Lecture 29

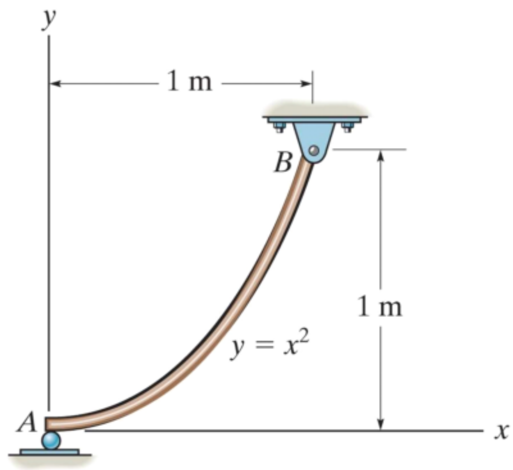
Wednesday, November 8, 2017 12:22

To do ...

- **CBTF Quiz 6** – next week!
- 211 students **DO NOT TAKE** 210 final, or you will get a zero on 211 final

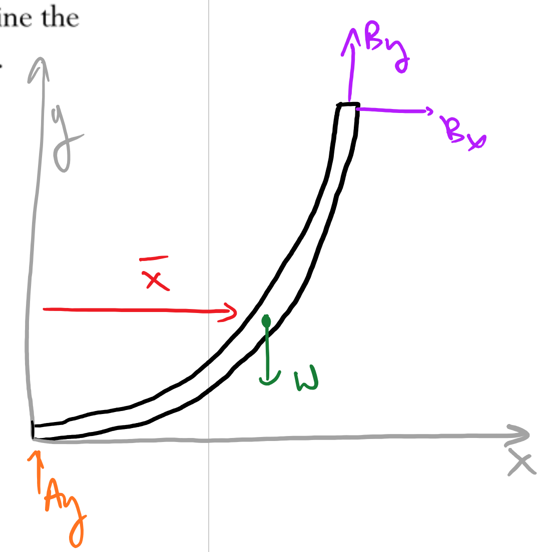
- NO HW 20

- HW 21 due **Thurs**
- HW 22 due **Tues**



If the rod has a weight per unit length of 100 N/m, determine the reaction supports at A and B.

DRAW FBD



Sum the forces and moments.

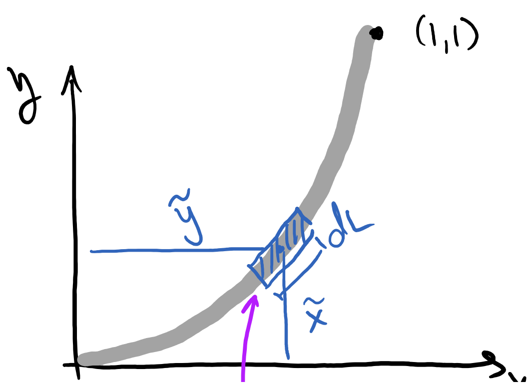
$$\Sigma F_x: \ B_x = 0$$

$$\Sigma F_y: \ A_y + B_y - W = 0$$

5 unknowns!

$$\Sigma M_B: -(1)A_y + (1-\bar{x})W = 0$$

Solve for $\bar{x}$ and W:

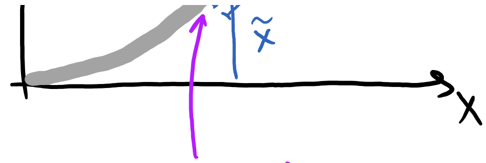


$$y = x^2 \rightarrow dy = 2x\,dx$$

$$\tilde{x} = x$$

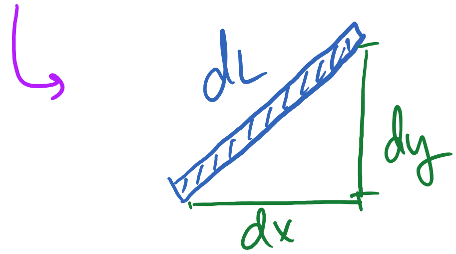
$$\tilde{y} = y$$

$$dL = \sqrt{dx^2 + dy^2}$$



$$dL = \sqrt{dx^2 + dy^2}$$

(zoom in)



$$dL = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

both expressions are valid but since $y = x^2$

$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{1 + 4x^2}\, dx$$

now apply equations for centroid of a line:

$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL} = \frac{\int x\sqrt{4x^2+1}\, dx}{\int \sqrt{4x^2+1}\, dx} = \underbrace{\frac{2\int_0^1 x\sqrt{x^2 + 1/4}\, dx}{2\int_0^1 \sqrt{x^2 + 1/4}\, dx}}_{\text{* everything expressed with a single variable}}$$

useful integrals:

Useful integrals:

$$\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2+a^2} + a^2\ln\left|x+\sqrt{x^2+a^2}\right|\right)$$

$$\int x\sqrt{x^2+a^2}\,dx = \frac{1}{3}\left(x^2+a^2\right)^{3/2}$$

$$\int x^2\sqrt{x^2+a^2}\,dx = \frac{1}{8}\left(x\sqrt{x^2+a^2}\,(2x^2+a^2) - a^4\ln\left(x+\sqrt{x^2+a^2}\right)\right)$$

$$\underline{\bar{x}} = \frac{\int_0^1 x\sqrt{x^2+1/4}\,dx}{\int\sqrt{x^2+1/4}\,dx} = \frac{0.8484\text{ m}^2}{1.478\text{ m}} = \boxed{0.574\text{ m}}$$

the total weight is

$$\underline{W} = \left(100\,\frac{N}{m}\right)\left(\int_0^1 dL\right) = \left(100\,\text{N/m}\right)\left(1.478\text{ m}\right) = \boxed{147.8\text{ N}}$$

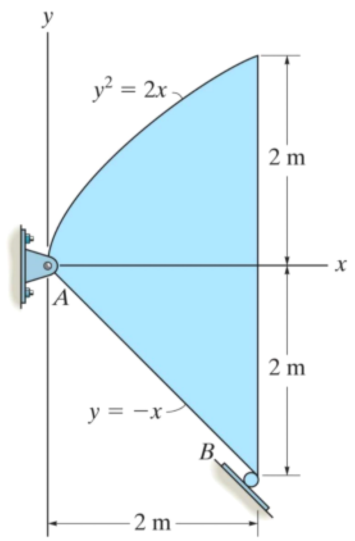
now we can solve for the reaction supports.

$$\Sigma M_B: \quad (-1)A_y + (1-\bar{x})W = 0$$

$$\underline{A_y} = (1-0.574)(147.8) = \boxed{\underline{63.1\text{ N}}}$$

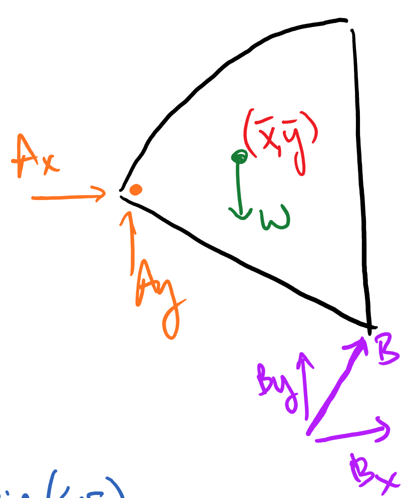
ΣF_y: $A_y + B_y - W = 0$

$B_y = W - A_y = 147.8 - 63.1 = \boxed{84.84 \text{ N}}$



The steel plate is 0.3 m thick and has a density of 7850 kg/m^3. Find the reactions at the pin and roller support.

DRAW A FBD:

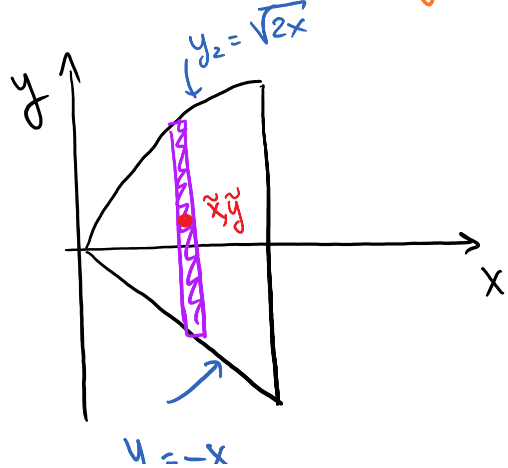


$$\Sigma F_x: \quad A_x + B\sin(45) = 0$$

$$\Sigma F_y: \quad A_y + B\cos(45) - W = 0$$

$$\Sigma M_A: \quad -\bar{x}W + 2B\sin(45) + 2B\cos(45) = 0$$

Solve for the AREA of the plate:



the differential element:

$$dA = (y_2 - y_1)dx = (\sqrt{2x} + x)dx$$

the centroid is:

$$(\tilde{x}, \tilde{y}) = \left(x, \frac{y_1 + y_2}{2}\right) = \left(x, \sqrt{2x} - x\right)$$

$y_1 = -x$

the centroid

$$(\tilde{x},\tilde{y}) = \left(x, \frac{y_1+y_2}{2}\right) = \left(x, \frac{\sqrt{2x}-x}{2}\right)$$

the eqns for the centroid:

$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA} = \frac{\int_0^2 x(\sqrt{2x}+x)\,dx}{\int_0^2 (\sqrt{2x}+x)\,dx} = \frac{\int_0^2 \sqrt{2}\,x^{3/2}+x^2\,dx}{\int_0^2 \sqrt{2}\,x^{1/2}+x\,dx}$$

$$\underline{\bar{x}} = \frac{\left[\frac{2\sqrt{2}}{5}x^{5/2}+\frac{1}{3}x^3\right]_0^2}{\left[\frac{2\sqrt{2}}{3}x^{3/2}+\frac{1}{2}x^2\right]_0^2} = \boxed{1.26\text{ m}}$$

$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA} = \frac{\int_0^2 \frac{\sqrt{2x}-x}{2}(\sqrt{2x}+x)\,dx}{\int_0^2 (\sqrt{2x}+x)\,dx}$$

$$\underline{\bar{y}} = \frac{\left[\frac{x^2}{2}-\frac{1}{6}x^3\right]_0^2}{\left[\frac{2\sqrt{2}}{3}x^{3/2}+\frac{1}{2}x^2\right]_0^2} = \boxed{0.143\text{ m}}$$

the Area of the plate is:

$$A = \int_A dA = \int_0^2 \sqrt{2x} + x \, dx = 4.667 \text{ m}^2$$

the weight is given by:

$$W = \rho V g = \rho A \cdot t g = (7850)(4.667)(0.3)(9.8)$$

$$\boxed{W = 107.81 \text{ kN}}$$

now use eqns. of equilibrium:

$\Sigma M_A = 0$

$$-\bar{x} W + 2B\cos(45) + 2B\sin(45) = 0$$

$$-\bar{x} W + 2\sqrt{2} B = 0$$

$$B = \frac{\bar{x} W}{2\sqrt{2}} = \frac{(1.26)(107.81)}{2\sqrt{2}} = \boxed{47.9 \text{ kN}}$$

$\Sigma F_x = 0$

$$A_x + B\sin(45) = 0$$

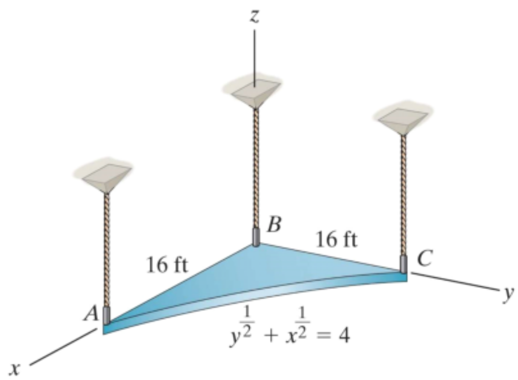
$$\boxed{A_x = -33.9 \text{ kN}}$$

$$\boxed{A_x = -33.9\ \text{kN}}$$

$$\Sigma F_y = 0$$

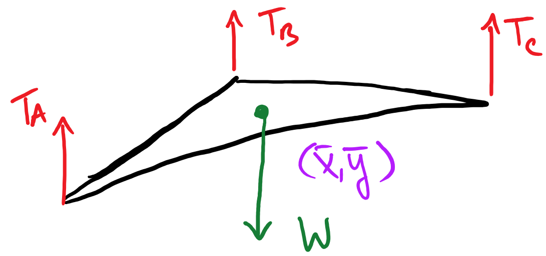
$$A_y + B\cos(45) - W = 0$$

$$A_y = W - B\frac{\sqrt{2}}{2} = \boxed{73.9\ \text{kN}}$$



The plate has a thickness of 0.25 ft and a specific weight of 180 lb/ft^3. Find the tension in each of the cords used to support it.

DRAW A FBD:



$$\Sigma\_ \; T_A + T + T_C - W = 0$$

6 unknowns!

$$\Sigma M_x: -\bar{y}W + (16)T_C = 0$$

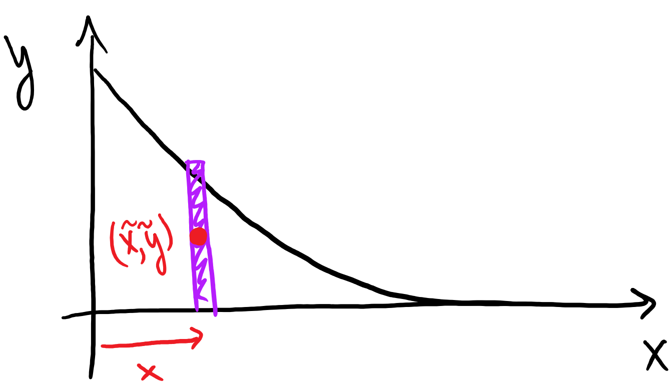
Area

$$W = \gamma \cdot V = \gamma \cdot A \cdot t$$

$$\Sigma M_y: \bar{x}W - (16)T_A = 0$$

$$(\bar{x},\bar{y}) = \frac{\left(\int \tilde{x}\,dA, \int \tilde{y}\,dA\right)}{\int dA}$$

need to find W and $(\bar{x},\bar{y})$:



the differential element:

$$dA = y\,dx$$

with centroid:

$$(\tilde{x},\tilde{y}) = (x, y/2)$$

bounded by:

$$y^{1/2} + x^{1/2} = 4 \quad \text{or} \quad y = \left(4 - x^{1/2}\right)^2$$

using eqns. for centroid:

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA} = \frac{\int x\, y dx}{\int y dx} = \frac{\int_0^{16} x\left(4 - x^{1/2}\right)^2 dx}{\int_0^{16} \left(4 - x^{1/2}\right)^2 dx} = \cdots$$

$$\underline{\bar{x} = 3.20 \text{ ft}}$$

$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA} = \frac{\int \frac{y}{2}\, y dA}{\int y dA} = \frac{\frac{1}{2}\int y^2 dA}{\int y dA} = \frac{\frac{1}{2}\int \left(4 - x^{1/2}\right)^4 dx}{\int \left(4 - x^{1/2}\right)^2 dx} = \cdots$$

$$\underline{\bar{y} = 3.20 \text{ ft}}$$

the weight is given by:

$$\underline{W} = \gamma \cdot V = \gamma \cdot A \cdot t = (180)\underbrace{(42.67)}_{\int dA = \int_0^{16} \left(4 - x^{1/2}\right)^2 dx}(0.25) = \underline{1920 \text{ lb}}$$

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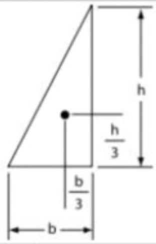
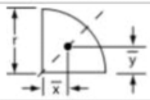
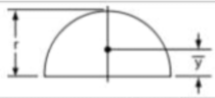
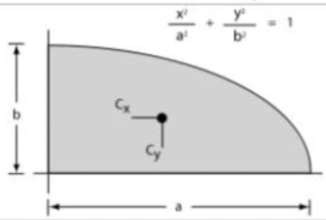
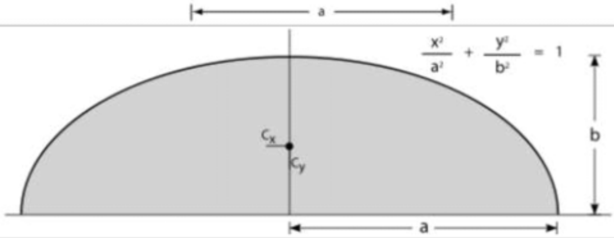
Centroid of typical 2D shapes

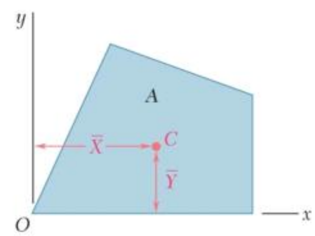
Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Right-triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

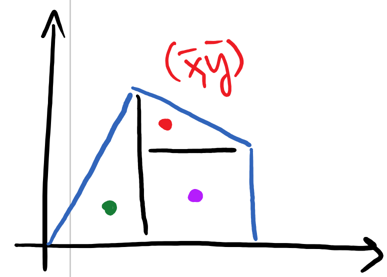
http://en.wikipedia.org/wiki/List_of_centroids

Composite bodies

A composite body consists of a series of connected simpler shaped bodies.

Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.





$(\bar{x},\bar{y})$ $(\bar{x},\bar{y})$

$$\bar{X} = \frac{\sum \tilde{x}W}{\sum W} \rightarrow \frac{\sum \tilde{x}M}{\sum M} \rightarrow \frac{\sum \tilde{x}V}{\sum V} \rightarrow \frac{\sum \tilde{x}A}{\sum A}$$

weight } mass } Volume } Area