wc32lect

Wednesday, November 15, 2017 12:00 PM

wc32lect

Announcements

- CBTF Quiz 6 this week
- No class on Friday ☺ (But Friday discussions still meet)
- Written Assignment #4 – Due Friday, Dec. 1

❑ Upcoming deadlines:

- Wednesday (11/15)
  - PL HW22
- Thursday (11/16)
  - ME HW23

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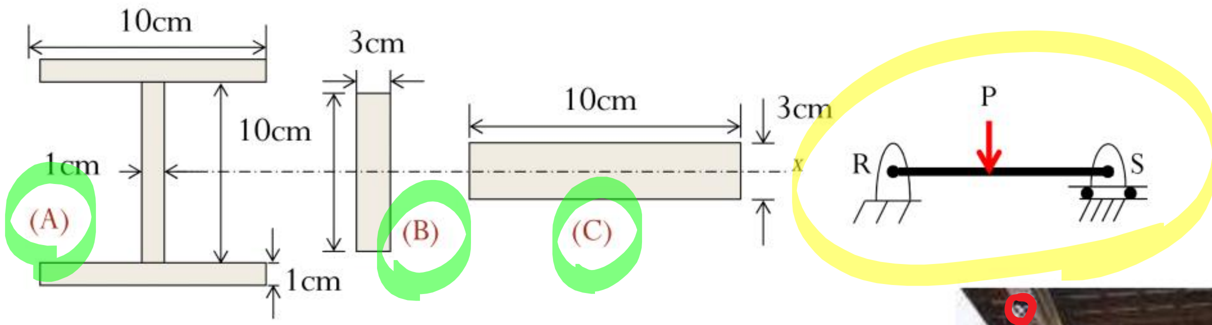
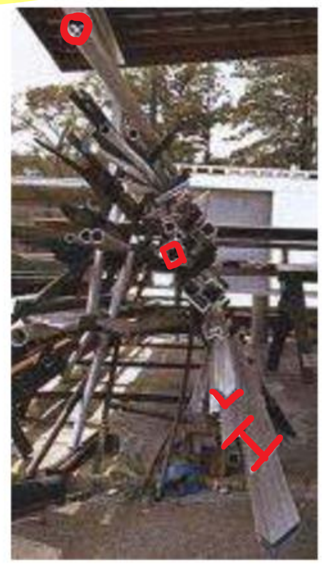
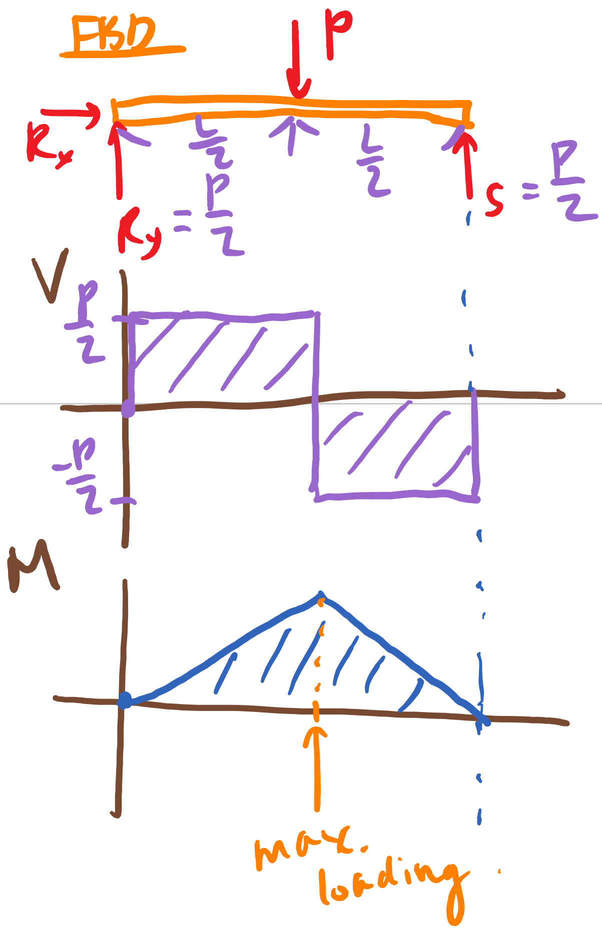
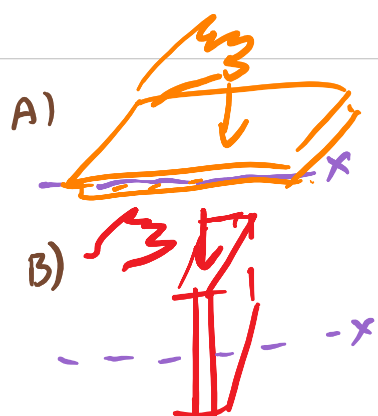
Moment of Inertia for Areas



Consider three different possible cross sectional shapes and areas for the beam RS. For the given vertical loading P on the beam, which shape will develop less internal stress and deflection?







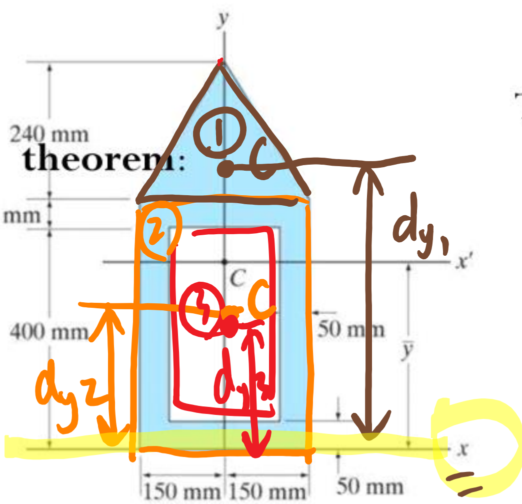
→ (A) is easier to break since its MoI about x is smaller.

Moment of inertia of composite

- If individual bodies making up a **composite** body have individual areas *A* and moments of inertia *I* computed through their centroids, then the **composite area** and **moment of inertia** is a sum of the individual component contributions.

$$I_x = \Sigma I_{x_i} = I_{x1} + I_{x2} - I_{x3}$$

This requires the **parallel axis theorem:**



$$I_{x1} = I_{x'} + d_{y1}^2 A_1$$

$$I_{x2} = I_{x'} + d_{y2}^2 A_2$$

$$I_{x3} = I_{x'} + d_{y3}^2 A_3$$

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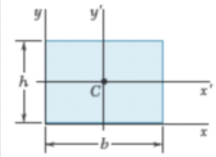
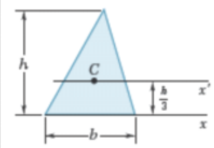
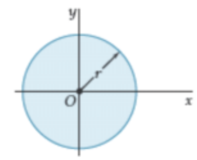
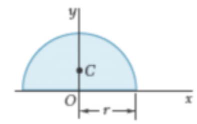
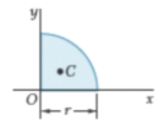
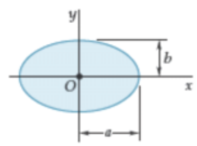
Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

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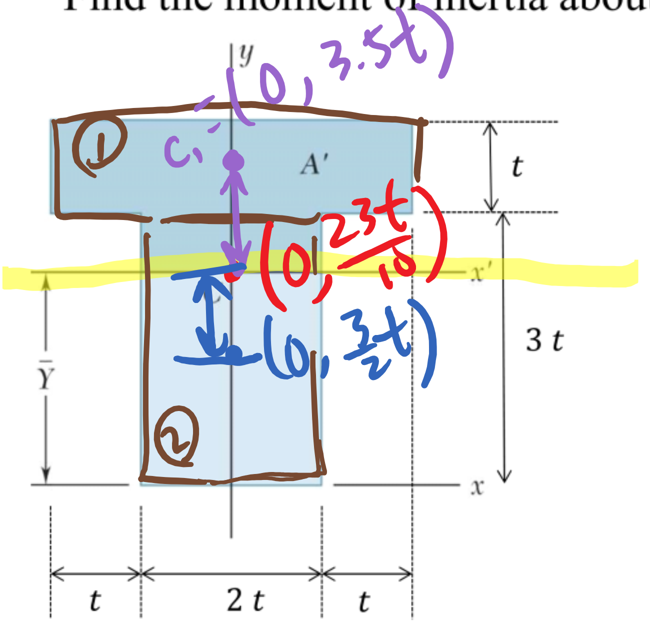
Find the moment of inertia about its centroid: (x')



$$\bar{Y} = \frac{4t^2\,(3.5t) + 6t^2\,(1.5t)}{4t^2 + 6t^2} = \frac{23t}{10}$$

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A}.$$

$$d_{y_1} = 3.5t - \frac{23t}{10}.$$

$$\boxed{I_{x'} = I_{x'_1} + I_{x'_2}} = \frac{12t}{10}.$$

$$I_{x'_1} = I_{x_1} + d_y^{\,2} A_1$$

$$= \frac{bh^3}{12} + \left(\frac{12t}{10}\right)^2 (4t)(t)$$

$$= \frac{(4t)(t)^3}{12} + \left(\frac{12t}{10}\right)^2 (4t)(t)$$

$$I_{x'_2} = I_{x'} + d_y^2 A$$

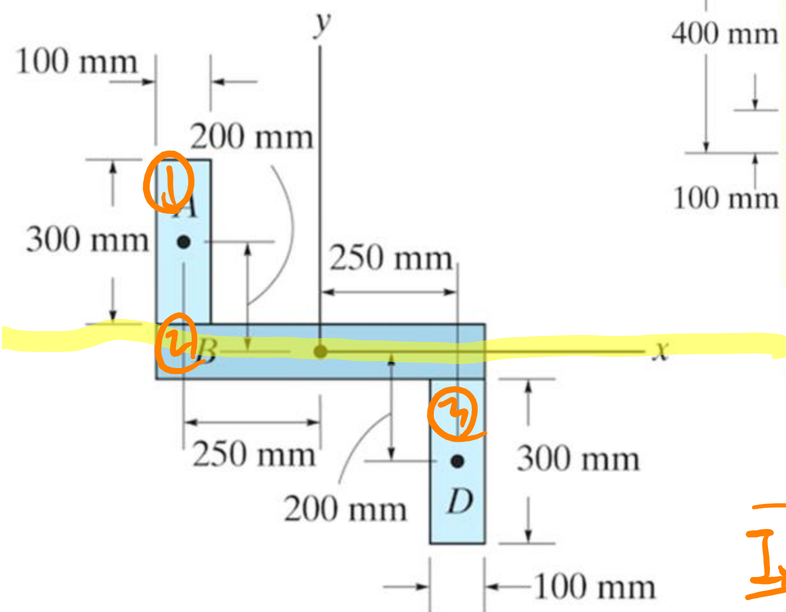
$$d_y = \frac{23t}{10} - \frac{3t}{2} = \frac{8t}{10}$$

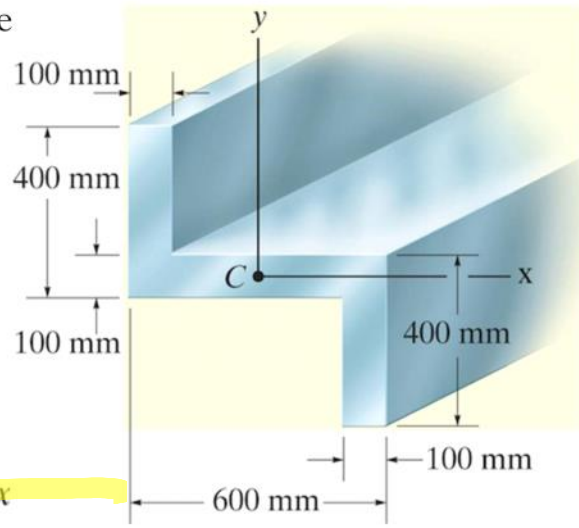
$$\Rightarrow \frac{bh^3}{12} + \left(\frac{8t}{10}\right)(2t)(3t)$$

$$= \frac{(2t)(3t)^3}{12} + \left(\frac{8t}{10}\right)(2t)(3t)$$

$$\rightarrow \underline{I_{x'} = 14.4t^4}.$$

Determine the moment of inertia for the cross-sectional area about the x and y centroidal axes.



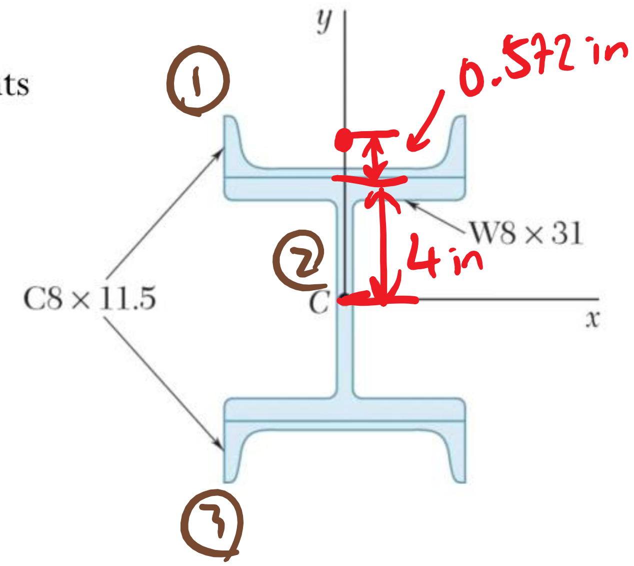


	①	②	③
$I_{x'}$	$2.25\times10^{8}\ mm^4$	$5\times10^{7}\ mm^4$	$2.25\times10^{8}\ mm^4$
d_y	200 mm	0	200 mm
A	$30000\ mm^2$	$60000\ mm^2$	$30000\ mm^2$

$$I_x = (I_{x'_1} + d_{y_1}^2 A_1) + I_{x'_2} + (I_{x'_3} + d_{y_3}^2 A_3)$$
$$= 2.9\times10^{9}\ mm^4 \ \checkmark$$

$$I_y = 5.6\times10^{9}\ mm^4 \ \checkmark$$

Two channels are welded to a rolled W section as shown. Determine the moments of inertia of the combined section with respect to the centroidal x and y axes.



$$I_x = I_{x_1} + I_{x_2} + I_{x_3}$$

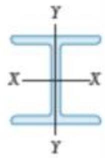
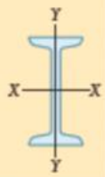
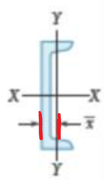
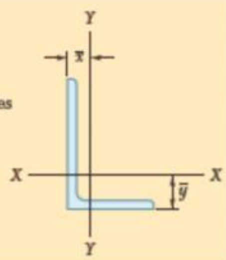
$$I_{x_1} = [1.31 + (4.572)^2(3.37)]\ in^4 = 71.8\ in^4$$

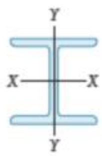
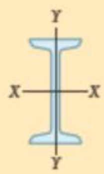
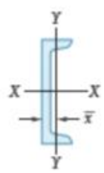
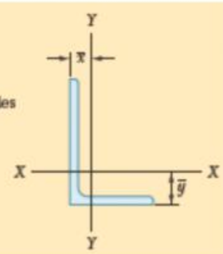
$$I_{x_2} = 110\ in^4$$

$$I_{x_3} = I_{x_1}$$

$$\Rightarrow I_x = \underline{254\ in^4.}$$

$$I_y = I_{y_1} + I_{y_2} + I_{y_3} = 32.5\ in^4 + 37.1\ in^4 + 32.5\ in^4 = \underline{102.1\ in^4}$$

	Designation	Area in²	Depth in.	Width in.	Axis X-X: $\bar{I}_x$, in⁴	Axis X-X: $\bar{k}_x$, in.	Axis X-X: $\bar{y}$, in.	Axis Y-Y: $\bar{I}_y$, in⁴	Axis Y-Y: $\bar{k}_y$, in.	Axis Y-Y: $\bar{x}$, in.
W Shapes (Wide-Flange Shapes) 	W18 × 76†	22.3	18.2	11.0	1330	7.73		152	2.61	
	W16 × 57	16.8	16.4	7.12	758	6.72		43.1	1.60	
	W14 × 38	11.2	14.1	6.77	385	5.87		26.7	1.55	
	W8 × 31	9.12	8.00	8.00	110	3.47		37.1	2.02	
S Shapes (American Standard Shapes) 	S18 × 54.7†	16.0	18.0	6.00	801	7.07		20.7	1.14	
	S12 × 31.8	9.31	12.0	5.00	217	4.83		9.33	1.00	
	S10 × 25.4	7.45	10.0	4.66	123	4.07		6.73	0.950	
	S6 × 12.5	3.66	6.00	3.33	22.0	2.45		1.80	0.702	
C Shapes (American Standard Channels) 	C12 × 20.7†	6.08	12.0	2.94	129	4.61		3.86	0.797	0.698
	C10 × 15.3	4.48	10.0	2.60	67.3	3.87		2.27	0.711	0.634
	C8 × 11.5	3.37	8.00	2.26	32.5	3.11		1.31	0.623	0.572
	C6 × 8.2	2.39	6.00	1.92	13.1	2.34		0.687	0.536	0.512
Angles 	L6 × 6 × 1‡	11.0			35.4	1.79	1.86	35.4	1.79	1.86
	L4 × 4 × $\frac{1}{2}$	3.75			5.52	1.21	1.18	5.52	1.21	1.18
	L3 × 3 × $\frac{1}{4}$	1.44			1.23	0.926	0.836	1.23	0.926	0.836
	L6 × 4 × $\frac{1}{2}$	4.75			17.3	1.91	1.98	6.22	1.14	0.981
	L5 × 3 × $\frac{1}{2}$	3.75			9.43	1.58	1.74	2.55	0.824	0.746
	L3 × 2 × $\frac{1}{4}$	1.19			1.09	0.953	0.980	0.390	0.569	0.487

	Designation	Area mm^2	Depth mm	Width mm	Axis X-X $\bar{I}_x$ 10^6 mm^4	$\bar{k}_x$ mm	$\bar{y}$ mm	Axis Y-Y $\bar{I}_y$ 10^6 mm^4	$\bar{k}_y$ mm	$\bar{x}$ mm
W Shapes (Wide-Flange Shapes) 	W460 × 113†	14400	462	279	554	196		63.3	66.3	
	W410 × 85	10800	417	181	316	171		17.9	40.6	
	W360 × 57.8	7230	358	172	160	149		11.1	39.4	
	W200 × 46.1	5880	203	203	45.8	88.1		15.4	51.3	
S Shapes (American Standard Shapes) 	S460 × 81.4†	10300	457	152	333	180		8.62	29.0	
	S310 × 47.3	6010	305	127	90.3	123		3.88	25.4	
	S250 × 37.8	4810	254	118	51.2	103		2.80	24.1	
	S150 × 18.6	2360	152	84.6	9.16	62.2		0.749	17.8	
C Shapes (American Standard Channels) 	C310 × 30.8†	3920	305	74.7	53.7	117		1.61	20.2	17.7
	C250 × 22.8	2890	254	66.0	28.0	98.3		0.945	18.1	16.1
	C200 × 17.1	2170	203	57.4	13.5	79.0		0.545	15.8	14.5
	C150 × 12.2	1540	152	48.8	5.45	59.4		0.286	13.6	13.0
Angles 	L152 × 152 × 25.4‡	7100			14.7	45.5	47.2	14.7	45.5	47.2
	L102 × 102 × 12.7	2420			2.30	30.7	30.0	2.30	30.7	30.0
	L76 × 76 × 6.4	929			0.512	23.5	21.2	0.512	23.5	21.2
	L152 × 102 × 12.7	3060			7.20	48.5	50.3	2.59	29.0	24.9
	L127 × 76 × 12.7	2420			3.93	40.1	44.2	1.06	20.9	18.9
	L76 × 51 × 6.4	768			0.454	24.2	24.9	0.162	14.5	12.4

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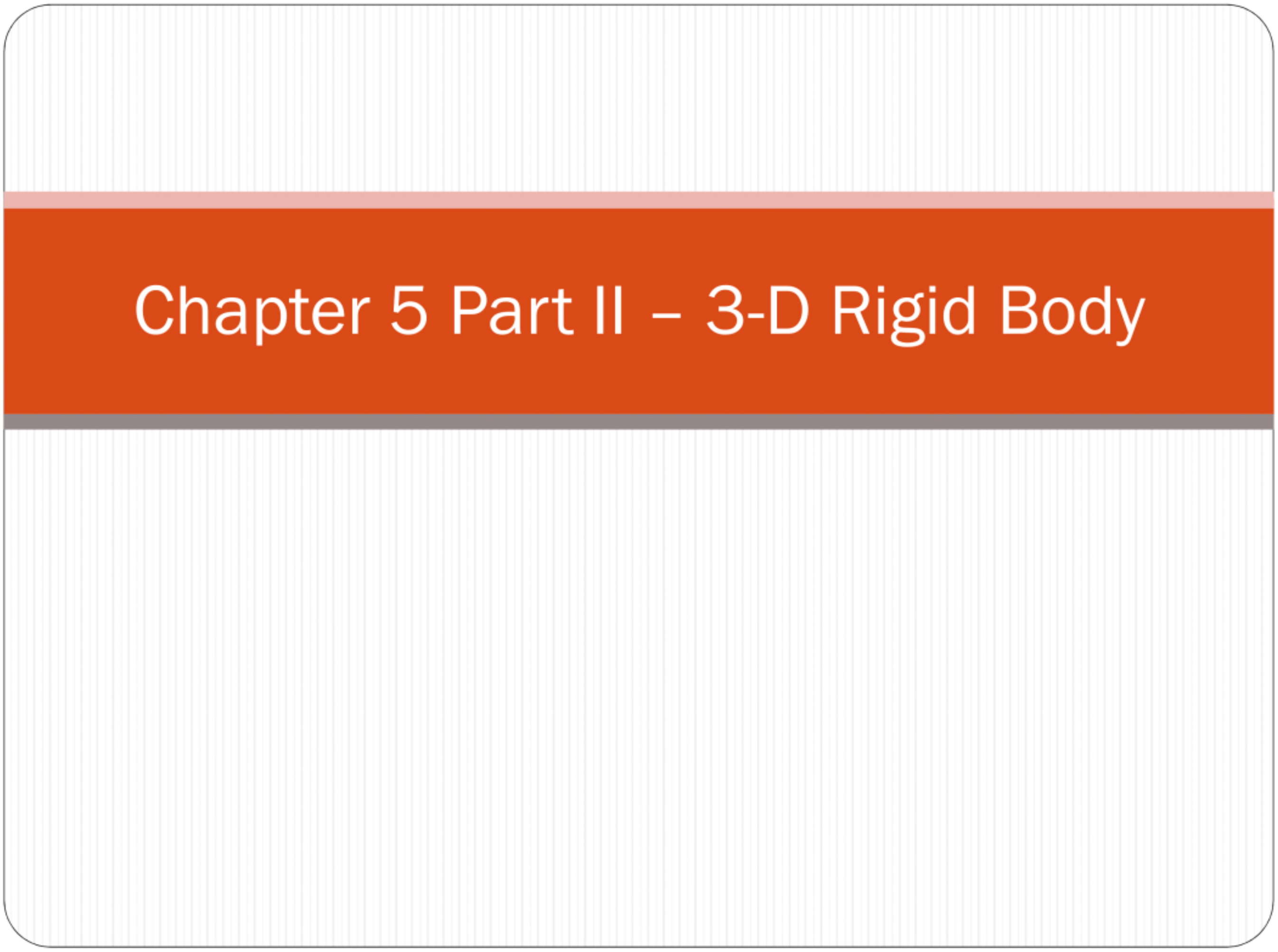

Equilibrium of a rigid body

Now we add the z-axis to the coordinate system!

6 Equations of Equilibriums:

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TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		
(2)  smooth surface support		
(3)  roller		

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

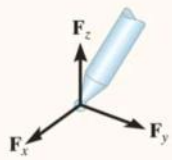
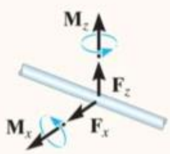
Types of Connection	Reaction	Number of Unknowns
(4)  ball and socket		
(5)  single journal bearing		

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TABLE 5–2 Continued

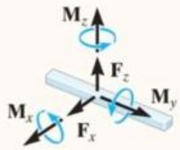
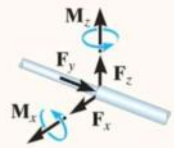
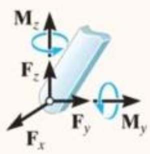
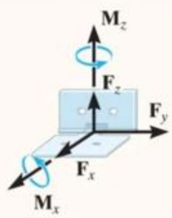
Types of Connection	Reaction	Number of Unknowns
(6)  single journal bearing with square shaft		
(7)  single thrust bearing		
(8)  single smooth pin		

TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(9)  single hinge		
(10)  fixed support