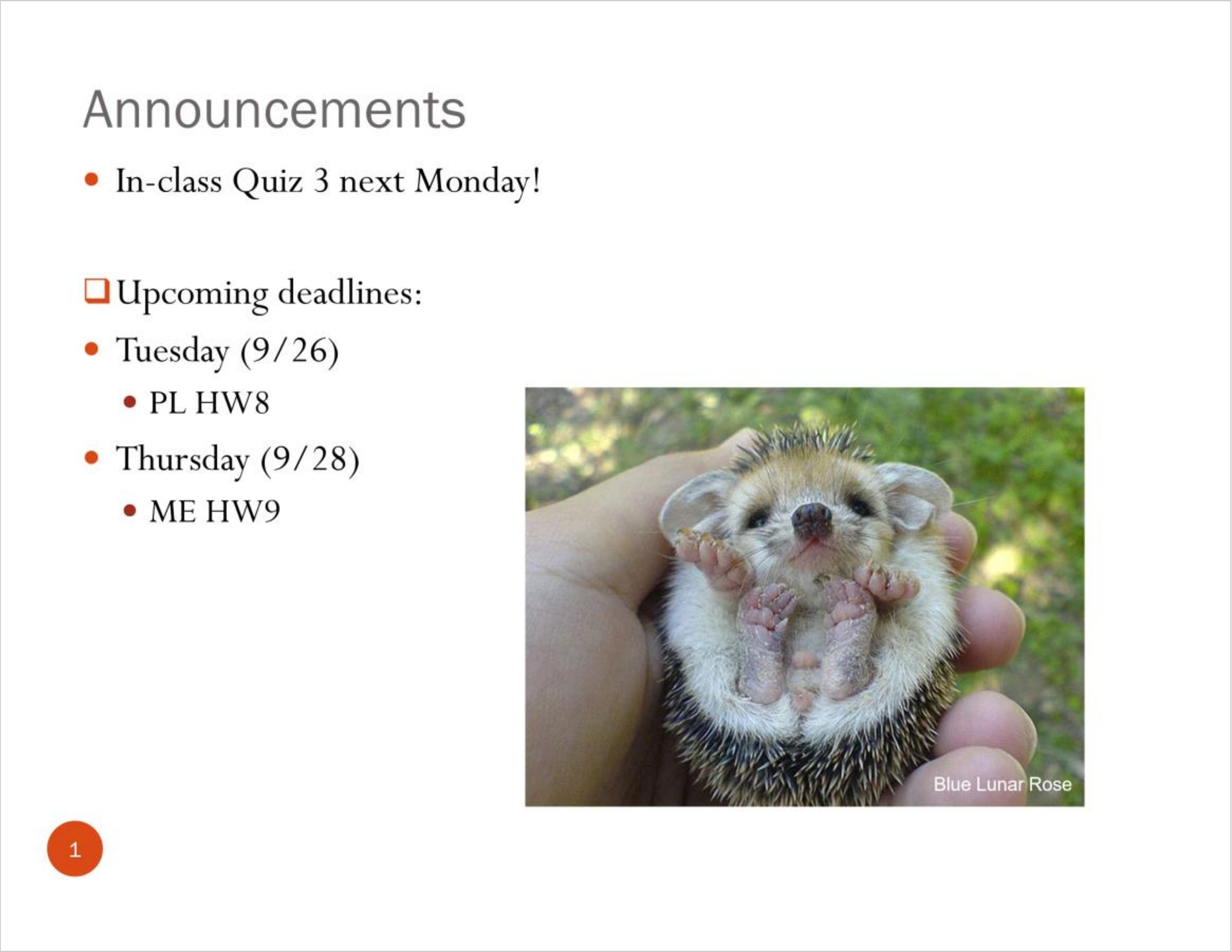
wc12lect



Announcements

- In-class Quiz 3 next Monday!

☐ Upcoming deadlines:

- Tuesday (9/26)
  - PL HW8
- Thursday (9/28)
  - ME HW9

Blue Lunar Rose

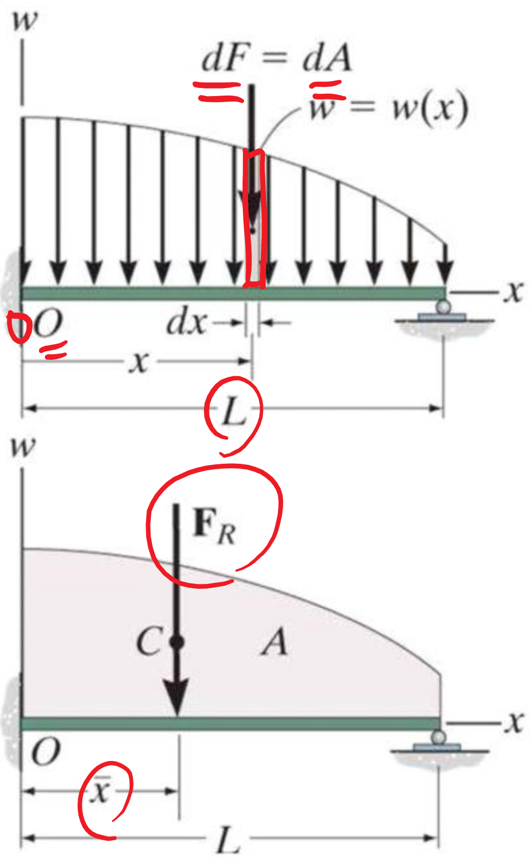
Recap

- Equivalent force system for distributed loading

$$F_R = \int dF = \int_0^L w(x)\,dx$$

$$M_o = \int_0^L \underset{\substack{\uparrow \\ d}}{x}\, \underbrace{w(x)\,dx}_{\substack{\uparrow \\ F}}$$

$$M_o = \bar{x} F_R \Rightarrow \bar{x} = \frac{M_o}{F_R}$$



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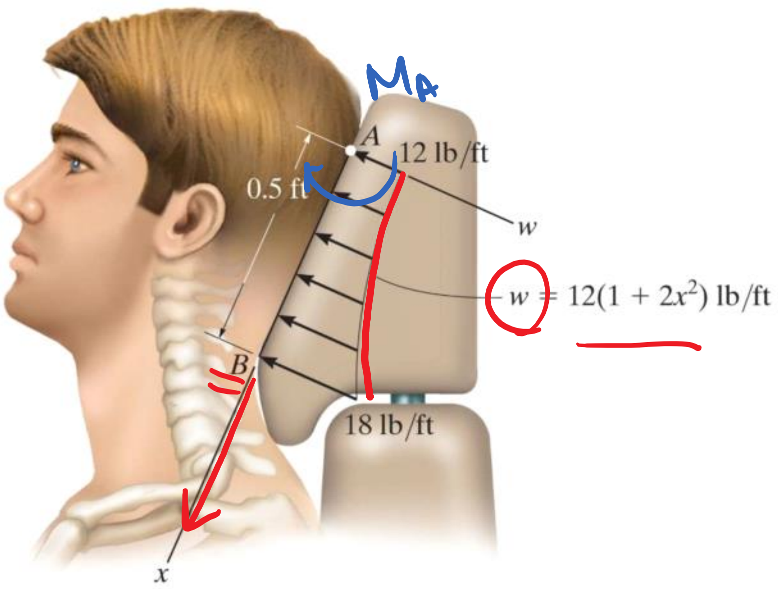
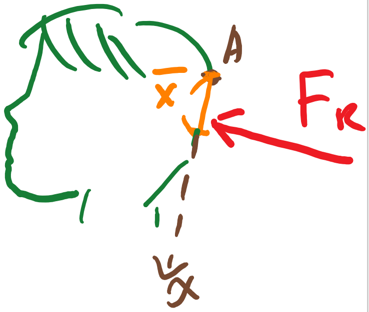
Example

Find the equivalent force and its location from point A for the loading on the headrest as shown.





$$F_R = \int_0^L w(x)\,dx$$

$$= \int_0^{0.5} 12(1+2x^2)\,dx$$

$$\bar{x} = \frac{M_A}{F_R} = \frac{\int_0^L x\,w(x)\,dx}{\int_0^L w(x)\,dx} = \frac{\int_0^{0.5} x\left[12(1+2x^2)\right]dx}{\int_0^{0.5}\left[12(1+2x^2)\right]dx}$$

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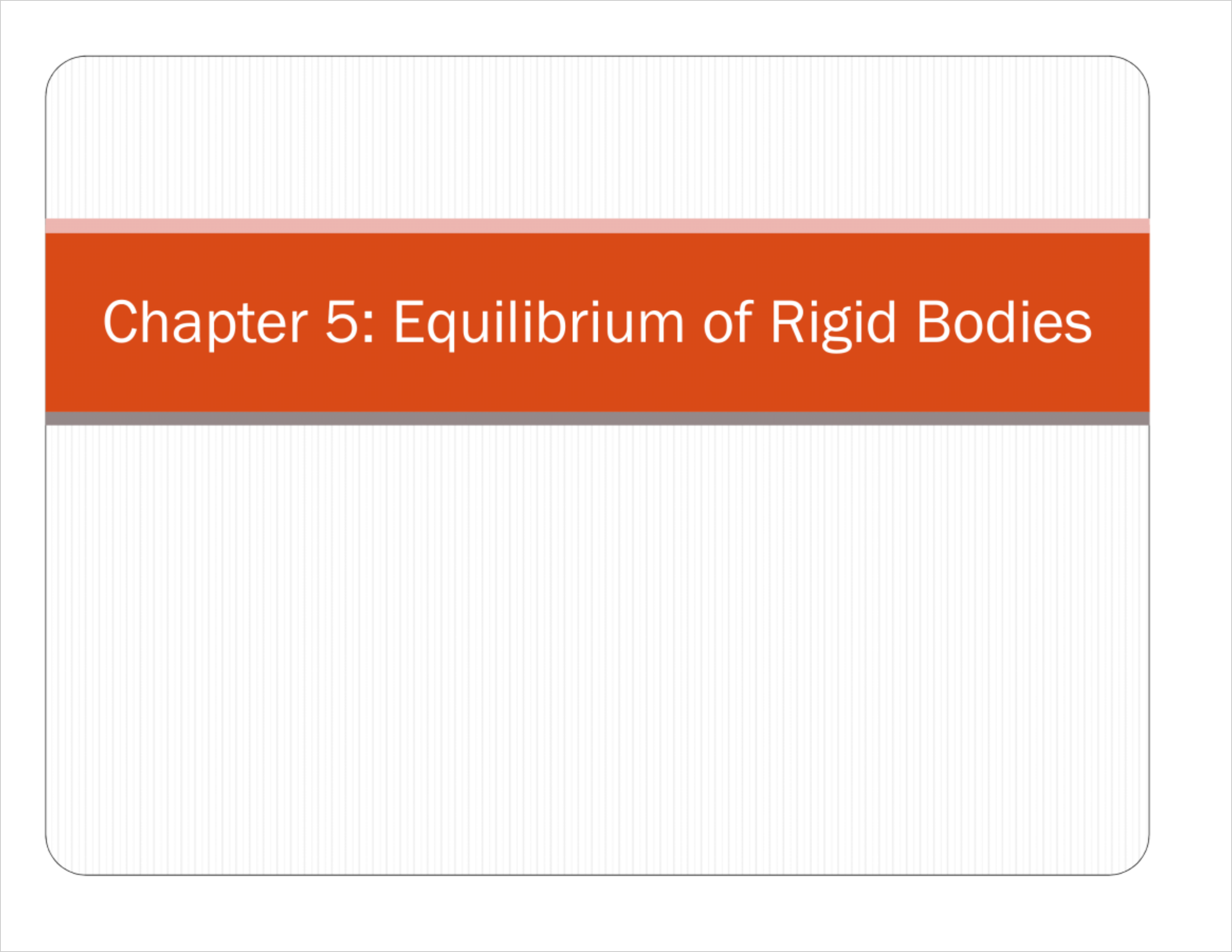

Chapter 5: Equilibrium of Rigid Bodies

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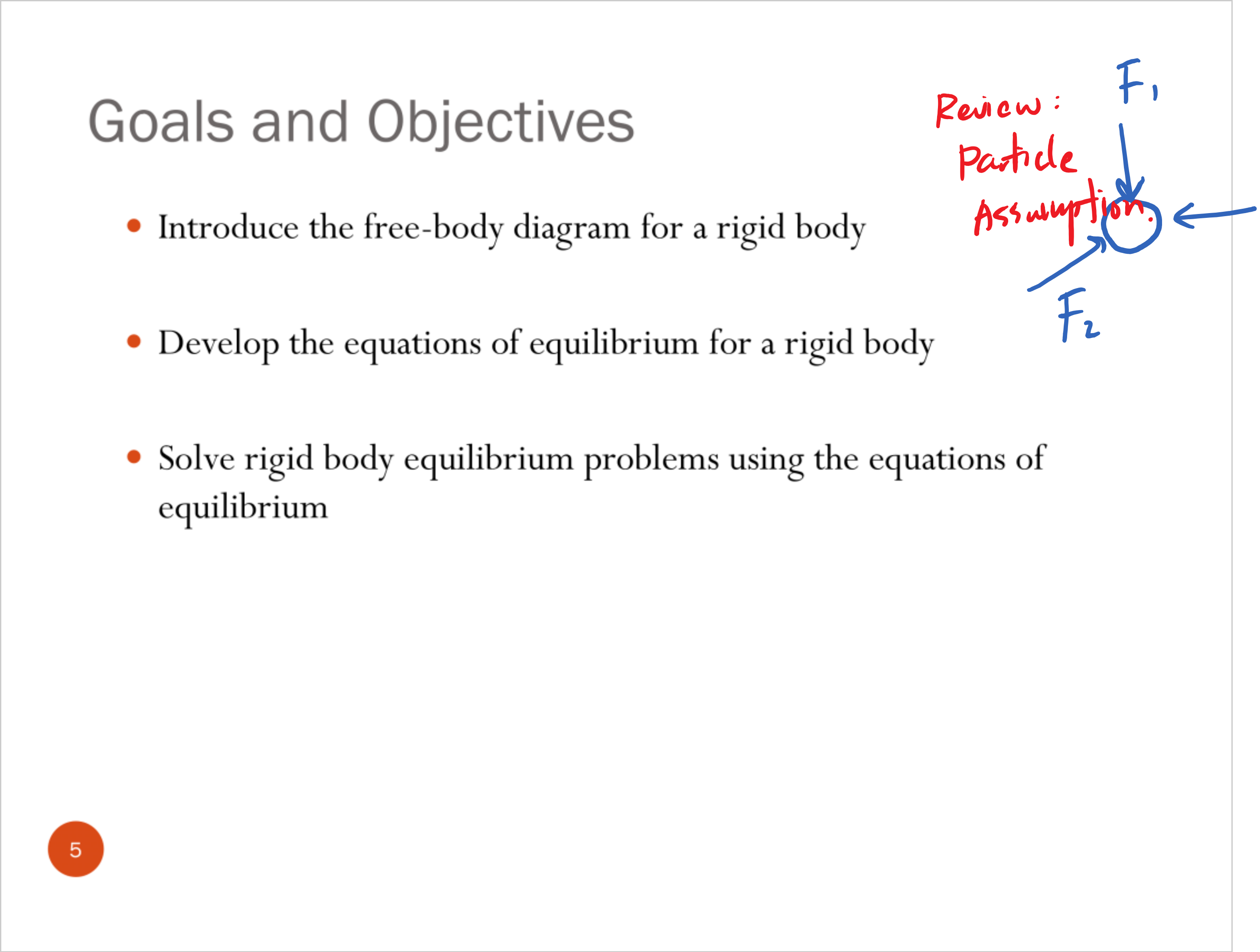

Goals and Objectives

- Introduce the free-body diagram for a rigid body
- Develop the equations of equilibrium for a rigid body
- Solve rigid body equilibrium problems using the equations of equilibrium

Review:
Particle
Assumption.

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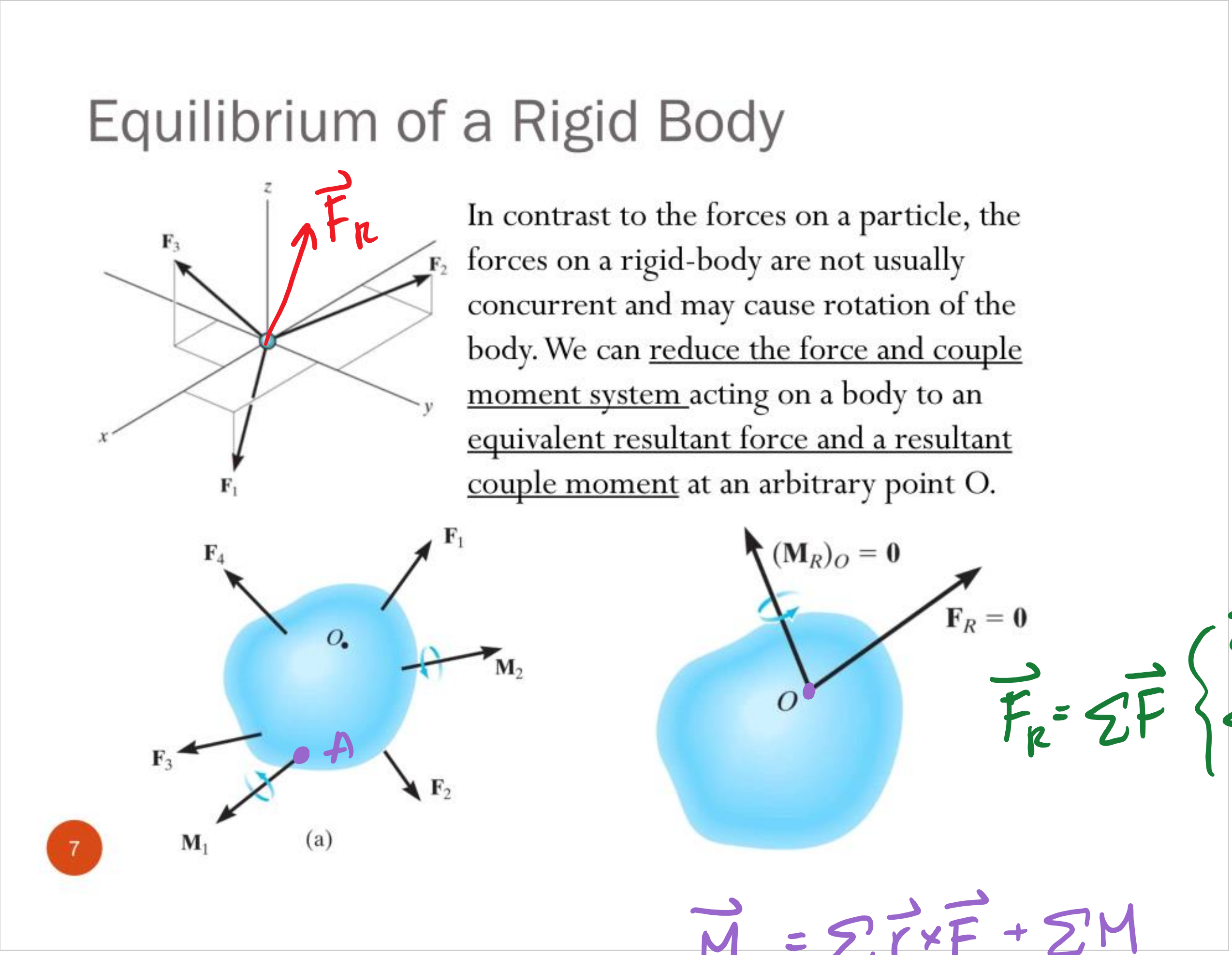
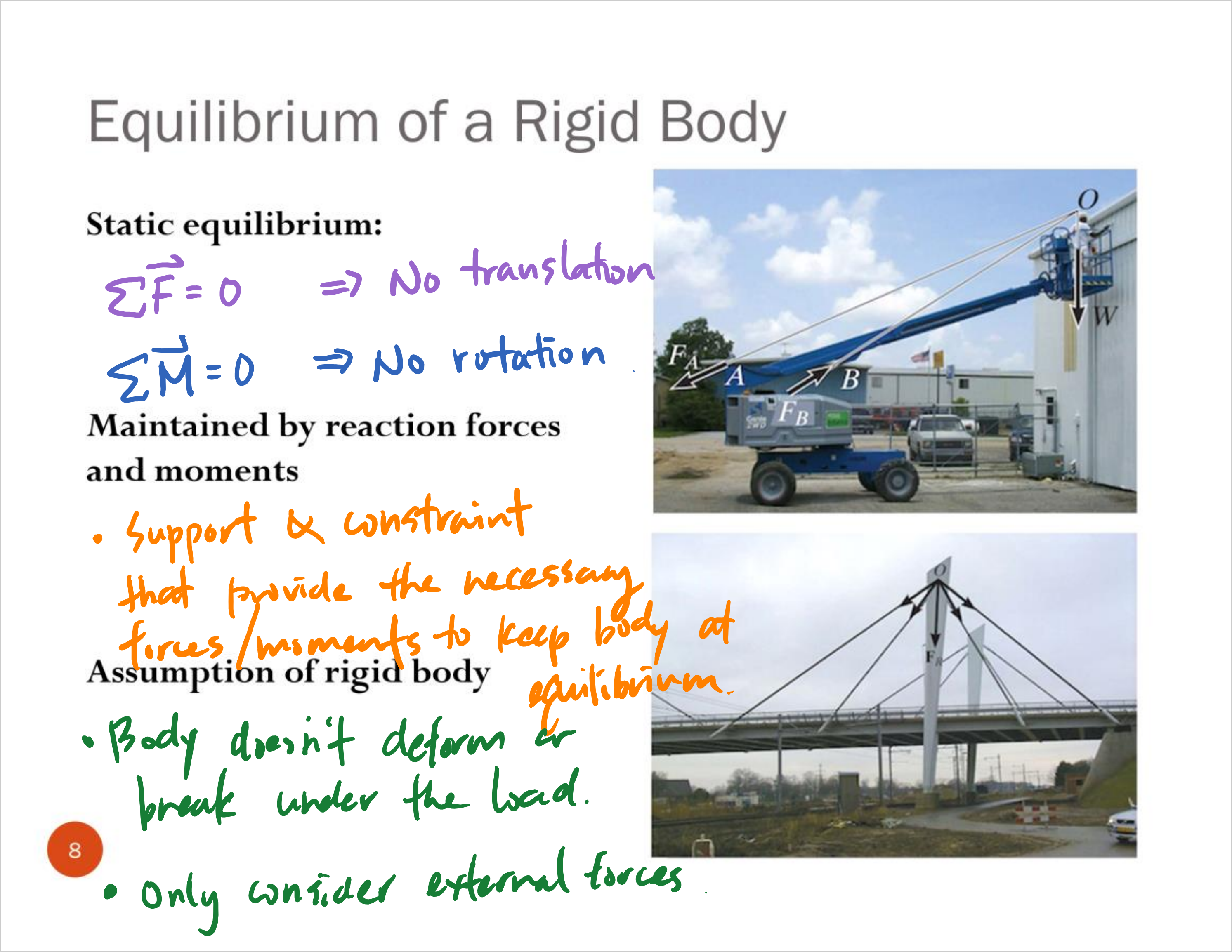


Equilibrium of a Rigid Body

In contrast to the forces on a particle, the forces on a rigid-body are not usually concurrent and may cause rotation of the body. We can reduce the force and couple moment system acting on a body to an equivalent resultant force and a resultant couple moment at an arbitrary point O.

$$\vec{F}_R = \sum \vec{F} \quad \begin{cases} \sum F_x = F_{Rx} \\ \sum F_y = F_{Ry} \\ \sum F_z = F_{Rz} \end{cases}$$

$$\vec{M}_{RO} = \underbrace{\sum \underline{\underline{\vec{r}}} \times \vec{F}}_{} + \underbrace{\sum M}_{}$$



Equilibrium of a Rigid Body

Static equilibrium:

$\Sigma \vec{F} = 0 \Rightarrow$ No translation

$\Sigma \vec{M} = 0 \Rightarrow$ No rotation.

Maintained by reaction forces and moments

- Support & constraint that provide the necessary forces/moments to keep body at equilibrium.

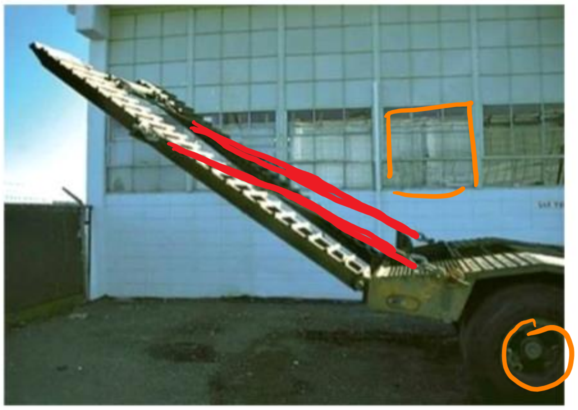
Assumption of rigid body

- Body doesn't deform or break under the load.

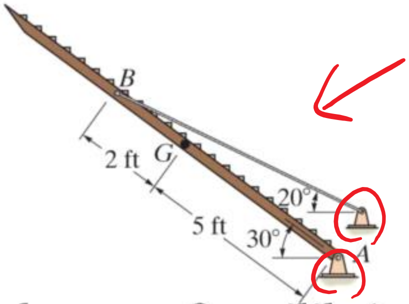
- Only consider external forces.

Process of solving rigid body equilibrium problems

The uniform truck ramp has weight 400 lb and is pinned to the body of the truck at each side and held in the position shown by the two side cables. Determine the reaction forces at the pins and the tension in the cables.

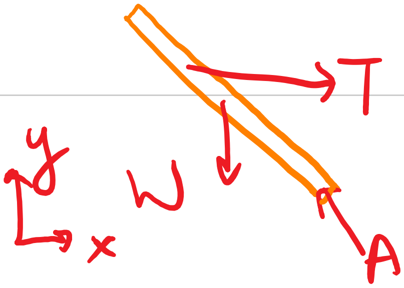


1. Create idealized model (modeling and assumptions)



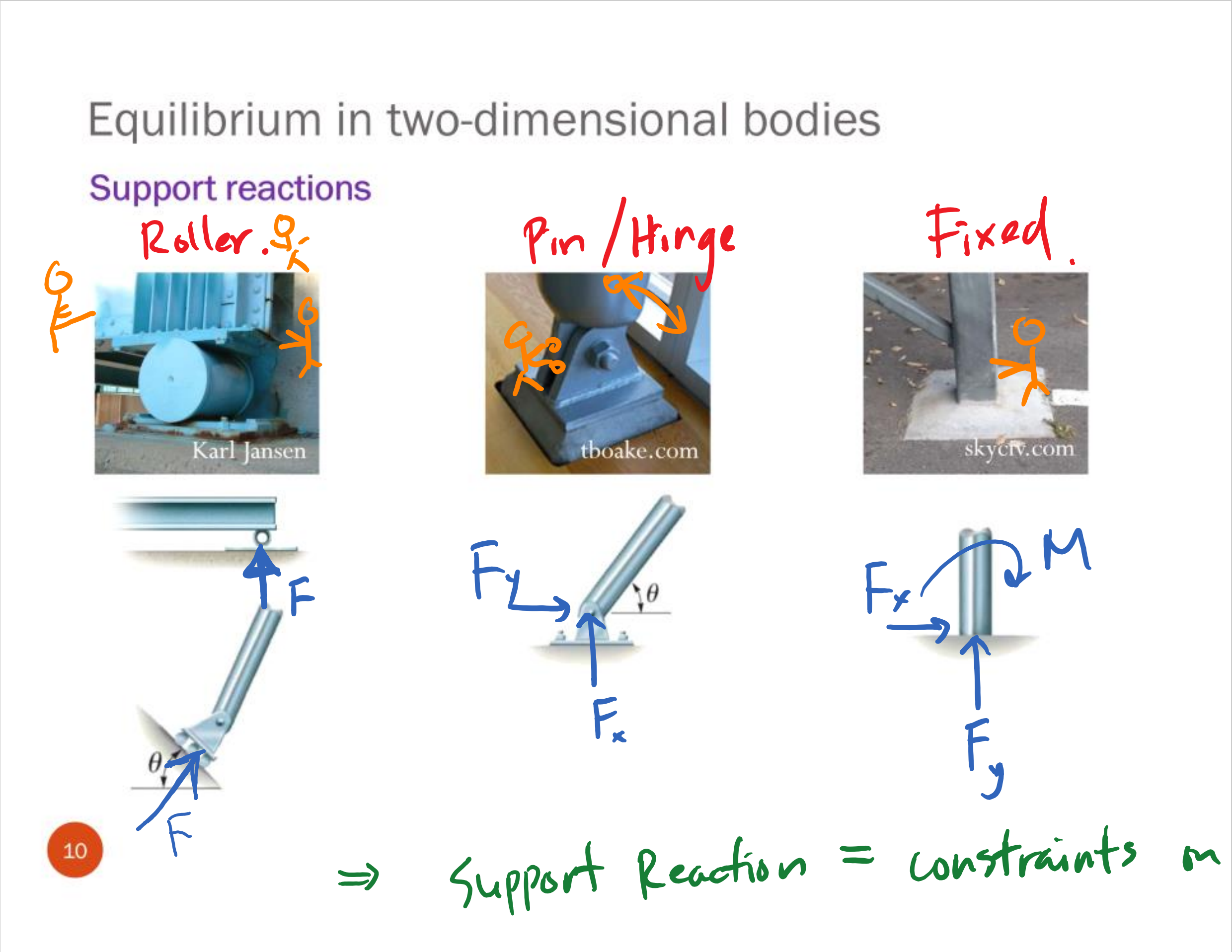
2. Draw free body diagram showing ALL the external (applied loads and supports)

3. Apply eqns of equilibrium



$\sum \vec{F} = 0$

$\sum \vec{M}_o = 0.$



Equilibrium in two-dimensional bodies

Support reactions

⇒ Support Reaction = constraints on the system.

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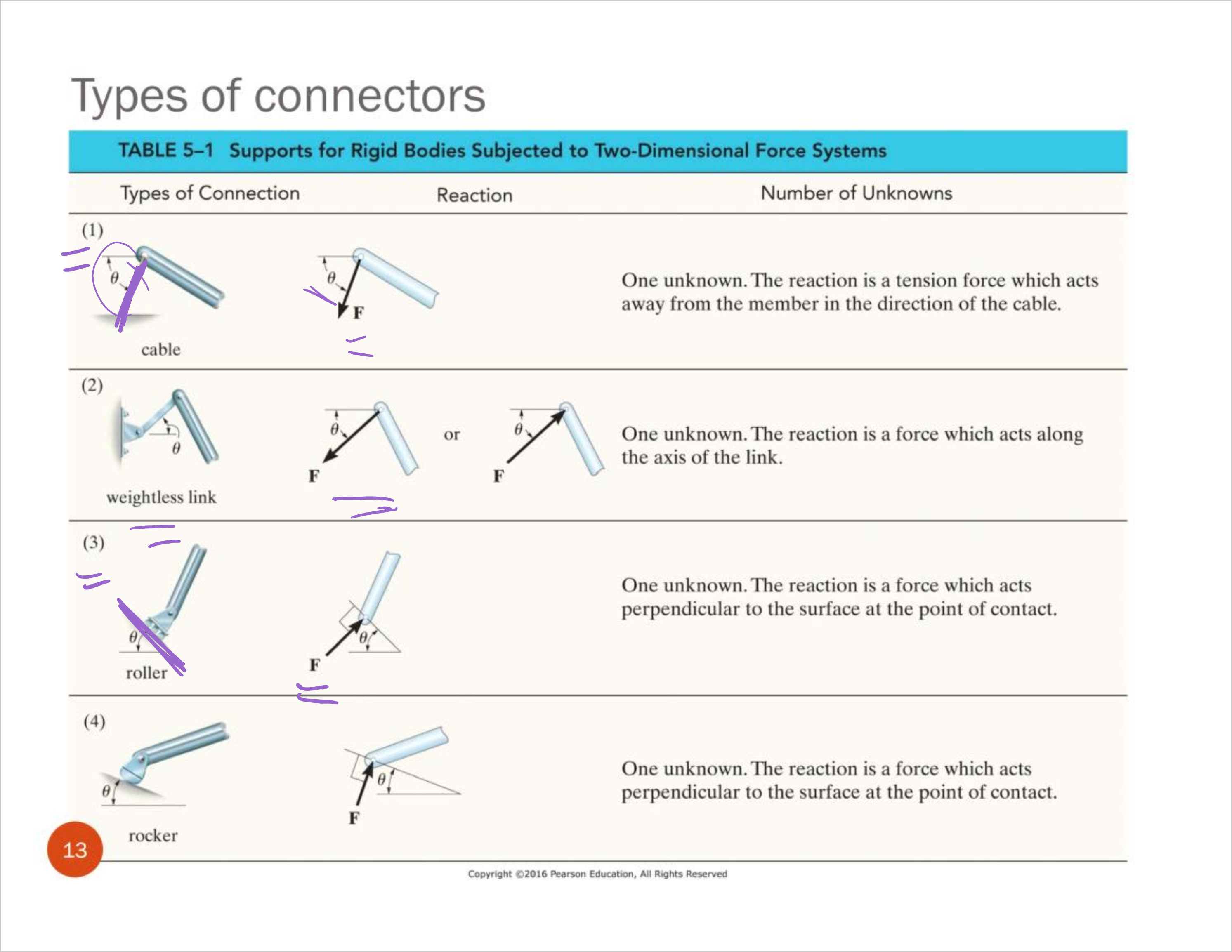

Types of connectors

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1) cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2) weightless link	or	One unknown. The reaction is a force which acts along the axis of the link.
(3) roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4) rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

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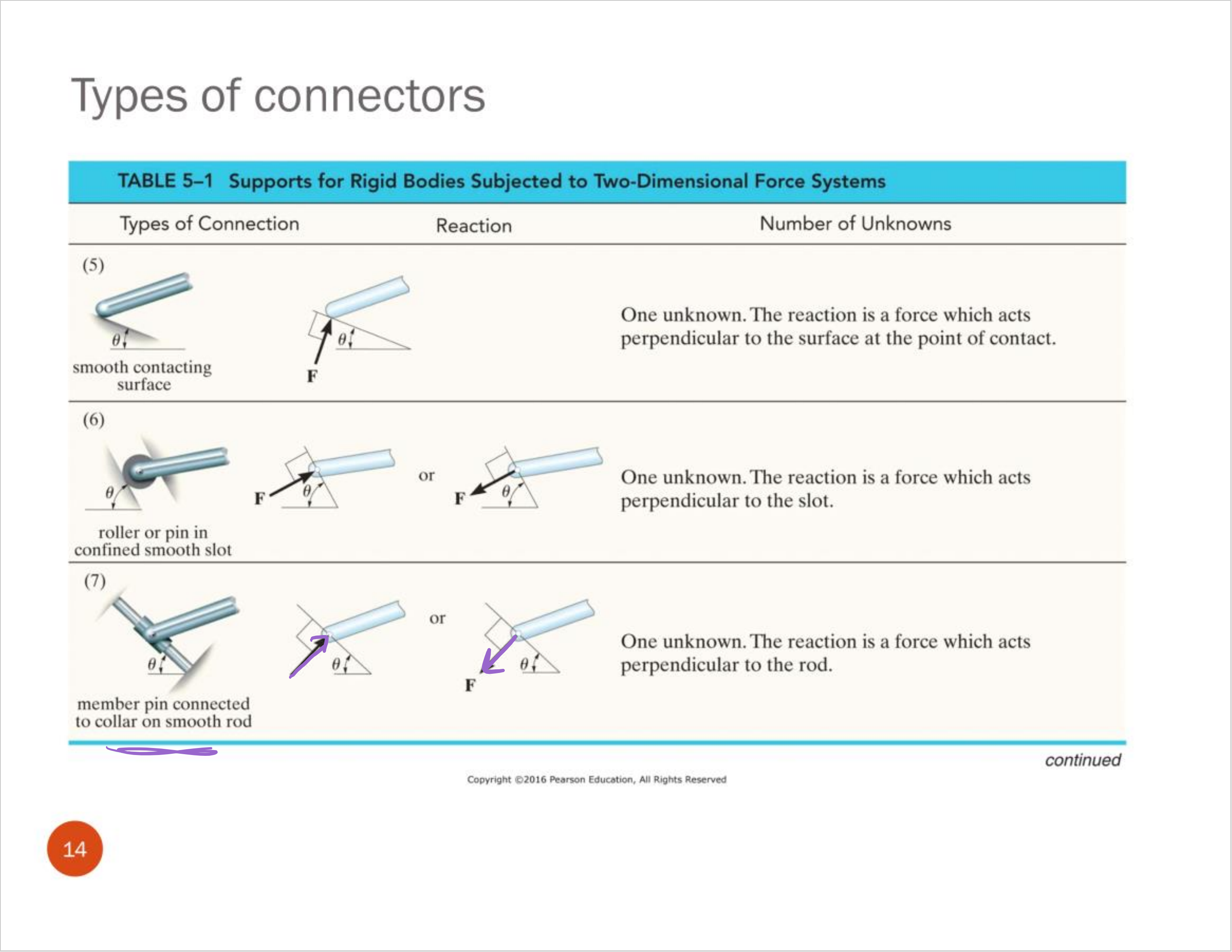

Types of connectors

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(5) smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6) roller or pin in confined smooth slot	or	One unknown. The reaction is a force which acts perpendicular to the slot.
(7) member pin connected to collar on smooth rod	or	One unknown. The reaction is a force which acts perpendicular to the rod.

continued

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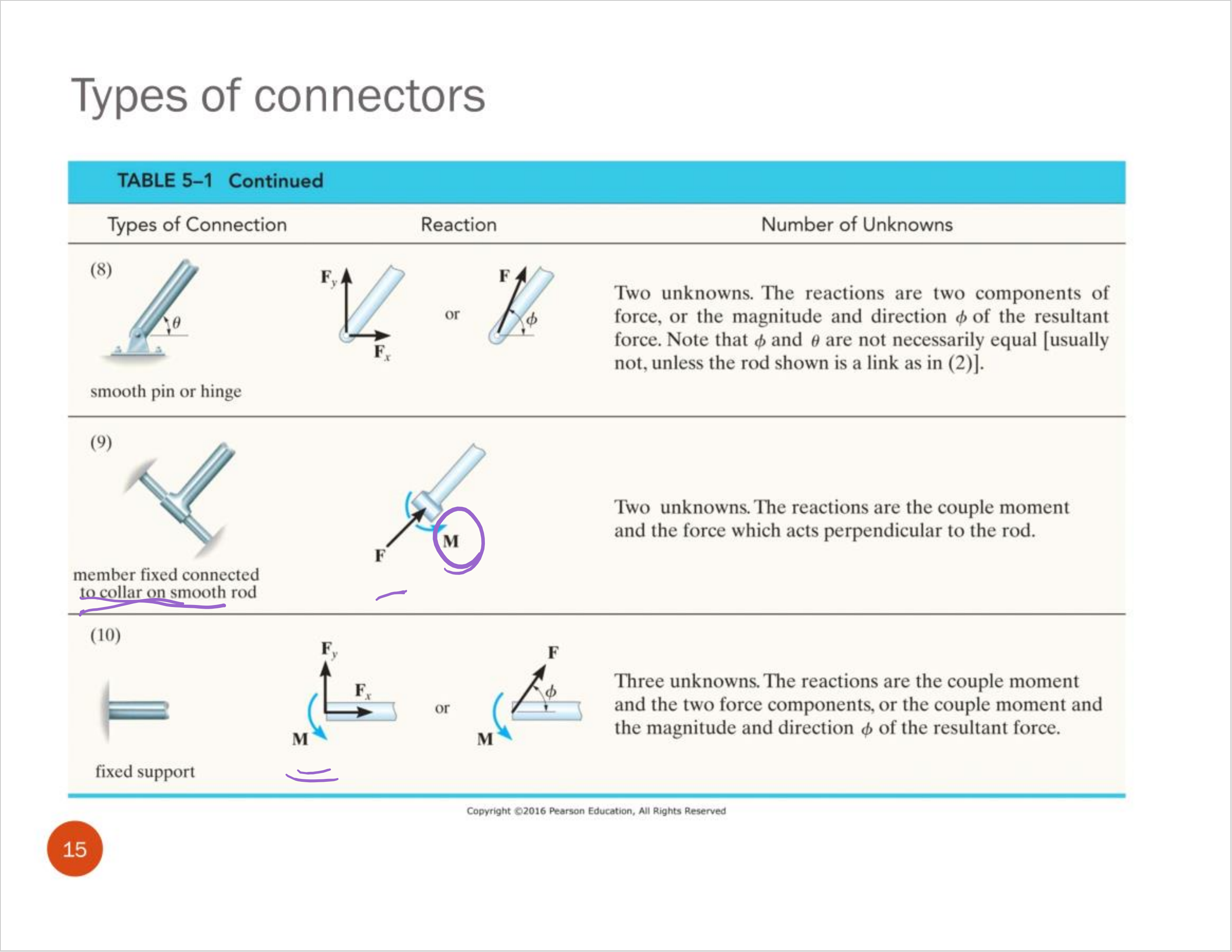

Types of connectors

TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(8) smooth pin or hinge	F_y, F_x or F, ϕ	Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9) member fixed connected to collar on smooth rod	F, M	Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10) fixed support	F_y, F_x, M or F, ϕ, M	Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

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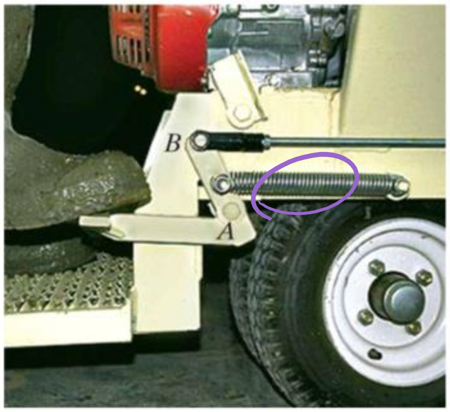
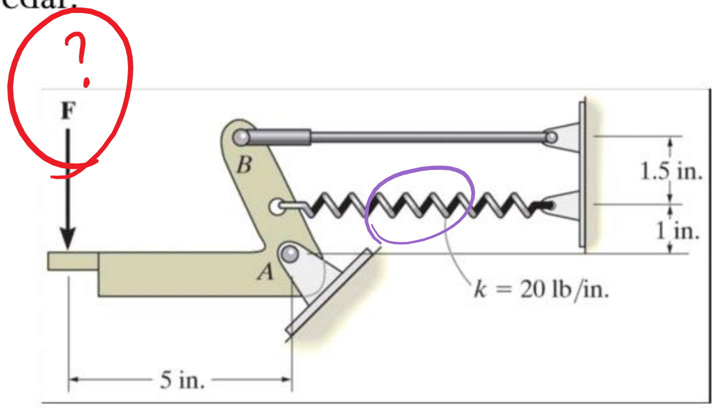
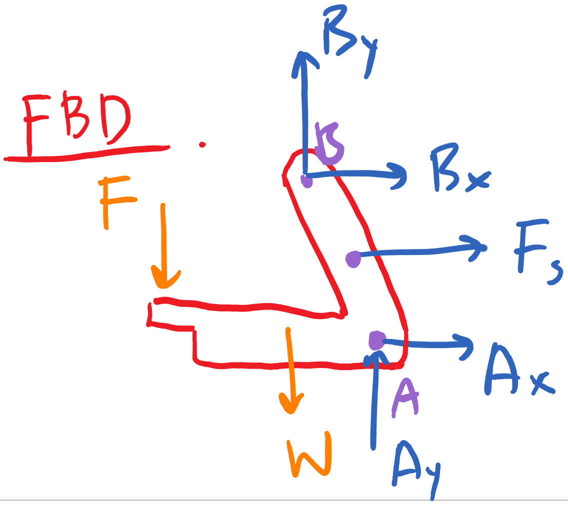
The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at B is 20 lb. Determine the vertical force applied to the pedal.







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Find the tension in cable B given the weight of the cage.