wc04lect

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wc04lect

Announcements

Get out your iClicker!

- Sign-up for Quiz 1
  - Practice quiz available on PL
  - Be familiar with MATLAB commands

- Upcoming deadlines:
- Thursday (9/7)
  - Mastering Engineering HW3
- Friday (9/8)
  - Quiz 1 sign up
- Tuesday (9/12)
  - PL HW4

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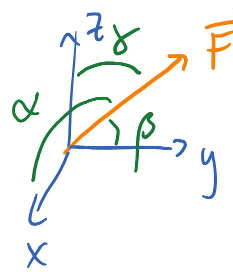
Recap

- A force can be treated as a vector, since forces obey all the rules that vectors do.
- Vector representations
  - Rectangular components
  - Cartesian vectors
  - Unit vector
  - Direction cosines
- Position vectors

$$\vec{F} = F\hat{u} = \vec{F}_x + \vec{F}_y + \vec{F}_z = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\hat{u} = \frac{\vec{r}}{r} \text{ (unitless)} = \frac{F_x}{F}\hat{i} + \frac{F_y}{F}\hat{j} + \frac{F_z}{F}\hat{k} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$$



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L3 - Force Vectors

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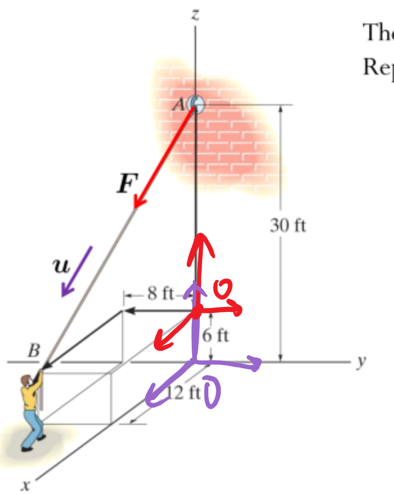
Force vector directed along a line



The man pulls on the cord with a force of 70 lb. Represent the force F as a Cartesian vector.

$$\vec{r}_{AB} = \vec{r}_{OB} - \vec{r}_{OA}$$

A: (0, 0, 24) B: (12, -8, 0)

$$\vec{r}_{BA} = (12-0)\hat{i} + (-8-0)\hat{j} + (0-24)\hat{k}$$

A: (0, 0, 30) B: (12, -8, 6)

$$\vec{r}_{BA} = (12-0)\hat{i} + (-8-6)\hat{j} + (6-30)\hat{k}$$

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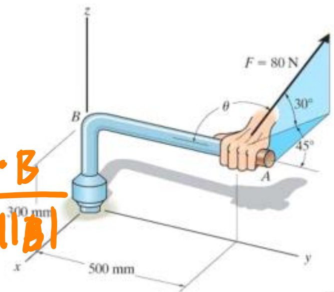
Dot (or scalar) product

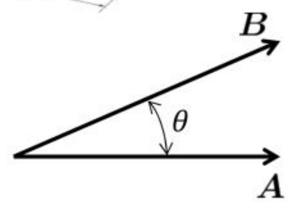


The dot product of vectors **A** and **B** is defined as such

$$A \cdot B = |A||B|\cos\theta \Rightarrow \cos\theta = \frac{A \cdot B}{|A||B|}$$

Cartesian vector formulation:

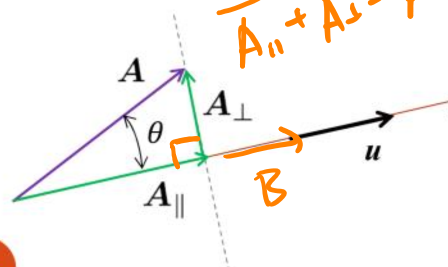
$$\boldsymbol{A} \cdot \boldsymbol{B} = A_x B_x + A_y B_y + A_z B_z$$



Projections

The scalar component $A_\parallel$ of a vector $\boldsymbol{A}$ along (parallel to) a line with unit vector $\boldsymbol{u}$ is given by:

$$\vec{A}_\parallel + \vec{A}_\perp = \vec{A}$$

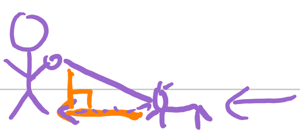


$$A_\parallel = A\cos\theta = |A|\left(\frac{A \cdot B}{|\cancel{A}||B|}\right)$$

And thus the vector components $\boldsymbol{A}_\parallel$ and $\boldsymbol{A}_\perp$ are given by:

$$\vec{\boldsymbol{A}}_\parallel = \frac{A_x B_x + A_y B_y + A_z B_z}{|B|} \quad \hat{u}_B = \frac{\vec{A} \cdot \vec{B}}{|B|} \cdot \underbrace{\frac{\vec{B}}{|B|}}_{\hat{u}_B}$$

$$\boldsymbol{A}_\perp = \vec{A} - \vec{A}_\parallel$$



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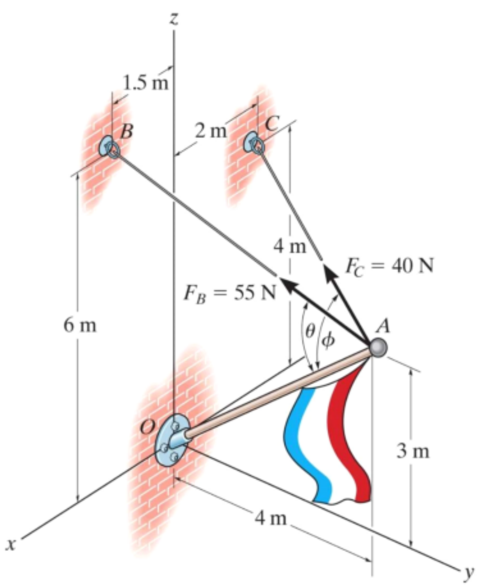
Example

a) Determine the angle between *AB* and the flag pole.



$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}$$

A: (0, 4, 3) m

B: (1.5, 0, 6) m

$$\vec{r}_{AO} = (4\hat{j} - 3\hat{k})\ \text{m}$$

$$\vec{r}_{AB} = (1.5\hat{i} - 4\hat{j} + 3\hat{k})$$

$$\theta = \cos^{-1}\left(\frac{0(1.5) + 4(-4) + (-3)(3)}{\sqrt{4^2+3^2}\cdot\sqrt{1.5^2+4^2+3^2}}\right)$$

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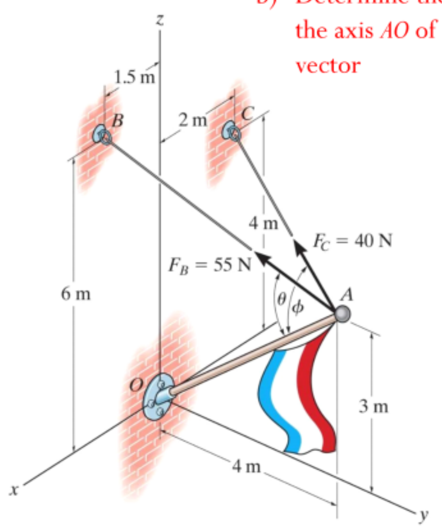
L3 - Force Vectors

Example

a) Determine the angle between *AB* and the flag pole.

b) Determine the projected component of the force vector $\mathbf{F}_B$ along the axis *AO* of the flag pole. Express your result as a Cartesian vector



$$\text{proj}(\vec{A},\vec{B}) = \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|}\hat{u}_B$$

$$\vec{A} = \vec{F}_B \qquad \vec{B} = \vec{r}_{AO}$$

$$\text{proj}(\vec{F}_B,\vec{r}_{AO}) = \underbrace{\left(\frac{F_{Bx}r_{AOx} + F_{By}r_{AOy} + F_{Bz}r_{AOz}}{\sqrt{r_{AOx}^2 + r_{AOy}^2 + r_{AOz}^2}}\right)}_{A\cos\theta}\hat{u}_{AO}.$$

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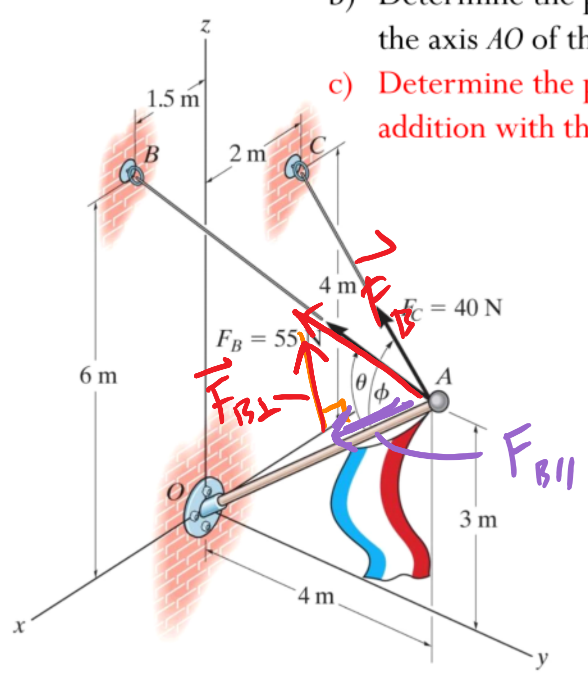
Example

a) Determine the angle between *AB* and the flag pole.

b) Determine the projected component of the force vector $\boldsymbol{F}_C$ along the axis *AO* of the flag pole.

c) Determine the perpendicular component such at its vector addition with the projected component equals $\boldsymbol{F}_C$.

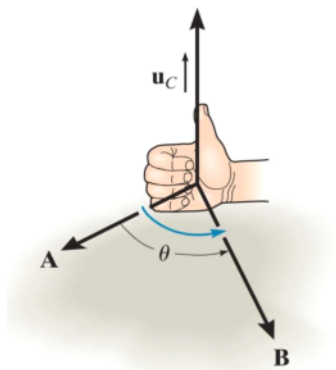


$$\vec{F}_{B\perp} = \vec{F}_B - \vec{F}_{B\parallel}$$

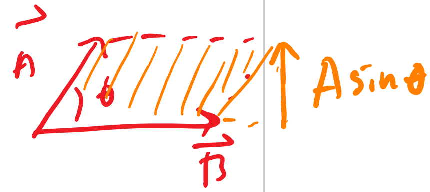
part B

Cross (or vector) product

The cross product of vectors **A** and **B** yields the vector **C**, which is written



$$\boxed{\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}}$$



The magnitude of vector **C** is given by:

$C = |A||B|\sin\theta$

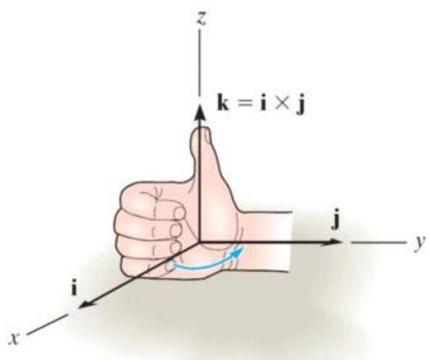
The vector **C** is perpendicular to the plane containing **A** and **B** (specified by the **right-hand rule**). Hence,

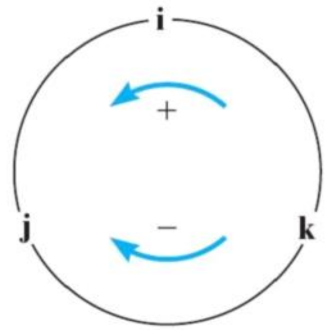
$\vec{C} \perp \vec{A}$

$\vec{C} \perp \vec{B}$

Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $i \times i = 0$





Considering the cross product in Cartesian coordinates

$$\boldsymbol{A} \times \boldsymbol{B} = (A_x\,\boldsymbol{i} + A_y\,\boldsymbol{j} + A_z\,\boldsymbol{k}) \times (B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k})$$

$\hat{i} \times \hat{j} = +\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$-\hat{k} \times \hat{i} = -\hat{j}$

$\hat{j} \times -\hat{i} = +\hat{k}$

Cross (or vector) product

Also, the cross product can be written as a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Each component can be determined using 2 × 2 determinants.

$$+(A_y B_z) - (B_y A_z)\hat{i} - [A_x B_z - B_x A_z]\hat{j} + (A_x B_y - B_x A_y)\hat{k}$$