wc09lect

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Announcements

- Quiz 2 starts tomorrow!

❑ Upcoming deadlines:

- Tuesday (9/19)
  - PL HW6
- Thursday (9/21)
  - ME HW7
- Friday (9/22)
  - Writing Assignment 1 (FBD only)

Pangolin!

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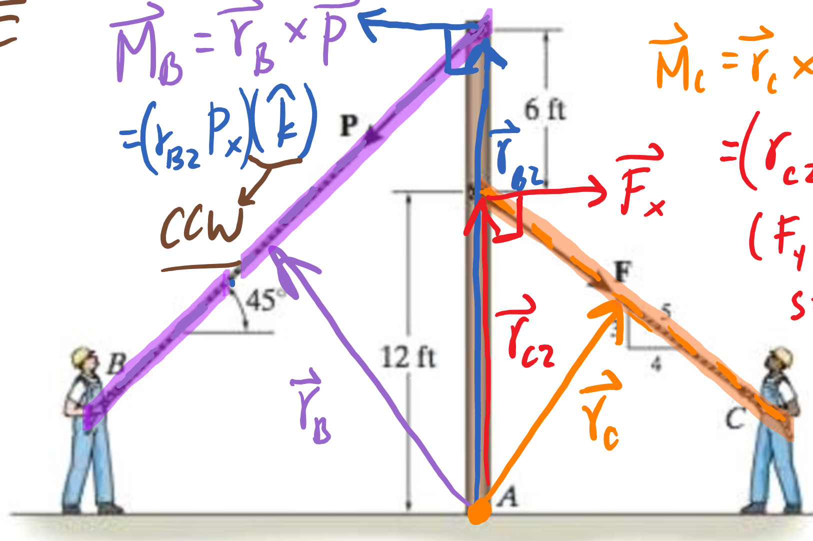
Recap

- Moment of a force
  - **About a point**

$\vec{M} = \vec{r} \times \vec{F}$

$\vec{M}_B = \vec{r}_B \times \vec{P}$

$= (r_{Bz} P_x)(\hat{k})$ — CCW



$\vec{M}_c = \vec{r}_c \times \vec{F}$

$= (r_{cz} F_x)(-k)$ — CW.

(F_y is // to r_{cz}, so it won't contribute to moment about A)

(same for P_y and r_{Bz})

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Recap

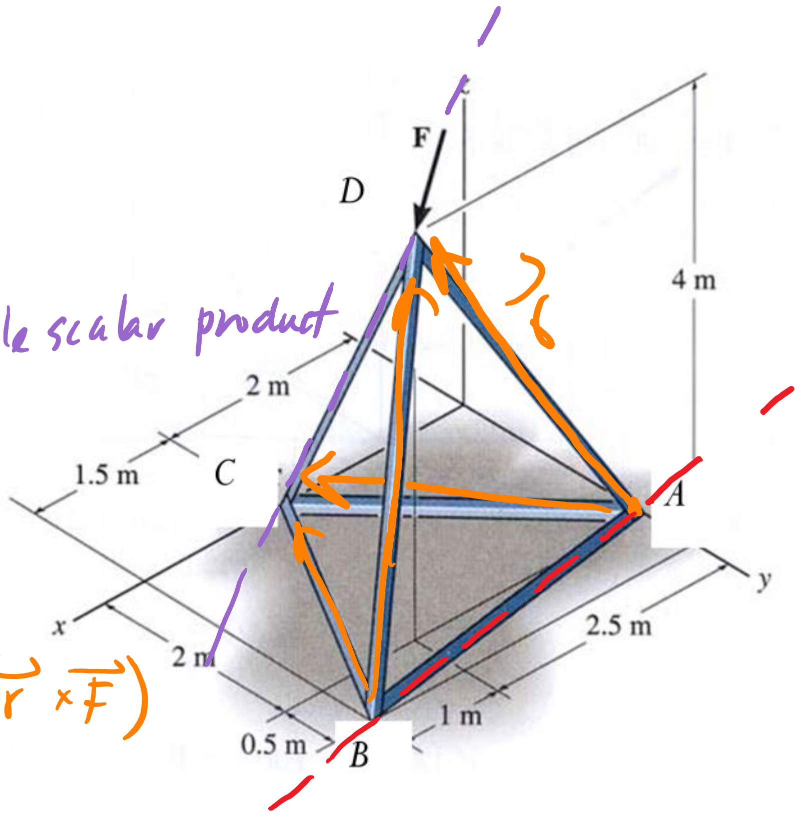
- Moment of a force
  - About a point
  - **About an axis**

Magnitude : triple scalar product

$M = \hat{u} \cdot (\vec{r} \times \vec{F})$

Direction : $\hat{u}$.

$\Rightarrow \overrightarrow{M}_{BA} = \hat{u}_{BA} \cdot (\vec{r} \times \vec{F})$



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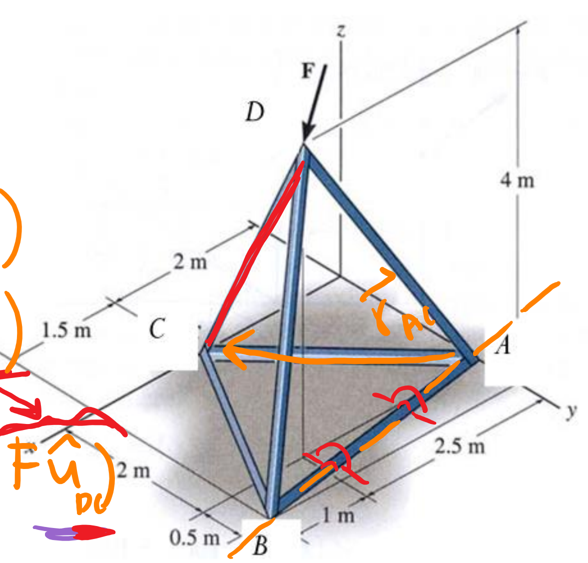
The force F = 10 N is acting along *DC*. Determine the moment of F about the bar BA.

Given : $\vec{F}$

Find : M_{BA}



$$\vec{M} = \hat{u}\cdot(\vec{r}\times\vec{F})$$
$$= \hat{u}_{BA}\cdot(\vec{r}_{AC}\times\vec{F})$$
$$= \hat{u}_{BA}\cdot(\vec{r}_{AC}\times F\hat{u}_{DC})$$

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A: $(0,2,0)$ m

B: $(3.5,2.5,0)$ m

C: $(2,0,0)$ m

D: $(2.5,2,4)$ m

$$\hat{u}_{BA} = \frac{\vec{r}_A - \vec{r}_B}{|\vec{r}_A - \vec{r}_B|}$$

$$\vec{r}_{AC} = \vec{r}_C - \vec{r}_A$$

$$\hat{u}_{DC} = \frac{\vec{r}_C - \vec{r}_D}{|\vec{r}_C - \vec{r}_D|}$$

$$\begin{cases} \hat{u}_{BA} = \left(\frac{-3.5}{3.54}\right)\hat{i} + \left(\frac{-0.5}{3.54}\right)\hat{j} \\ \vec{r}_{AC} = (2\hat{i} - 2\hat{j})\text{ m} \\ \hat{u}_{DC} = \left(\frac{-0.5}{4.5}\right)\hat{i} + \left(\frac{-2}{4.5}\right)\hat{j} + \left(\frac{-4}{4.5}\right)\hat{k} \end{cases}$$

Use triple scalar product to find moment about $\vec{BA}$:

$$M_{BA} = \begin{vmatrix} \frac{-3.5}{3.54} & \frac{-0.5}{3.54} & 0 \\ 2 & -2 & 0 \\ (10)\left(\frac{-0.5}{4.5}\right) & (10)\left(\frac{-2}{4.5}\right) & (10)\left(\frac{-4}{4.5}\right) \end{vmatrix} = -20.1\ \text{N}\cdot\text{m}$$

$$\Rightarrow \boxed{\vec{M}_{BA} = M_{BA}\,\hat{u}_{BA}}$$

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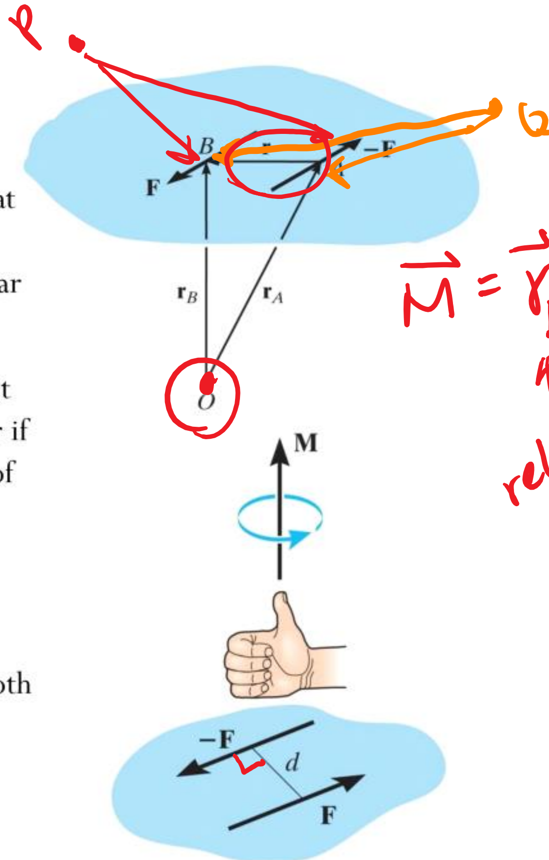
Moment of a couple

A **couple** is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d.

Since the resultant force is zero, the only effect of a couple is to produce an actual rotation, or if no movement is possible, there is a tendency of rotation in a specified direction.

The moment produced by a couple is called **couple moment**.

Let's determine the sum of the moments of both couple forces about **any** arbitrary point:



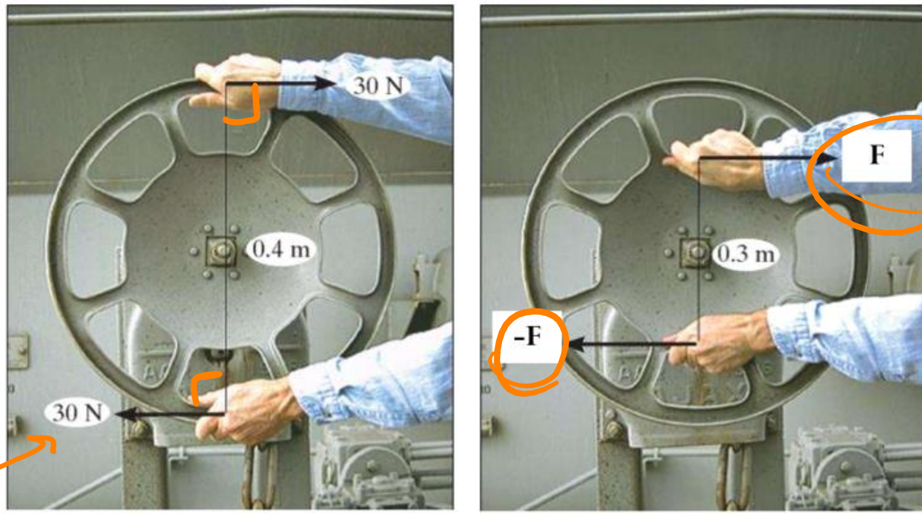
$\vec{M} = \vec{r}_{AB} \times \vec{F}$

relative position vector between the couple.

$M = Fd$

$\vec{M} = \vec{r} \times \vec{F}$

$F_1 = 30N$ F_2



30N

A torque or moment of 12 N·m is required to rotate the wheel. Would F be greater or less than 30 N?

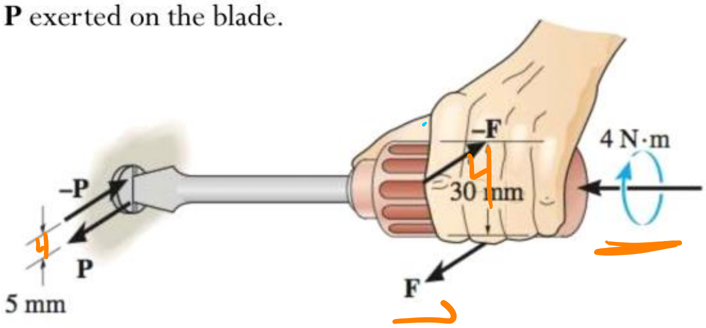
?

$M = Fd$

$12 N\cdot m = (30 N)(0.4 m) = F_2 (0.3 m)$

$F_2 = 40 N > F_1$

A twist of 4 N-m is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces **F** exerted on the handle and **P** exerted on the blade.



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Given: $\vec{M}$

Find: F, P.

Governing Eqn: $M = Fd$.

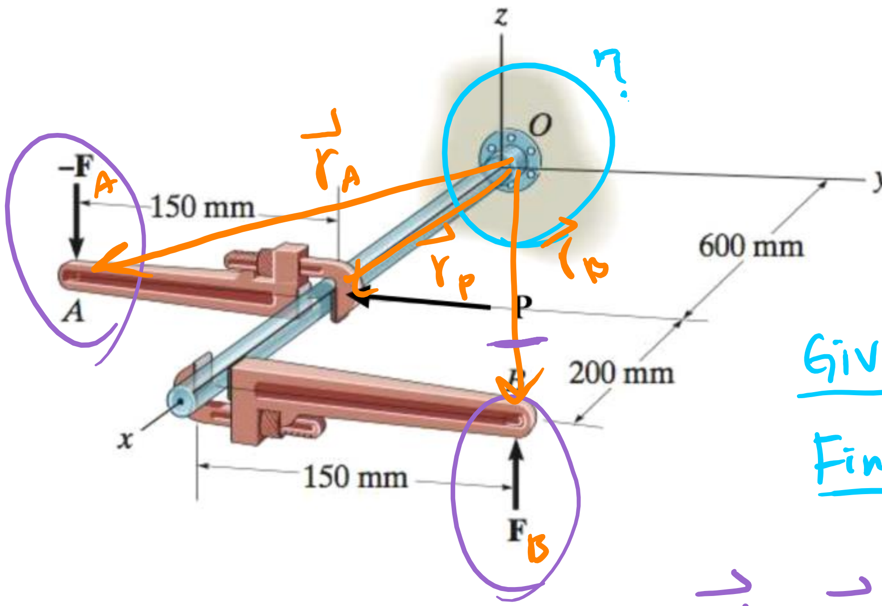
For F

$$M = 4\,N\cdot m = F(30\,mm) \Rightarrow F = \frac{4\,N\cdot \cancel{m}}{30\,\cancel{mm}}\left(\frac{10^3\,\cancel{mm}}{\cancel{m}}\right)$$

For P

$$M = 4\,N\cdot m = P(5\,mm) \Rightarrow P = \frac{4\,N\cdot \cancel{m}}{5\,\cancel{mm}}\left(\frac{10^3\,\cancel{mm}}{\cancel{m}}\right)$$

Find the moment about the support at *O*? F = 125 N, P = 100 N.



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Given: $\vec{F}, -\vec{F}, \vec{P}$

Find: $\vec{M}_O$

$$\vec{M}_O = \vec{r}_A \times \vec{F}_A + \vec{r}_B \times \vec{F}_B + \vec{r}_P \times \vec{P}$$

$$= \underbrace{\vec{r}_P \times \vec{P}}_{\vec{M}_P} + \underbrace{\vec{r}_{AB} \times \vec{F}_B}_{\vec{M}_{AB}}$$

$$\vec{M}_{AB} = \vec{r}_{AB} \times \vec{F}_B$$

$$\vec{r}_{AB} = (200\hat{i} + 300\hat{j})\text{ mm}$$

$$\vec{F}_B = (125\hat{k})\text{ N}$$

$$\vec{M}_{AB} = \overbrace{(200\text{ mm})(125\text{ N})(\underbrace{\hat{i} \times \hat{k}}_{=-\hat{j}})}^{①} + \overbrace{(300\text{ mm})(125\text{ N})(\underbrace{\hat{j} \times \hat{k}}_{=\hat{i}})}^{②}$$

Alternatively,

$$\vec{M}_{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 200 & 300 & 0 \\ 0 & 0 & 125 \end{vmatrix} = (300\text{ mm})(125\text{ N})\hat{i} - (200\text{ mm})(125\text{ N})\hat{j}$$

Same!

Moment about O from P:

$\vec{r}_p = (600\ mm)\hat{i}$ $\qquad$ $\vec{P} = (-100\ N)\hat{j}$

$\vec{M}_p = (600\hat{i} \times -100\hat{j})\ N\cdot mm = -60000\hat{k}\ N\cdot mm.$

Total moment about O:

$\vec{M}_o = \vec{M}_{AB} + \vec{M}_p = (37500\hat{i} - 25000\hat{j} - 66000\hat{k})\ N.mm$

or

$$\boxed{\vec{M}_o = (37.5\hat{i} - 25\hat{j} - 60\hat{k})\ N\cdot m}$$