat{i} + 0.3\cos\theta\,\hat{j}$

4.) $\delta\vec{S}_B = 0.3\cos\theta\,\delta\theta\,\hat{i} - 0.3\sin\theta\,\delta\theta\,\hat{j}$ $\quad \vec{P} = P\hat{i}$

$\delta\vec{S}_D = 0.9\cos\theta\,\delta\theta\,\hat{i} - 0.3\sin\theta\,\delta\theta\,\hat{j}$ $\quad \vec{F}_s = -ks\,\hat{i}$

$s = (0.3\text{ m})(\sin 60^\circ - \sin 30^\circ)$

5) $\delta U = \vec{P}\cdot\delta\vec{S}_D + \vec{F}_s\cdot\delta\vec{S}_B$

$= P(0.9)\cos\theta\,\delta\theta + (-ks)(0.3)\cos\theta\,\delta\theta = 0$

$$\boxed{P = \frac{ks}{3}}$$