wc07lect

Wednesday, September 13, 2017 10:26 AM

wc07lect

Announcements

- Morning Office Hours: Mon/Wed, 9–10am in MEB 220H
- Quiz 2 sign-ups are now open
  - The scope of the exam will cover up to the end of today's lecture (Lecture 7)
  - Same format as Quiz 1

❑ Upcoming deadlines:

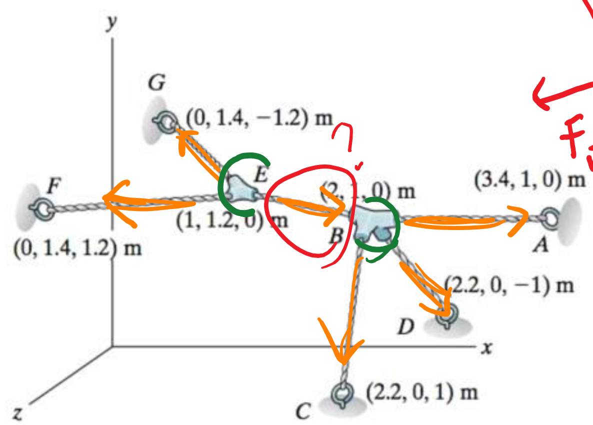
- Thursday (9/14)
  - ME HW5
- Tuesday (9/19)
  - PL HW6
- Due next week
  - Writing Assignment 1

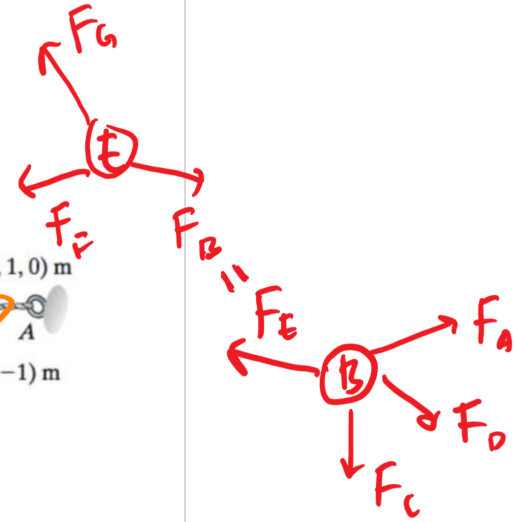
1

Recap

- Equilibrium of a particle in 2D and 3D
- Equilibrium of a system of particles
- Free body diagram
- Equation of equilibrium

2D: $\Sigma F_x = 0$
$\Sigma F_y = 0$



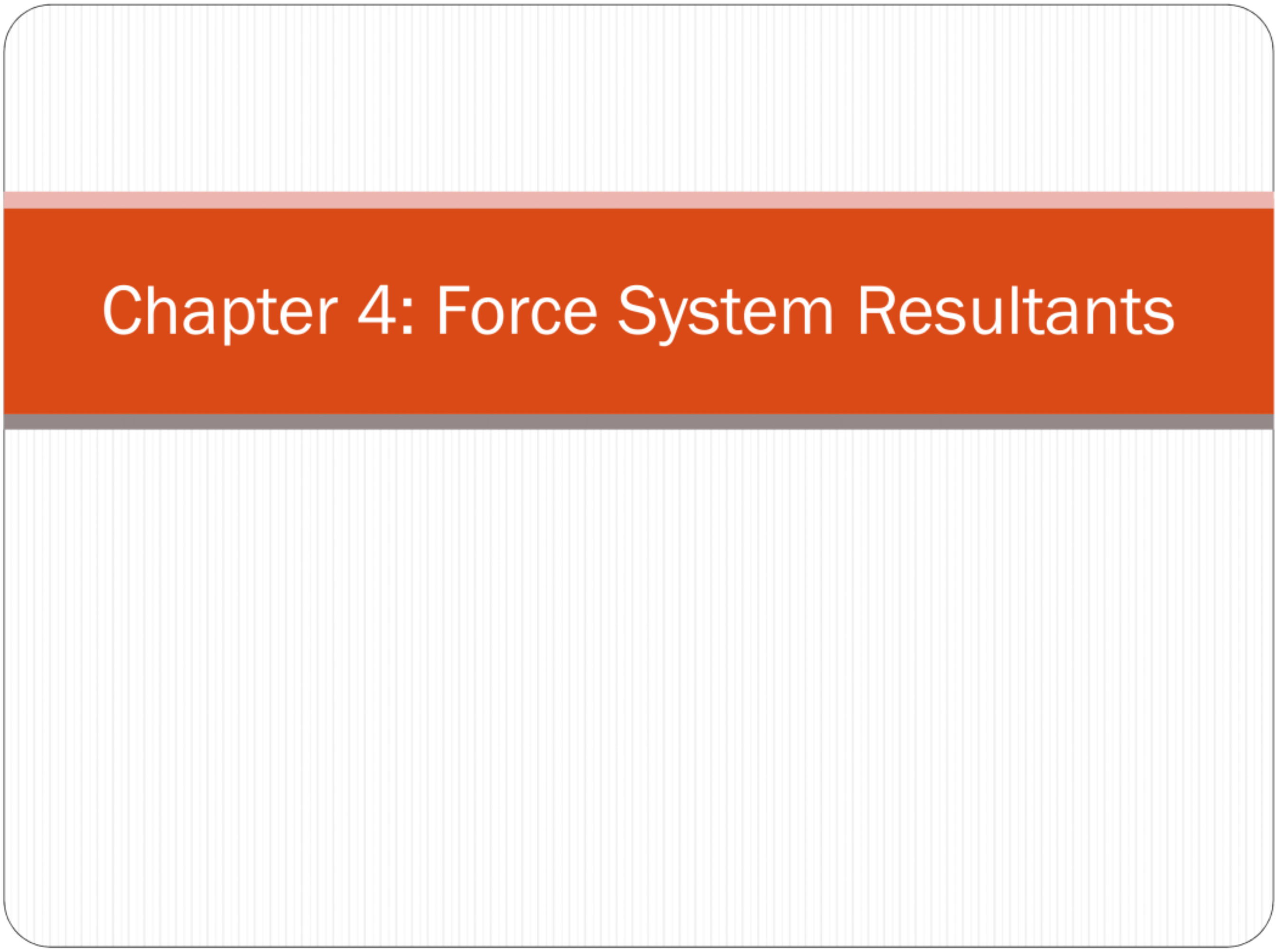


2

3D: $\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma F_z = 0$

Page 5

Wednesday, September 13, 2017 10:26 AM

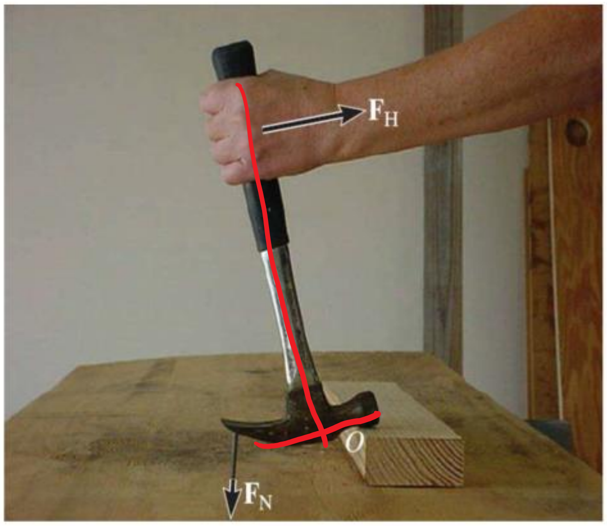


Goals and Objectives

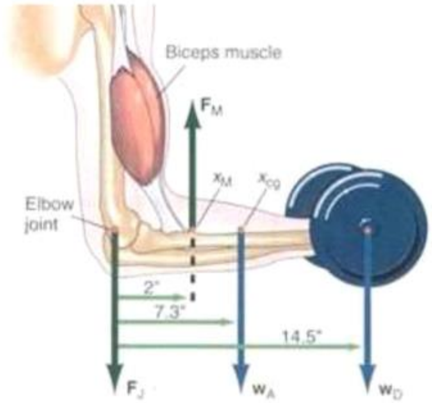
- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- How to find the moment about a specified axis
- Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading

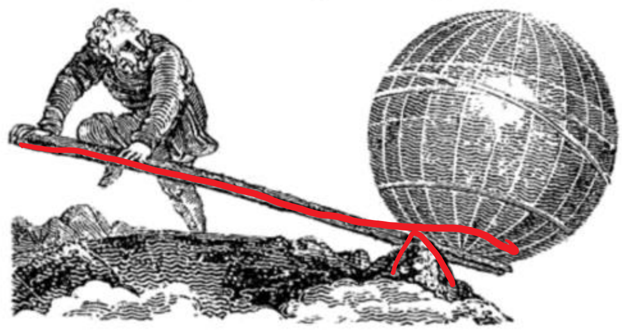
6

Applications



Carpenters often use a hammer in this way to pull a stubborn nail. Through what sort of action does the force F_H at the handle pull the nail? How can you mathematically model the effect of force F_H at point O?



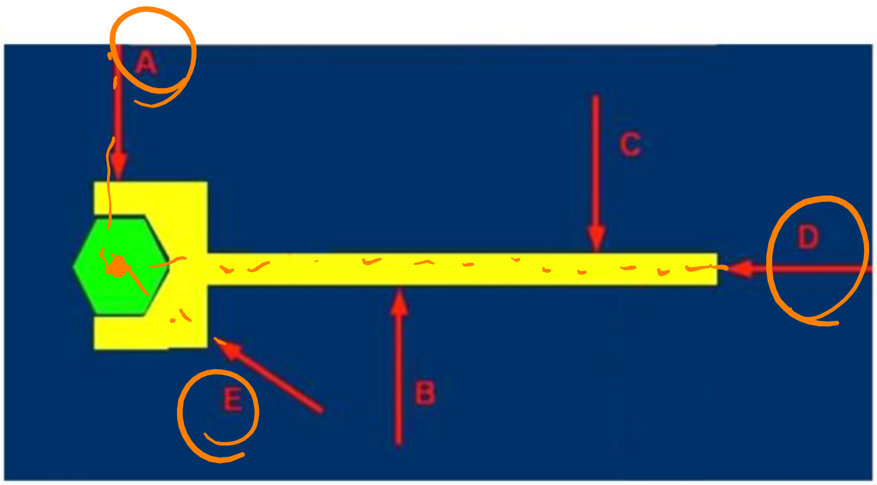


Moment 1.a very brief period of time. An Exact point in time. 2. importance. 3. A turning Effect produced by a force acting at a distance on An object.

Page 9

Wednesday, September 13, 2017 10:26 AM

Moment of a Force



Which force(s) have NO turning effect?

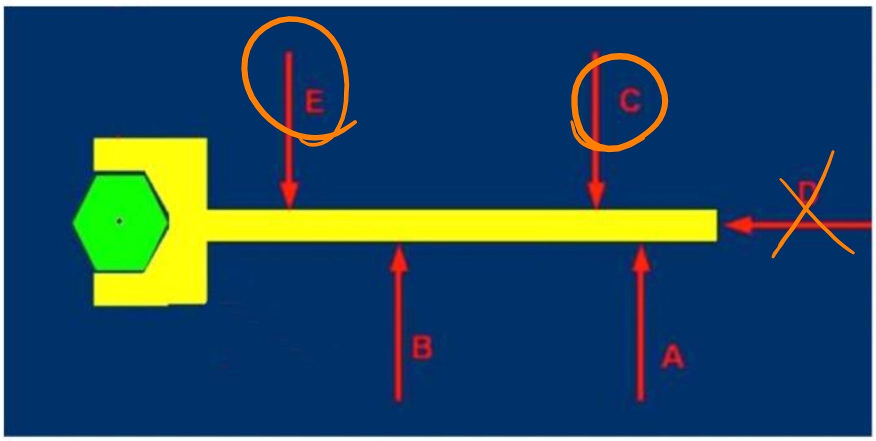
A, D, E

9

Page 10

Wednesday, September 13, 2017 10:26 AM

Moment of a Force



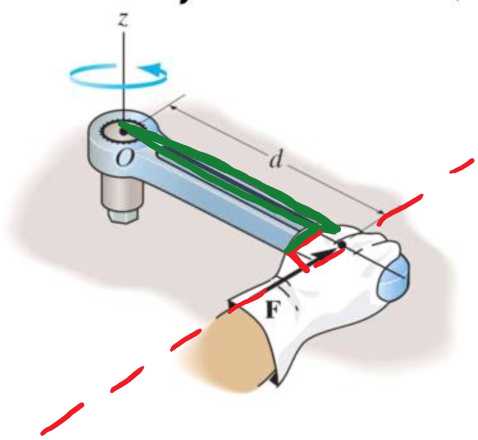
1) Which force(s) yields a "tighty" effect? C, E

~~1~~2) Which force(s) yields a "loosey" effect? A, B

10

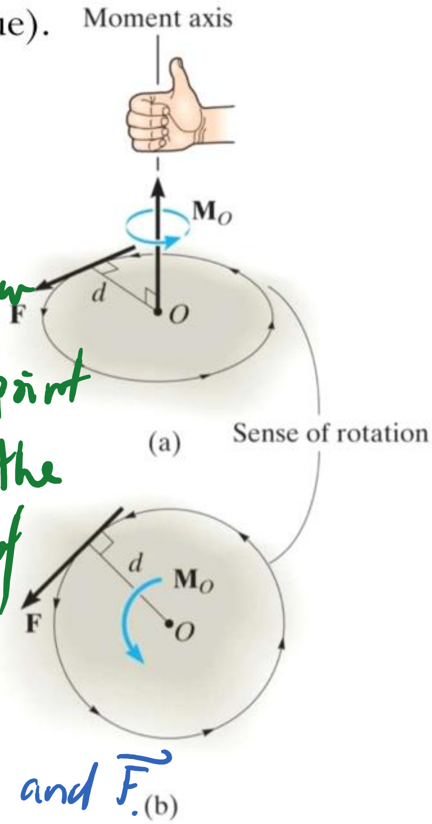
Moment of a force – scalar formulation

The **moment of a force about a point** provides a measure of the **tendency for rotation** (sometimes called a torque).



$M = Fd$

d: perpendicular distance from point of reference to the line of action of force.

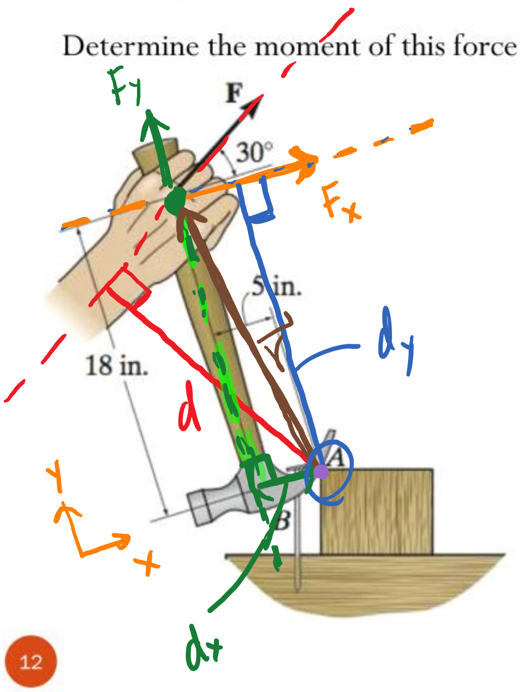


Direction of $\vec{M}$: $\perp$ to both $\vec{d}$ and $\vec{F}$.

11

Example – Scalar Formulation

Determine the moment of this force about the point A as a function of **F**.



12

$M_A = Fd$

$\vec{r} = d_x\hat{i} + d_y\hat{j}$

$= -5\hat{i} + 18\hat{j}$ in.

$M_{Ax} = F_x d_y = -F\cos 30°(18)$

F_x d_y CW: (−)

$M_{Ay} = F_y d_x = -F\sin 30°(5)$

F_y d_x CW: (−)

CCW: (+)

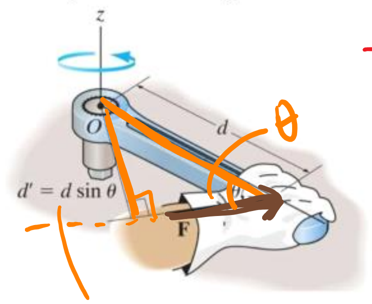
CW: (−)

$M_A = M_{Ax} + M_{Ay} = -F\cos 30°(18) - F\sin 30°(5)$

Wednesday, September 13, 2017 10:26 AM

Moment of a force – vector formulation

The moment of a force **F** about point **O**, or actually about the moment axis passing through O and perpendicular to the plane containing **O** and **F**, can be expressed using the cross (vector) product, namely:



$\vec{r}$: position vector from reference point to line of action of force $\vec{F}$.

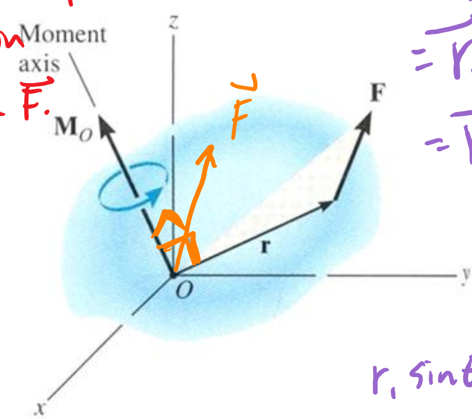
$$\boxed{\vec{M} = \vec{r} \times \vec{F}}$$

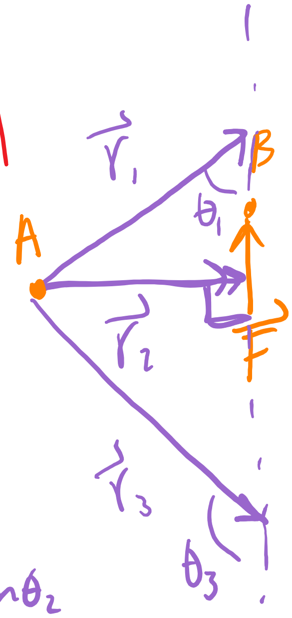
$$= \vec{r}_1 \times \vec{F}$$

$$= \vec{r}_2 \times \vec{F}$$

$$= \vec{r}_3 \times \vec{F}$$

$\vdots$





$d_\perp = d\sin\theta$

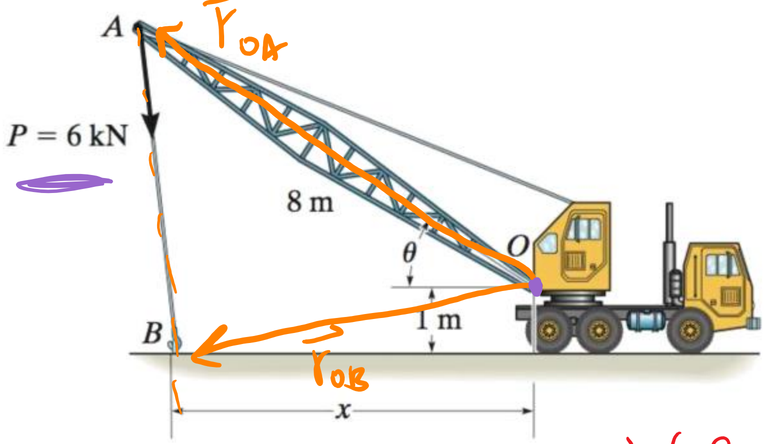
$M = F d \sin\theta = |\vec{r} \times \vec{F}|$

$r_1 \sin\theta_1 = r_2 \sin\theta_2$

$= r_3 \sin\theta_3 = \dots$

13

Example – Vector Formulation



Given: The angle $\theta = 30°$ and $x = 10$ m.

Find: The moment by **P** about point O.

15

$$\vec{M}_O = \vec{r}\times\vec{P} = \begin{cases}\vec{r}_{OA}\times\vec{P}\\ \vec{r}_{OB}\times\vec{P}\end{cases}$$

$$\vec{r}_{OA} = (-8\cos\theta\,\hat{i} + 8\sin\theta\,\hat{j})\,m$$

$$\vec{r}_{OB} = (-10\,\hat{i} - \hat{j})\,m$$

$$\vec{P} = P\hat{u}_{AB}$$

$$\hat{u}_{AB} = \frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_{AB}|} = \frac{(-10+8\cos\theta)\hat{i} - (-1-8\sin\theta)\hat{j}}{r_{AB}} = \frac{(-3.07\hat{i} - 5\hat{j})\,m}{5.87\,m}$$

$$\vec{r}_{OA}\times\vec{P} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ r_{OAx} & r_{OAy} & 0\\ P_x & P_y & 0\end{vmatrix}$$

$$= (r_{OAx}P_y - r_{OAy}P_x)\hat{k}$$

$$= (r_{OAx}Pu_{ABy} - r_{OAy}Pu_{ABx})\hat{k}$$

$$\Rightarrow \boxed{M_O \approx 48.0\ kN\cdot m}$$

$$\vec{r}_{OB}\times\vec{P} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ r_{OBx} & r_{OBy} & 0\\ P_x & P_y & 0\end{vmatrix}$$

$$= (r_{OBx}P_y - r_{OBy}P_x)\hat{k}$$

$$= r_{OBx}Pu_{ABy} - r_{OBy}Pu_{ABx}$$

$$\Rightarrow \boxed{M_O \approx 48.0\ kN\cdot m}$$

Same!