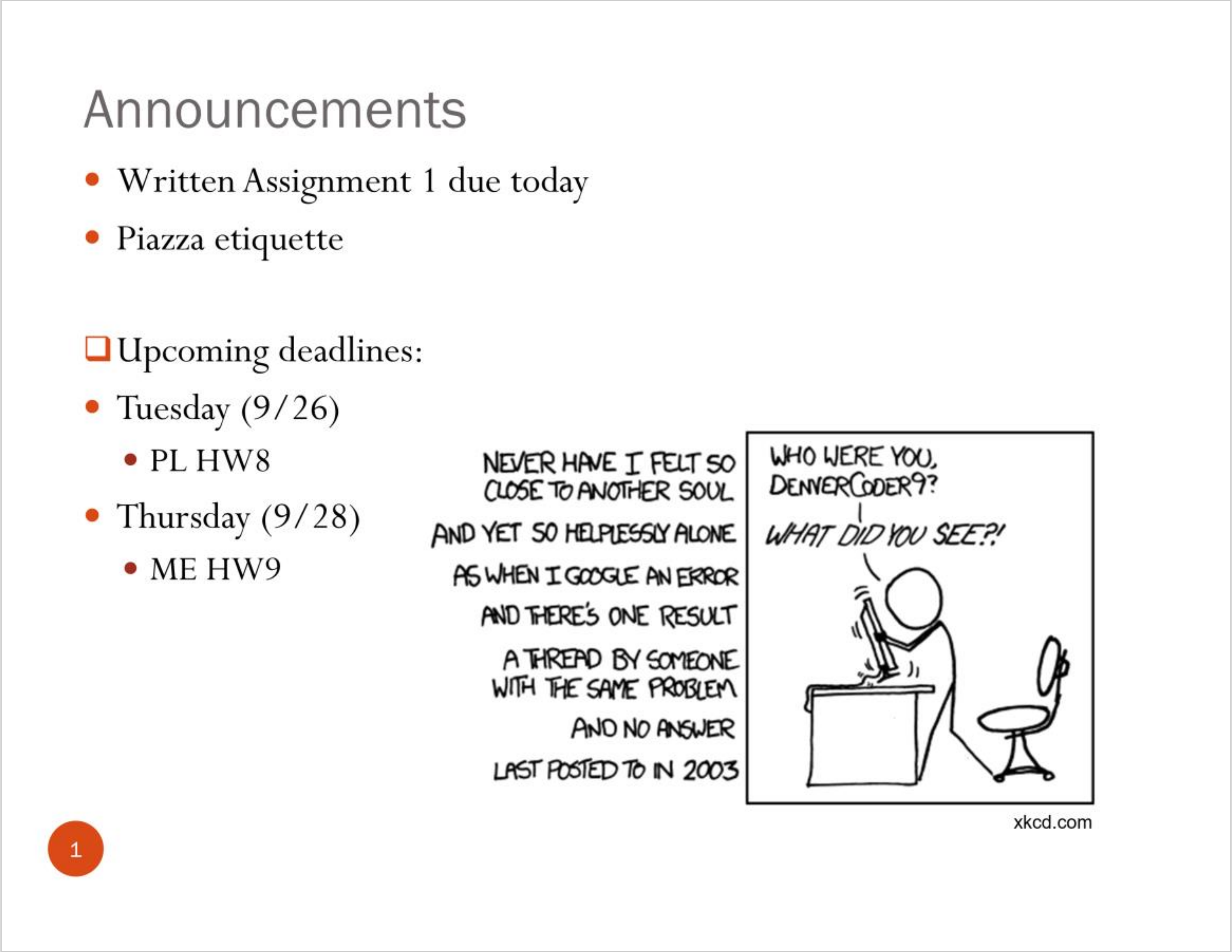
wc11lect



Announcements

- Written Assignment 1 due today
- Piazza etiquette

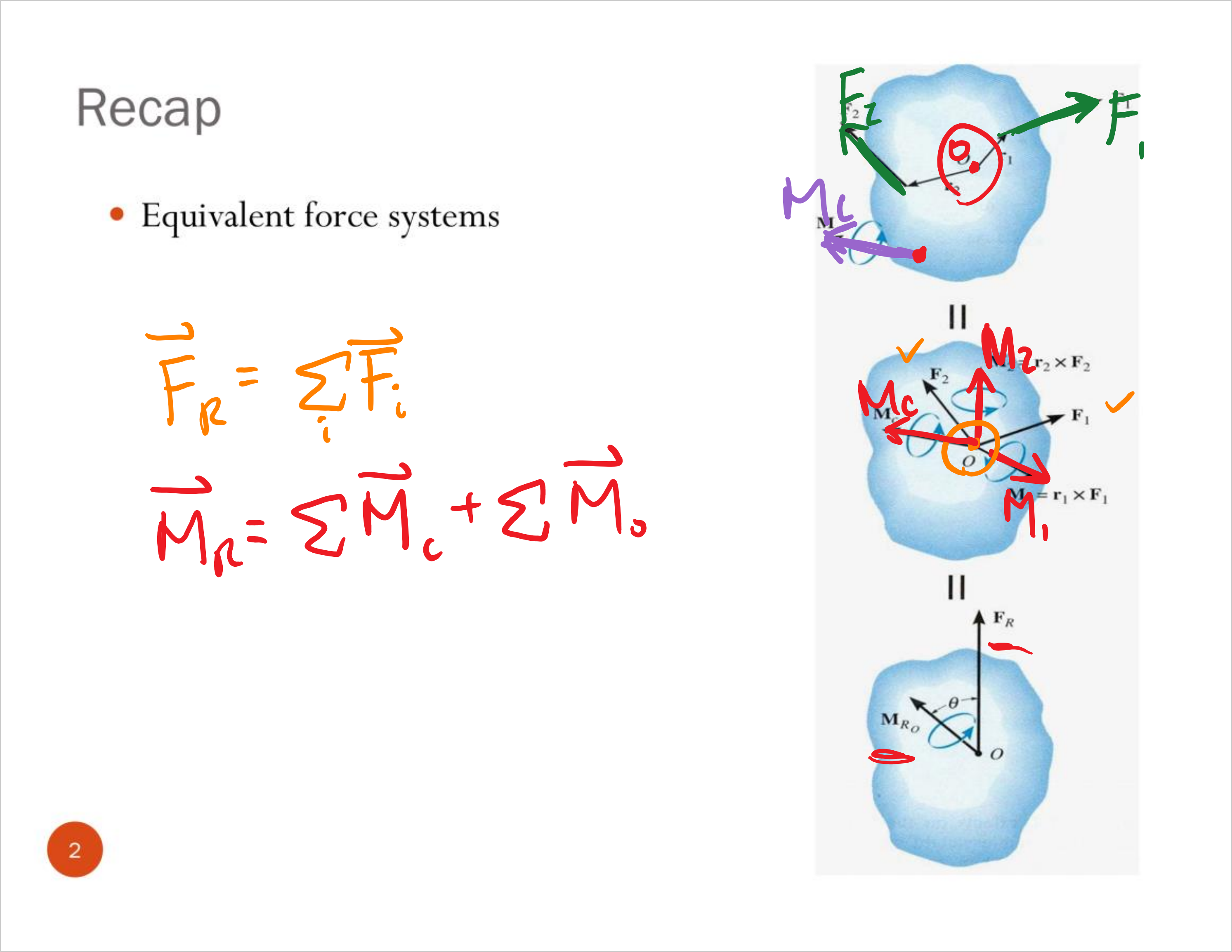
☐ Upcoming deadlines:

- Tuesday (9/26)
  - PL HW8
- Thursday (9/28)
  - ME HW9

xkcd.com

Page 2

Friday, September 22, 2017 11:32 AM



Recap

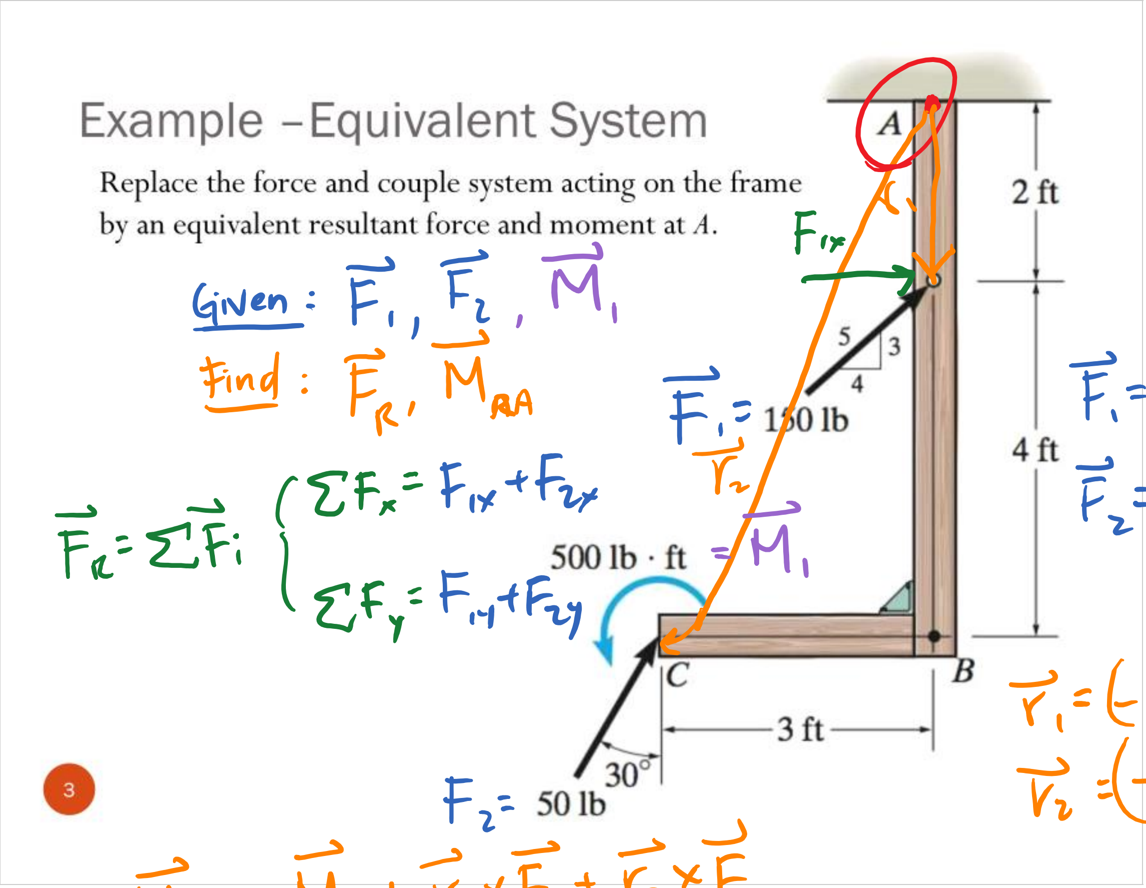
- Equivalent force systems

$$\vec{F}_R = \sum_i \vec{F}_i$$

$$\vec{M}_R = \sum \vec{M}_c + \sum \vec{M}_o$$

Page 3

Friday, September 22, 2017 11:32 AM



Example –Equivalent System

Replace the force and couple system acting on the frame by an equivalent resultant force and moment at *A*.

Given : $\vec{F}_1 , \vec{F}_2 , \vec{M}_1$

Find : $\vec{F}_R , \vec{M}_{RA}$

$$\vec{F}_R = \sum \vec{F}_i \begin{cases} \sum F_x = F_{1x} + F_{2x} \\ \sum F_y = F_{1y} + F_{2y} \end{cases}$$

$\vec{F}_1 = 150 lb\left(\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right)$

$\vec{F}_2 = 50 lb\left(\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j}\right)$

$\vec{r}_1 = (-2\hat{j})$ ft

$\vec{r}_2 = (-3\hat{i} - 6\hat{j})$ ft.

$$\vec{M}_{RA} = \vec{M}_1 + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$

$\Rightarrow$ $F_{Rx} = 120 lb + 25 lb = 145 lb$

$F_{Ry} = 90 lb + 25\sqrt{3} lb \simeq 133 lb$

$\boxed{\vec{F}_R = (145\hat{i} + 133\hat{j}) lb}$

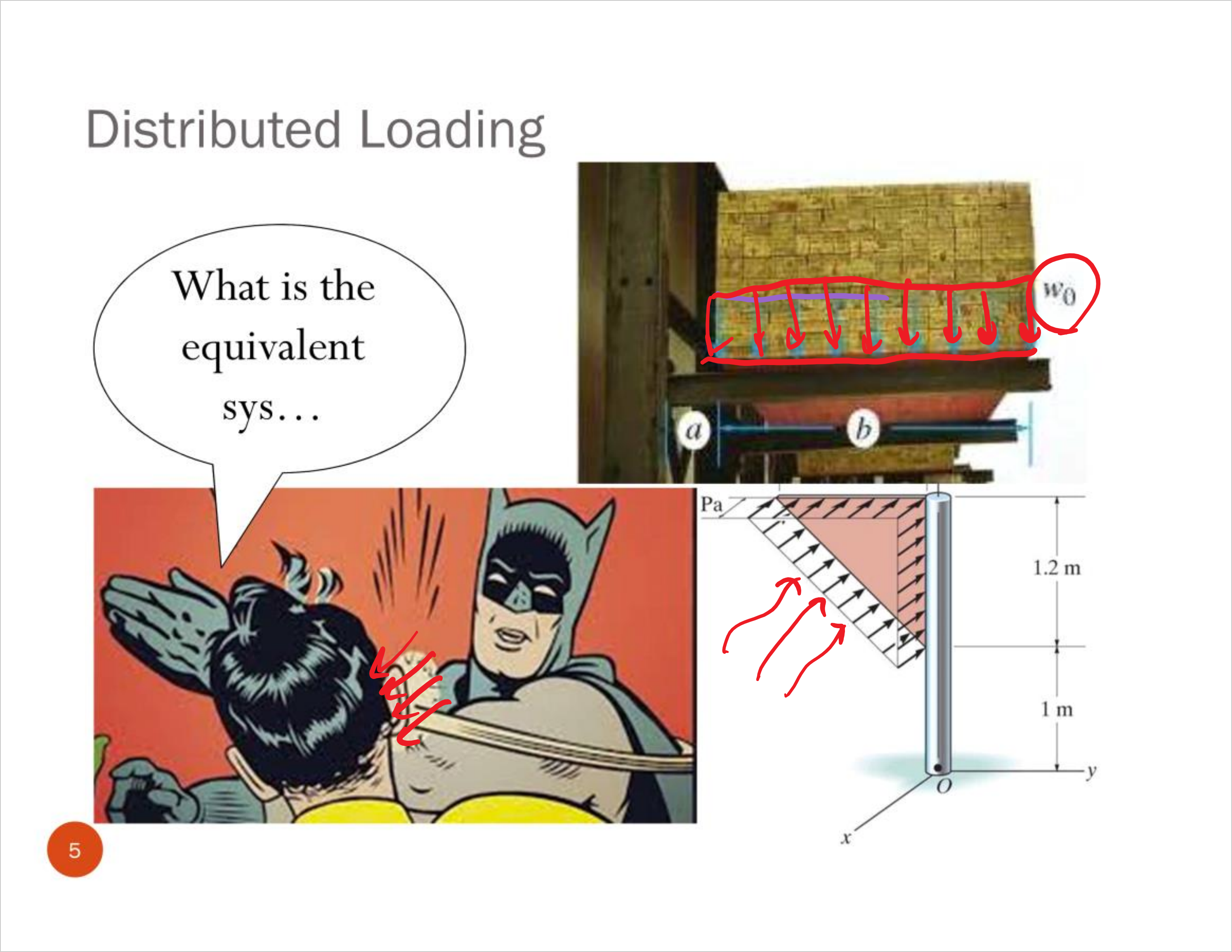
$M_{RA} = M_1 + r_1 F_{1x} + r_{2y} F_{2x} - r_{2x} F_{2y}$

$= 500 lb \cdot ft + (2 ft)(120 lb) + (6 ft)(25 lb) - (3 ft)(133 lb)$

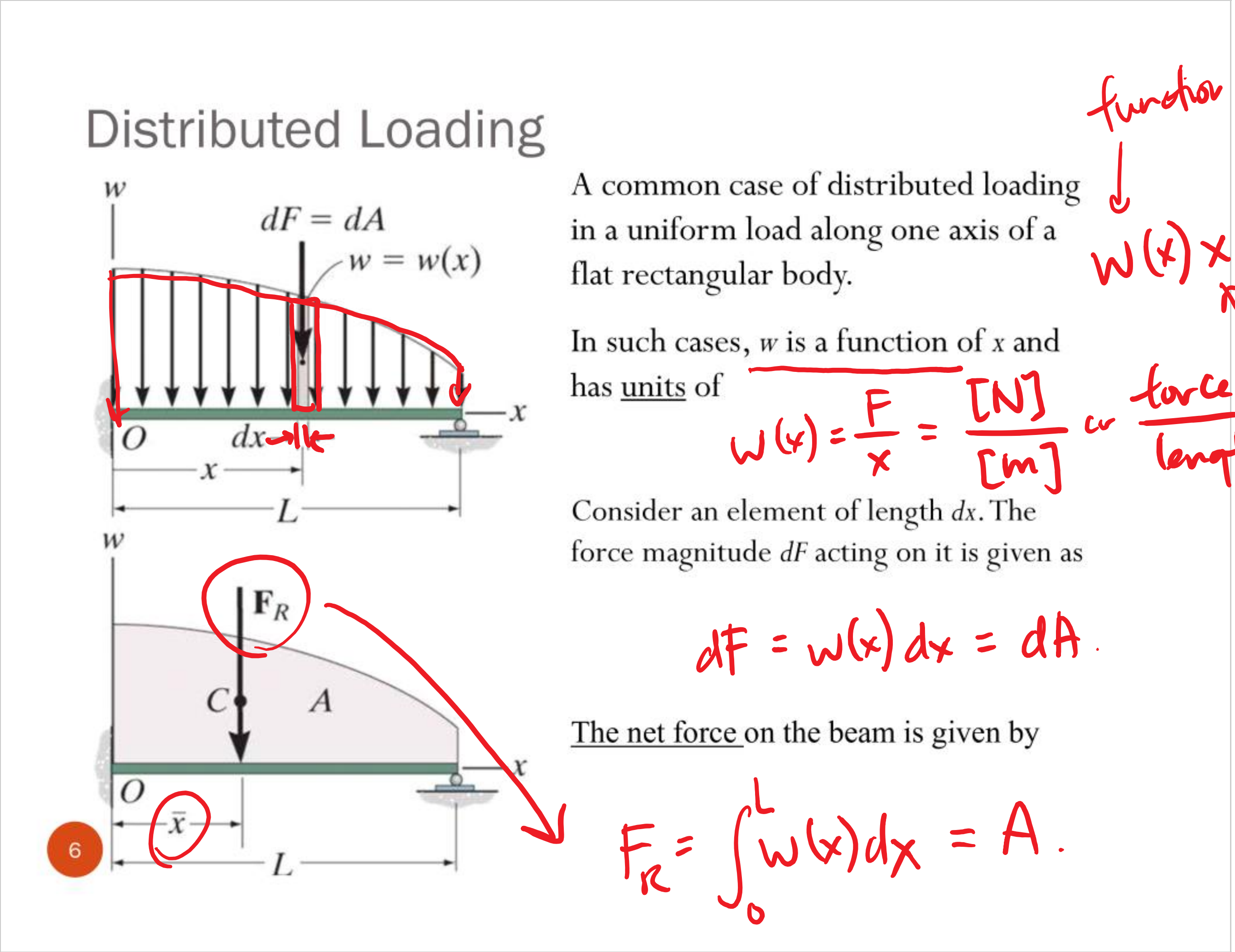
$\Rightarrow \boxed{M_{RA} = 490 lb \cdot ft}$

Page 5

Friday, September 22, 2017 11:32 AM



Distributed Loading



Distributed Loading

A common case of distributed loading in a uniform load along one axis of a flat rectangular body.

function

$$W(x)\,x = F.$$

length

In such cases, *w* is a function of *x* and has units of

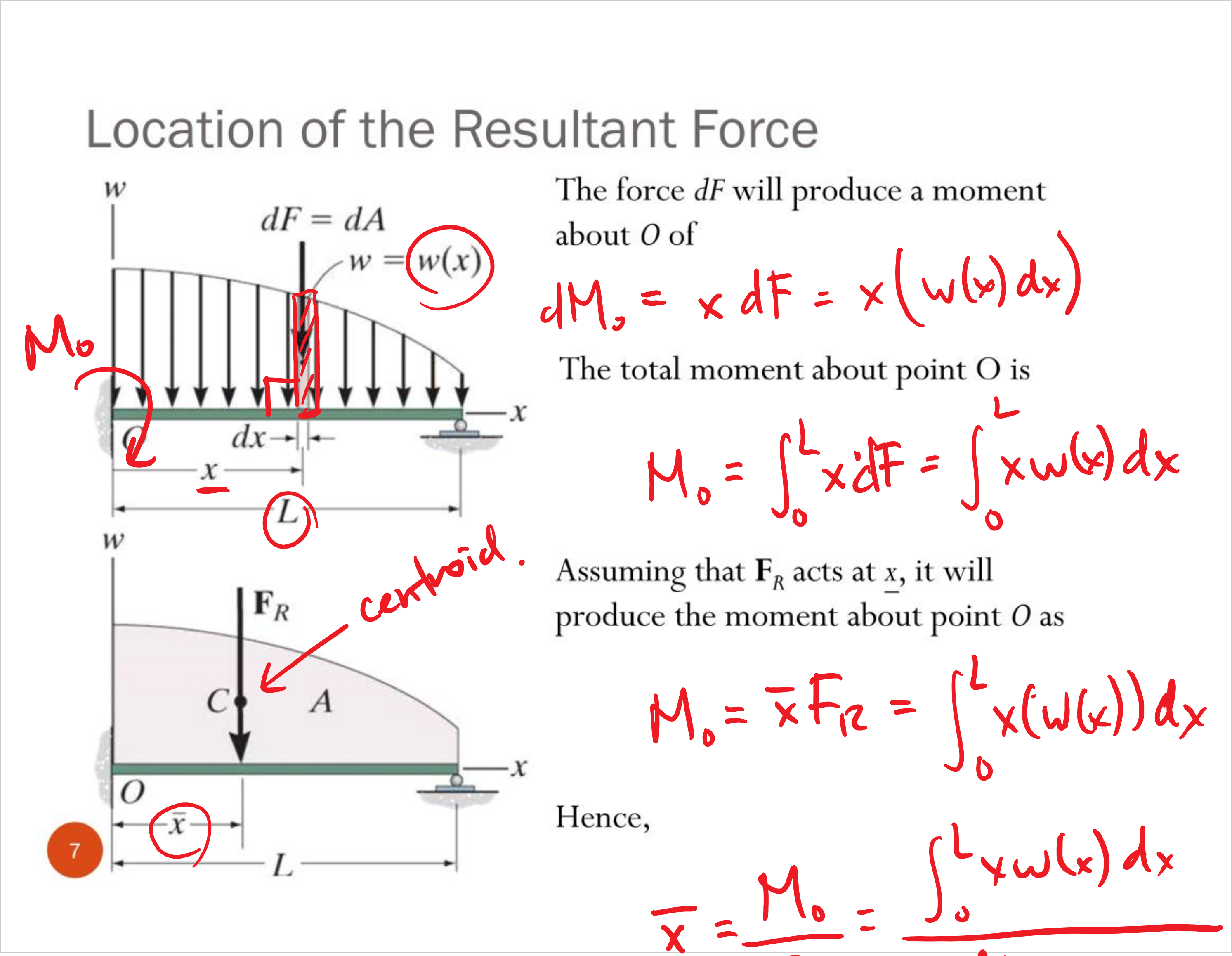
$$w(x) = \frac{F}{x} = \frac{[N]}{[m]} \text{ or } \frac{\text{force}}{\text{length}}.$$

Consider an element of length *dx*. The force magnitude *dF* acting on it is given as

$$dF = w(x)\,dx = dA.$$

The net force on the beam is given by

$$F_R = \int_0^L w(x)\,dx = A.$$



Location of the Resultant Force

The force *dF* will produce a moment about *O* of

$$dM_o = x\,dF = x(w(x)\,dx)$$

The total moment about point O is

$$M_o = \int_0^L x\,dF = \int_0^L x\,w(x)\,dx$$

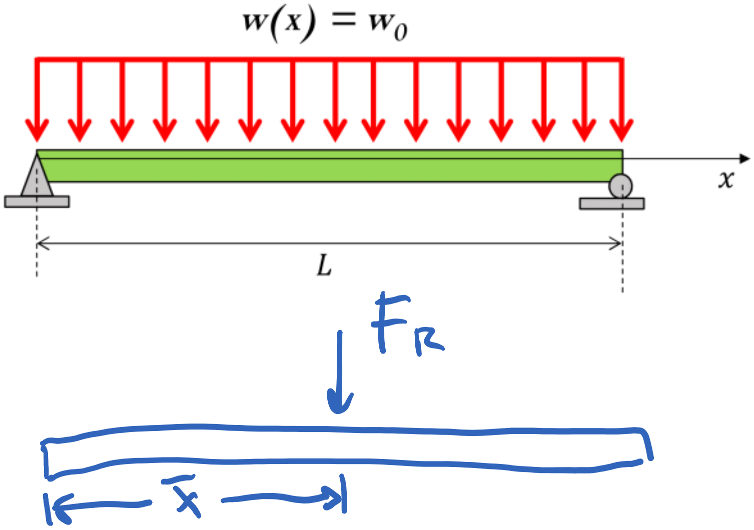
Assuming that $\mathbf{F}_R$ acts at $\underline{x}$, it will produce the moment about point *O* as

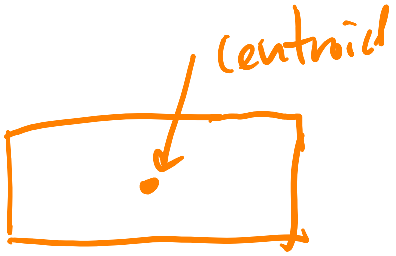
$$M_o = \bar{x} F_R = \int_0^L x(w(x))\,dx$$

Hence,

$$\bar{x} = \frac{M_o}{F_R} = \frac{\int_0^L x\,w(x)\,dx}{\int_0^L w(x)\,dx}$$

Rectangle Loading



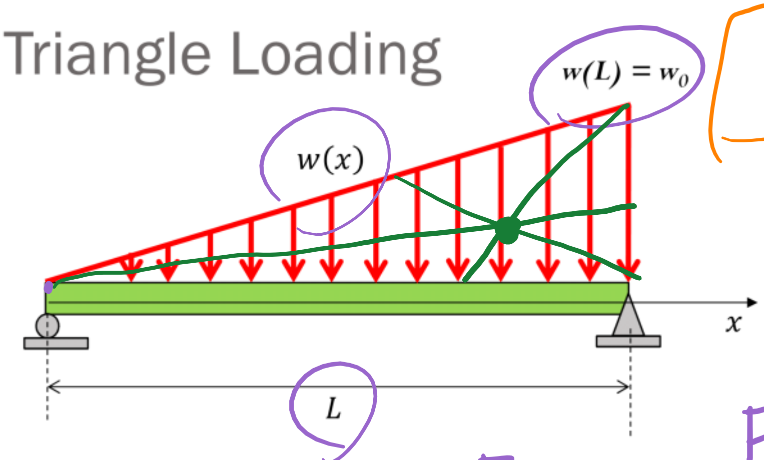


$$F_R = A = (\text{base})(\text{height})$$

$$\boxed{F_R = L(w_0)}$$

$$\bar{x} = \frac{M_0}{F_R} = \text{centroid} = \boxed{\frac{L}{2} = \bar{x}}$$

Triangle Loading

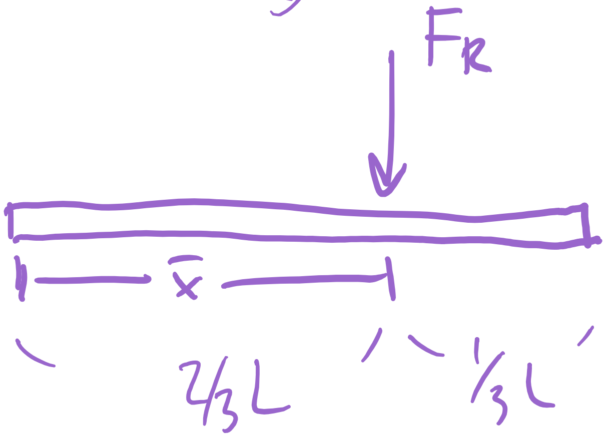


constant

$\boxed{w(x) = ax}$

• linear function with zero y-int.

$F_R = \frac{bh}{2} \Rightarrow \boxed{F_R = \frac{L(w_0)}{2}}$



$\bar{x}$ = centroid = $\boxed{\frac{2}{3}L = x}$ ✓

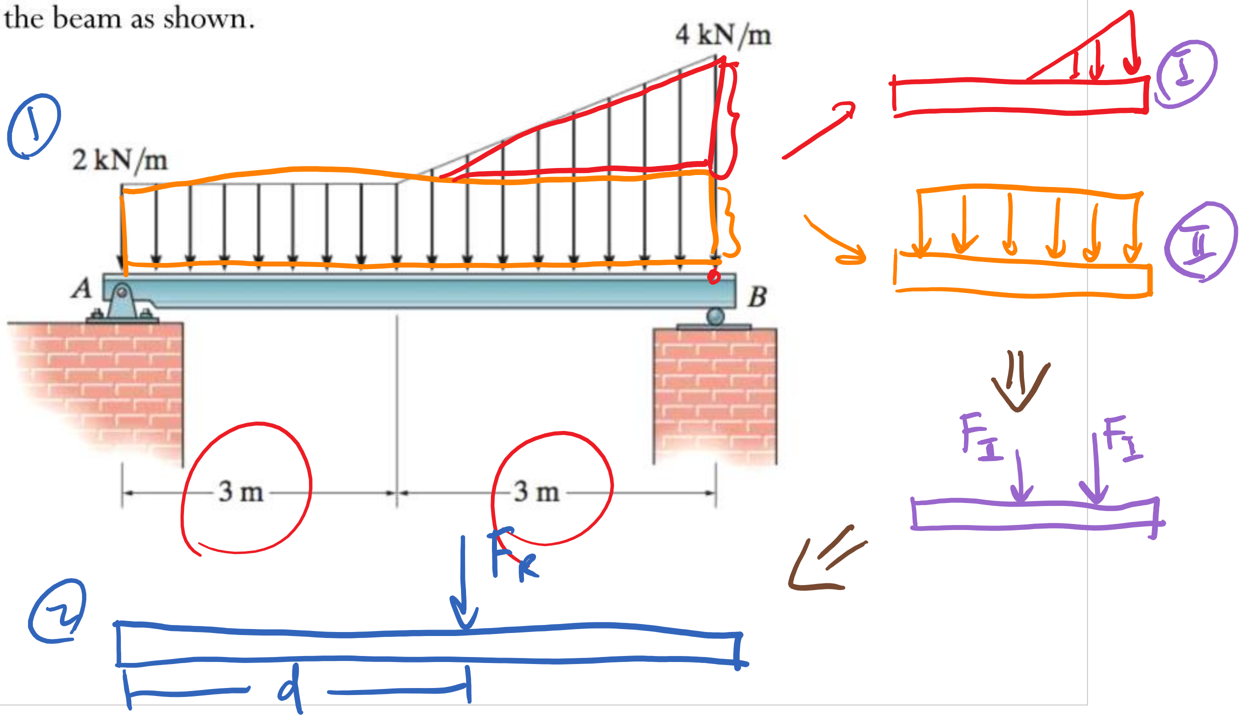
$$\bar{x} = \frac{M_0}{F_R} = \frac{\int_0^L x(w(x))dx}{\int_0^L w(x)dx} = \frac{\int_0^L x(ax)\,dx}{\int_0^L (ax)dx}$$

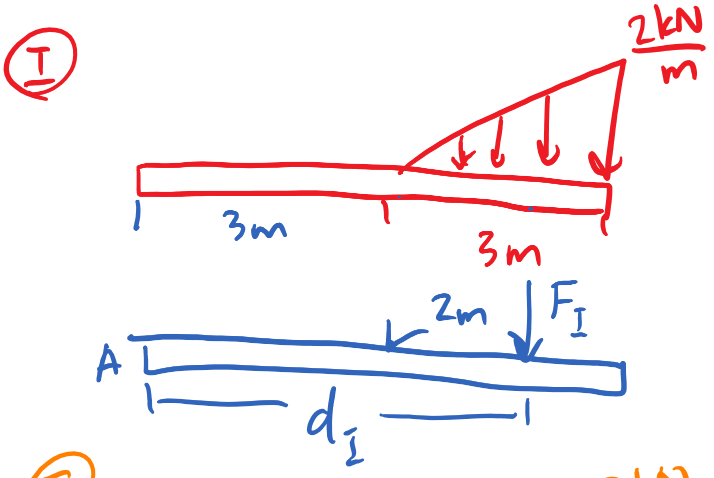
$$= \frac{\frac{1}{3}\cancel{a}x^3\Big|_0^L}{\frac{1}{2}\cancel{a}x^2\Big|_0^L} = \frac{2}{3}x\Big|_0^L$$

$\therefore \boxed{\bar{x} = \frac{2}{3}L}$ for triangles.

Example

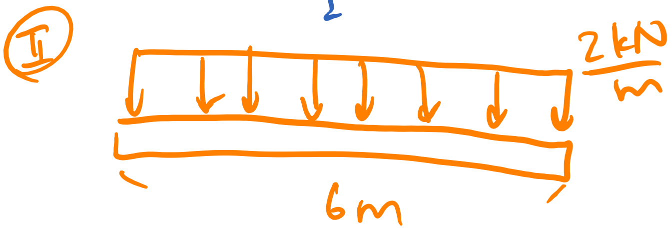
Find the equivalent force and its location from point *A* for the loading on the beam as shown.





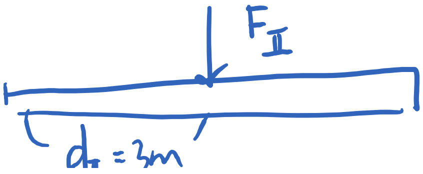
$$\underline{\underline{F_I}} = \frac{bh}{2} = \frac{(3m)(2\frac{kN}{m})}{2} = \underline{\underline{3\,kN}}$$

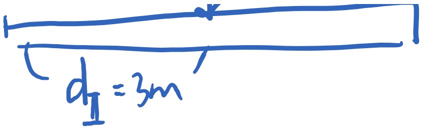
$$d_I = (3m)+(2m) = \underline{5m}$$

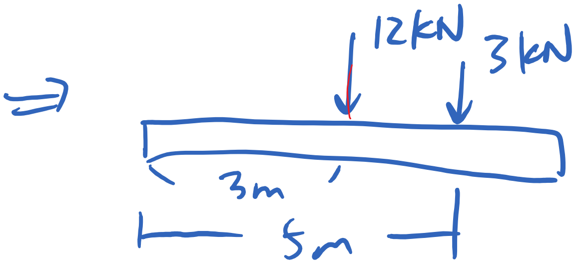


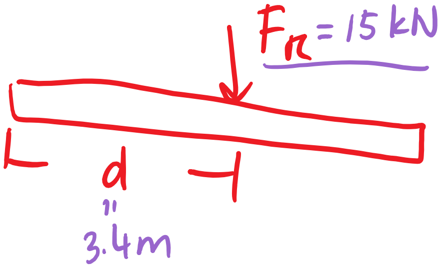
$$F_{II} = bh = (6m)(2\frac{kN}{m}) = 12\,kN$$

$$d_{II} = \frac{1}{2}L = \frac{1}{2}(6m) = 3m$$









$$\boxed{F_R = F_I + F_{II} = 15\,kN}$$

$$M_A = F_I d_I + F_{II} d_{II}$$

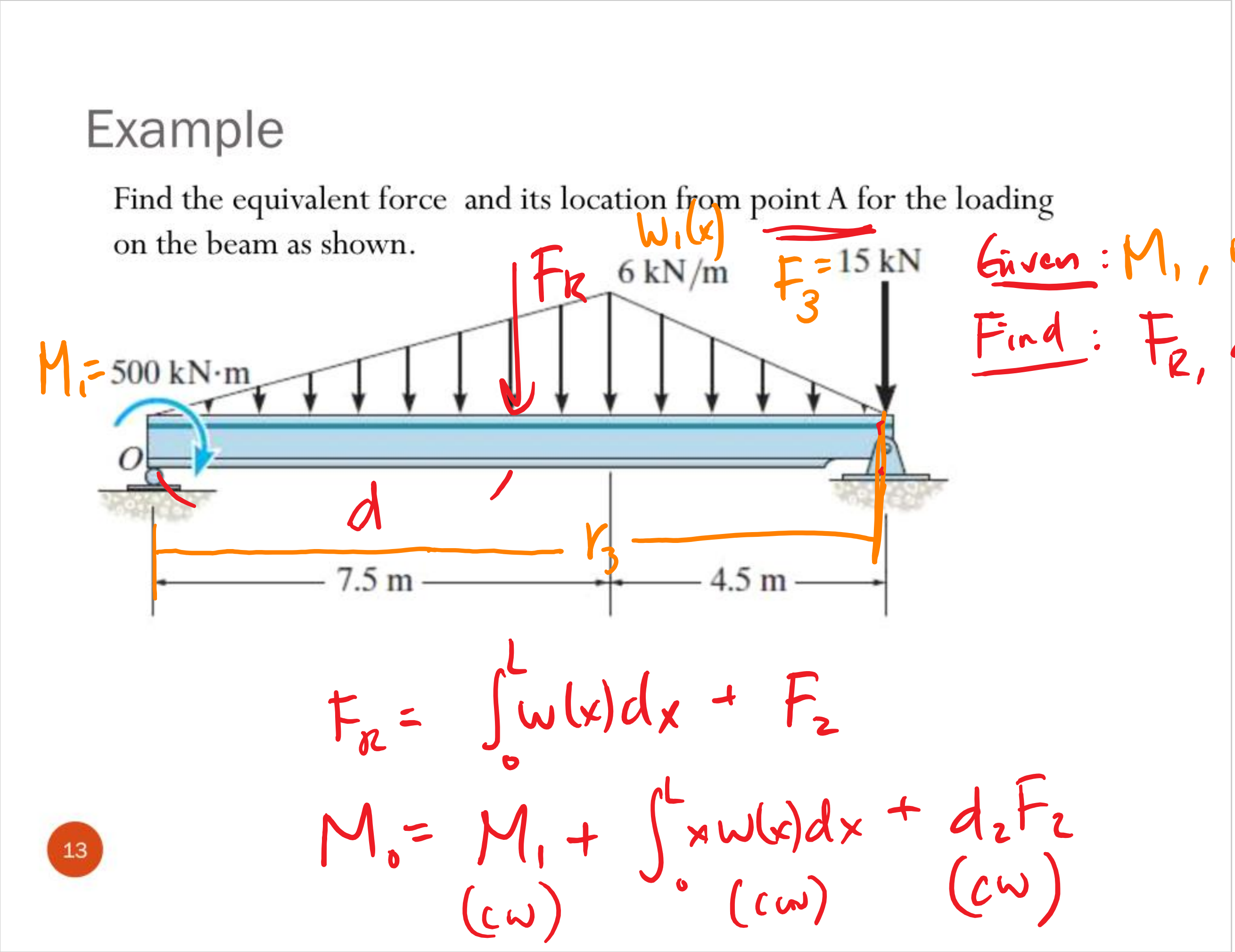
$$= (3\,kN)(5m) + (12\,kN)(3m)$$

$$= 51\,kN \cdot m$$

$$d = \frac{M_A}{F_R} = \frac{51\,\cancel{kN}\cdot \textcircled{m}}{15\,\cancel{kN}}$$

$$=$$

$$\boxed{d \approx 3.4\,m}$$



Example

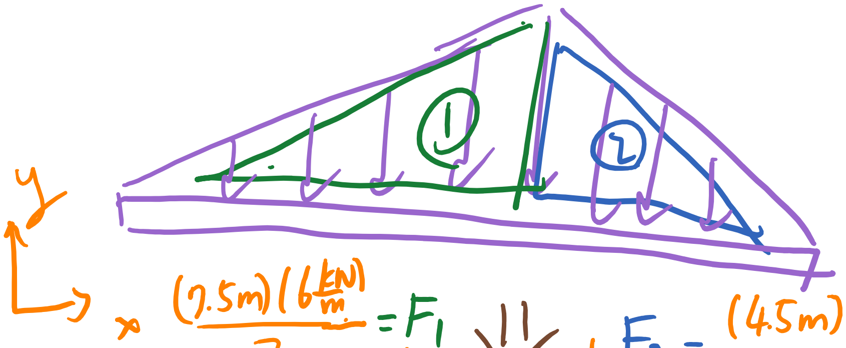
Find the equivalent force and its location from point A for the loading on the beam as shown.

Given : M_1, $w_1(x)$, F_3

Find : F_R, d

$$F_R = \int_0^L w(x)\,dx + F_2$$

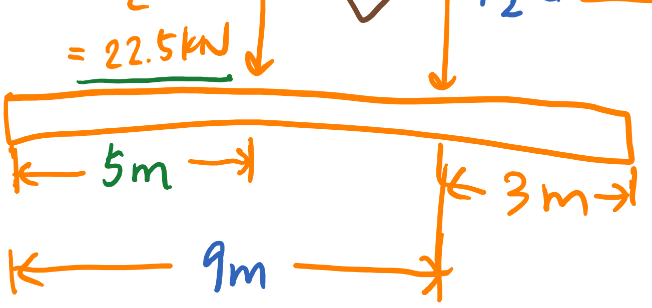
$$M_0 = \underset{(cw)}{M_1} + \underset{(cw)}{\int_0^L x\,w(x)\,dx} + \underset{(cw)}{d_2 F_2}$$



$\frac{(7.5m)(6\frac{kN}{m})}{2} = F_1$

$= 22.5 kN$

$F_2 = \frac{(4.5m)(6\frac{kN}{m})}{2} = 13.5 kN$



$F_R = F_1 + F_2 + F_3$

$= (22.5 + 13.5 + 15)$ kN

$\boxed{F_R = 51 \text{ kN}}$

$M_A = r_1 F_1 + r_2 F_2 + r_3 F_3 + M_1$

$$M_A = r_1 F_1 + r_2 F_2 + r_3 F_3 + M_1$$

$$\boxed{F_R = 51\ kN}$$

$$= (5m)(22.5kN) + (9m)(13.5kN) + (12m)(15kN) + 500\ kN \cdot m$$

$$\boxed{M_A = 914\ kN \cdot m}\ (CW)$$

$$d = \frac{M_{RA}}{F_R} = \frac{914\ kN \cdot m}{51\ kN} \Rightarrow \boxed{d = 17.9m}$$