wc03lect

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Announcements

- MATLAB Clinics – Today (Fri) 9am – 5pm
- Upcoming deadlines:
- Friday (9/1)
  - Prairie Learn HW0
- Sunday (9/3)
  - Mastering Engineering HW1
- Tuesday (9/5)
  - Prairie Learn HW2
- Thursday (9/7)
  - Mastering Engineering HW3

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Recap

- Pay attention to units!
- Solve problem symbolically
- Equations must be dimensionally homogenous
- 1% accuracy
- Scalar – defined by magnitude (negative/positive)
- Vector – defined by magnitude and direction
- Vector operations – addition/subtraction

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L3 - Force Vectors

Force vectors

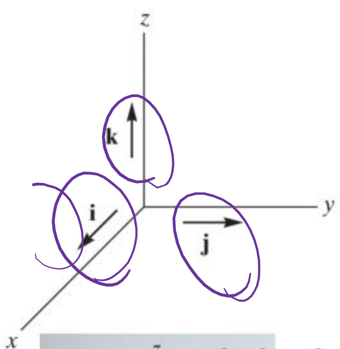
A force—the action of one body on another—can be treated as a vector, since forces obey all the rules that vectors do.

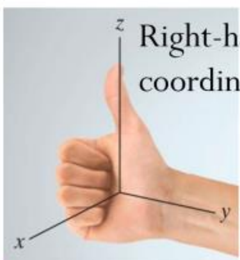
Cartesian vectors

Rectangular coordinate system: formed by 3 mutually perpendicular axes, the *x, y, z* axes, with unit vectors $\hat{i}, \hat{j}, \hat{k}$ in these directions.

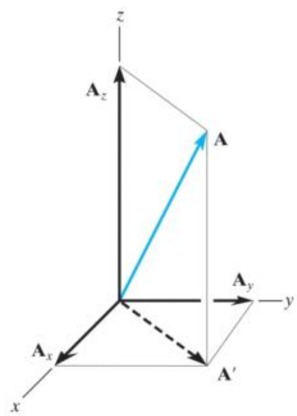
Note that we use the special notation "^" to identify *basis vectors* (instead of "~" or "→")

$(\hat{i}, \hat{j}, \hat{k})$ or $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$



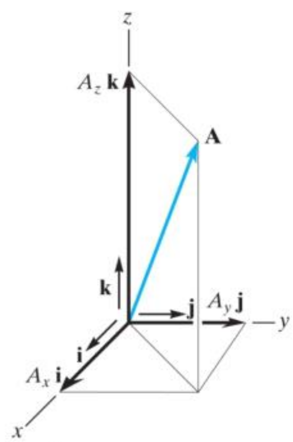


Right-handed coordinate system



Rectangular components of a vector

$$\boldsymbol{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$



Cartesian vector representation

$$\boldsymbol{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

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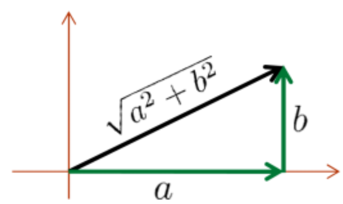
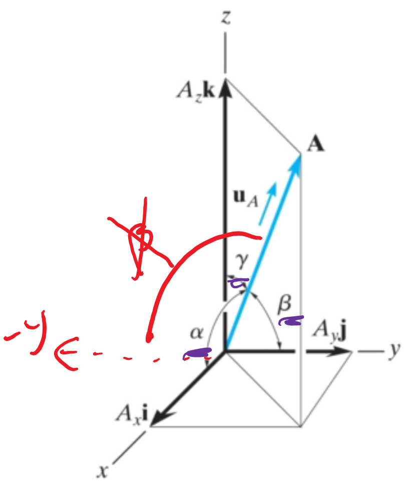
scalar

Magnitude of Cartesian vectors

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



Direction of Cartesian vectors



Expressing the direction using a unit vector:

$$\boldsymbol{u}_A = \frac{\boldsymbol{A}}{A}$$

Direction cosines are the components of the unit vector:

$$\cos\alpha = \frac{A_x}{A}$$

$$\cos\beta = \frac{A_y}{A}$$

$$\cos\gamma = \frac{A_z}{A}$$

Addition of Cartesian vectors

$$\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

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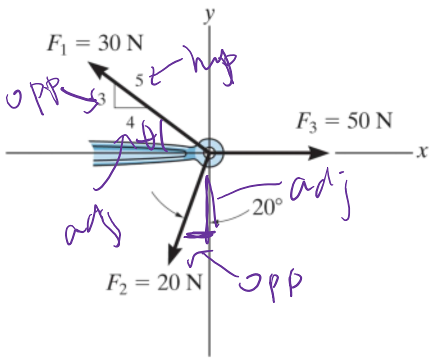
Example



Express each force vector using the Cartesian vector form (components form).

$-\cos\theta = \frac{adj}{hyp.}$

$+\sin\theta = \frac{opp}{hyp.}$

$$\vec{F}_1 = F_1\hat{u}_1 = 30N\left(-\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right)$$

$$\vec{F}_2 = F_2\hat{u}_2 = 20N\left(-\sin 20^\circ\hat{i} - \cos 20^\circ\hat{j}\right)$$

$$\vec{F}_3 = F_3\hat{u}_3 = 50N(\hat{i})$$

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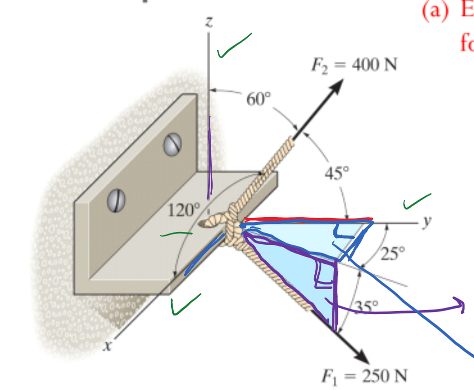
Example

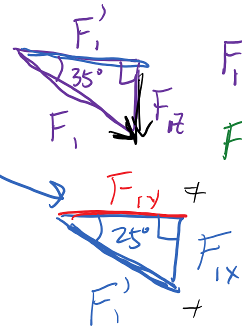
The bracket is subjected to the two forces on the ropes.

(a) Express each force vector using the Cartesian vector form (components form).



magnitude

$$\vec{F}_1 = F_{1x}\hat{i} + F_{1y}\hat{j} + F_{1z}\hat{k} = F_1\underbrace{\left(\cos 35° \sin 25° \hat{i} + \cos 35° \cos 25° \hat{j} - \cancel{+} \sin 35° \hat{k}\right)}_{\text{direction}}$$



$F_{1z} = -F_1 \sin 35°$

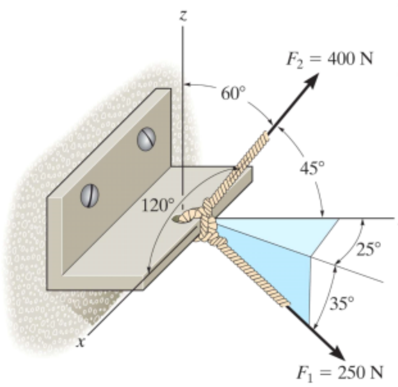
$F_1' = F_1 \cos 35°$

$F_{1x} = F_1' \sin 25° = (F_1 \cos 35°) \sin 25°$

$F_{1y} = F_1' \cos 25° = (F_1 \cos 35°) \cos 25°$

$$\vec{F}_2 = F_2\left(\cos 120° \hat{i} + \cos 45° \hat{j} + \cos 60° \hat{k}\right)$$

Example



The bracket is subjected to the two forces on the ropes.

(a) Express each force vector using the Cartesian vector form (components form).

(b) Determine the magnitude of the resultant force vector

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

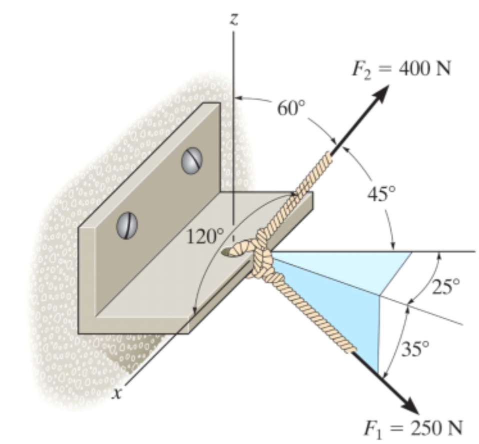
$$\vec{R} = \vec{F}_1 + \vec{F}_2 = (F_{1x} + F_{2x})\hat{i} + (F_{1y} + F_{2y})\hat{j} + (F_{1z} + F_{2z})\hat{k}$$

$$= \underbrace{(F_1 \cos 35° \sin 25° + F_2 \cos 120°)}_{R_x}\hat{i}$$

$$+ \underbrace{(F_1 \cos 35° \cos 25° + F_2 \cos 45°)}_{R_y}\hat{j}$$

$$+ \underbrace{(-F_1 \sin 35° + F_2 \cos 60°)}_{R_z}\hat{k}$$

Example



The bracket is subjected to the two forces on the ropes.

(a) Express each force vector using the Cartesian vector form (components form).

(b) Determine the magnitude of the resultant force vector

(c) Determine the direction cosines of the resultant force vector

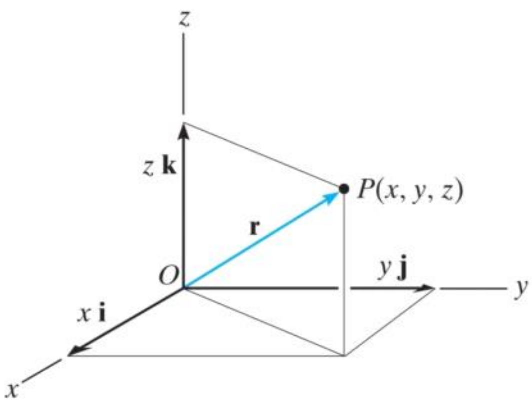
$$\cos\alpha = \frac{A_x}{A} \Rightarrow \alpha = \cos^{-1}\left(\frac{A_x}{A}\right)$$

$$\alpha = \frac{R_x}{R} = \frac{(F_1 \cos 35 \sin 25° + F_2 \cos 25°)}{\sqrt{R_x^2 + R_y^2 + R_z^2}}$$

$$\beta = \frac{R_y}{R}$$

$$\gamma = \frac{R_z}{R}$$

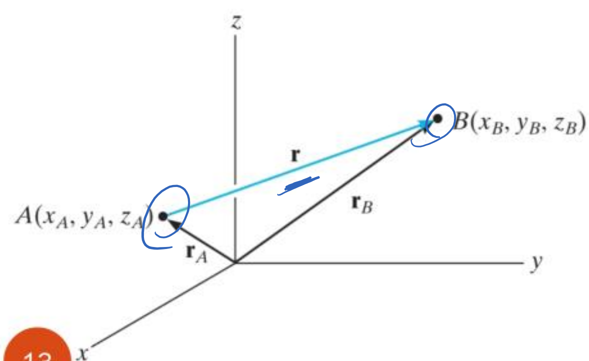
Position vectors



A position vector $\boldsymbol{r}$ is defined as a fixed vector which locates a point in space relative to another point. For example,

$$\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$$

expresses the position of point $P(x,y,z)$ with respect to the origin O.



The position vector $\boldsymbol{r}$ of point $\boldsymbol{B}$ with respect to point $\boldsymbol{A}$ is obtained from

$$\vec{r}_A + \vec{r} = \vec{r}_B$$

$$\Rightarrow \vec{r} = \vec{r}_B - \vec{r}_A$$

Thus, the $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ components of the positon vector $\boldsymbol{r}$ may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).

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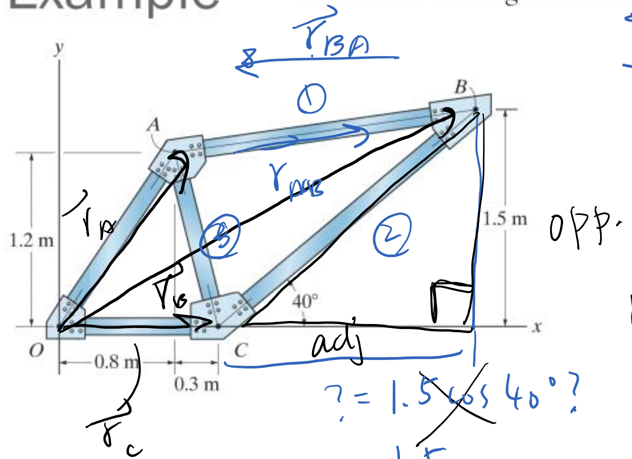
Example

Determine the lengths of bars AB, BC and AC.

1 2 3



$? = 1.5\cancel{\cos 40°}$?

$$? = \frac{1.5}{\tan 40°} \quad ?$$

$$\tan 40 = \frac{1.5}{?} \Rightarrow$$

$$\vec r_{AB} = \vec r_B - \vec r_A$$

$$= \left[\left(0.3 + \frac{1.5}{\tan 40°}\right)\hat i + 0.3\hat j\right] m$$

$$r_{AB} = \sqrt{\left(0.3 + \frac{1.5}{\tan 40°}\right)^2 + 0.3^2}\ m$$

$$\vec r_B = \left[\left(1.1 + \frac{1.5}{\tan 40°}\right)\hat i + (1.5)\hat j\right] m$$

$$\vec r_A = (0.8\hat i + 1.2\hat j)\,m$$

$$\vec r_C = (1.1\hat i)\,m$$

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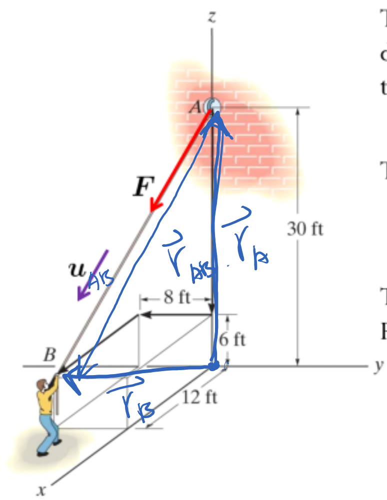
L3 - Force Vectors

Force vector directed along a line



The force vector $\boldsymbol{F}$ acting a long the rope can be defined by the unit vector $\boldsymbol{u}$ (defined the direction of the rope) and the magnitude of the force.

$$\boldsymbol{F} = F\,\boldsymbol{u}$$

The unit vector $\boldsymbol{u}$ is specified by the position vector:

$$\hat{u} = \frac{\vec{r}}{r}$$

The man pulls on the cord with a force of 70 lb. Represent the force F as a Cartesian vector.

$F = 70\text{ lb}$

$$\hat{u}_{AB} = \frac{\vec{r}_{AB}}{r_{AB}}$$

$\vec{r}_A = 30\text{ft}\cdot\hat{k}$

$\vec{r}_B = (12\hat{i} - 8\hat{j} + 6\hat{k})\text{ ft.}$

$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = (12\hat{i} - 8\hat{j} - 24\hat{k})\text{ ft}$

$r_{AB} = \sqrt{12^2 + 8^2 + 24^2}$

$$\vec{F} = \left[70\left(\frac{12}{r_{AB}}\right)\hat{i} + 70\left(\frac{-8}{r_{AB}}\right)\hat{j} + 70\left(\frac{-24}{r_{AB}}\right)\hat{k}\right]$$

N? lb! lb/s^2

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L3 - Force Vectors