To do ...

- **CBTF Quiz 6** – this week!
- 211 students **DO NOT TAKE** 210 final, or you will get a zero on 211 final
- HW20 PL for practice

- HW 21 due **Thurs**

Chapter 9: Center of gravity and centroid

Main goals and learning objectives

- Discuss the concept of the center of gravity, center of mass, and centroid

- Determine the location of the center of gravity and centroid for a system of discrete particles and a body of arbitrary shape

Center of gravity

To design the structure for supporting a water tank, we will need to know the weight of the tank and water as well as the locations where the resultant forces representing these distributed loads act.

How can we determine these resultant weights and their lines of action?
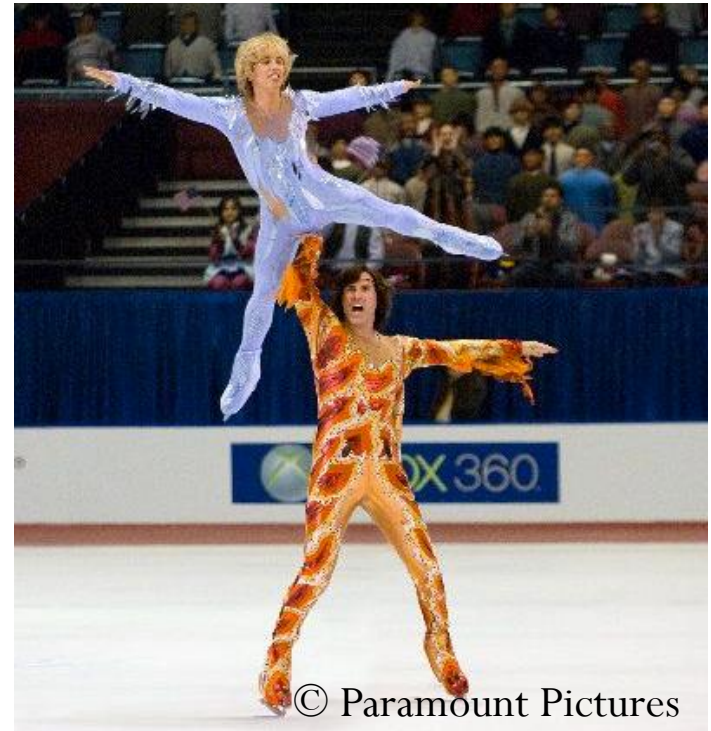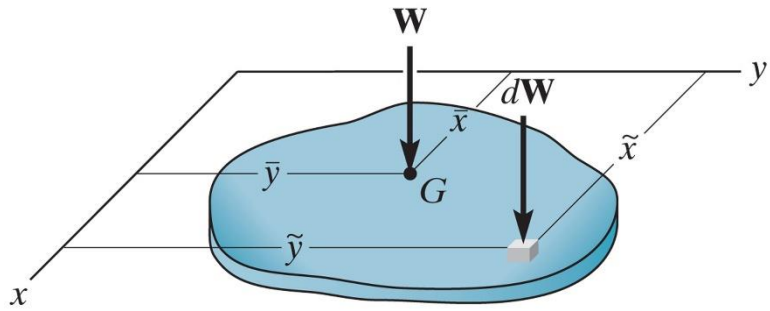
Center of gravity

teachersource.com

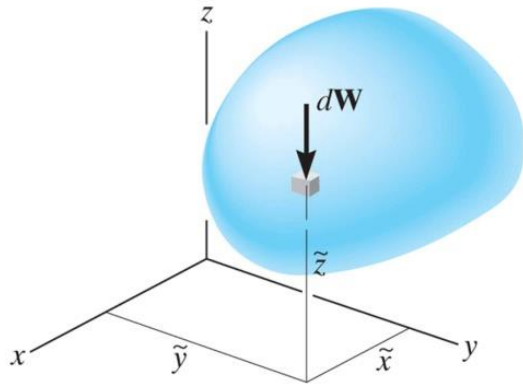© Paramount Pictures

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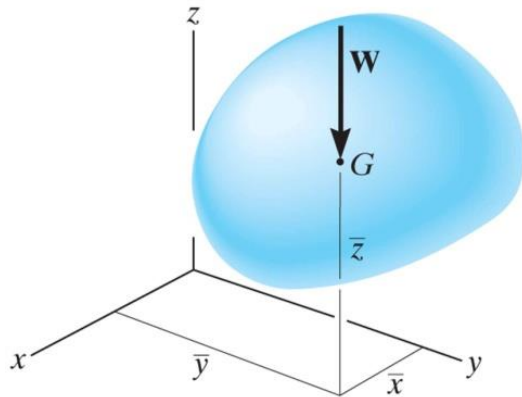thegadgetflow.com

Center of gravity

Center of gravity

A body is composed of an infinite number of particles, and so if the body is located within a gravitational field, then each of these particles will have a weight dW.

The **center of gravity (CG)** is a point, often shown as G, which locates the resultant weight of a system of particles or a solid body.

From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G.

If dW is located at point $(\tilde{x}, \tilde{y}, \tilde{z})$ then
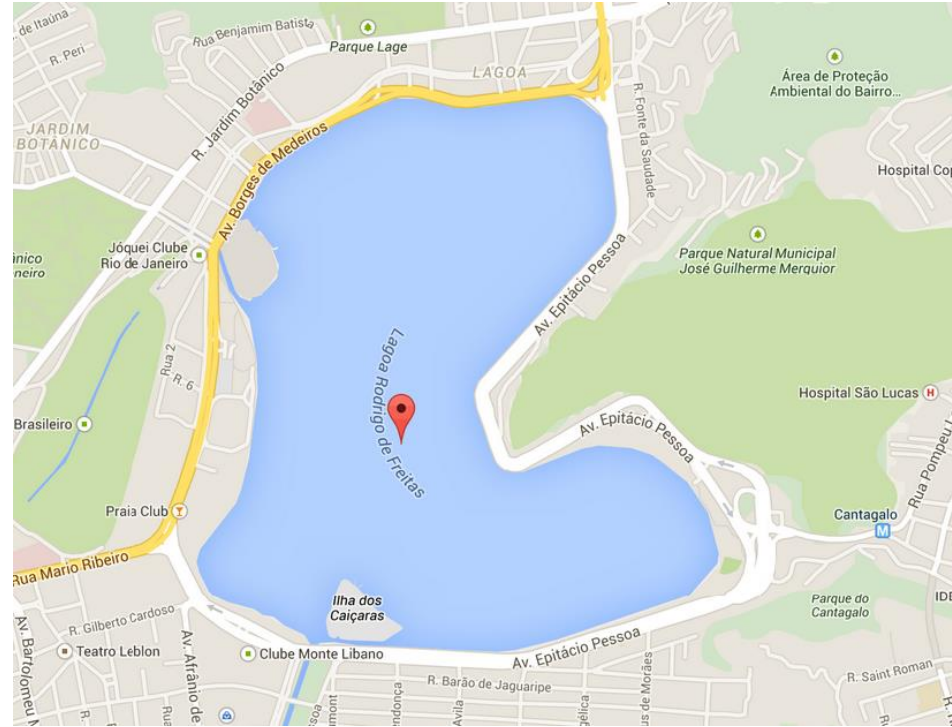
$$\bar{x}\, W = \int \tilde{x}\, \mathrm{d}W$$
$$\bar{y}\, W = \int \tilde{y}\, \mathrm{d}W$$
$$\bar{z}\, W = \int \tilde{z}\, \mathrm{d}W$$

Center of gravity

Center of Area

Center of Mass

$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm}$$

$$\bar{y} = \frac{\int \tilde{y}\, dm}{\int dm}$$

$$\bar{z} = \frac{\int \tilde{z}\, dm}{\int dm}$$

Center of Volume

$$\bar{x} = \frac{\int \tilde{x}\, dV}{\int dV}$$

$$\bar{y} = \frac{\int \tilde{y}\, dV}{\int dV}$$

$$\bar{z} = \frac{\int \tilde{z}\, dV}{\int dV}$$

Center of Area

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA}$$

$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$$\bar{z} = \frac{\int \tilde{z}\, dA}{\int dA}$$

Centroid

The centroid, C, is a point defining the geometric center of an object.

The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogeneous (density or specific weight is constant throughout the body).

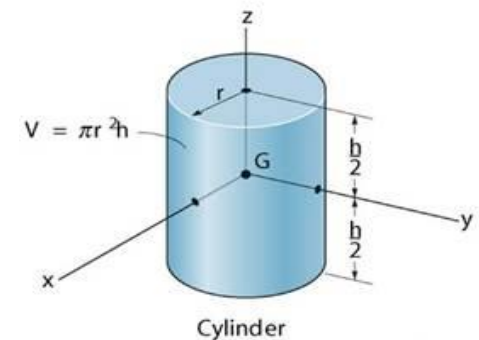If an object has an axis of symmetry, then the centroid of object lies on that axis.

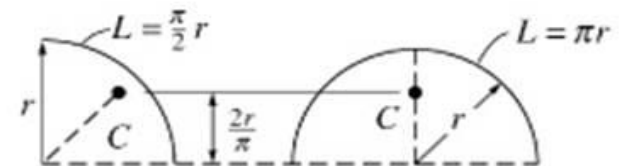In some cases, the centroid may not be located on the object.
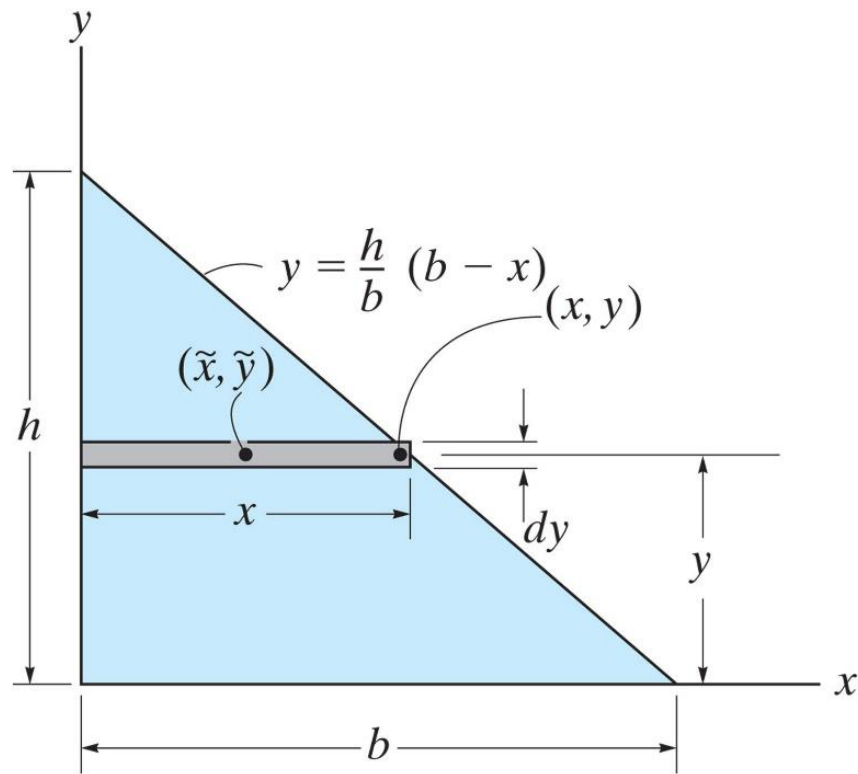
$A = bh$

Rectangular area

$A = \frac{1}{2}bh$

Triangular area

$V = \pi r^2 h$

Cylinder

$L = \frac{\pi}{2}r$
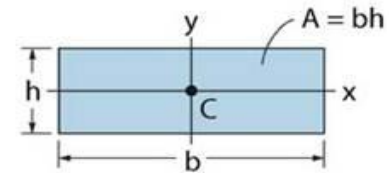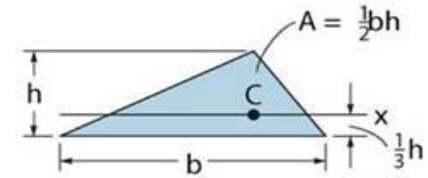
$L = \pi r$

Quarter and semicircle arcs

$y = \dfrac{h}{b}(b - x)$

(x, y)

$(\tilde{x}, \tilde{y})$

h

x

dy

y

b
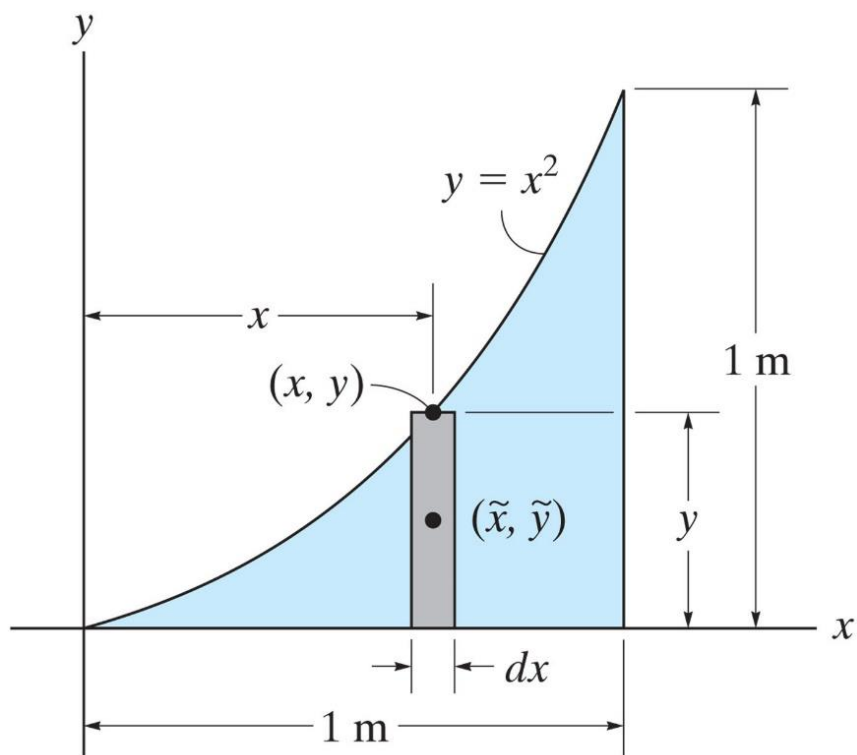
Determine the distance y measured from the x axis to the centroid of the area of the triangle.

Locate the centroid of the area.

$y = x^2$

(x, y)

dy

1 m

$(\tilde{x}, \tilde{y})$

y

x

$(1 - x)$

1 m

Locate the centroid of the area.