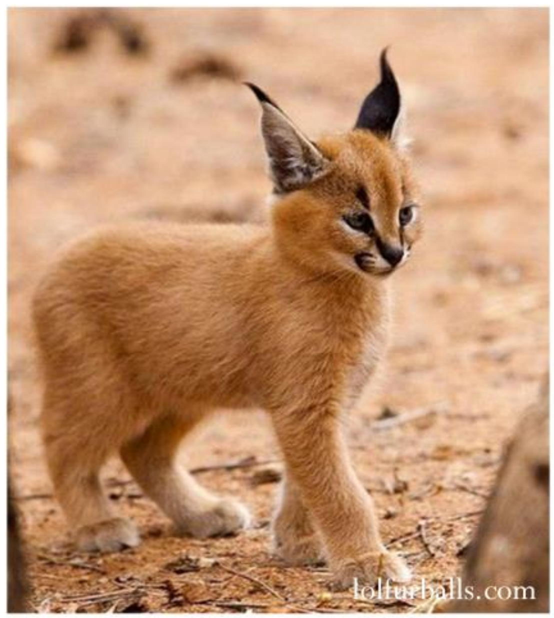
wc30lect

Announcements

- CBTF Quiz 6 next week
- 211 students **DO NOT** take 210 final, or you will get a **zero** on 211 final!!!



Upcoming deadlines:

- Tuesday (11/14)
  - PL HW22
- Thursday (11/16)
  - ME HW23

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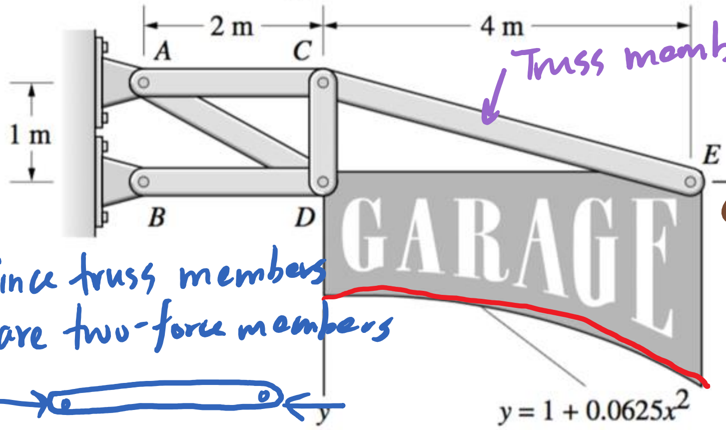
Center of Gravity Application

The suspended sign is a homogeneous flat plate that has a mass of 130 kg. Determine the support reaction at B.

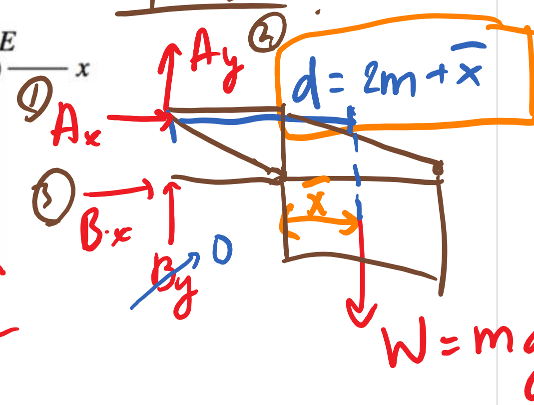


Truss member: assume negligible mass

• Since truss members are two-force members

$\Rightarrow B_y = 0$.

FBD



$$\boxed{d = 2m + \bar{x}}$$

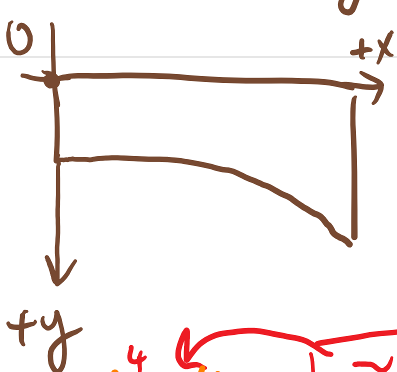
$$\sum M_A = B_x(1m) - Wd = 0.$$

$$\Rightarrow \boxed{B_x = \frac{Wd}{(1m)} = W(2 + \bar{x})}$$

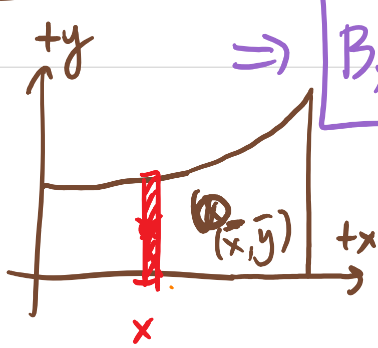
need $\bar{x}$ only for B_x.

$B_x = 5420$ N

CoG Analysis for $\bar{x}$



$\Rightarrow$



$$\bar{x} = \frac{\int_0^4 \tilde{x}\, dA}{\int_0^4 dA}.$$

$$\boxed{\tilde{x} = x}$$

$$dA = y\, dx$$

$$\boxed{dA = (1 + 0.0625x^2)\, dx}$$

$$\Longrightarrow \frac{\int_0^4 x(1 + 0.0625x^2)\, dx}{\int_0^4 1 + 0.0625x^2\, dx} = 2.25 \text{ ft}.$$

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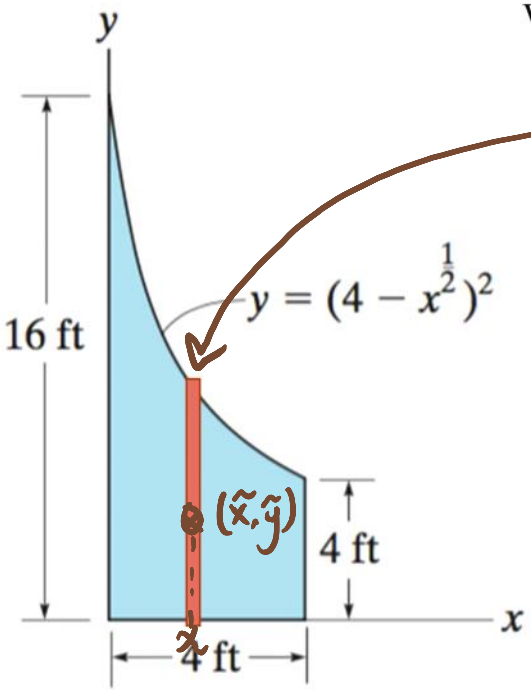
Example



Where is the centroid of the area?

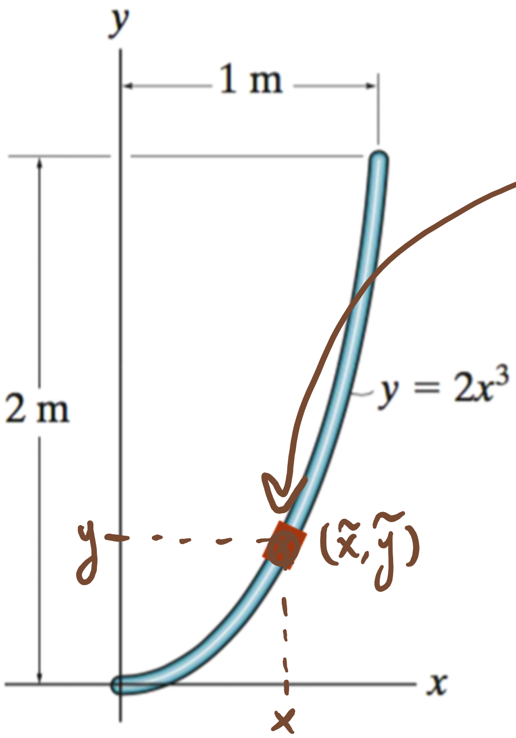
For the element:

$$\tilde{x} = x$$

$$\tilde{y} = \frac{y}{2} = \frac{(4 - x^{1/2})^2}{2}$$

$$dA = y\,dx = (4 - x^{1/2})^2\,dx$$

Example



Where is the centroid of the bar?

For the element:

$\tilde{x} = x$

$\tilde{y} = y = 2x^3$

$\sqrt[3]{\frac{y}{2}}$

$dL = \sqrt{dx^2 + dy^2}$

$\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

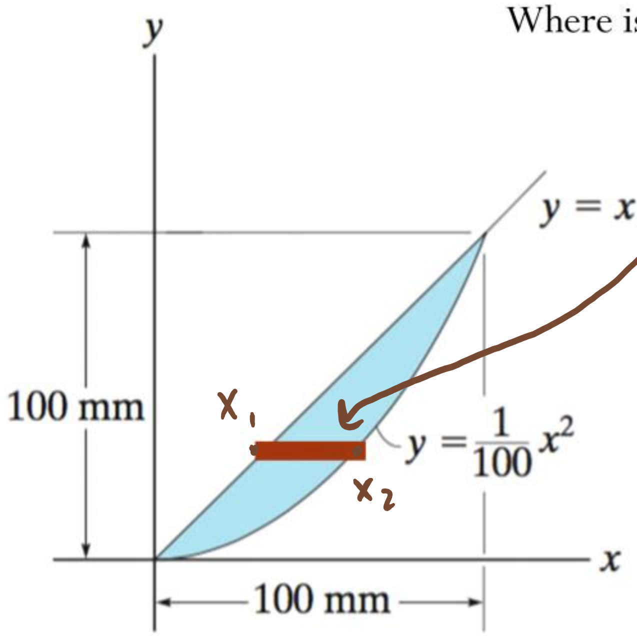
$= \sqrt{1 + 36x^4}\, dx$

$\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\, dy$

$= \sqrt{\left(\frac{y^{-2/3}}{3\sqrt[3]{2}}\right)^2 + 1}\, dy$

Example

Where is the centroid of the area?



For the element:

$$\tilde{x} = \frac{x_2 + x_1}{2} = \frac{\sqrt{100y} + y}{2}$$

$$\tilde{y} = y$$

$$dA = (x_2 - x_1)dy = (\sqrt{100y} - y)dy$$

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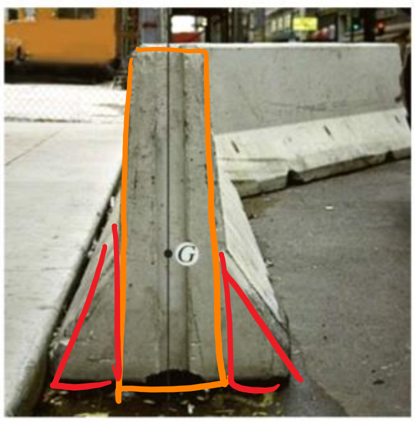
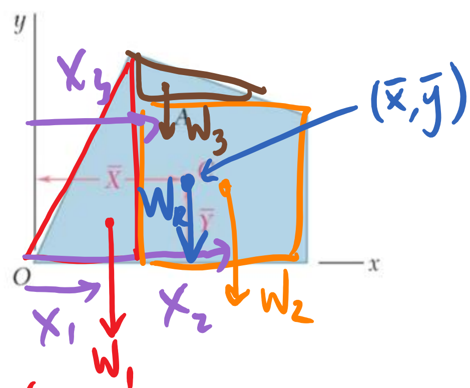
Composite bodies

A composite body consists of a series of connected simpler shaped bodies.

Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.





$$W_R = W_1 + W_2 + W_3$$

$$M_R = X_1 W_1 + X_2 W_2 + X_3 W_3$$

$$\bar{x} = \frac{M_R}{F_R} = \frac{\Sigma_i x_i W_i}{\Sigma_i W_i}$$

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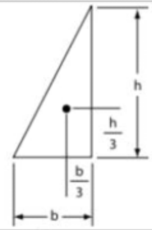
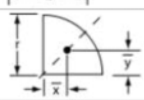
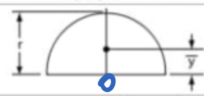
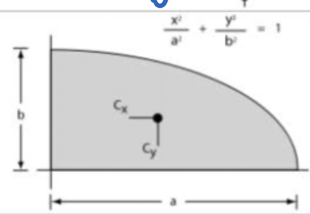
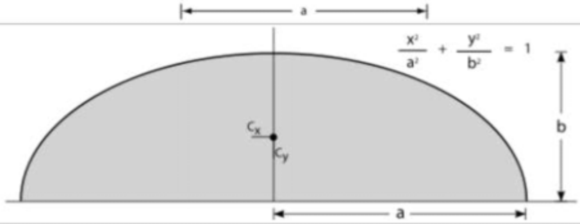
Centroid of typical 2D shapes

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Right-triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

http://en.wikipedia.org/wiki/List_of_centroids

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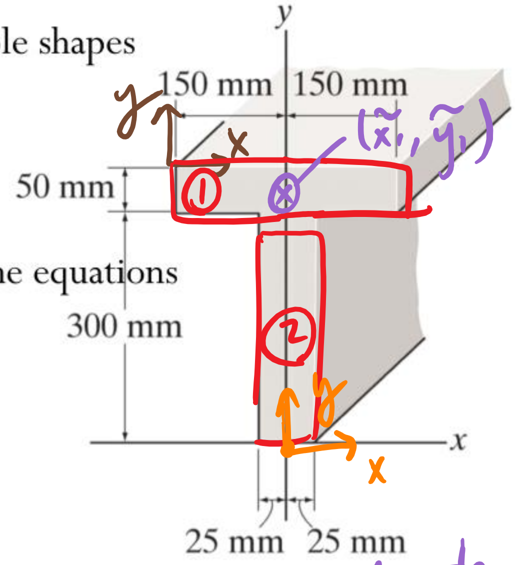
Composite bodies – Analysis Procedure

1. Divide the body into finite number of simple shapes
2. Consider "holes" as "negative" parts
3. Establish coordinate axes
4. Determine centroid location by applying the equations



$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W} \qquad \bar{x} = \frac{\sum \tilde{x}A}{\sum A}$$

$$\bar{y} = \frac{\sum \tilde{y}W}{\sum W} \qquad \bar{y} = \frac{\sum \tilde{y}A}{\sum A}$$

$$\bar{z} = \frac{\sum \tilde{z}W}{\sum W} \qquad \bar{z} = \frac{\sum \tilde{z}A}{\sum A}$$

$\tilde{x}, \tilde{y}$ depends on coordinate system

- for $\lfloor^{y}_{\to x}$, $\tilde{x}_1 = 150\text{ mm}$, $\tilde{y}_1 = -25\text{ mm}$
- for $\lfloor^{y}_{\to x}$, $\tilde{x}_1 = 0\text{ mm}$, $\tilde{y}_1 = 325\text{ mm}$

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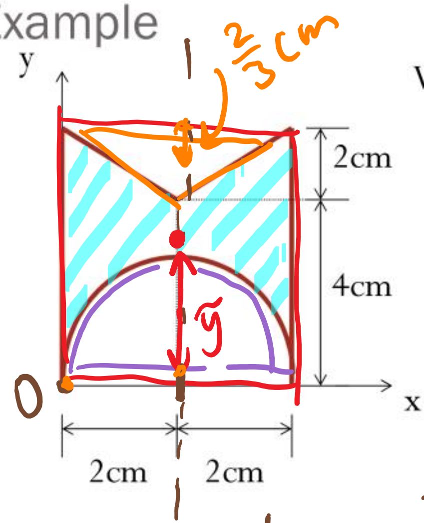
Example



What is the centroid of the resultant area?

	1	2	3
A	$24\,cm^2$	$4\,cm^2$	$\frac{\pi(2)^2}{2}\,cm^2$
$\tilde{y}$	$3\,m$	$5\frac{1}{3}\,cm$	$\frac{4(2)}{3\pi}\,cm.$

• use symmetry

$$\boxed{\bar{x} = 2\,cm}$$

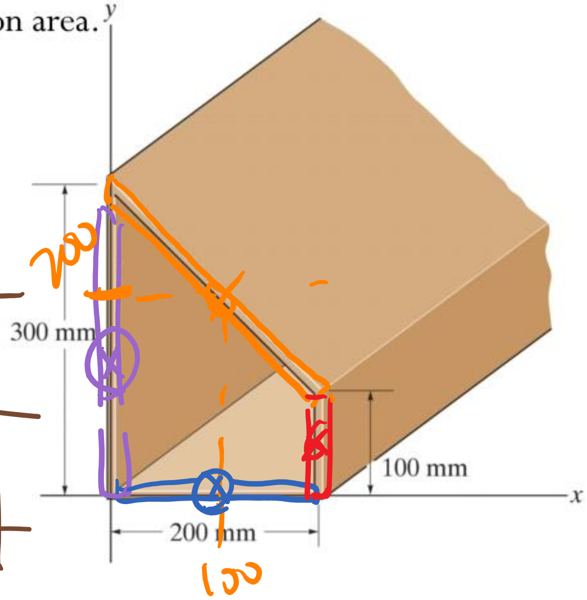
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A.} = \frac{(3\,cm)(24\,cm^2) - (5\frac{1}{3}\,cm)(4\,cm^2) - (\frac{8}{3\pi}\,cm)(2\pi\,cm^2)}{(24 - 4 - 2\pi)\,cm^2}$$

$$\Rightarrow \bar{y} =$$

Example

Locate the centroid of the cross section area.

L.

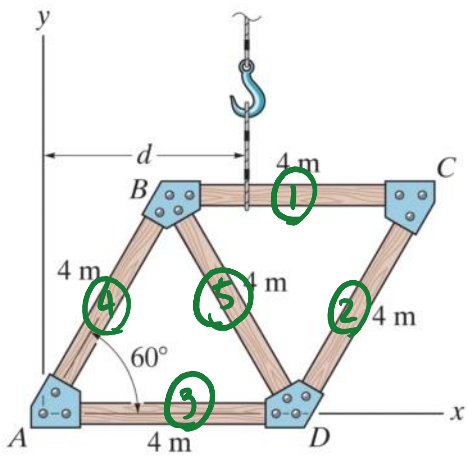


(mm)	1	2	3	4
L	100	200	300	$200\sqrt{2}$
$\tilde{x}$	200	100	0	100
$\tilde{y}$	50	0	150	200

$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{[200(100) + 100(200) + 0(300) + 100(200\sqrt{2})]\ mm^2}{(100 + 200 + 300 + 200\sqrt{2})\ mm}$$

$\bar{x} = 77.3\ mm$

$\bar{y} = 121\ mm$

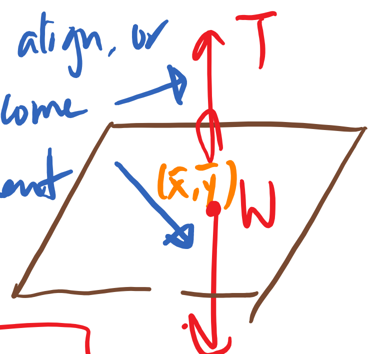


The truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. Determine the distance d to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.

$$W = \rho L g = 7\left(\frac{kg}{m}\right)(4m)\left(9.81\frac{m}{s^2}\right)$$

$$= 274.68\ N$$ ← same for all 5 members

- Must align, or they'll become couple moment



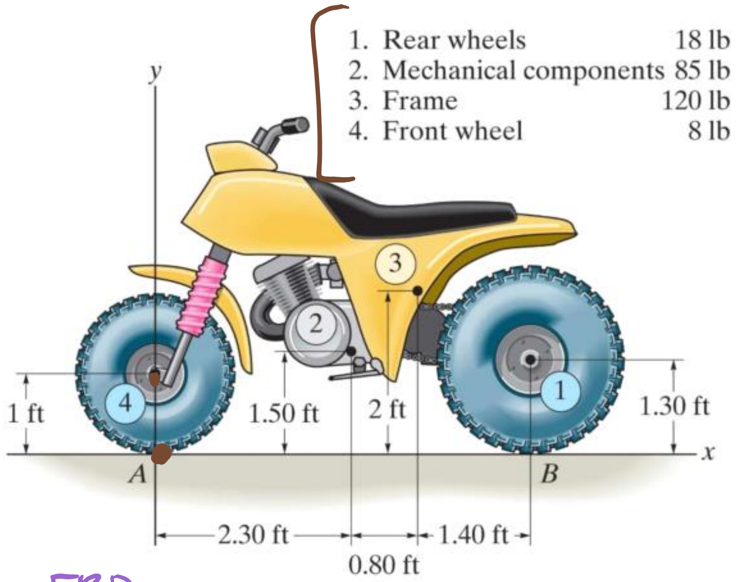
$$\Rightarrow \boxed{d = \bar{x}}$$

	1	2	3	4	5
W	W	W	W	W	W
$\tilde{x}$	4m	5m	2m	1m	3m

$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} = \frac{(4m)\cancel{W} + (5m)\cancel{W} + (2m)\cancel{W} + (1m)\cancel{W} + (3m)\cancel{W}}{5\cancel{W}}$$

$$\boxed{\bar{x} = 3m}$$

Determine the location of the center of gravity of the three-wheeler. If the three-wheeler is symmetrical with respect to the x-y plane, determine the normal reaction each of its wheels exerts on the ground.



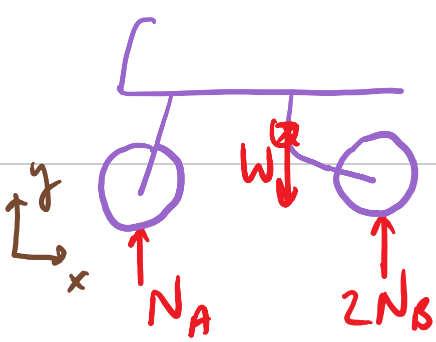
$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} \qquad \bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W}$$

	1	2	3	4
W	18 lb	85 lb	120 lb	8 lb
$\tilde{x}$	4.5 ft	2.3 ft	3.1 ft	0
$\tilde{y}$	1.3 ft	1.5 ft	2 ft	1 ft

$\Rightarrow \bar{x} = 2.81$ ft

$\bar{y} = 1.73$ ft.

FBD



EoE

$$\Sigma F_y = 0 = N_A + 2N_B - W = 0$$

$$\Sigma M_A = 0 = 2N_B(4.5\text{ ft}) - W\bar{x}$$

$\Rightarrow$ $N_A = 86.9$ lb, $N_B = 72.1$ lb.