wc29lect

Announcements

- CBTF Quiz 6 next week
- 211 students **<u>DO NOT</u>** take 210 final, or you will get a **<u>zero</u>** on 211 final!!!

- No PL 20 !

- Upcoming deadlines:
- Thursday (11/9)
  - ME HW21

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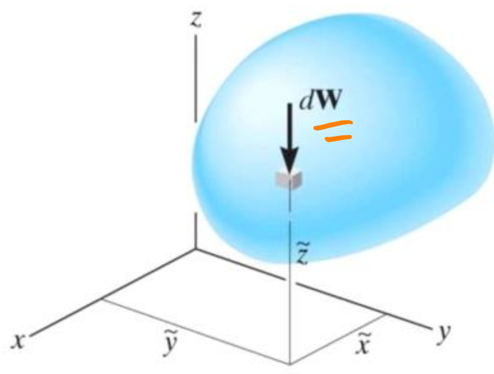
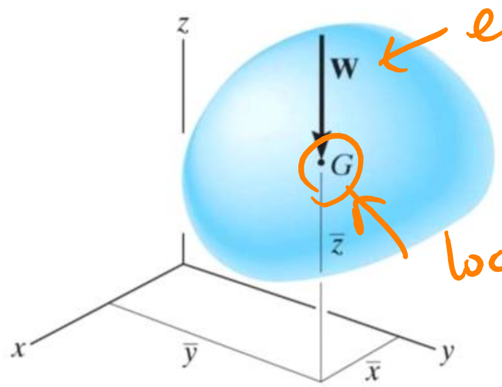
Wednesday, November 8, 2017 10:27 AM

Center of Gravity	Center of Volume
$\bar{x} = \dfrac{\int \tilde{x}\, dW}{\int dW}$	$\bar{x} = \dfrac{\int \tilde{x}\, dV}{\int dV}$
$\bar{y} = \dfrac{\int \tilde{y}\, dW}{\int dW}$	$\bar{y} = \dfrac{\int \tilde{y}\, dV}{\int dV}$
$\bar{z} = \dfrac{\int \tilde{z}\, dW}{\int dW}$	$\bar{z} = \dfrac{\int \tilde{z}\, dV}{\int dV}$

The same if g and ρ are constants.



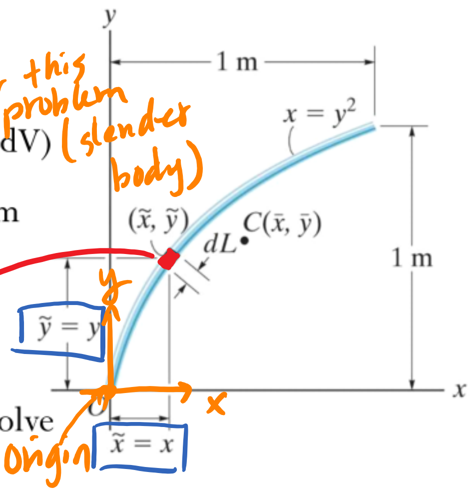


equivalent force

location

Centroid – Analysis Procedure

1. Select an appropriate coordinate system
2. Define the appropriate element (dL, dA, or dV) for this problem (slender body)
3. Express (2) in terms of the coordinate system
4. Identify any symmetry
5. Express the moment arms (centroid) of (2)
6. Substitute (3) and (4) into the integral and solve



3. dL, dy, dx

$$dL = \sqrt{dx^2 + dy^2} = \sqrt{dx^2\left(1 + \frac{dy^2}{dx^2}\right)} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

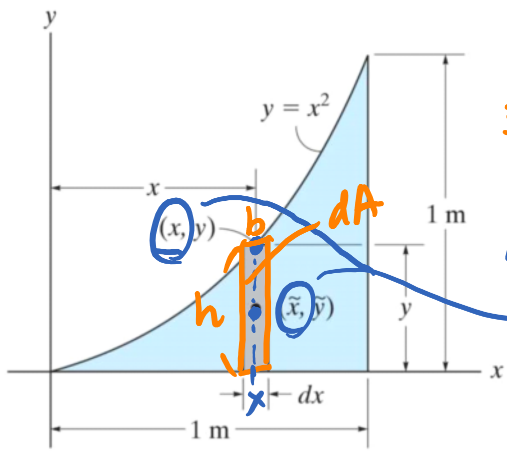
5. $\tilde{x}$ = moment arm of the element (or x-centroid) = x (location of the element for this problem)

$\tilde{y} = y = \sqrt{x}$

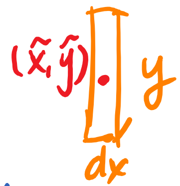
6. $$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL} = \frac{\int x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx}{\int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx} = \frac{\int_0^1 x \sqrt{1 + \frac{1}{4x}}\, dx}{\int_0^1 \sqrt{1 + \frac{1}{4x}}\, dx} \approx \boxed{0.410\text{ m} = \bar{x}}$$

Note: $\frac{dy}{dx}$ = derivative of $y = \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2}x^{-1/2}$

$$\bar{y} = \frac{\int \tilde{y}\, dL}{\int dL} = \frac{\int_0^1 \sqrt{x} \cdot \sqrt{1 + \frac{1}{4x}}\, dx}{\int_0^1 \sqrt{1 + \frac{1}{4x}}\, dx} \approx \boxed{0.574\text{ m} = \bar{y}}$$



Locate the centroid of the area.



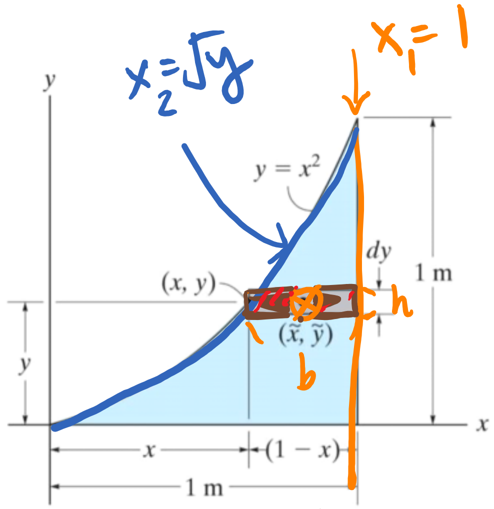
3.) $dA = bh = y\,dx = x^2 dx$

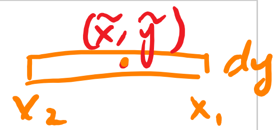
5.) $\tilde{x} = x$. (as $dx \to 0$, $\tilde{x}$ is the same as x)

$\tilde{y} = \frac{1}{2}y$ (midpoint of the rectangular element dA)

6.) $$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA} = \frac{\int_0^1 x(x^2)\,dx}{\int_0^1 x^2\,dx} = \frac{\left.\frac{x^4}{4}\right|_0^1}{\left.\frac{x^3}{3}\right|_0^1} = \boxed{\frac{3}{4}\text{ m} = \bar{x}}$$

$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA} = \frac{\int \frac{1}{2}y\,(x^2dx)}{\int x^2dx} = \frac{\int_0^1 \frac{1}{2}x^2(x^2dx)}{\int_0^1 x^2dx} = \frac{\left.\frac{x^5}{10}\right|_0^1}{\left.\frac{x^3}{3}\right|_0^1} = \boxed{\frac{3}{10}\text{ m} = \bar{y}}$$





Locate the centroid of the area.

3.) $dA = bh = (x_1 - x_2)\,dy = (1 - \sqrt{y})\,dy$

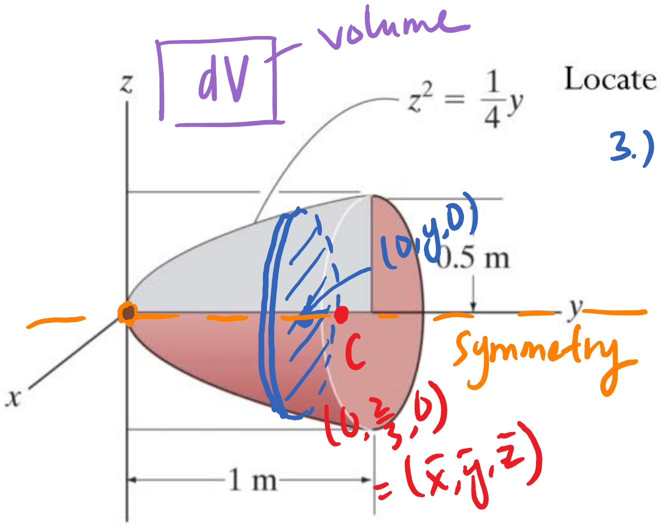
5.) $\tilde{x} = \frac{x_1 + x_2}{2} = \frac{1 + \sqrt{y}}{2}$ (½ way between x_1 & x_2)

$\tilde{y} = y$

6.) $$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA} = \frac{\int_0^1 \frac{1+\sqrt{y}}{2}(1-\sqrt{y})\,dy}{\int_0^1 (1-\sqrt{y})\,dy} = \frac{\frac{1}{2}\left(y - \frac{y^2}{2}\right)\Big|_0^1}{y - \frac{2y^{3/2}}{3}\Big|_0^1} = \frac{\frac{1}{4}}{\frac{1}{3}} = \boxed{\frac{3}{4}\,m = \bar{x}}$$

$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA} = \frac{\int_0^1 y(1-\sqrt{y})\,dy}{\int_0^1 (1-\sqrt{y})\,dy} = \frac{\frac{y^2}{2} - \frac{2}{5}y^{5/2}\Big|_0^1}{y - \frac{2}{3}y^{3/2}\Big|_0^1} = \frac{\left(\frac{1}{10}\right)}{\left(\frac{1}{3}\right)} = \boxed{\frac{3}{10}\,m = \bar{y}}$$

Same as before, just different approach of using horizontal elements and dy.



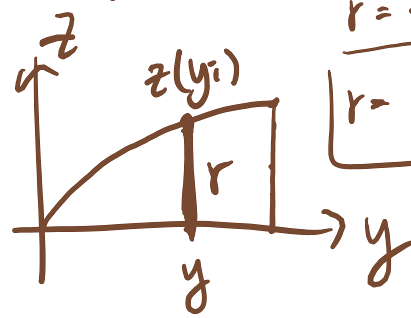
Locate the centroid of the volume.

3.) $dV = \pi r^2 dy = \pi\left(\frac{\sqrt{y}}{2}\right)^2 dy = \frac{\pi y}{4} dy$

• use y-z plane

$r = z = \sqrt{\frac{y}{4}}$

$\boxed{r = \frac{\sqrt{y}}{2}}$

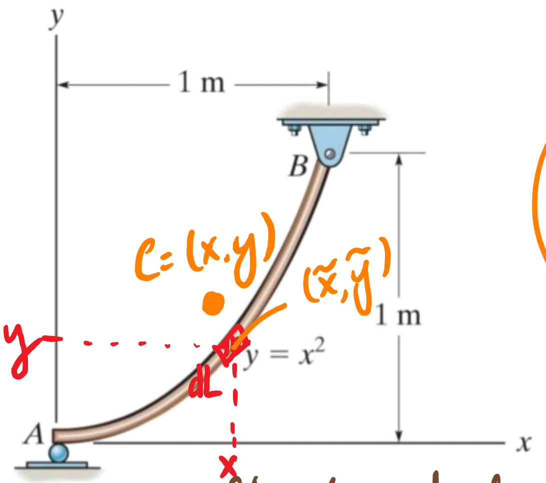


4.) symmetry: y-axis

∴ $\boxed{\bar{x} = 0}$ $\boxed{\bar{z} = 0}$. ~ centroid lies on axis of symmetry.

5.) $\tilde{y} = y$ $(\tilde{x} = 0, \tilde{z} = 0)$

6.) $$y = \frac{\int \tilde{y}\, dV}{\int dV} = \frac{\int_0^1 y\left(\frac{y\pi}{4}\right) dy}{\int_0^1 \frac{y\pi}{4}\, dy} = \frac{\frac{\cancel{\pi} y^3}{12}\Big|_0^1}{\frac{\cancel{\pi} y^2}{8}\Big|_0^1} = \boxed{\frac{2}{3}\, m = \bar{y}}$$



Locate the center of gravity of the homogeneous rod. If the rod has a weight per unit length of 100 N/m, determine the reaction supports at A and B.

Center of gravity and centroid are the same! (ρ is constant)

~ Slender body: use $dL = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ $\frac{dy}{dx} = 2x$.

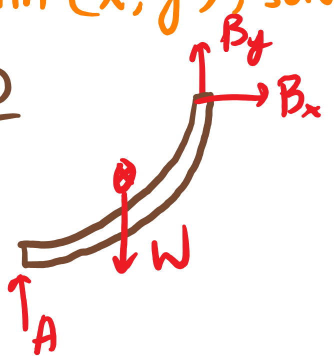
5.) $\tilde{x} = x$, $\tilde{y} = y = x^2$.

6.) $$\bar{x} = \frac{\int \tilde{x}\,dL}{\int dL} = \frac{\int_0^1 x\sqrt{1+(2x)^2}\,dx}{\int_0^1 \sqrt{1+(2x)^2}\,dx} \approx 0.574\text{m}$$

$$\bar{y} = \frac{\int \tilde{y}\,dL}{\int dL} = \frac{\int_0^1 x^2\sqrt{1+4x^2}\,dx}{\int \sqrt{1+4x^2}\,dx} \approx 0.410\text{m}$$

$\Rightarrow$ with $(\bar{x},\bar{y})$, solve for reactions at A & B.

FBD



EoE

$\Sigma F_x = \boxed{B_x = 0}$

$\Sigma F_y = A + B_y - W = 0 \rightarrow \boxed{A = W(1-\bar{x})}$

$\Sigma M_A = -W\bar{x} + B_y(1\text{m}) = 0$

$\Rightarrow \boxed{B_y = W\bar{x}}$

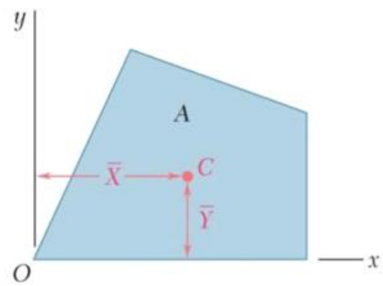
$W = \rho L = \left(100\frac{N}{m}\right)(1.48\text{m})$

≈ 148N

Composite bodies

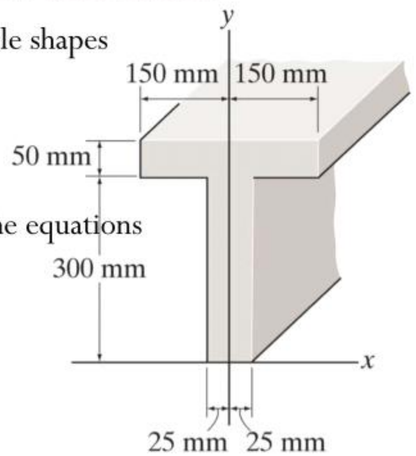
A composite body consists of a series of connected simpler shaped bodies.

Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.



Composite bodies – Analysis Procedure

1. Divide the body into finite number of simple shapes
2. Consider "holes" as "negative" parts
3. Establish coordinate axes
4. Determine centroid location by applying the equations



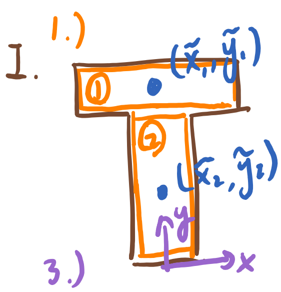
$$\bar{x} = \frac{\sum \tilde{x} W}{\sum W}$$ or L, A, V for centroid

$$\bar{y} = \frac{\sum \tilde{y} W}{\sum W}$$

$$\bar{z} = \frac{\sum \tilde{z} W}{\sum W}$$

$\bar{x}$ example:

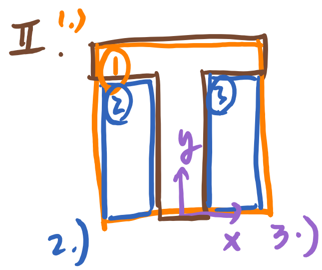
I. 1.)



4.) $\bar{x} = \dfrac{\Sigma x_i A_i}{\Sigma A_i}$

$= \dfrac{x_1 A_1 + x_2 A_2}{A_1 + A_2}$

II. 1.)



4.) $\bar{x} = \dfrac{\Sigma \tilde{x}_i A_i}{\Sigma A_i}$ negative for "holes"

$= \dfrac{x_1 A_1 - x_2 A_2 - x_3 A_3}{A_1 - A_2 - A_3}$

"holes"