# Announcements

- CBTF Quiz 7 next week
- 3D rigid body problems: PL HW10

❑ Upcoming deadlines:

- Friday (12/1)
  - WA #4
- Saturday (12/2)
  - ME HW25

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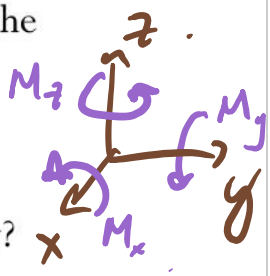
# Chapter 5 Part II – 3-D Rigid Body

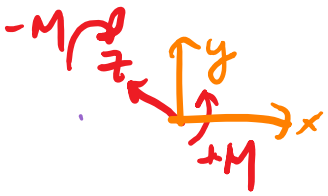
# Equilibrium of a rigid body


Now we add the z-axis to the coordinate system!

What are the possible movements for a 3-D body?




Moment < translate (F) : 2D → x, y ; 3D → x, y, z
rotate (M) : z → x, y, z




6 equations of equilibrium

$\Sigma F_x = 0$ $\Sigma M_x = 0$
$\Sigma F_y = 0$ $\Sigma M_y = 0$
$\Sigma F_z = 0$ $\Sigma M_z = 0$

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**TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems**

| Types of Connection | Reaction | Number of Unknowns |
|---|---|---|
| (1)  cable |   |  |
| (2)  smooth surface support |   |  |
| (3)  roller |   |  |

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**TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems**

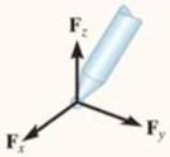
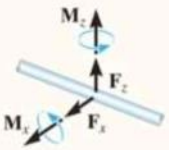
| Types of Connection | Reaction | Number of Unknowns |
|---|---|---|
| (4)  ball and socket |   |  |
| (5)  single journal bearing |   |  |

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**TABLE 5–2 Continued**

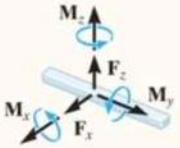
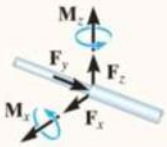
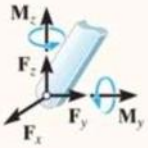
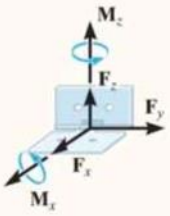
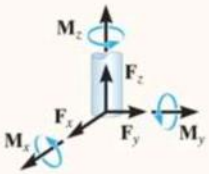
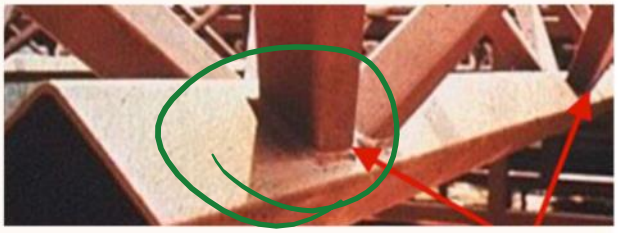
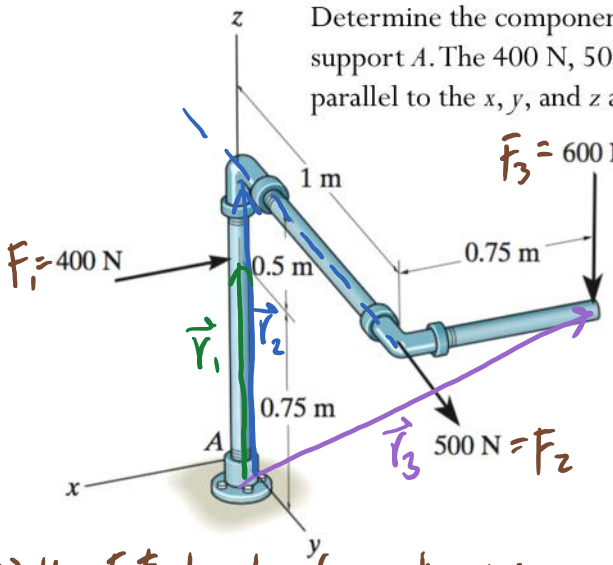
| Types of Connection | Reaction | Number of Unknowns |
|---|---|---|
| (6)  single journal bearing with square shaft |   |  |
| (7)  single thrust bearing |   |  |
| (8)  single smooth pin |   |  |

**TABLE 5–2 Continued**

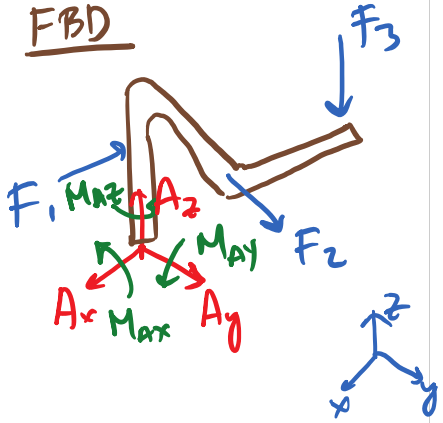
| Types of Connection | Reaction | Number of Unknowns |
|---|---|---|
| (9)  single hinge |   |  |
| (10)  fixed support |   |  |

Determine the components of reaction at the fixed support $A$. The 400 N, 500 N, and 600 N forces are parallel to the $x$, $y$, and $z$ axes, respectively.




1) Draw FBD of system

FBD




2.) Use EoE to solve for unknowns.

EoE

$\Sigma F_x = A_x - F_1 = 0 \Rightarrow \boxed{A_x = F_1 = 400 \text{ N}}$

$\Sigma F_y = A_y + F_2 = 0 \Rightarrow \boxed{A_y = -F_2 = -500 \text{ N}}$

$\Sigma F_z = A_z - F_3 = 0 \Rightarrow \boxed{A_z = F_3 = 600 \text{ N}}$

Get from vector notations

$\Sigma M_x = 0$

$\Sigma M_y = 0$

$\Sigma M_z = 0$

A) Equilant moment about A from $\vec{F}_1$

$\Rightarrow \vec{M}_{A1} = \vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & .75 \\ -400 & 0 & 0 \end{vmatrix} = -300\hat{j} \text{ N·m}$

B.) Moment about A from $\vec{F}_2$.

$\vec{M}_{A2} = -500(1.25)\hat{i} = -625\hat{i} \text{ N·m}$

C.) Moment about A from $\vec{F}_3$

$\vec{M}_{A3} = \vec{r}_3 \times \vec{F}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ .75 & 1.25 & \\ 0 & 0 & -600 \end{vmatrix} = -600\hat{i} - 450\hat{j} \text{ N·m}$

$\Rightarrow \Sigma \vec{M}_A = 0 = \vec{M}_{A1} + \vec{M}_{A2} + \vec{M}_{A3} + \vec{M}_A$ ← support moment at A

in comp.

$\Sigma M_{Ax} = M_{2x} + M_{3x} + M_{Ax} = 0 \Rightarrow M_{Ax} = (625 + 600) \text{ N·m} = \boxed{1225 \text{ N·m} = M_{Ax}}$

$\Sigma M_{Ay} = M_{1y} + M_{3y} + M_{Ay} = 0 \Rightarrow M_{Ay} = (300 + 450) \text{ N·m} = \boxed{750 \text{ N·m} = M_{Ay}}$

in components:

$\Sigma M_{Ay} = M_{1y} + M_{3y} + M_{Ay} = 0 \Rightarrow M_{Ay} = (300 + 450)\,N \cdot m = \boxed{750\,N \cdot m = M_{Ay}}$

$\Sigma M_{Az} = \boxed{M_{Az} = 0}$

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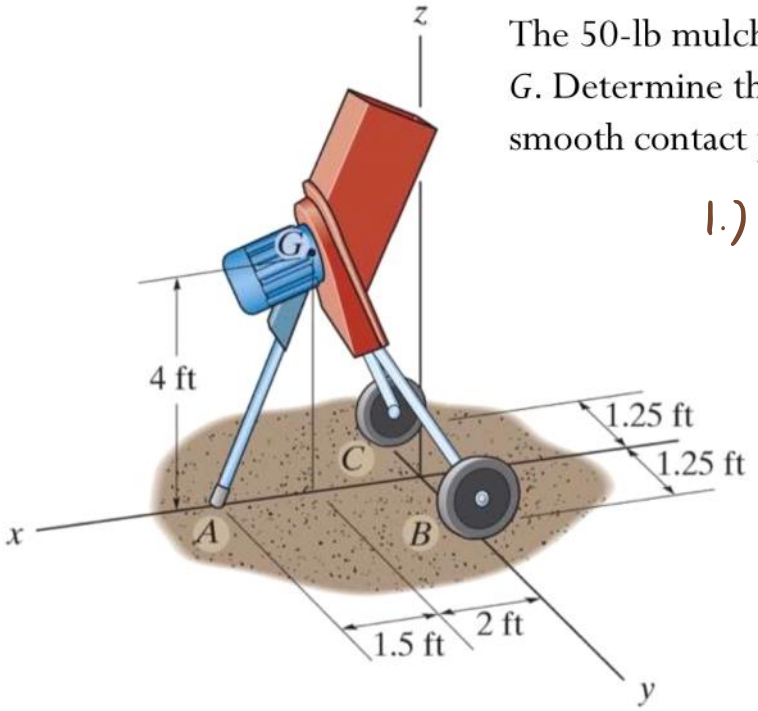
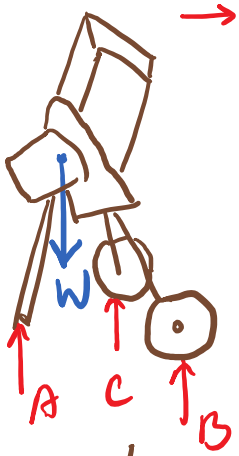



The 50-lb mulching has a center of gravity at G. Determine the vertical reactions at the smooth contact point A.

1.) Draw FBD of the system

→ 3 unknowns from supports




2) Quickest way to find $\vec{A}$ is moment about y-axis. ($\vec{B}$ & $\vec{C}$ intercept y-axis)

- Moment of $\vec{W}$ about y:

$$M_{wy} = \hat{j} \cdot (\vec{r}_A \times \vec{W}) = \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 4 \\ 0 & 0 & -50 \end{vmatrix} = 100 \text{ lb} \cdot \text{ft}$$

- Moment of $\vec{A}$ about y:

$$M_{Ay} = \hat{j} \cdot (\vec{r}_G \times \vec{A}) = \begin{vmatrix} 0 & 1 & 0 \\ 3.5 & 0 & 0 \\ 0 & 0 & A \end{vmatrix} = -3.5A$$

$$\rightarrow \Sigma M_y = M_{wy} + M_{Ay} = 100 \text{ lb} \cdot \text{ft} - 3.5A = 0$$

$$\Rightarrow \boxed{A = \frac{100}{3.5} \text{ lb}}$$