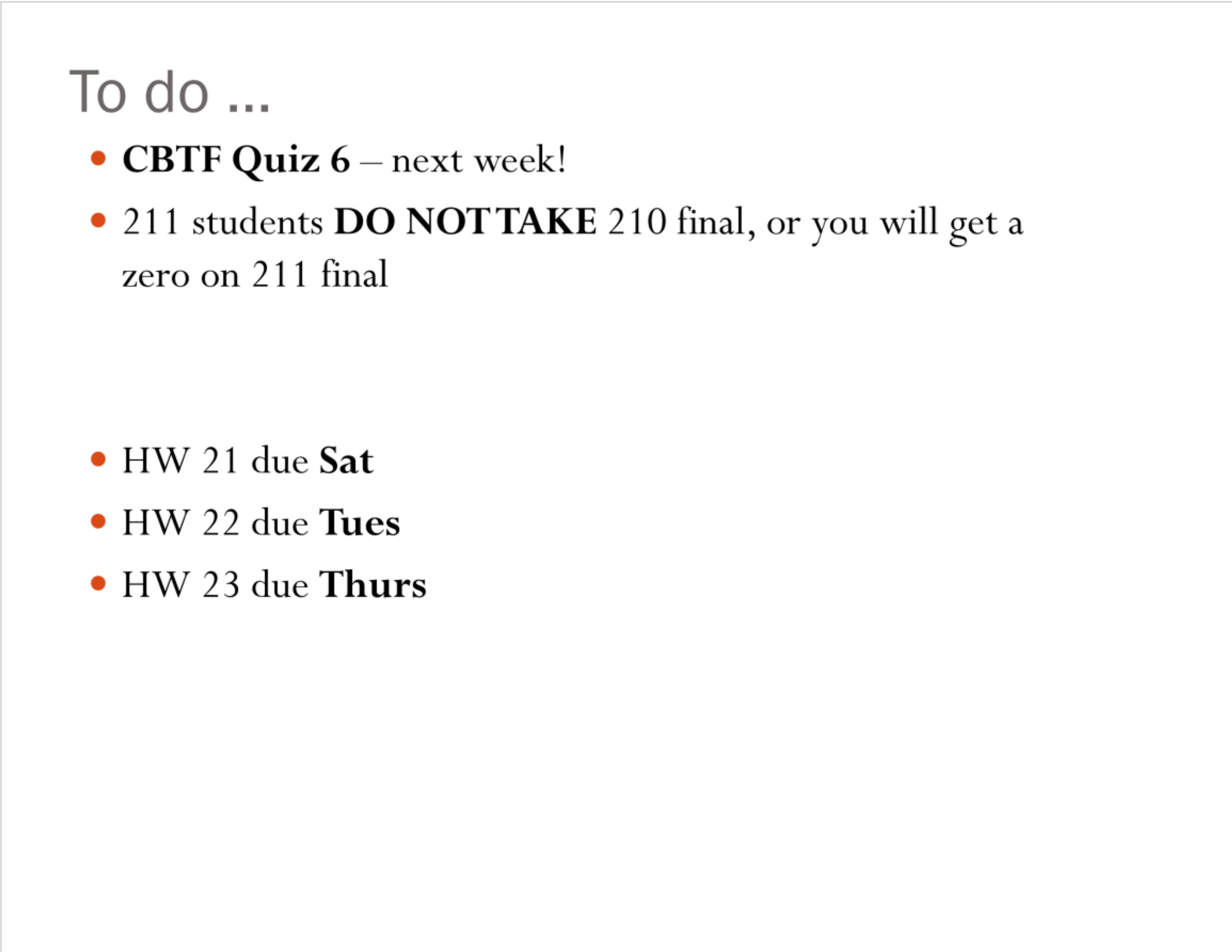
# Lecture 30

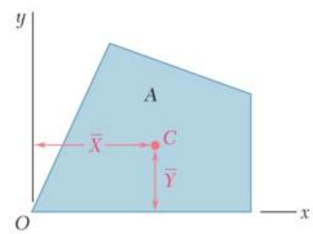
Friday, November 10, 2017 12:16



## Composite bodies

A composite body consists of a series of connected simpler shaped bodies.

Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.



$(\bar{x}\bar{y})$ $(\bar{x},\bar{y})$

$$\bar{X} = \frac{\sum \tilde{x} W}{\sum W} \rightarrow \frac{\sum \tilde{x} M}{\sum M} \rightarrow \frac{\sum \tilde{x} V}{\sum V} \rightarrow \frac{\sum \tilde{x} A}{\sum A}$$

Weight | Mass | Volume | Area

PROCEDURE FOR ANALYSIS

- Divide the body into pieces that are known shapes.
  - ↳ Holes are considered pieces with negative weight or size

- MAKE A table

| SEGMENT # | weight, mass, volume, AREA | moment Arm $\tilde{x}, \tilde{y}, \tilde{z}$ | calculations $\tilde{x}A, \tilde{y}V, \tilde{z}W$ |
|---|---|---|---|
| | ↓ | | ↓ |
| | $\Sigma W, \Sigma A$ | | $\Sigma \tilde{x}A, \Sigma \tilde{y}W$ |

- use Appropriate eqns.

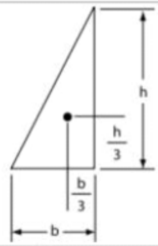
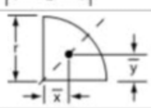
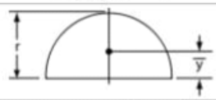
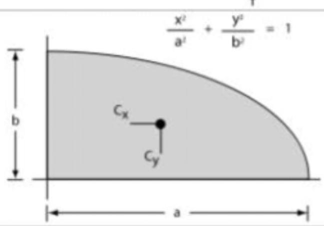
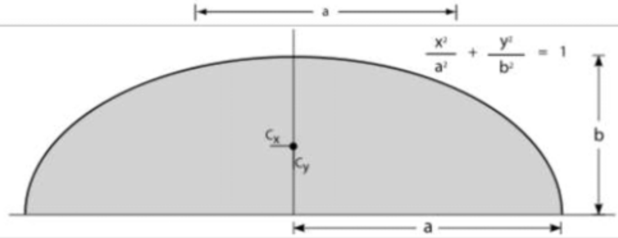
$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} = \frac{\Sigma \tilde{x} M}{\Sigma M} = \frac{\Sigma \tilde{x} V}{\Sigma V} = \frac{\Sigma \tilde{x} A}{\Sigma A}$$

$$\bar{y} = \dots$$

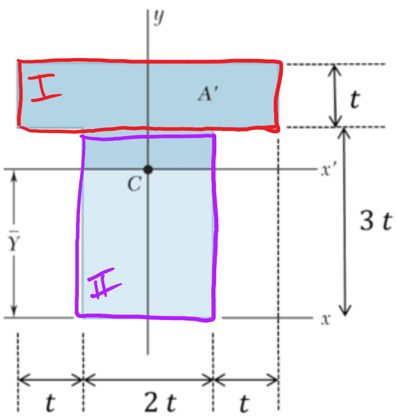
# Page 3

Friday, November 10, 2017 12:16

## Centroid of typical 2D shapes

| Shape | Figure | $\bar{x}$ | $\bar{y}$ | Area |
|---|---|---|---|---|
| Right-triangular area |   | $\frac{b}{3}$ | $\frac{h}{3}$ | $\frac{bh}{2}$ |
| Quarter-circular area |   | $\frac{4r}{3\pi}$ | $\frac{4r}{3\pi}$ | $\frac{\pi r^2}{4}$ |
| Semicircular area |   | 0 | $\frac{4r}{3\pi}$ | $\frac{\pi r^2}{2}$ |
| Quarter-elliptical area |   | $\frac{4a}{3\pi}$ | $\frac{4b}{3\pi}$ | $\frac{\pi ab}{4}$ |
| Semielliptical area |   | 0 | $\frac{4b}{3\pi}$ | $\frac{\pi ab}{2}$ |

http://en.wikipedia.org/wiki/List_of_centroids

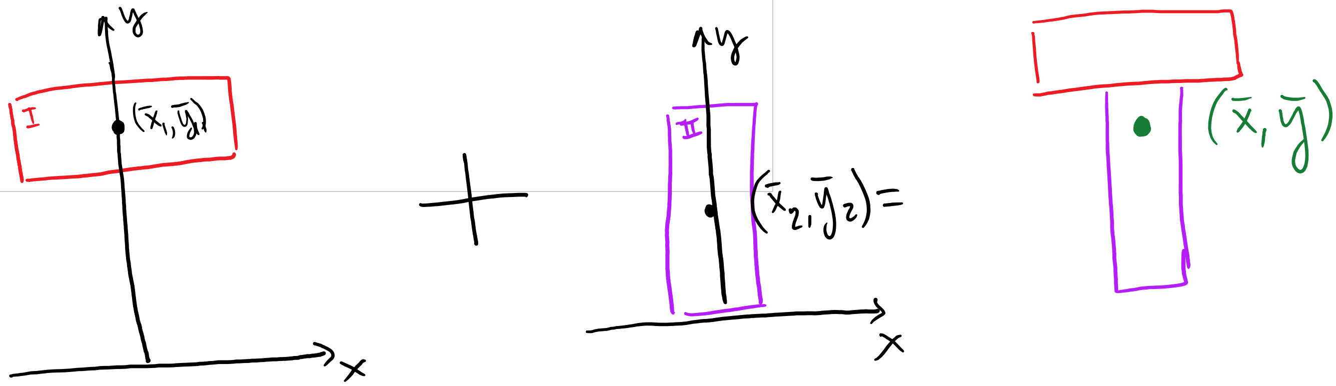



Find the centroid of the area below.

Q: How many sections?
– As many as you want

Q: Axis of symmetry? – yes, ∴

$\bar{x} = 0 \quad \bar{y} = \dfrac{\Sigma \tilde{y} A}{\Sigma A}$



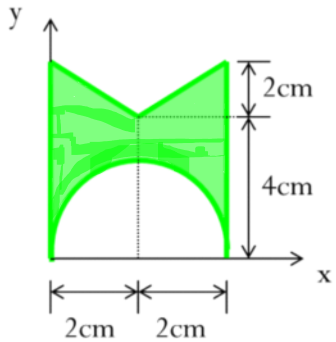

| Segment | Area | $\tilde{x}$ | $\tilde{y}$ | $\tilde{x}A$ | $\tilde{y}A$ |
|---|---|---|---|---|---|
| I | $4t^2$ | 0 | $\frac{7}{2}t$ | 0 | $14t^3$ |
| II | $6t^2$ | 0 | $\frac{3}{2}t$ | 0 | $9t^3$ |

$\Sigma A = 10t^2$

$\Sigma \tilde{y} A = 23t^3$

$\boxed{\bar{x} = 0}$

$\bar{y} = \dfrac{\Sigma \tilde{y} A}{\Sigma A} = \dfrac{23t^3}{10t^2} = \boxed{2.3t}$



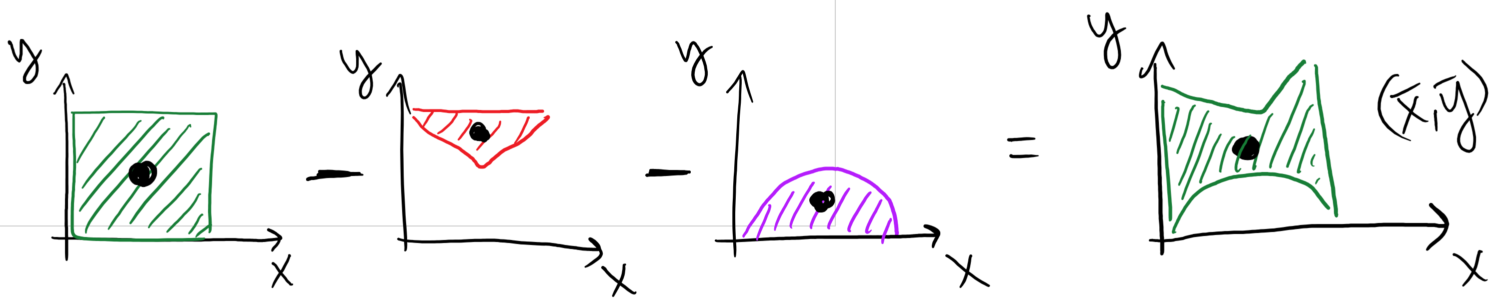

What is the centroid of the resultant area?

Q: How many sections? – 3

Q: Symmetry? – yes, ∴

$\boxed{\bar{x} = 2\text{cm}}$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$



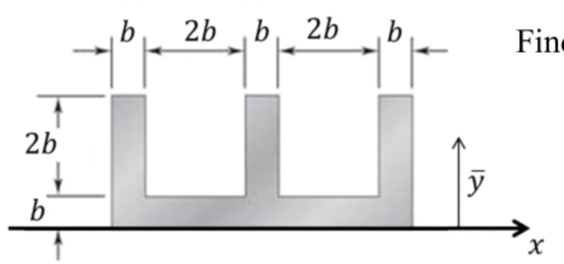

| Segment | Area ($cm^2$) | $\tilde{x}$ | $\tilde{y}$ | $\tilde{x}A$ | $\tilde{y}A$ |
|---|---|---|---|---|---|
| I | 24 | 2 | 3 | 48 | 72 |
| II | $-4$ | 2 | $\frac{16}{3}$ | $-8$ | $\frac{-64}{3}$ |
| III | $-2\pi$ | 2 | $\frac{8}{3\pi}$ | $-4\pi$ | $\frac{-16}{3}$ |

$\Sigma A = 13.72 \text{ cm}^2$

$\Sigma \tilde{y} A = 45.33 \text{ cm}^3$

$\boxed{\bar{x} = 2\text{cm}}$

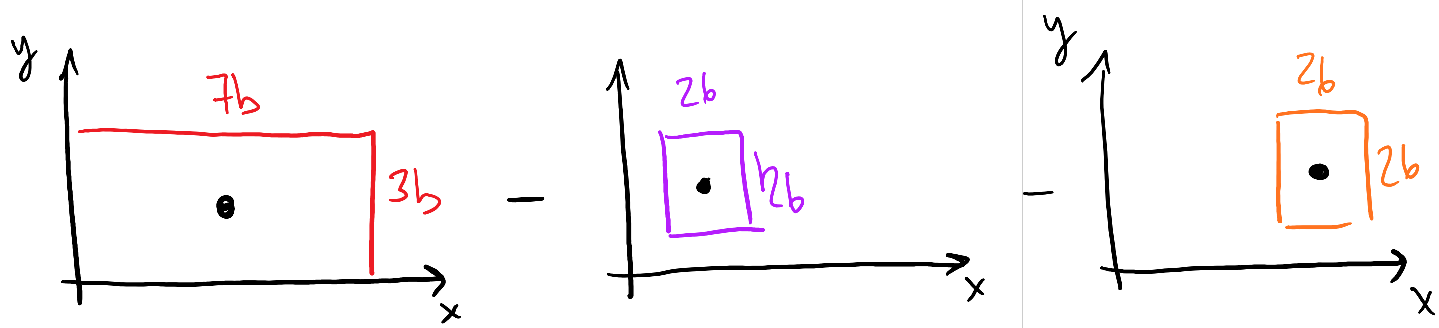
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{45.33 \text{ cm}^3}{13.72 \text{ cm}^2} = \boxed{3,3 \text{ cm}}$$




Find the centroid of the area below.

Q: number of segments?

Q: Axis of symmetry? - yes!



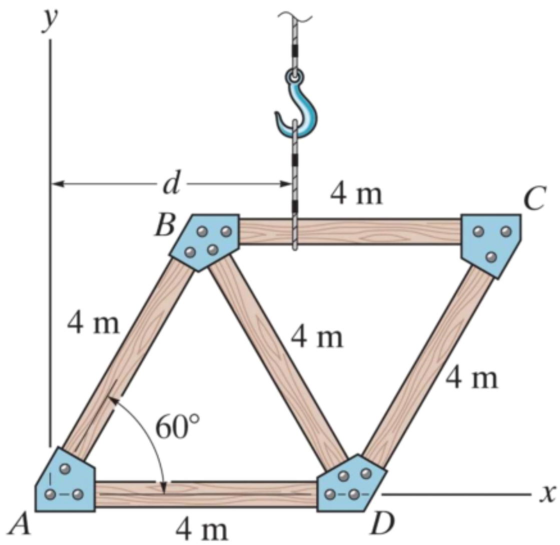

| Segment | Area | $\tilde{x}$ | $\tilde{y}$ | $\tilde{x}A$ | $\tilde{y}A$ |
|---|---|---|---|---|---|
| I | $21b^2$ | $\frac{7}{2}b$ | $\frac{3}{2}b$ | $\frac{147}{2}b^3$ | $\frac{63}{2}b^3$ |
| II | $-4b^2$ | $2b$ | $2b$ | $-8b^3$ | $-8b^3$ |
| III | $-4b^2$ | $5b$ | $2b$ | $-20b^3$ | $-8b^3$ |

$\Sigma A = 13b^2$ $\Sigma \tilde{y}A = 15.5b^3$

$\boxed{\bar{x} = \frac{7}{2}b}$ (Symmetry)

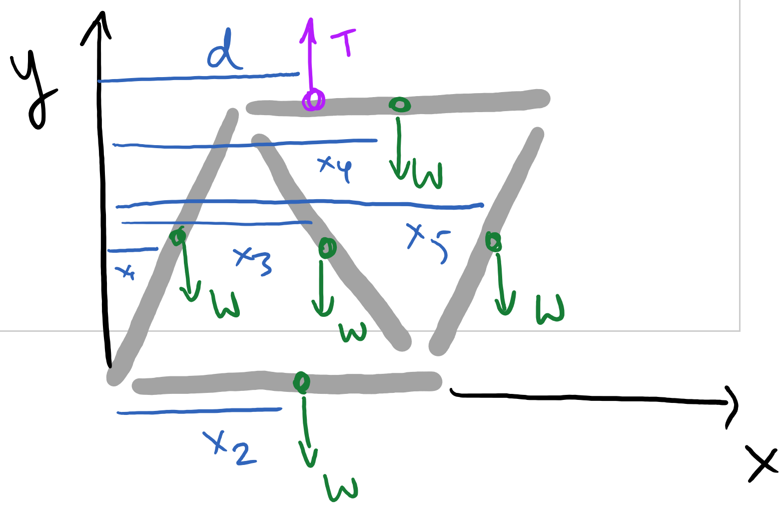
$\Sigma \tilde{y}A$ $15.5b^3$ $\boxed{1.191}$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{15.5b^3}{13b^2} = \boxed{1.19b}$$




The truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. Determine the distance $d$ to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.

DRAW the FBD of the truss:




Sum moments?

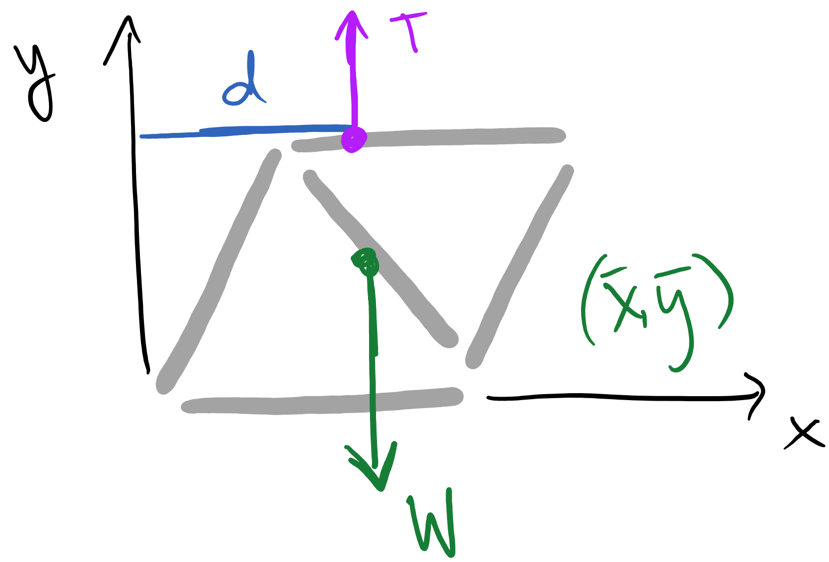
or find center of MASS!

the mass of each member is

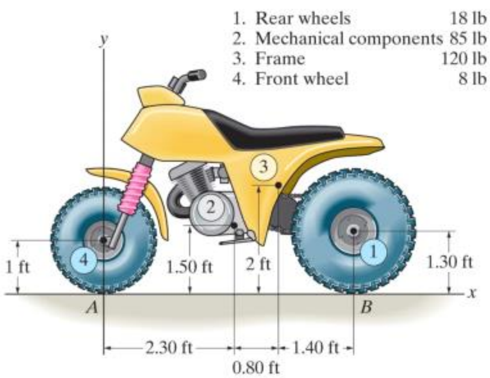
$$m = (4m)(7\ kg/m) = 28\ kg$$

| Segment | mass (kg) | $\tilde{x}$ (m) | $\tilde{y}$ | $\tilde{x}M$ | $\tilde{y}m$ |
|---|---|---|---|---|---|
| 1 | 28 | 1 | | 28 | |
| 2 | 28 | 2 | | 2(28) | |
| 3 | 28 | 3 | | 3(28) | |
| 4 | 28 | 4 | | 4(28) | |
| 5 | 28 | 5 | | 5(28) | |
| $\Sigma m =$ | 5(28) | | | 28(1+2+3+4+5) | |

$$\bar{x} = \frac{\Sigma \tilde{x} M}{\Sigma m} = \frac{28(1+2+3+4+5)}{5(28)} = \frac{15}{5} = \boxed{3\,m}$$




$$d = \bar{x} = 3\,m$$




Determine the location of the center of gravity of the three-wheeler. If the three-wheeler is symmetrical with respect to the x-y plane, determine the normal reaction each of its wheels exerts on the ground.

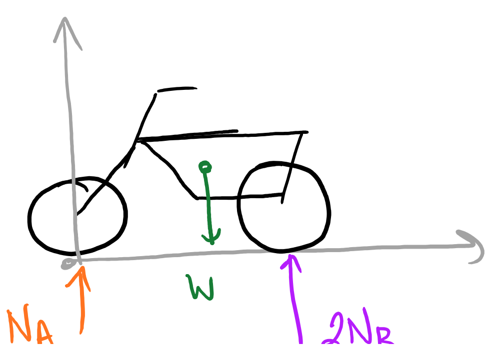
| Segment | W (lb) | $\tilde{x}$ | $\tilde{y}$ | $\tilde{x}W$ | $\tilde{y}W$ |
|---|---|---|---|---|---|
| 1 | 18 | 4.5 | 1.3 | 4.5(18) | 1.3(18) |
| 2 | 85 | 2.3 | 1.5 | 2.3(85) | 1.5(85) |
| 3 | 120 | 3.1 | 2 | 3.1(120) | 2(120) |
| 4 | 8 | 0 | 1 | 0 | 8 |

$\Sigma W =$ 231 lb  648.5 lb·ft  398.9 lb·ft

$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} = \frac{648.5 \text{ lb·ft}}{231 \text{ lb}} = \boxed{2.81 \text{ ft}}$$

$$\bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} = \frac{398.9 \text{ lbft}}{231 \text{ lb}} = \boxed{1.73 \text{ ft}}$$

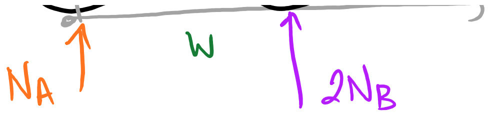
DRAW the FBD of the three-wheeler:




Sum the moment About A.

$\Sigma M_A$: $2N_B(4.5) - 231(2.81) = 0$

$231(2.81)$




$\Sigma M_A:$

$$N_B = \frac{231(2.81)}{2(4.5)}$$

$$\boxed{N_B = 72.1 \text{ lb}}$$

* EACH REAR WHEEL!

now sum the forces in y:

$\Sigma F_y$: $N_A + 2N_B - W = 0$

$$N_A = W - 2N_B = 231 - 2(72.1) = \boxed{86.9 \text{ lb}}$$