Announcements

- MATLAB Clinics – Today (Fri) 9am – 5pm
- Upcoming deadlines:
- Friday (9/1)
  - Prairie Learn HW0
- Sunday (9/3)
  - Mastering Engineering HW1
- Tuesday (9/5)
  - Prairie Learn HW2
- Thursday (9/7)
  - Mastering Engineering HW3

Recap

- Pay attention to units!
- Solve problem symbolically
- Equations must be dimensionally homogenous
- 1% accuracy
- Scalar – defined by magnitude (negative/positive)
- Vector – defined by magnitude and direction
- Vector operations – addition/subtraction

Force vectors

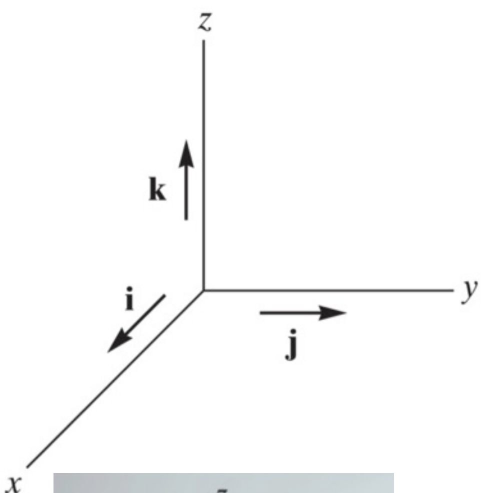
A force—the action of one body on another—can be treated as a vector, since forces obey all the rules that vectors do.

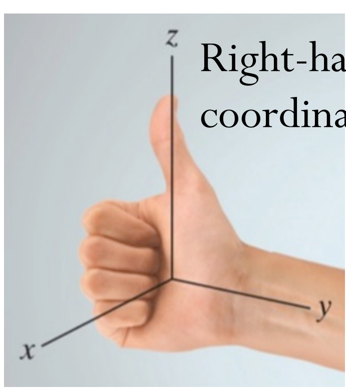
Cartesian vectors

Rectangular coordinate system: formed by 3 mutually perpendicular axes, the *x, y, z* axes, with unit vectors $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ in these directions.

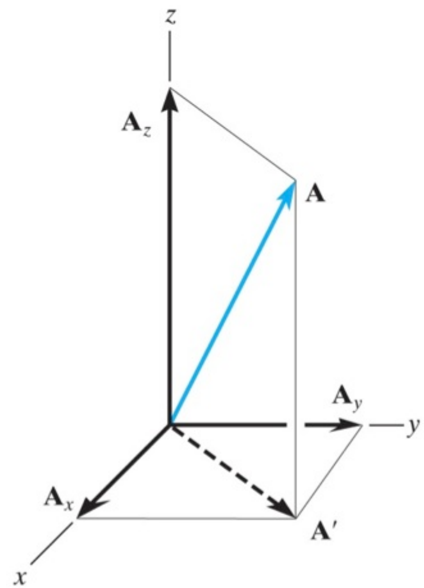
Note that we use the special notation "^" to identify *basis vectors* (instead of the "~" or "→" notation)

$(\hat{\imath},\, \hat{\jmath},\, \hat{k})$ or $(\boldsymbol{i},\, \boldsymbol{j},\, \boldsymbol{k})$



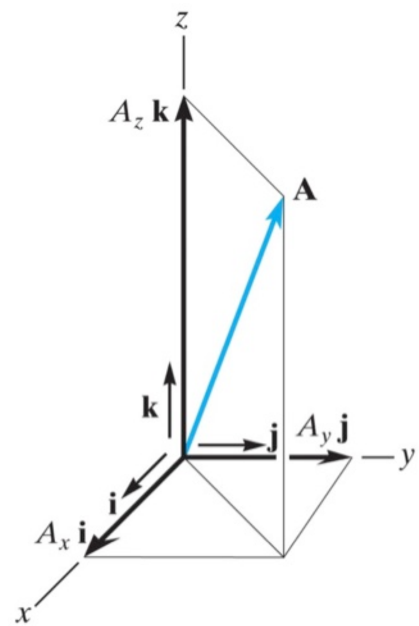


Right-handed coordinate system



Rectangular components of a vector

$\boldsymbol{A} =$

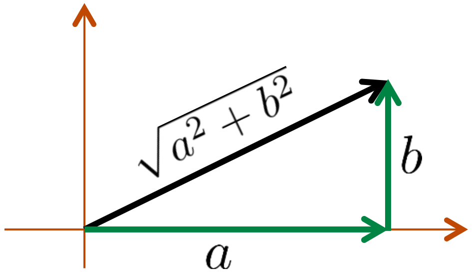


Cartesian vector representation

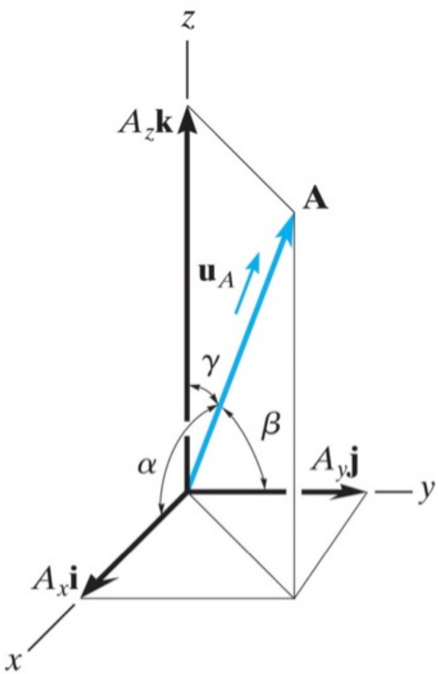
$\boldsymbol{A} =$

Magnitude of Cartesian vectors

$$A = |\boldsymbol{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



Direction of Cartesian vectors



Expressing the direction using a unit vector:

$$\boldsymbol{u}_A = \frac{\boldsymbol{A}}{A}$$

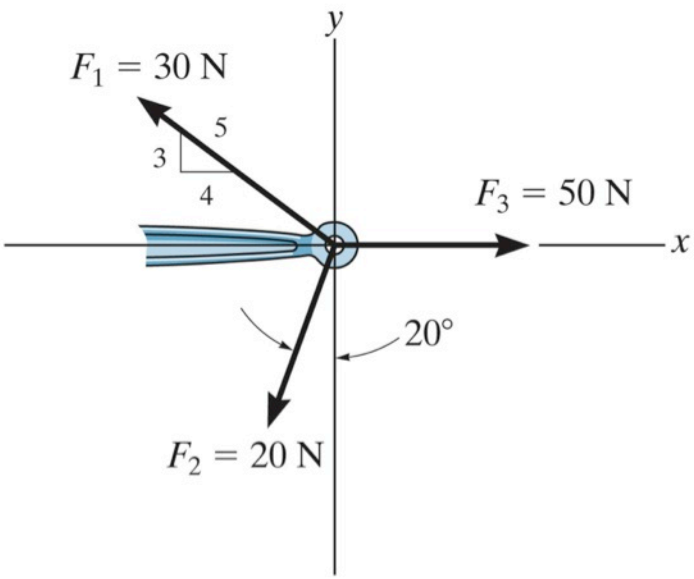
Direction cosines are the components of the unit vector:

Addition of Cartesian vectors

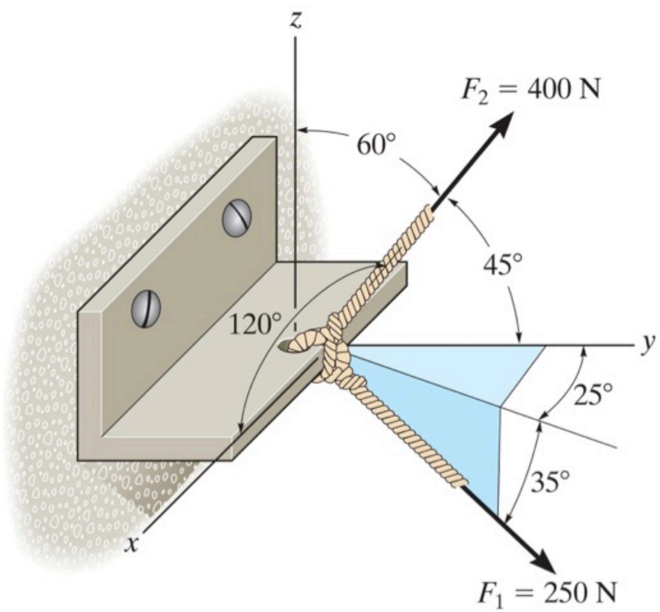
$$\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B} =$$

Example

Express each force vector using the Cartesian vector form (components form).



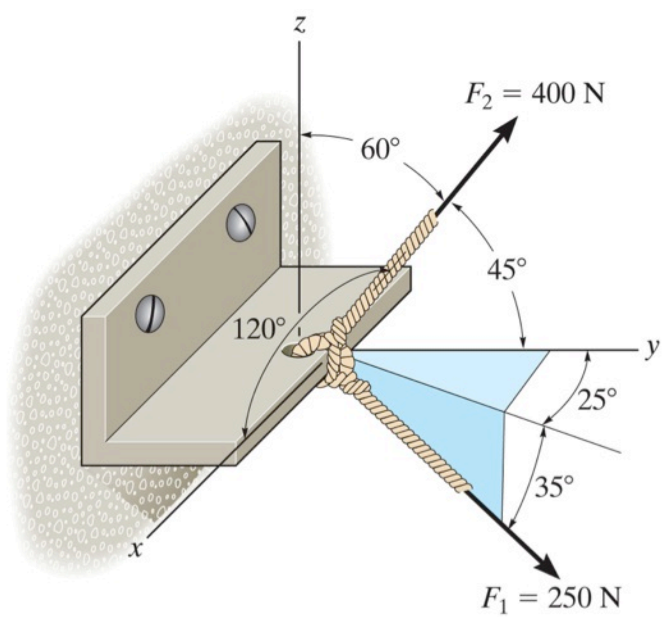
Example



The bracket is subjected to the two forces on the ropes.

(a) Express each force vector using the Cartesian vector form (components form).

Example

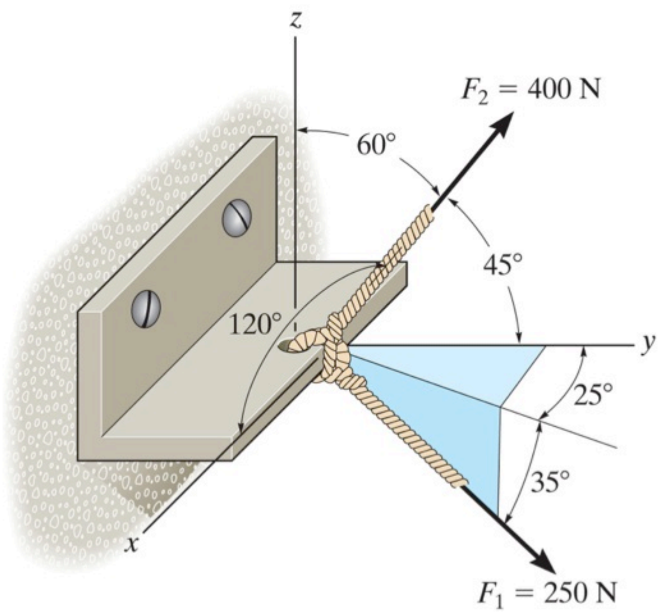


The bracket is subjected to the two forces on the ropes.

(a) Express each force vector using the Cartesian vector form (components form).

(b) Determine the magnitude of the resultant force vector

Example



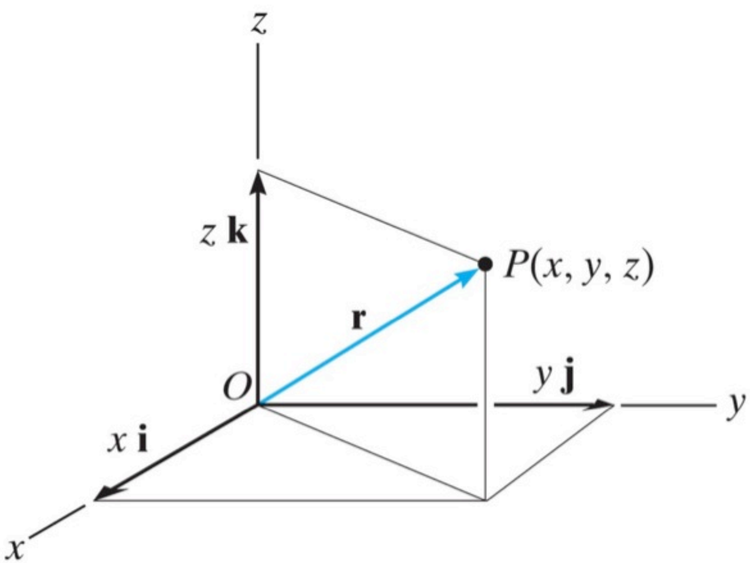
The bracket is subjected to the two forces on the ropes.

(a) Express each force vector using the Cartesian vector form (components form).

(b) Determine the magnitude of the resultant force vector

(c) Determine the direction cosines of the resultant force vector

Position vectors



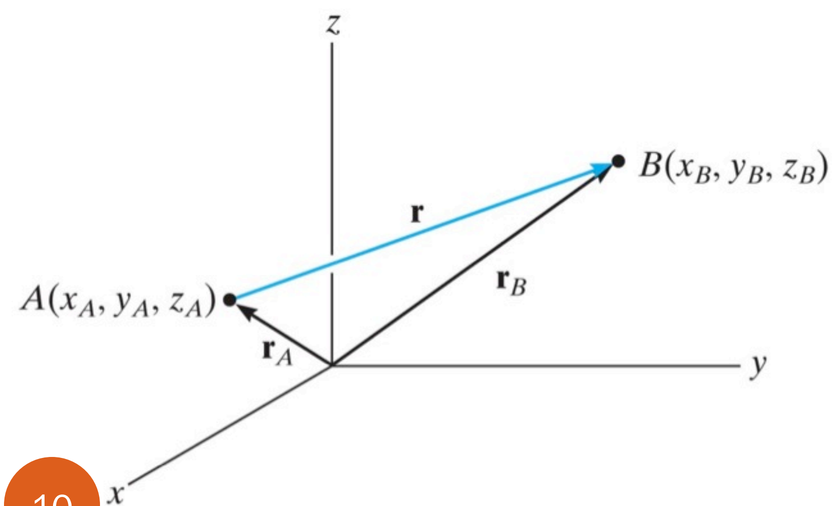
A position vector $\boldsymbol{r}$ is defined as a fixed vector which locates a point in space relative to another point. For example,

$$\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$$

expresses the position of point $P(x,y,z)$ with respect to the origin O.

The position vector $\boldsymbol{r}$ of point $\boldsymbol{B}$ with respect to point $\boldsymbol{A}$ is obtained from

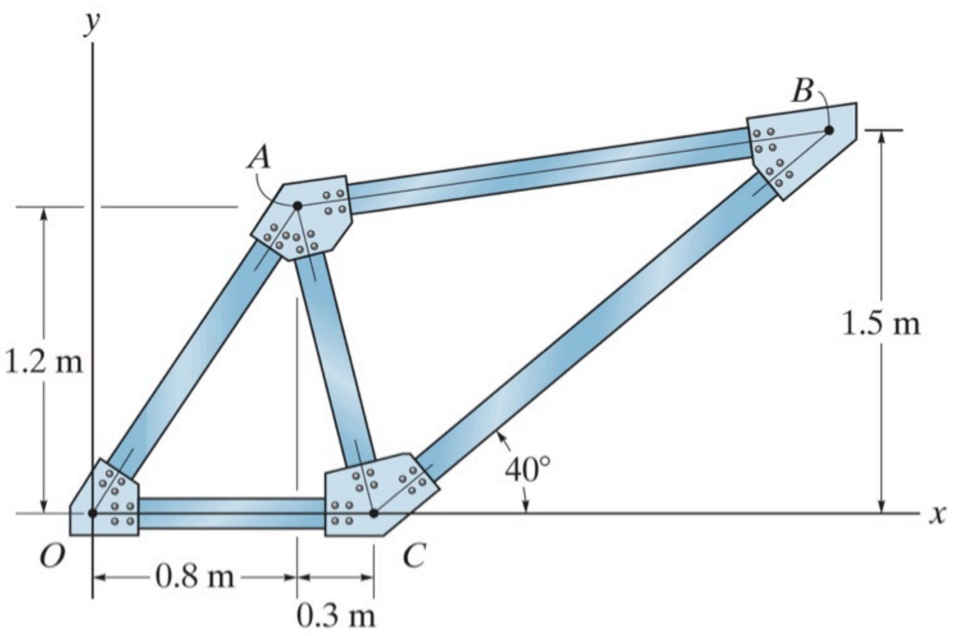
Hence,



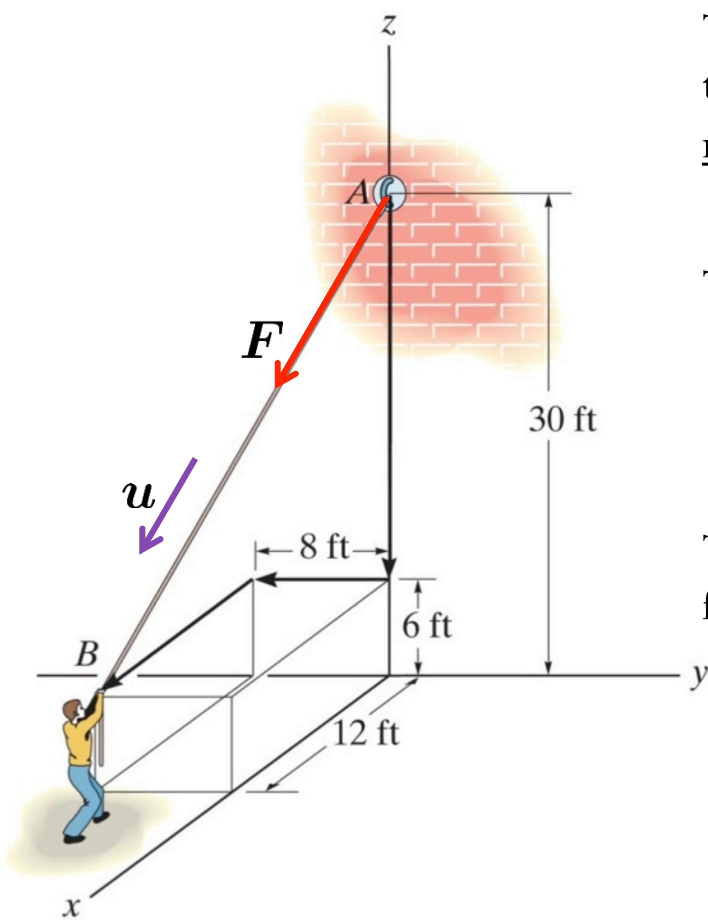
Thus, the $(\boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k})$ components of the positon vector $\boldsymbol{r}$ may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).

Example

Determine the lengths of bars AB, BC and AC.



Force vector directed along a line



The force vector $\boldsymbol{F}$ acting a long the rope can be defined by the unit vector $\boldsymbol{u}$ (defined the <u>direction</u> of the rope) and the <u>magnitude</u> of the force.

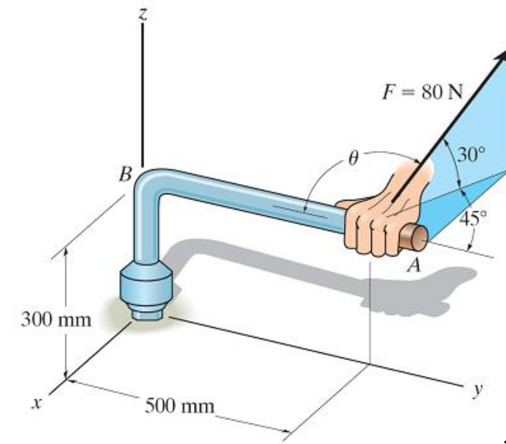
$$\boldsymbol{F} = F\,\boldsymbol{u}$$

The unit vector $\boldsymbol{u}$ is specified by the position vector:

The man pulls on the cord with a force of 70 lb. Represent the force F as a Cartesian vector.

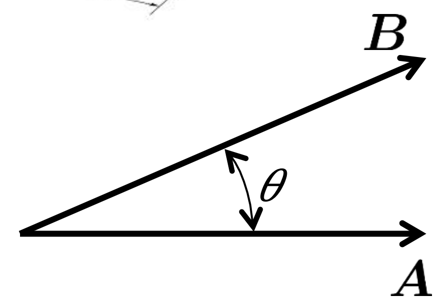
Dot (or scalar) product

The dot product of vectors **A** and **B** is defined as such



Cartesian vector formulation:

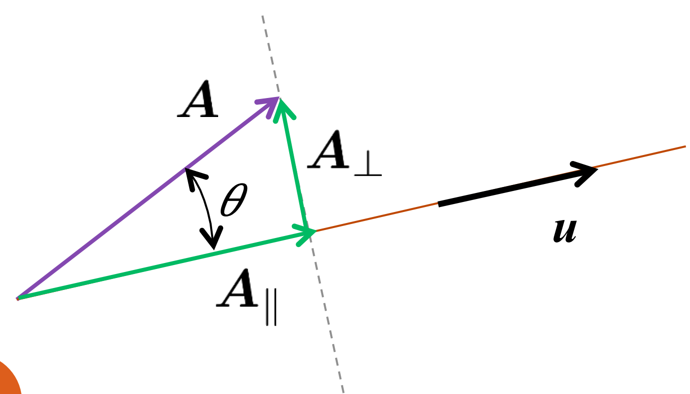
$$\boldsymbol{A} \cdot \boldsymbol{B} \;\; =$$



Note that:

Projections

The scalar component $A_{||}$ of a vector $\boldsymbol{A}$ along (parallel to) a line with unit vector $\boldsymbol{u}$ is given by:



$$A_{||} =$$

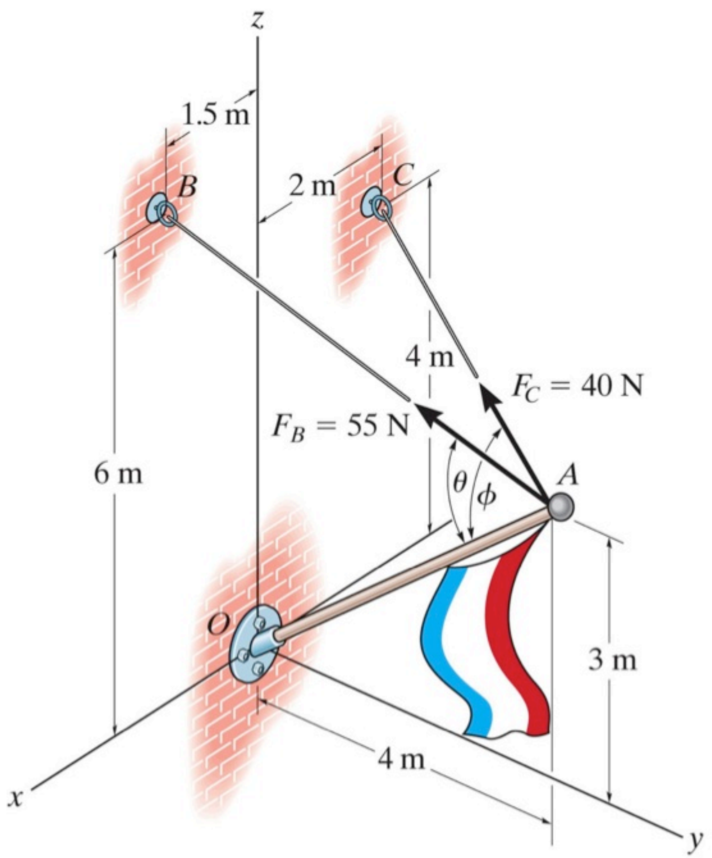
And thus the <u>vector</u> components $\boldsymbol{A}_{||}$ and $\boldsymbol{A}_\perp$ are given by:

$$\boldsymbol{A}_{||} =$$

$$\boldsymbol{A}_\perp =$$

Example

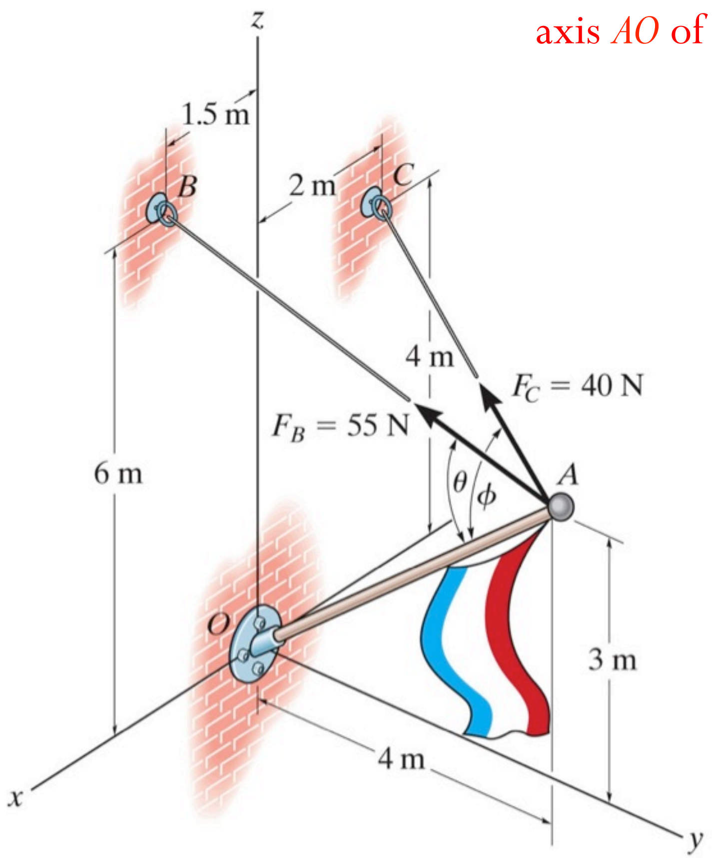
a) Determine the angle between *AB* and the axis *AO* of the flag pole.



Example

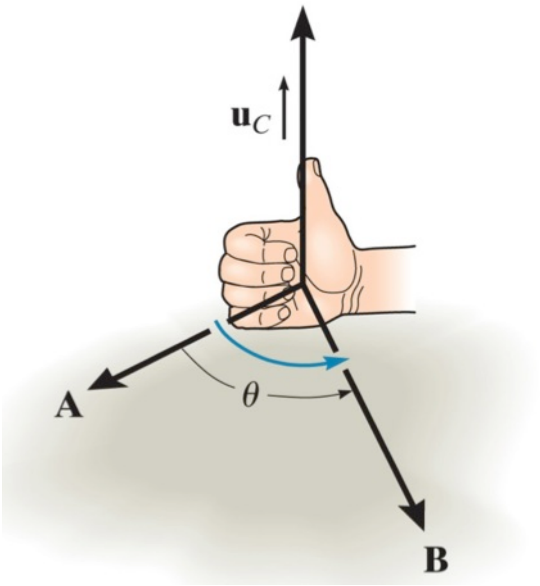
a) Determine the angle between AB and the axis AO of the flag pole.

b) Determine the projected component of the force vector $\boldsymbol{F}_C$ along the axis AO of the flag pole. Express your result as a Cartesian vector



Cross (or vector) product

The cross product of vectors **A** and **B** yields the vector **C**, which is written



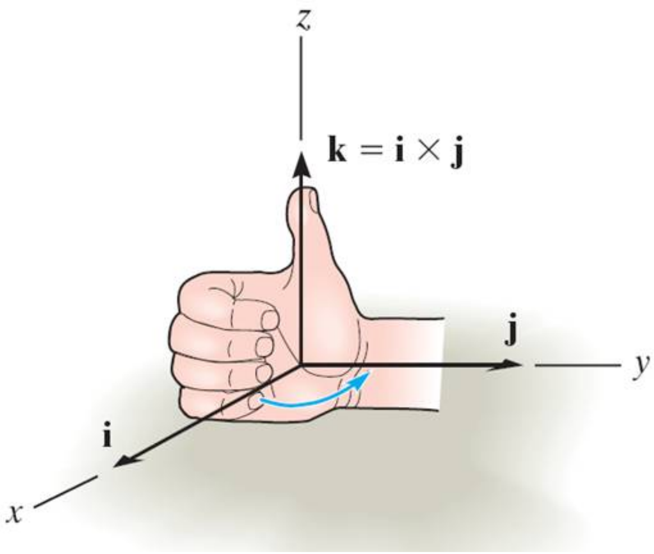
$$\boxed{\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}}$$

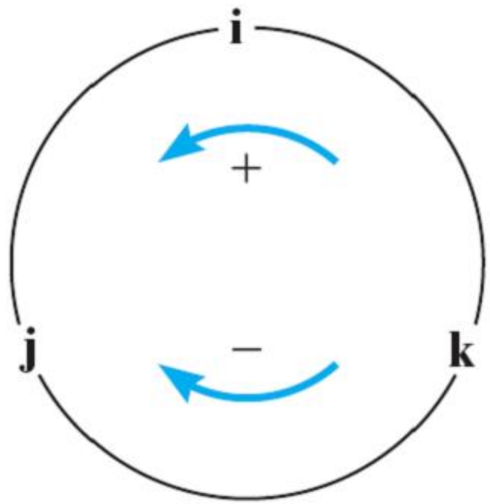
The magnitude of vector **C** is given by:

The vector **C** is perpendicular to the plane containing **A** and **B** (specified by the **right-hand rule**). Hence,

Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{0}$





Considering the cross product in Cartesian coordinates

$$\boldsymbol{A} \times \boldsymbol{B} \quad = \quad (A_x\,\boldsymbol{i} + A_y\,\boldsymbol{j} + A_z\,\boldsymbol{k}) \times (B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k})$$

Cross (or vector) product

Also, the cross product can be written as a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be determined using 2 × 2 determinants.

Examples

Given the vectors

$$\begin{aligned} \boldsymbol{A} &= 2\,\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k} \\ \boldsymbol{B} &= 15\,\boldsymbol{i} - 20\,\boldsymbol{j} + 18\,\boldsymbol{k} \\ \boldsymbol{C} &= \boldsymbol{i} + 7\,\boldsymbol{k} \end{aligned}$$

Determine:

1. $\boldsymbol{A} + \boldsymbol{B}$
2. $\boldsymbol{B} - \boldsymbol{C}$
3. $\boldsymbol{A} \cdot \boldsymbol{B}$
4. $\boldsymbol{B} \times \boldsymbol{C}$
5. a unit vector in the direction of $\boldsymbol{C}$
6. the direction cosines of $\boldsymbol{B}$