Lecture 35

Friday, December 1, 2017 12:21

To do ...

- Quiz 7 next week
- WA 4 due **TODAY**
- HW 25 ME due **Sat**
- HW 24 PL due **Tues**

Fluids

Pascal's law: A fluid at rest creates a pressure p at a point that is the *same* in *all* directions

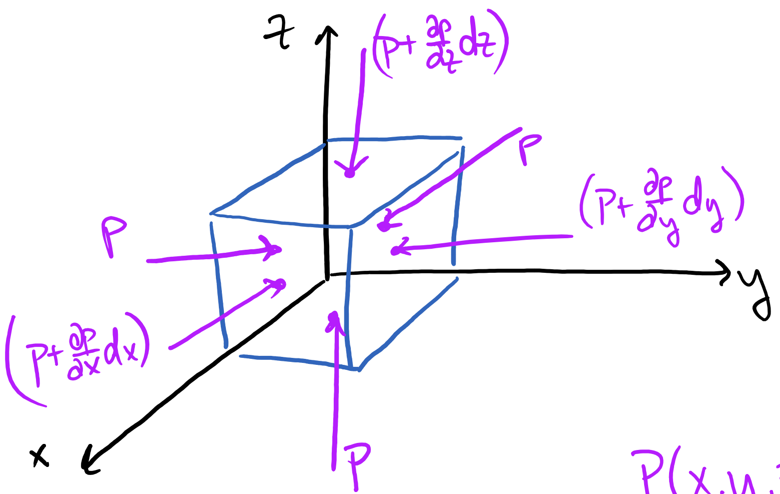
pressure

$$p \equiv \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

$$\frac{\text{force}}{\text{Area}} \rightarrow \frac{N}{m^2} \rightarrow \text{Pascal}$$

Incompressible: An incompressible fluid is one for which the mass density ρ is independent of the pressure p. Liquids are generally considered incompressible. Gases are compressible, but may be approximated as incompressible if the pressure variations are relatively small.

Consider the control volume:



$p(x,y,z)$

Sum of forces:

$$\Sigma F_x = 0$$

$$p\,dy\,dz - \left(p + \frac{\partial p}{\partial x}dx\right)dy\,dz = 0$$

$\Sigma F_y = 0$

$$p\,dx\,dz - \left(p + \frac{\partial p}{\partial y}dy\right)dx\,dz = 0$$

$\Sigma F_z = 0$

$$p\,dx\,dy - \left(p + \frac{\partial p}{\partial z}dz\right)dx\,dy - \rho g\,dx\,dy\,dz = 0$$

Simplify $\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$ ...

$\Sigma F_x = 0$

$$p\,dy\,dz - p\,dy\,dz - \frac{\partial p}{\partial x}dx\,dy\,dz = 0$$

$$\therefore \quad \frac{\partial p}{\partial x} = 0$$

$\Sigma F_y = 0$

$$p\,dx\,dz - p\,dx\,dz - \frac{\partial p}{\partial y}dy\,dx\,dz = 0$$

$$\therefore \quad \frac{\partial p}{\partial y} = 0$$

$\Sigma F_z = 0$

$$p\,dx\,dy - p\,dx\,dy - \frac{\partial p}{\partial z}dz\,dx\,dy - \rho g\,dx\,dy\,dz = 0$$

$$\therefore \quad \frac{\partial p}{\partial z} = -\rho g$$

Since $P(x,y,z) \to P(z)$ only,

$$\frac{dP}{dz} = -\rho g$$

integrate...

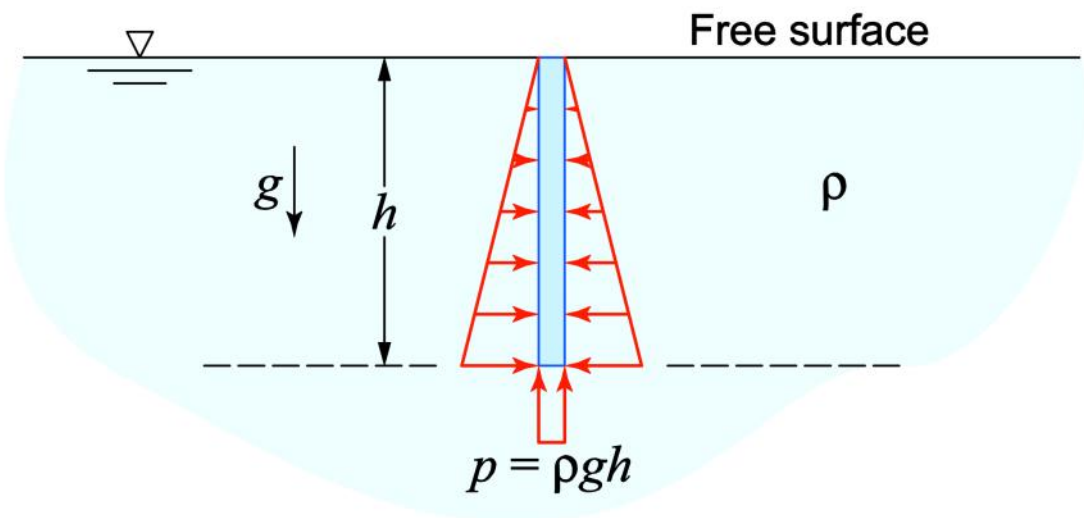
$$\int_{P_1}^{P_2} dP = -\int_{z_1}^{z_2} \rho g \, dz$$

$$\boxed{P_2 - P_1 = -\rho g (z_2 - z_1)}$$

$$\{ \Delta P = \rho g \Delta z \}$$ Pascal's LAW!

Recap: Fluid Pressure

For an incompressible fluid at rest with mass density , the pressure varies linearly with depth z, and is *constant* along any horizontal plane (since h is constant):



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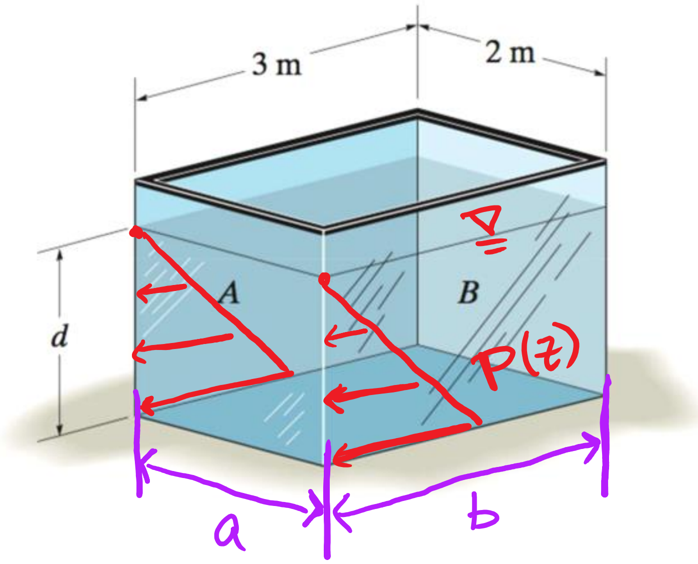
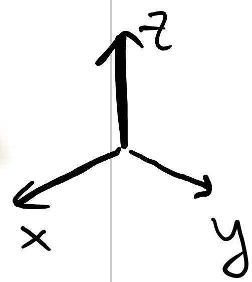
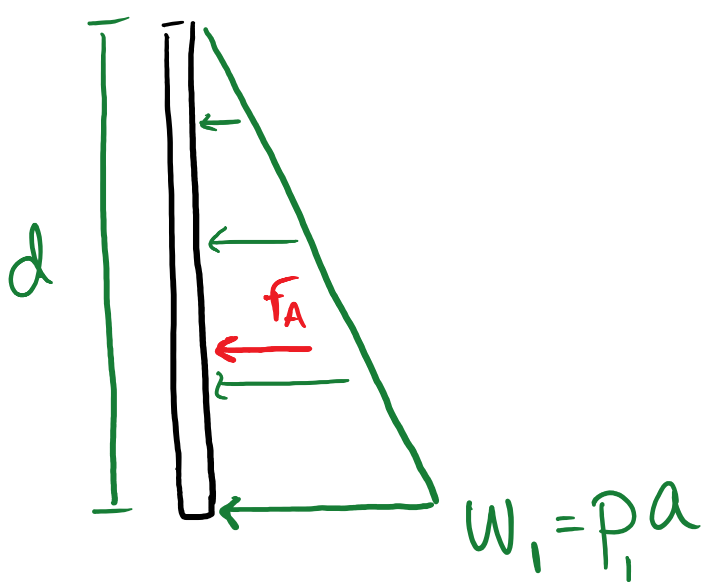
The tank is filled with water to a depth of $d = 4$ m. Determine the resultant force the water exerts on side A of the tank. ($\rho = 1000\ \text{kg/m}^3$)





At side A:

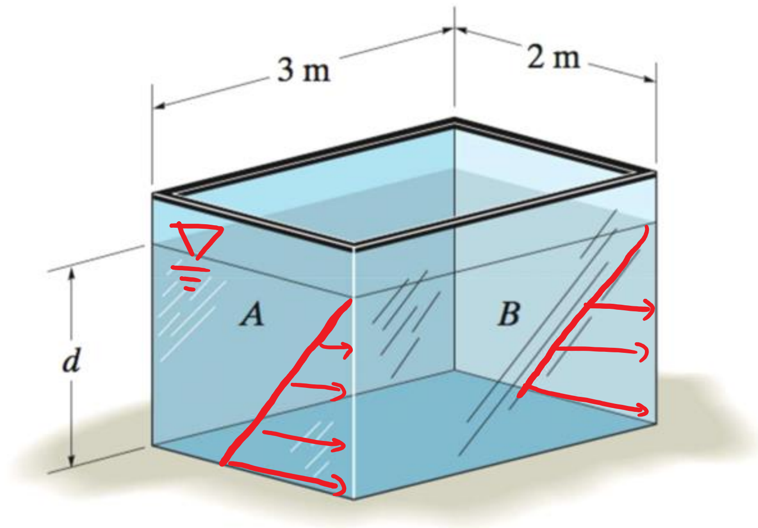
the FBD of side A:



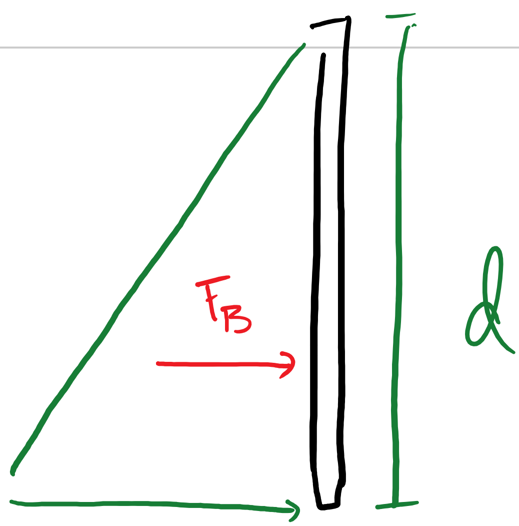
$$F_A = \frac{1}{2} d w_1 = \frac{1}{2} d p_1 a = \frac{1}{2} d \rho g d a$$

$$F_A = \frac{1}{2} \rho g a d^2 = \boxed{157 \text{ kN}}$$

The tank is filled with water to a depth of $d = 4$ m. Determine the resultant force the water exerts on side B of the tank. ($\rho = 1000$ kg/m^3)



At side B:



$W_2 = P_2 b$

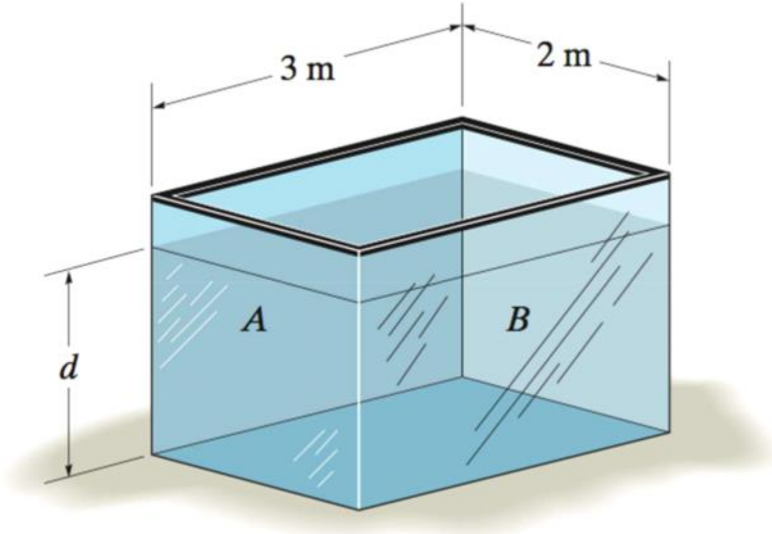
$$F_B = \frac{1}{2} d W_2 = \frac{1}{2} d P_2 b$$

$$F_B = \frac{1}{2} d \rho g d b$$

$$F_B = \frac{1}{2} \rho g b d^2$$

$$\boxed{F_B = 235\ \text{kN}}$$

If the tank is filled with oil instead, what depth d should it reach so that it creates the same resultant forces on side A. ($\rho = 900\ \mathrm{kg/m^3}$)

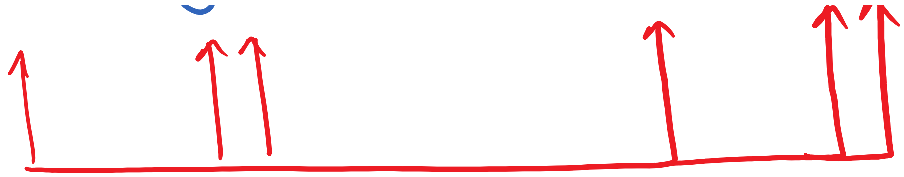


Forces on side A:

$$F_A = \frac{1}{2}\rho g a d^2$$

water: oil:

$$\frac{1}{2}\rho_w g a d_w^2 = \frac{1}{2}\rho_o g a d_o^2$$

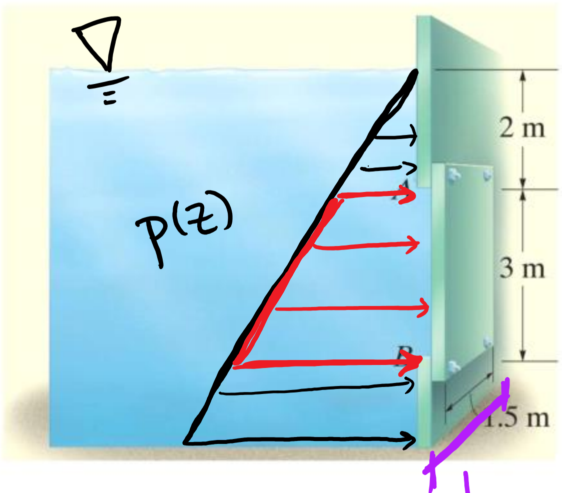


(not changing) $\therefore$

$$\rho_w d_w^2 = \rho_o d_o^2$$

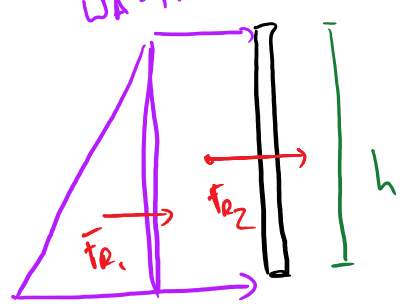
$$d_o = \sqrt{\frac{\rho_w}{\rho_o}}\, d_w$$

$$\boxed{d_o = 4.21\ \text{m}}$$



Determine the magnitude and location of the resultant hydrostatic force acting on the submerged rectangular plate AB. The plate has width 1.5m. The density of the water is 1000 kg/m^3

First calculate Resultant force:



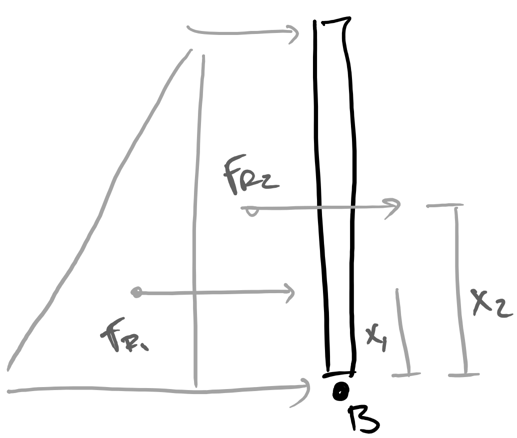
$$F_{R_2} = w_A h = p_A b h = \rho g z_A b h = \underline{88.3 \text{ kN}}$$

$$F_{R_1} = \frac{1}{2} h (w_B - w_A) = \frac{1}{2} h b (p_B - p_A)$$

$$F_{R_1} = \frac{1}{2} \rho g h b (z_B - z_A) = \underline{66.2 \text{ kN}}$$

$$F_R = F_{R_1} + F_{R_2} = \rho g h b \left(\frac{z_A + z_B}{2}\right) = \underline{154.5 \text{ kN}}$$

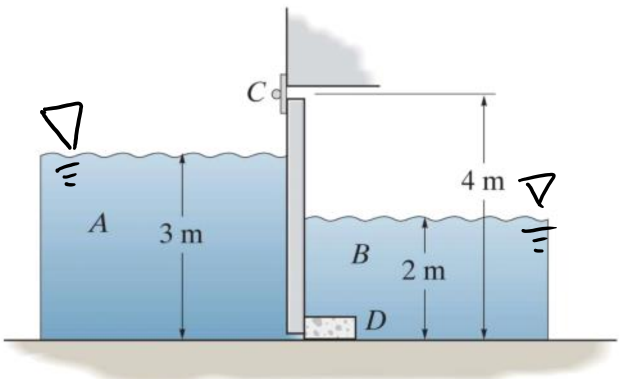
Sum the moments About B:



$$(\Sigma M_R)_B = \Sigma M_B$$

$$\bar{x} F_R = x_1 F_{R_1} + x_2 F_{R_2}$$

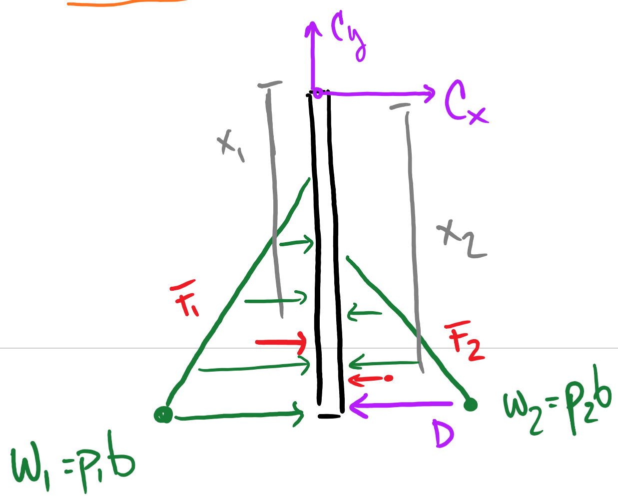
$$\bar{x} = \frac{x_1 F_{R_1} + x_2 F_{R_2}}{F_{R_1} + F_{R_2}} = \underline{1.29\ m}$$



For the condition of high tide shown, determine the reactions developed at the hinge C and stop block. The length of the gate is 6 m and its height is 4 m. The density of the water is 1000 kg/m^3

} $b = 6\,m$

DRAW the FBD of the gate:



Sum forces, & ℰ:

$$\Sigma F_y = 0 \quad \therefore \quad C_y = 0$$

$$\Sigma F_x = 0 \quad \therefore$$

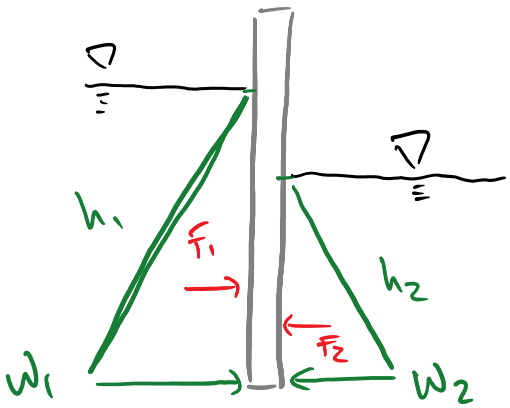
$$C_x - D + F_1 - F_2 = 0$$ (2 unknowns)

$$\Sigma M_c = 0 \quad \therefore$$

$$x_1 F_1 - x_2 F_2 - (4m) D_x = 0$$ (1 unknown)

Solve for locations and magnitude of resultant forces:

resultant forces:



$$F_1 = \frac{1}{2} h_1 w_1 = \frac{1}{2} h_1 p_1 b$$

$$\underline{F_1 = \frac{1}{2} \rho g b h_1^2}$$

$$F_2 = \frac{1}{2} h_2 w_2 = \frac{1}{2} h_2 p_2 b$$

$$\underline{F_2 = \frac{1}{2} \rho g b h_2^2}$$

use moment eqn to solve:

$$\left(1 + \frac{2}{3} h_1\right)\left(\frac{1}{2} \rho g b h_1^2\right) - \left(2 + \frac{2}{3} h_2\right)\left(\frac{1}{2} \rho g b h_2^2\right) - 4 D_x = 0$$

$$\frac{1}{2} \rho g b \left[h_1^2 \left(1 + \frac{2}{3} h_1\right) - h_2^2 \left(2 + \frac{2}{3} h_2\right) \right] = 4 D_x$$

$$\boxed{D_x = 101 \text{ kN}}$$

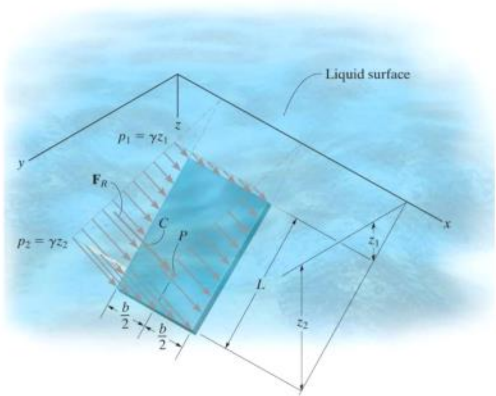
now sum forces:

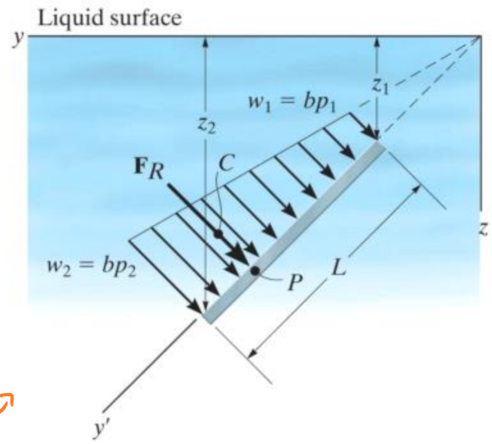
$$C_x + F_1 - F_2 - D_x = 0$$

$$\boxed{C_x = 46.6 \text{ kN}}$$

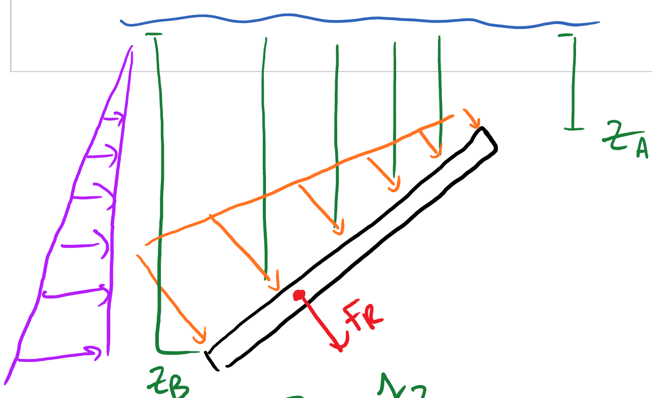
Fluid Pressure

For an incompressible fluid at rest with mass density , the pressure varies linearly with depth z





Reduce to 2D

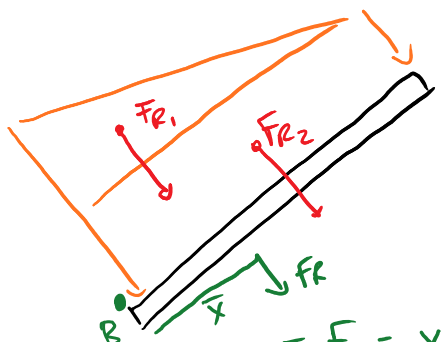


$P_A = \gamma z_A$

$W_A = P_A b = \gamma z_A b$

$P_B = \gamma z_B$

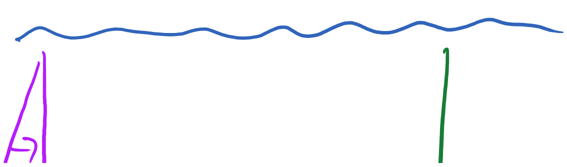
$W_B = P_B b = \gamma z_B b$

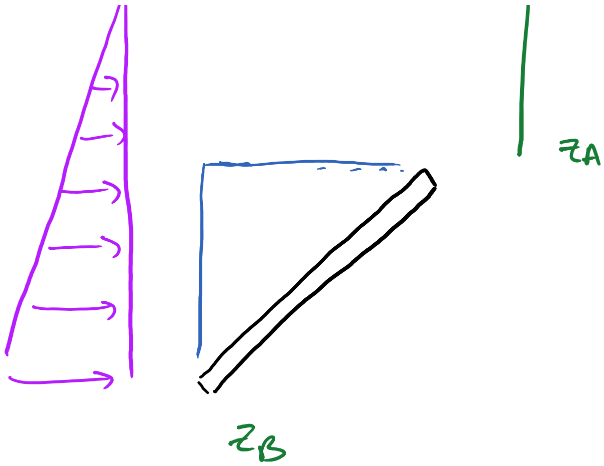


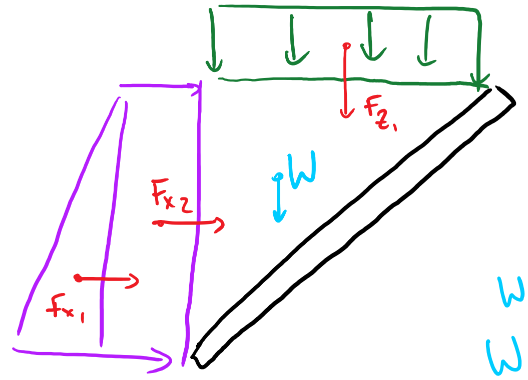
$\Sigma M_B = 0$

$\bar{x} F_R = x_1 F_{R_1} + x_2 F_{R_2}$

Separate into components.







W = Area * width

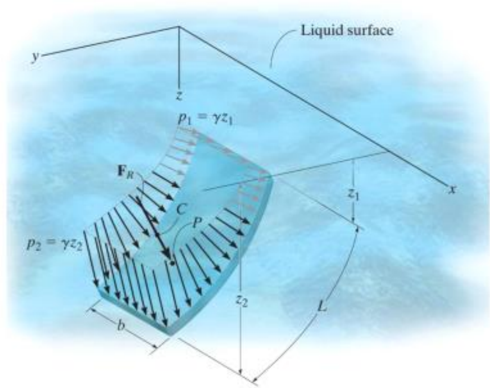
W = volume · γ

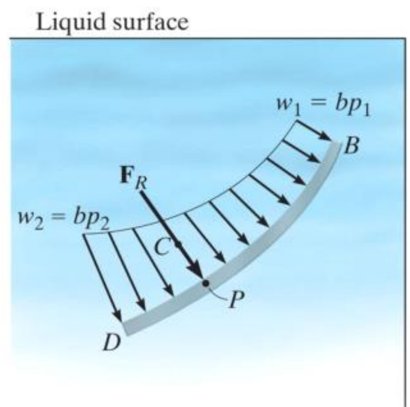
$$F_R = \sqrt{F_x^2 + F_z^2}$$

$$F_R = \sqrt{(F_{x_1} + F_{x_2})^2 + (F_z + W)^2}$$

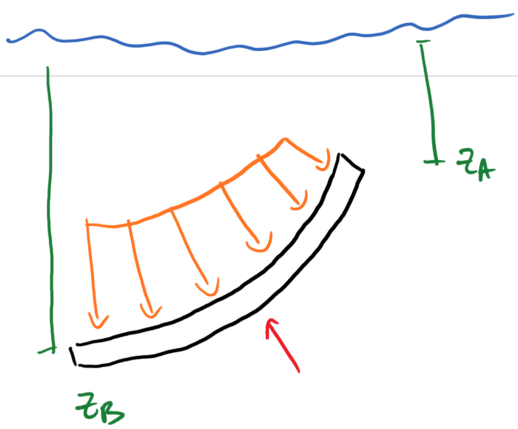
Fluid Pressure

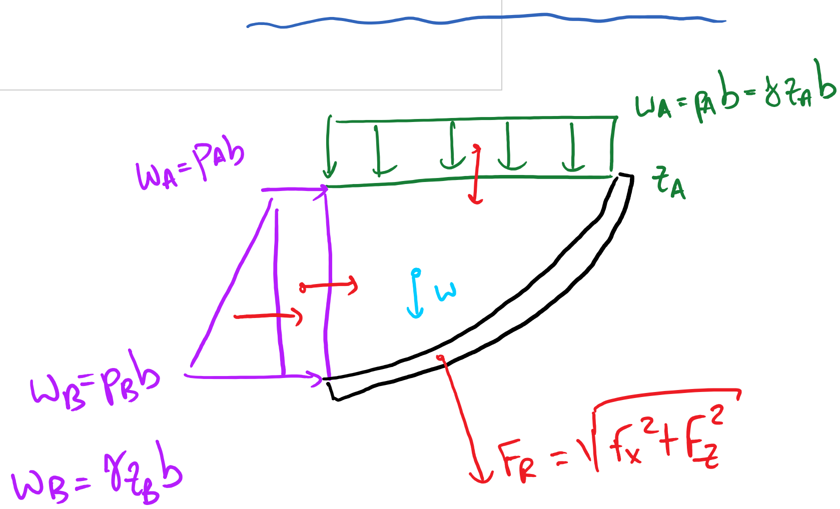
For an incompressible fluid at rest with mass density , the pressure varies linearly with depth z





Reduce to 2D.





Q: what is F on the other side?