wc23lect

Announcements

- TAM210 last lecture: Friday, Nov. 3rd
- TAM210 Final: 1 hour 50 minutes exam
  - Location: CBTF
  - Thursday, Nov. 9th through Sunday, Nov. 12th

❑ Upcoming deadlines:

- Tuesday (10/24)
  - PL HW16
- Thursday (10/26)
  - ME HW17

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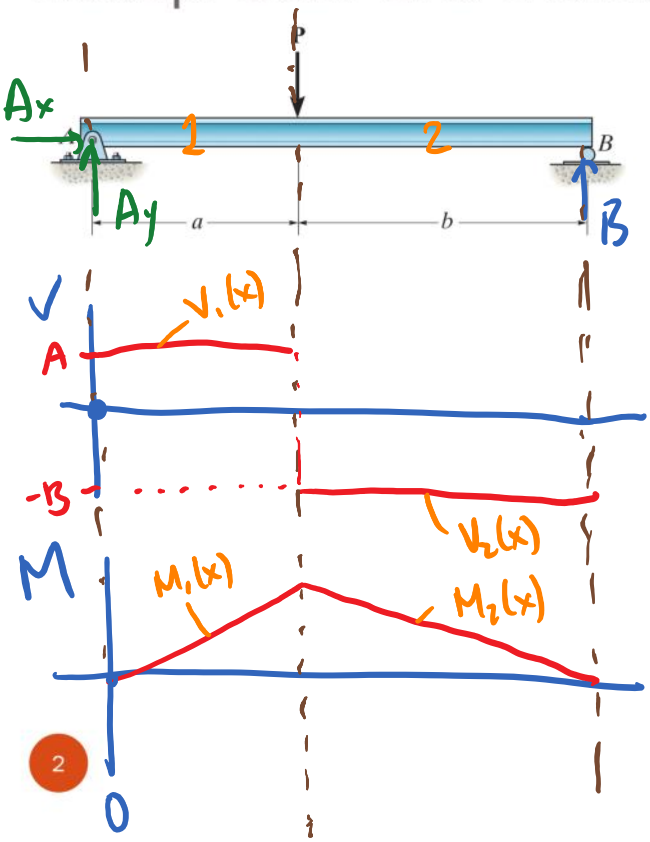
Recap: Sear and Moment Diagram (for beams with perpendicular loading & couple moments)



1.) ID ext. loads.

2) ID x-coordinate system

3.) Divide into sections according to loading

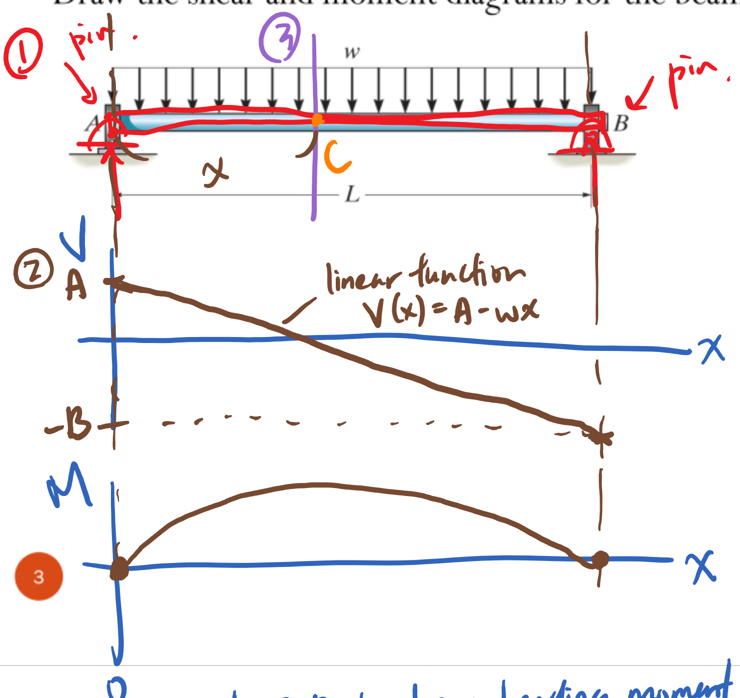
4.) Make cuts in the middle of each section and use FBD and EoE to derive V(x) and M(x).

Shear and Moment Diagram

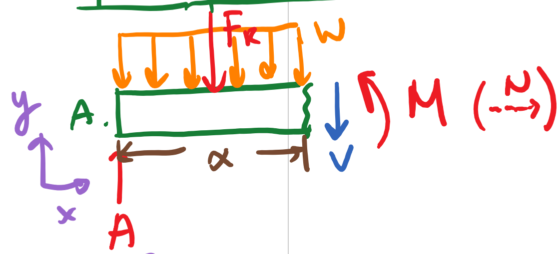
Draw the shear and moment diagrams for the beam.

① Support Reactions

$A = B = \frac{wL}{2}$.



④ FBD of section (left)



EoE

• Use the F_y EoE to derive the shear force function, $V(x)$

$$\Sigma F_y = A - V - F_R = 0.$$

F_R = eq. dist. load

$= \underline{xw}$ (area under the curve)

$$\Rightarrow \boxed{V = A - F_R = \frac{wL}{2} - wx}$$

$y = mx + b$ ← slope-intercept form

$m = -w$ ← slope

$b = A$. ← y-intercept, $V(0) = b$.

$y(L) = A - wL = -B$ ← find $V(x)$ value at the boundary

• Use the moment EoE to derive bending moment fxn, $M(x)$.

$$\Sigma M_A = -Vx + M - F_R\left(\frac{x}{2}\right) = 0.$$

$$\Rightarrow M = Vx + F_R\left(\frac{x}{2}\right) = Vx + xw\left(\frac{x}{2}\right)$$

$$\boxed{M = \left(\frac{w}{2}\right)x^2 + Vx.}$$

~ Substituting $V(x)$ in terms of ext. loading

$$M = \left(\frac{w}{2}\right)x^2 + (A - wx)(x)$$

$$\boxed{M = \left(\frac{wL}{2}\right)x - \frac{wx^2}{2}}$$

~ Find boundary conditions at $x=0$ and $x=L$.

$$M(0) = \frac{w}{2}(0)^2 + V(0) = 0$$

$\left(\frac{wL}{2}\right)$ from step 1.

$$M(L) = \frac{w}{2}(L)^2 + V(L) = \frac{w}{2}(L)^2 + (A - w(L))(L) = 0$$

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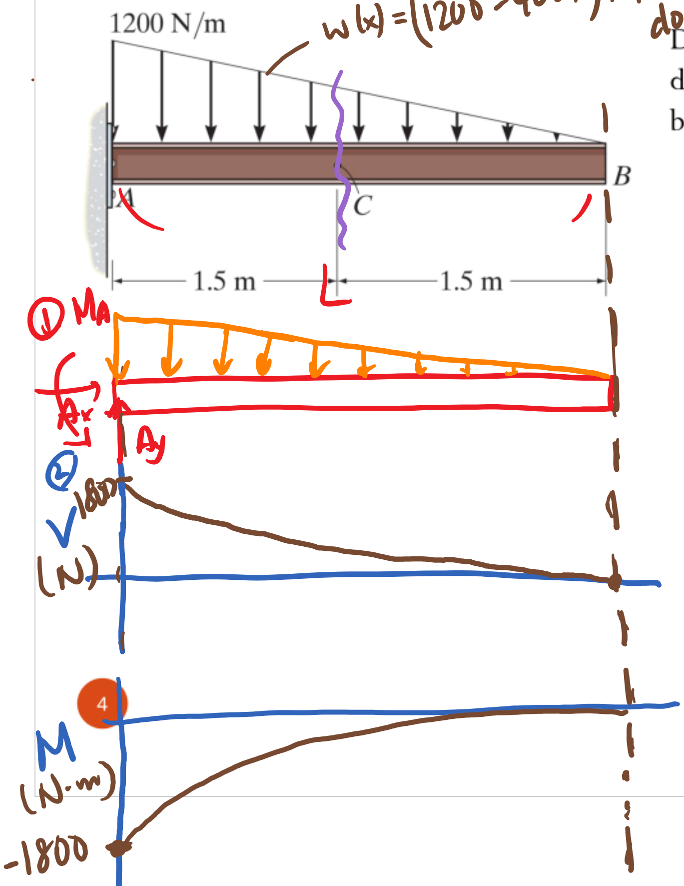
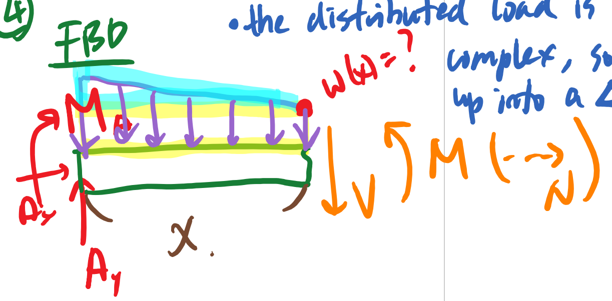
$w(x) = (1200 - 400x)$ N/m downward ↓



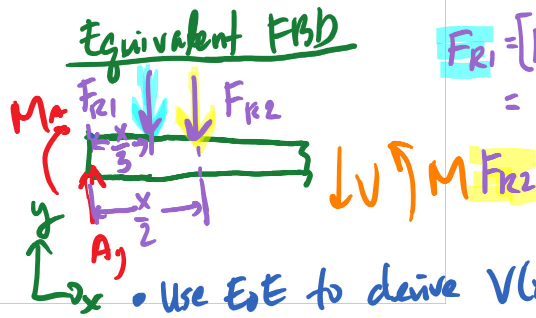
Draw the shear and moment diagrams for the cantilever beam.

① $A_y = 1800\,N$, $M_A = 1800\,N\cdot m$

④



• the distributed load is a bit complex, so break it up into a Δ and a □.



$F_{R1} = [1200 - w(x)]\,x\,(\frac{1}{2})$ ↓
$= 200x^2$ ↓

$F_{R2} = w(x) \cdot x$ ↓
$= 1200x - 400x^2$ ↓

• Use EoE to derive $V(x)$ and $M(x)$

~ for $V(x)$

$\Sigma F_y = A_y - F_{R1} - F_{R2} - V = 0$

$V = -F_{R1} - F_{R2} + A_y$

$-(200x^2) - (1200x - 400x^2) + 1800$

$\boxed{V = 200x^2 - 1200x + 1800}$ N

~ calculate boundary values, $V(0)$ & $V(L)$

$V(0) = 1800$ N

$V(L) = V(3) = 0$ N

~ for $M(x)$

$\Sigma M_A = -F_{R1}(\frac{x}{3}) - F_{R2}(\frac{x}{2}) - Vx + M + M_A = 0$

$M = F_{R1}(\frac{x}{3}) + F_{R2}(\frac{x}{2}) + Vx - M_A$

$= (200x^2)(\frac{x}{3}) + (1200x - \cancel{400x^2})(\frac{x}{2})$

$+ (\cancel{200x^2} - 1200x + 1800)x - 1800$

$\boxed{M = \frac{200}{3}x^3 - 600x^2 + 1800x - 1800}$

~ calculate boundary values, $M(0)$ & $M(L)$

$M(0) = -1800$ N·m

$M(L) = M(3) = 0$

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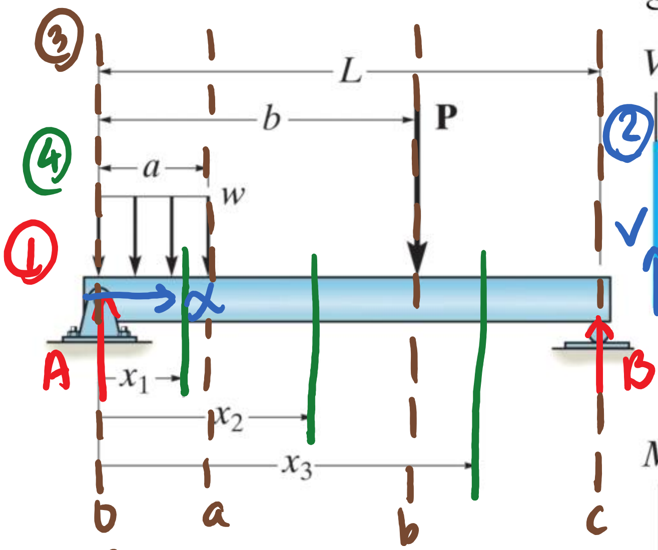
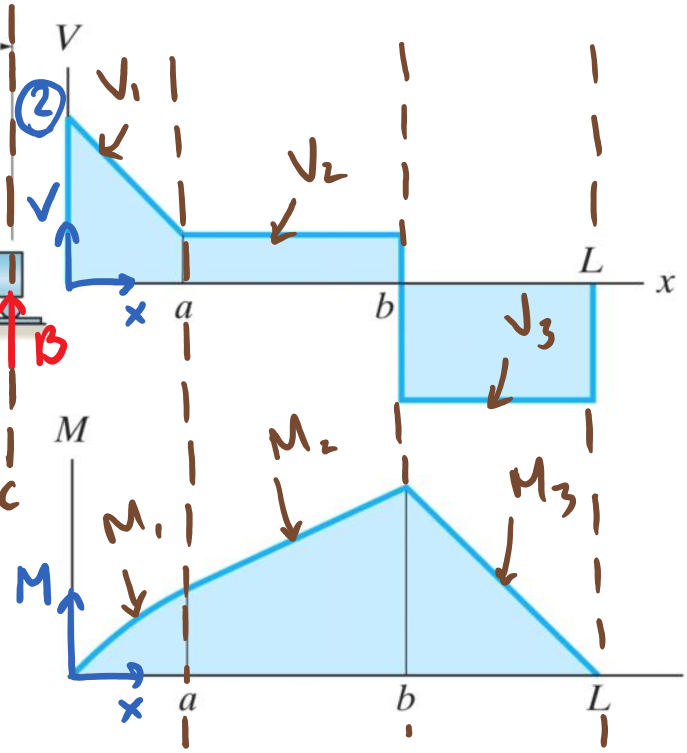
Shear and Moment Diagram

Draw the shear and moment diagrams for the beam.





3 regions:

1) $0 \leq x < a$

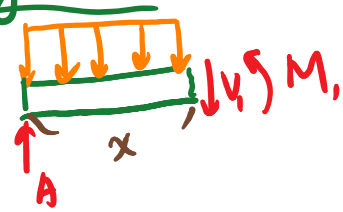
2.) $a \leq x < b$

3.) $b \leq x \leq c$

(4) FBD & EOE.

~ each region will have a different $V(x)$ and $M(x)$ functions

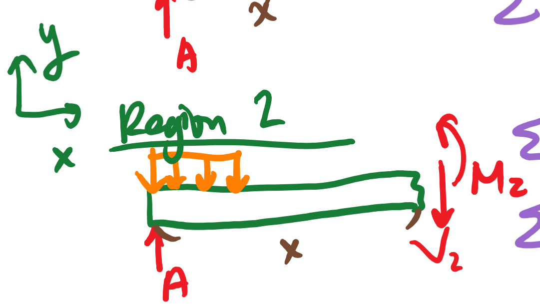
Region 1



$\Sigma F_y = 0 \Rightarrow V_1(x)$

$\Sigma M_A = 0 \Rightarrow M_1(x)$

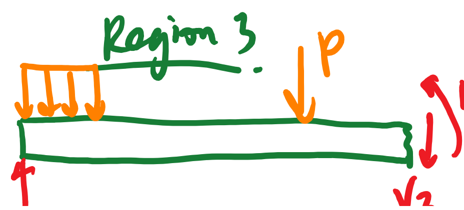
Region 2



$\Sigma F_y = 0 \Rightarrow V_2(x)$

$\Sigma M_A = 0 \Rightarrow M_2(x)$

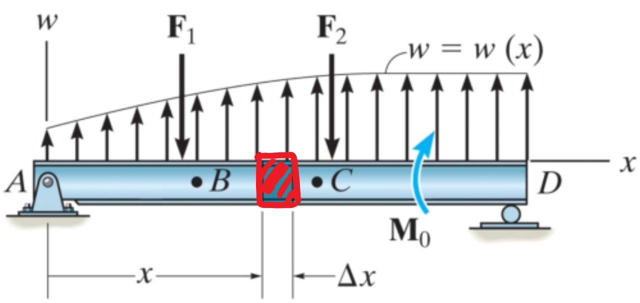
Region 3.

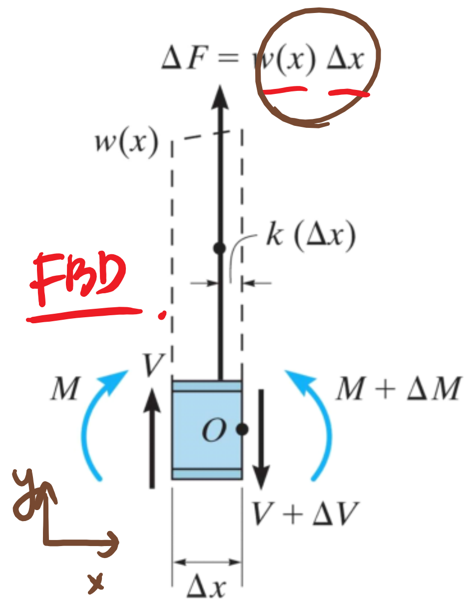


$\Sigma F_y = 0 \Rightarrow V_3(x)$

$\Sigma M_A = 0 \Rightarrow M_3(x)$

Relations Among Load, Shear and Bending Moments





Wherever there is an distributed force, the shear force function is:

EoE

$$\Sigma F_y = \cancel{V} + \Delta F - (\cancel{V} + \Delta V) = 0$$

8 $$\frac{\Delta V = \Delta F = w\Delta x}{\Delta x} \Rightarrow \frac{\Delta V}{\Delta x} = w$$

take $\Delta x \to 0$, $\boxed{\frac{dV}{dx} = w}$ ∴ Slope of shear diagram = $w(x)$

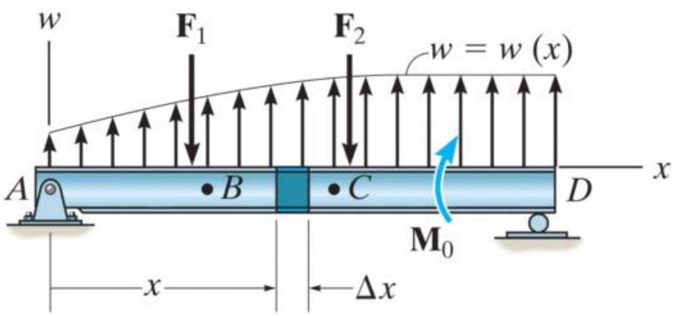
$$\Sigma M_O = -\cancel{M} + (\cancel{M} + \Delta M) - \Delta F(k\Delta x) - V\Delta x = 0$$

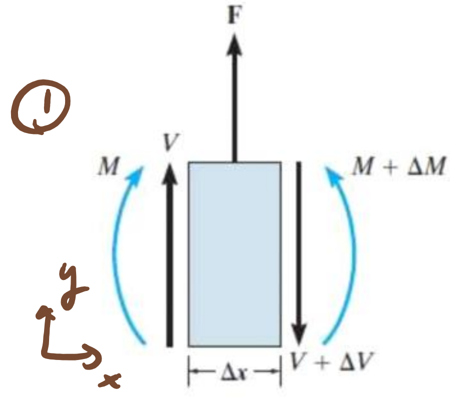
$$\Delta M = \Delta F(k\Delta x) + V\Delta x = w\Delta x(k\Delta x) + V\Delta x$$

$$\Rightarrow \frac{\Delta M = wk\Delta x^2 + V\Delta x}{\Delta x} \Rightarrow \frac{\Delta M}{\Delta x} = V + wk\Delta x$$

take $\Delta x \to 0$, $\boxed{\frac{dM}{dx} = V}$ ∴ slope of moment diagram = shear

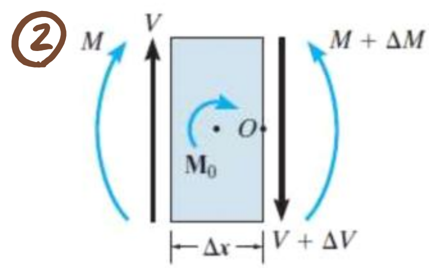
Relations Among Load, Shear and Bending Moments





(a)

Wherever there is an external concentrated force, or a concentrated moment, there will be a change (jump) in shear or moment respectively.



(b)

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EoE ① $\sum F_y = \cancel{V} + F - (\cancel{V} + \Delta V) = 0$

$\Rightarrow \Delta V = F$

$\therefore$ upward F = ⊕ jump in V diagram.

0 as $\Delta x \to 0$.

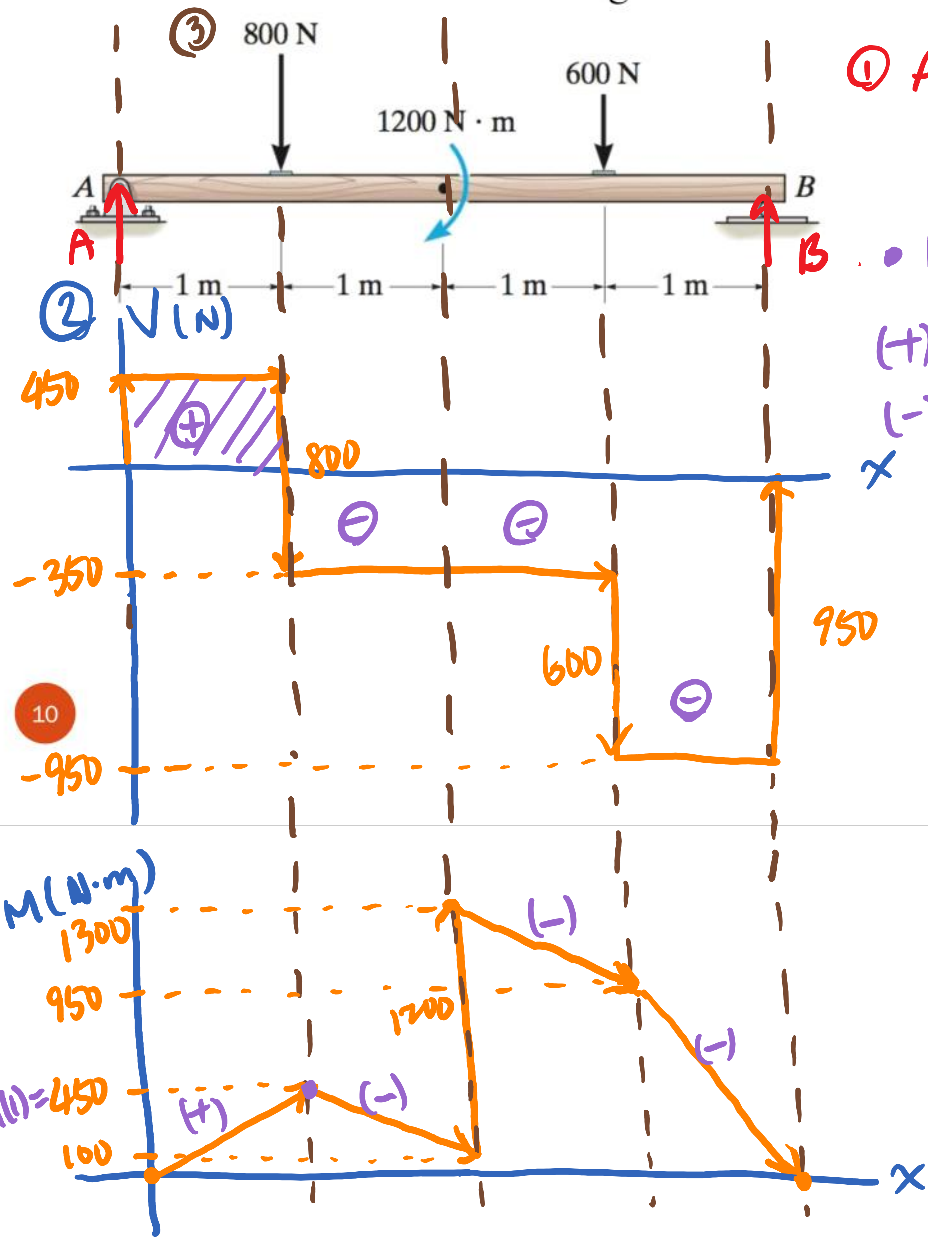
EoE ② $\sum M_O = -\cancel{M} + (\cancel{M} + \Delta M) - (\cancel{V} + \Delta V)\cancel{\left(\frac{\Delta x}{2}\right)} - \cancel{V}\left(\frac{\Delta x}{2}\right) - M_0 = 0$

$\Rightarrow \Delta M = M_0$.

$\therefore$ clockwise M_0 = ⊕ jump in M diagram.

Example

Draw the shear and moment diagrams for the beam.



① A = 450N, B = 950 N

• M = area under V curve

(+) constant V = ↗ M

(-) constant V = ↘ M

Example:

• for $0 < x < 1$, $V = 450N$,

so $M(1) = (1\,m)(450N)$

$= 450\ N\cdot m$

Example

Draw the shear and moment diagrams for the beam.