wc28lect

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Announcements

- 211 students **DO NOT** take 210 final, or you will get a **zero** on 211 final!!!
- PL HW20 – Practice only ☺
- The marathon continues… CBTF Quiz 6 next week

Upcoming deadlines:

- Thursday (11/9)
  - ME HW21

Chapter 9: Center of Gravity and Centroid

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Goals and Objectives

- Understand the concepts of center of gravity, center of mass, and centroid.
- Be able to determine the location of these points for a body.
- Explore the relationship between fluid pressure and force on a submerged surface.

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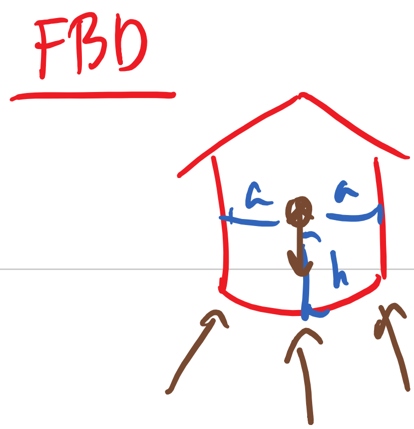
Center of gravity

To design the structure for supporting a water tank, we will need to know the weight of the tank and water as well as the locations where the resultant forces representing these distributed loads act.

How can we determine these resultant weights and their lines of action?



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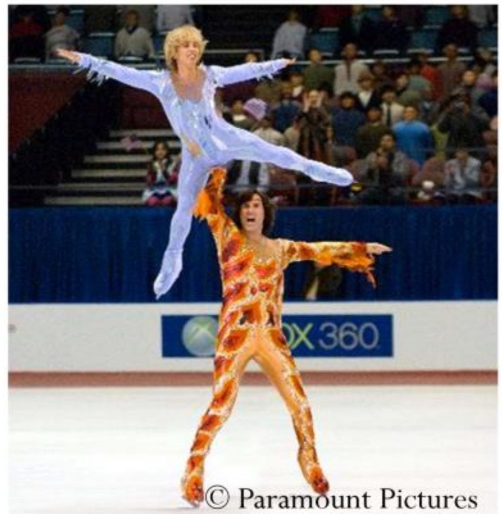
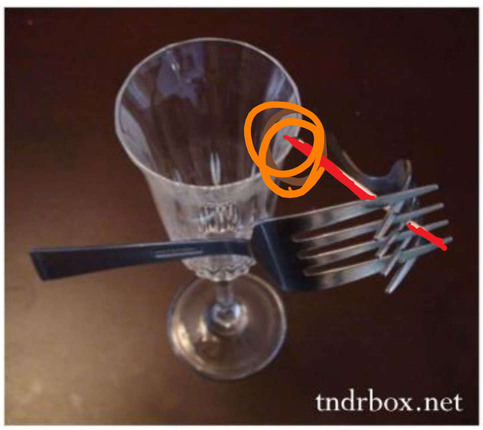
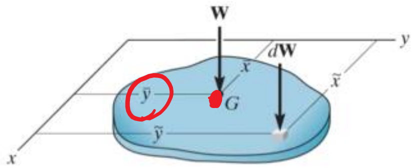
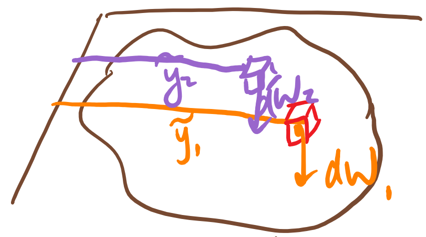
Center of gravity





Center of gravity – moment arms of equivalent W force





- Find equivalent M_R & F_R of the disk:

$$M_R = \sum_i \tilde{y}_i \, dW_i$$

$$F_R = \sum_i dW_i$$

- Relate M_R, F_R to moment arm $\bar{y}$

$$M_R = F_R d$$

$$\bar{y} = \frac{M_R}{F_R} = \frac{\sum \tilde{y}_i \, dW_i}{\sum dW_i}$$

- Take limit as $\Delta \to 0$

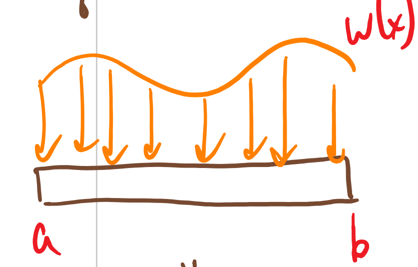
$$\bar{y} = \frac{\int \tilde{y} \, dW}{\int dW}$$

$$\bar{x} = \frac{\int \tilde{x} \, dW}{\int dW}$$

$$\bar{y} = \frac{\int \tilde{y} \, dW}{\int dW}$$

$$\bar{z} = \frac{\int \tilde{z} \, dW}{\int dW}$$

Review: Finding equivalent force.



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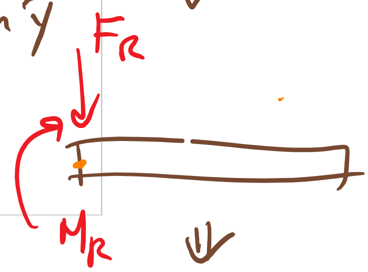

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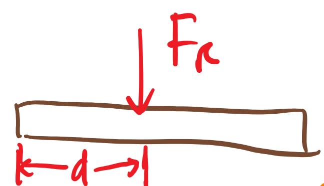

$$\Rightarrow d = \frac{M_R}{F_R}.$$

M_R ← equivalent moment

F_R ← equivalent force

d ← moment arm of equivalent force

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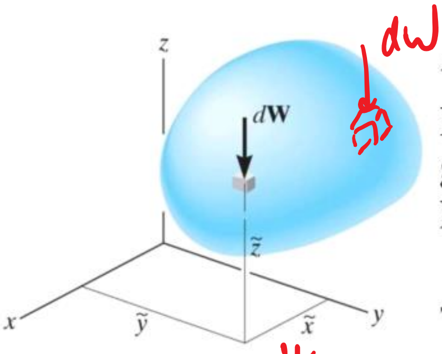
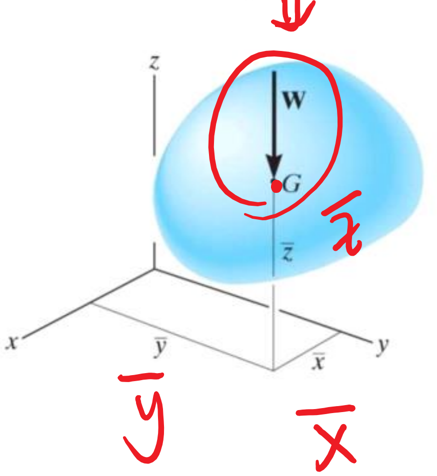
Center of gravity



A body is composed of an infinite number of particles, and so if the body is located within a gravitational field, then each of these particles will have a weight dW.



The **<u>center of gravity (CG)</u>** is a point, often shown as G, which locates the resultant weight of a system of particles or a solid body.

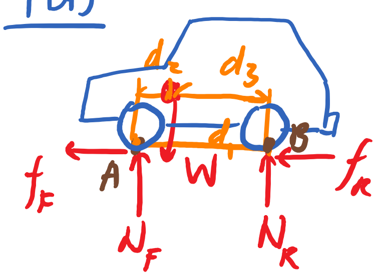
From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G.

$\tilde{x}$ = location of elemental pieces
= moment arm

Center of gravity

Front-wheel (FWD) or rear-wheel (RWD) drive is better for getting out?

FBD



Compare maximum friction force on front vs. rear wheels: ($f = \mu_k N$)

RWD: $\sum M_A = 0 = -Wd_2 + N_R d_1$

$$\Rightarrow N_R = \frac{Wd_2}{d_1}, \quad f_R = \mu_k \frac{Wd_2}{d_1}$$

FWD: $\sum M_B = 0 = Wd_3 - N_F d_1$

$$\Rightarrow N_F = \frac{Wd_3}{d_1}, \quad f_F = \mu_k \frac{Wd_3}{d_1}$$

Since $d_3 > d_2$, $f_F > f_R$

$\therefore$ FWD will be able to provide more force out of snow.

If mass distribution is given, then $W = mg$, so

$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} = \frac{\int \tilde{x}\, d(mg)}{\int d(mg)} \overset{g \text{ is constant}}{=} \frac{\cancel{g}\int \tilde{x}\, dm}{\cancel{g}\int dm}$$

$$\Rightarrow \bar{x} = \frac{\int \tilde{x}\, dm}{\int dm}$$ center of mass equation

If density of the object is constant, then $m = \rho V$, so

$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm} = \frac{\int \tilde{x}\, d(\rho V)}{\int d(\rho V)} = \frac{\cancel{\rho}\int \tilde{x}\, dV}{\cancel{\rho}\int dV}$$

$$\bar{x} = \frac{\int x\,dm}{\int dm} = \frac{\int x\,d(\rho V)}{\int d(\rho V)} = \frac{\rho \int x\,dV}{\rho \int dV}$$

$$\Rightarrow \bar{x} = \frac{\int \check{x}\,dV}{\int dV}$$ center of volume.

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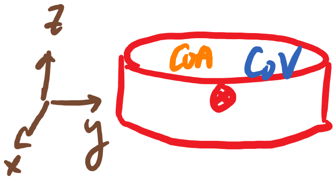
Center of Area

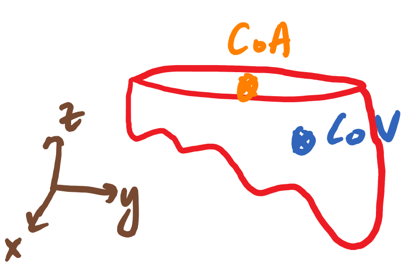
- Center of mass and center of volume will be the same if ρ is constant.
- If the lake has constant depth (h), center of volume can be simplified to center of area in x- and y-direction.

$$\bar{x} = \frac{\int \tilde{x}\, dV}{\int dV} = \frac{\int \tilde{x}\, d(Ah)}{\int d(Ah)} = \frac{\cancel{h} \int \tilde{x}\, dA}{\cancel{h} \int dA}$$

$$\rightarrow \quad \bar{x} = \frac{\int \tilde{x}\, dA}{\int dA} \quad \text{center of area.}$$



- constant depth: same x- and y-components of center of volume & area.



→ not true when depth changes.

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CENTROID

Center of Mass	Center of Volume	Center of Area
$\bar{x} = \dfrac{\int \tilde{x}\, dm}{\int dm}$	$\bar{x} = \dfrac{\int \tilde{x}\, dV}{\int dV}$	$\bar{x} = \dfrac{\int \tilde{x}\, dA}{\int dA}$
$\bar{y} = \dfrac{\int \tilde{y}\, dm}{\int dm}$	$\bar{y} = \dfrac{\int \tilde{y}\, dV}{\int dV}$	$\bar{y} = \dfrac{\int \tilde{y}\, dA}{\int dA}$
$\bar{z} = \dfrac{\int \tilde{z}\, dm}{\int dm}$	$\bar{z} = \dfrac{\int \tilde{z}\, dV}{\int dV}$	$\bar{z} = \dfrac{\int \tilde{z}\, dA}{\int dA}$

dV → $dx\,dy\,dz$

dA → $dx\,dy$

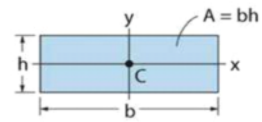
Centroid

The centroid, C, is a point defining the geometric center of an object.

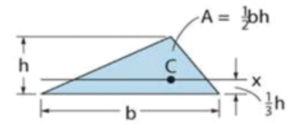
The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogeneous (density or specific weight is constant throughout the body).

If an object has an axis of symmetry, then the centroid of object lies on that axis.

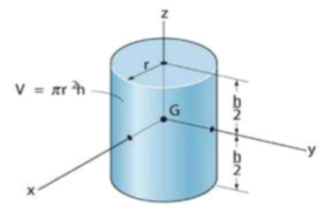
In some cases, the centroid may not be located on the object.



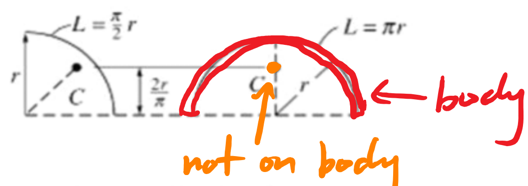
Rectangular area



Triangular area



Cylinder



Quarter and semicircle arcs