wc08lect

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Announcements

- Sign up for <u>next week's Quiz 2</u>! (If you haven't already)

❑ Upcoming deadlines:

- Tuesday (9/19)
  - PL HW6
- Thursday (9/21)
  - ME HW7
- Friday (9/22)
  - Writing Assignment 1

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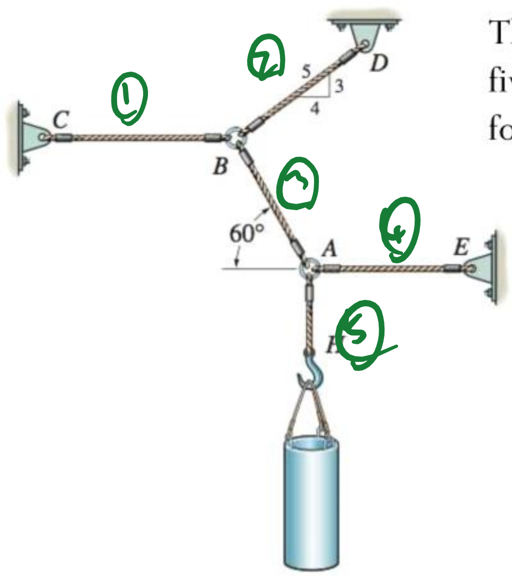
What unknowns?



The 30-kg pipe is supported at *A* by a system of five cords. Determine the force in each cord for equilibrium.

How many unknowns are associated with this problem?

Given: $\hat{u}_1, \hat{u}_2, \hat{u}_3, \hat{u}_4, \hat{u}_5, \vec{W}$

Find: F_1, F_2, F_3, F_4, F_5

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$$\begin{cases} x + y = 2 \\ 2x - 3y = 3 \end{cases}$$

Unknowns: x, y

$\vec{F}$: 2D = 2 unknowns

(F, θ) or (F_x, F_y)

3D = 3 unknowns

$(F, \alpha, \beta, \gamma(\alpha, \beta))$

or

(F_x, F_y, F_z)

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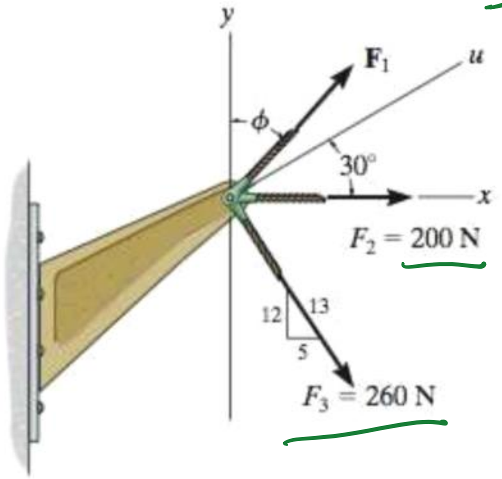
How many unknowns?

If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, find $\mathbf{F}_1$.



How many unknowns are associated with this problem?

A) 1

B) 2

C) 3

D) 4

E) 5

3

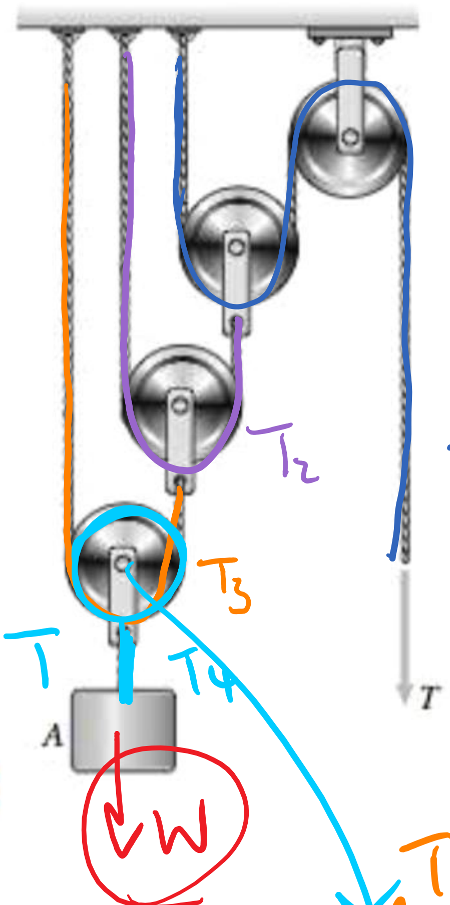
Given: $\vec{F_2}, \vec{F_3}, \vec{F_R}$

Find: $\vec{F_1}$

$$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$$

$$\begin{cases} F_{Rx} = F_{1x} + F_{2x} + F_{3x} \\ F_{Ry} = F_{1y} + F_{2y} + F_{3y} \end{cases}$$

How many unknowns?



The mass of the suspended object *A* is m_A and assume the masses of all pulleys are negligible, determine the force T necessary for the system to be in equilibrium.

Given

How many unknowns are associated with this problem?

A) 1

B) 2

C) 3

D) 4

E) 5

~~3~~ magnitudes.

4 !!!

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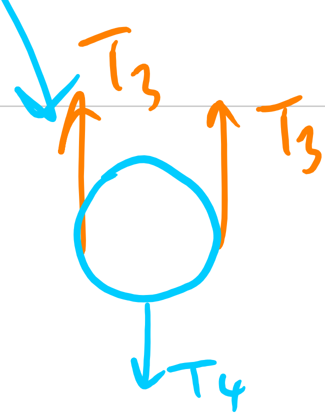
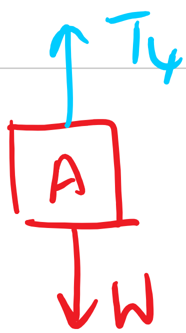
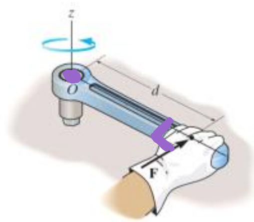




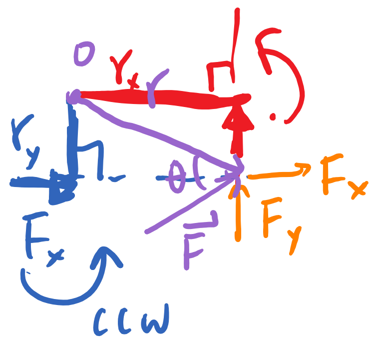
Recap

- Moment of a force
  - Scalar representation



$M = Fd.$

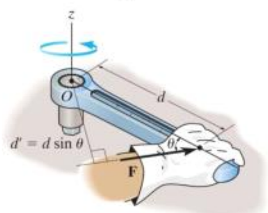
CW: $(-)$
CCW $= (+)$



$M = M_1 + M_2$

$= +F_x r_y + F_y r_x$

  - Vector representation



$\vec{M} = \vec{r} \times \vec{F}$

$M = rF\sin\theta$

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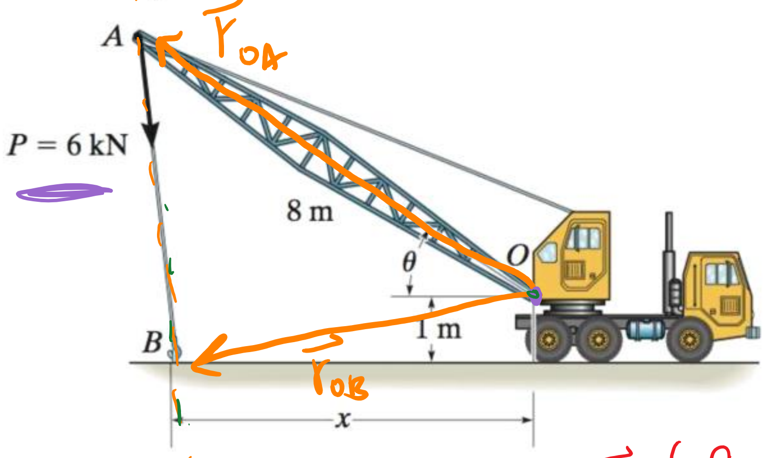
Example – Vector Formulation



Given: The angle $\theta = 30°$ and $x = 10$ m.

Find: The moment by **P** about point O.

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$$\vec{r}_{OA} = (-8\cos\theta\,\hat{i} + 8\sin\theta\,\hat{j})\,\text{m}$$

$$\vec{r}_{OB} = (-10\,\hat{i} - \hat{j})\,\text{m}$$

$$\vec{P} = P\,\hat{u}_{AB}$$

$$\hat{u}_{AB} = \frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_{AB}|} = \frac{(-10 + 8\cos\theta)\,\hat{i} - (-1 - 8\sin\theta)\,\hat{j}}{r_{AB}} = \frac{(-3.07\,\hat{i} - 5\,\hat{j})\,\text{m}}{5.87\,\text{m}}$$

$$\vec{M}_O = \vec{r} \times \vec{P} = \begin{cases} \vec{r}_{OA} \times \vec{P} \\ \vec{r}_{OB} \times \vec{P} \end{cases}$$

$$\vec{r}_{OA} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_{OAx} & r_{OAy} & 0 \\ P_x & P_y & 0 \end{vmatrix}$$

$$= (r_{OAx} P_y - r_{OAy} P_x)\,\hat{k}$$

$$= (r_{OAx} P u_{ABy} - r_{OAy} P u_{ABx})\,\hat{k}$$

$$\Rightarrow \boxed{M_O \approx 48.0\ \text{kN}\cdot\text{m}}$$

$$\vec{r}_{OB} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_{OBx} & r_{OBy} & 0 \\ P_x & P_y & 0 \end{vmatrix}$$

$$= (r_{OBx} P_y - r_{OBy} P_x)\,\hat{k}$$

$$= r_{OBx} P u_{ABy} - r_{OBy} P u_{ABx}$$

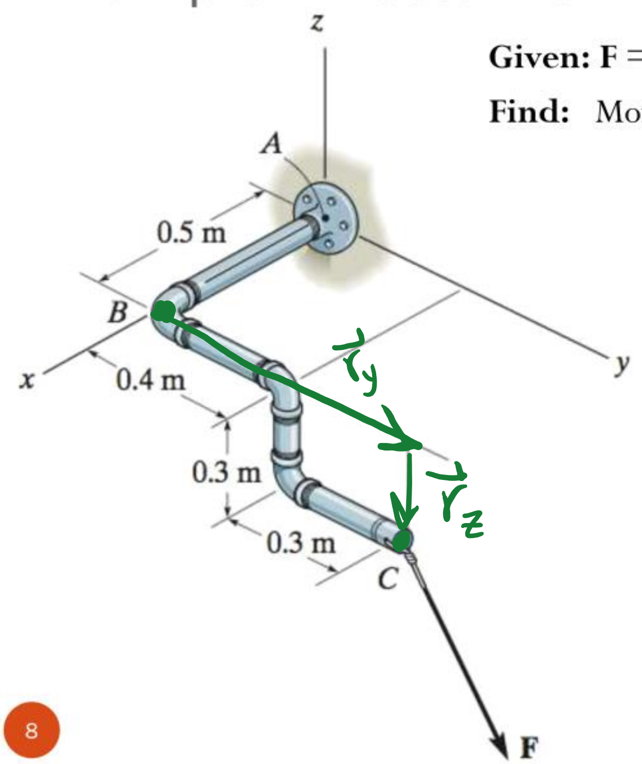
$$\Rightarrow \boxed{M_O \approx 48.0\ \text{kN}\cdot\text{m}}$$

Same!

Example – Vector Formulation

Given: $\mathbf{F} = \{600\mathbf{i} + 800\mathbf{j} - 500\mathbf{k}\}$ N

Find: Moment of the force about point B.



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$$\vec{M}_B = \vec{r}_{BC} \times \vec{F}$$

$$\vec{r}_{BC} = 0.7\,\text{m}\,\hat{j} - 0.3\,\text{m}\,\hat{k}$$

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.7 & -0.3 \\ 600 & 800 & -500 \end{vmatrix}$$

$$= \left[(0.7\,\text{m})(-500\,\text{N}) - (800\,\text{N})(-0.3\,\text{m})\right]\hat{i}$$

$$- \left[(0\,\text{m})(-500\,\text{N}) - (600\,\text{N})(-0.3\,\text{m})\right]\hat{j}$$

$$+ \left[(0\,\text{m})(800\,\text{N}) - (600\,\text{N})(0.7\,\text{m})\right]\hat{k}$$

$$\Rightarrow \boxed{\vec{M} = (-110\hat{i} - 180\hat{j} - 420\hat{k})\,\text{N}\cdot\text{m}}$$

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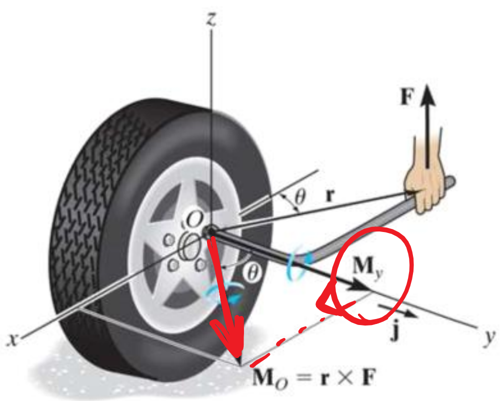
Moment about a Specific Axis

Remember, the component of a vector, **A**, along the direction of another, **B**, can be determined using the dot product:



$$\text{proj}(\vec{A}, \vec{B}) = (\vec{A} \cdot \hat{u}_B)\hat{u}_B.$$

$$\Rightarrow \text{proj}(\vec{M}_o, \hat{u})$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$\vec{M}_y = \vec{M} \cdot \hat{j} = \underbrace{\hat{j} \cdot \overbrace{(\vec{r} \times \vec{F})}^{\text{vector}}\, \hat{j}}_{\text{triple scalar product.}}$$

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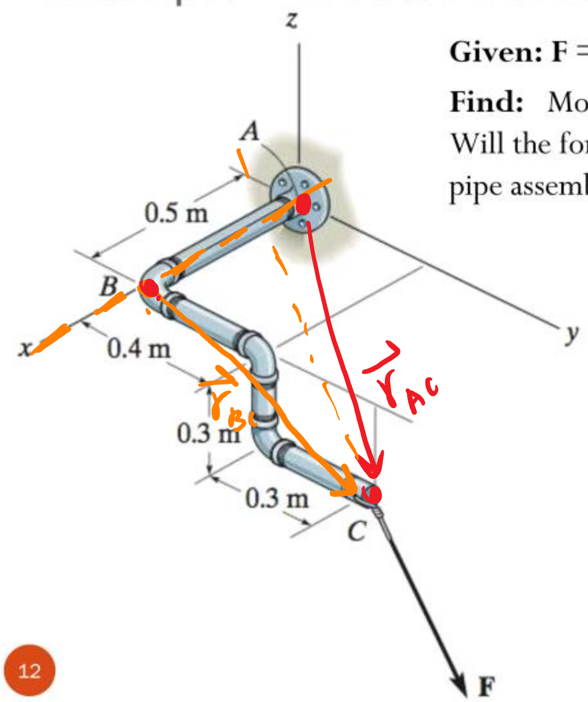
Example – Vector Formulation



Given: **F** = {600**i** + 800**j** - 500**k**} N

Find: Moment of the force about the *x*-axis. Will the force be tightening or loosening the pipe assembly at *A*?

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$$\vec{M}_x = [\hat{u}_x \cdot (\vec{r} \times \vec{F})]\,\hat{u}_x$$

$$\hat{u}_x = \hat{i}$$

$\vec{r}_{AC}$ or $\vec{r}_{BC}$ → both work!

$$M_x = \hat{i} \cdot (\vec{r} \times \vec{F})$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= r_y F_z - F_y r_z$$

$$(r_{AC}) = (0.5\hat{i} + 0.7\hat{j} - 0.3\hat{k})\,m$$

$$= (0.7m)(-500N) - (800N)(-0.3m)$$

$$M_x = -110N \Rightarrow \boxed{\vec{M}_x = (-110N)\,\hat{i}}$$

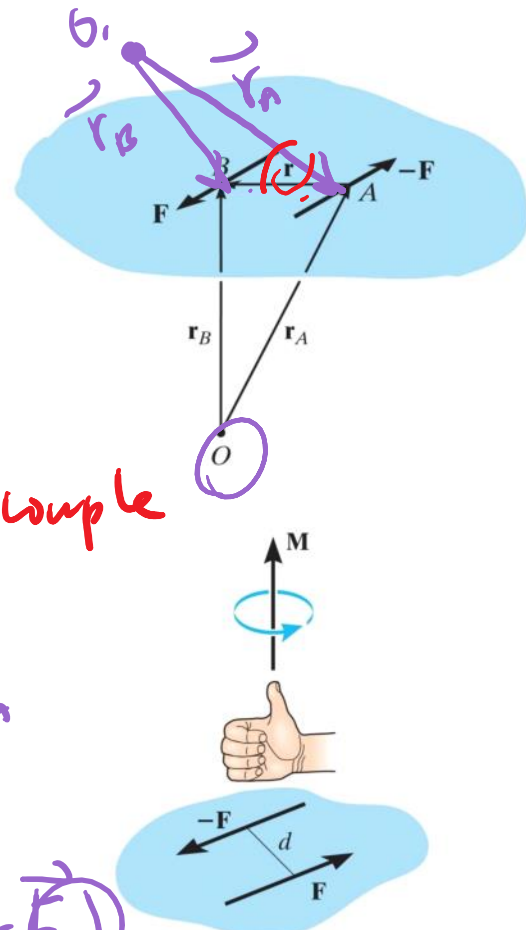
$$(r_{BC}) = (0.7\hat{j} - 0.3\hat{k})\,m$$

$$= (0.7m)(-500N) - (800N)(-0.3m)$$

$$M_x = -110\,N \Rightarrow \boxed{\vec{M}_x = (-110N)\,\hat{i}}$$

same!

Moment of a couple



$$\vec{M} = \vec{r} \times \vec{F}$$

$\vec{r}$: position vector between the couple

$$\vec{M} = \vec{r}_B \times \vec{F}_B + \vec{r}_A \times \vec{F}_A$$

$$\vec{F}_A = -\vec{F}_B$$

$$= \vec{r}_B \times \vec{F}_B + \vec{r}_A \times (-\vec{F}_B)$$

$$= (\vec{r}_B - \vec{r}_A) \times \vec{F}_B = \vec{r}_{AB} \times \vec{F}_B$$

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