Lecture 28

Monday, November 6, 2017 12:29

To do ...

- **CBTF Quiz 6** —next week
- 211 students **DO NOT TAKE** 210 final, or you will get a zero on 211 final
- HW20 PL for practice
- HW 21 due **Thurs**

Chapter 9: Center of gravity and centroid

Main goals and learning objectives

- Discuss the concept of the center of gravity, center of mass, and centroid
- Determine the location of the center of gravity and centroid for a system of discrete particles and a body of arbitrary shape

Center of gravity

To design the structure for supporting a water tank, we will need to know the weight of the tank and water as well as the locations where the resultant forces representing these distributed loads act.

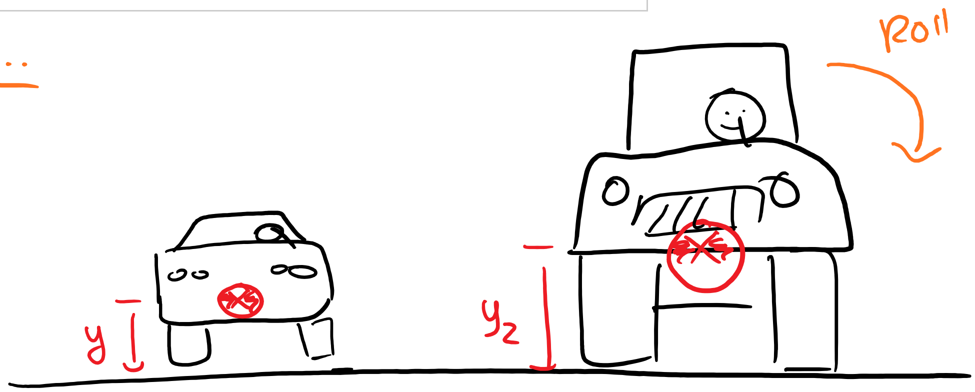
How can we determine these resultant weights and their lines of action?

think About ...

RACE CARS

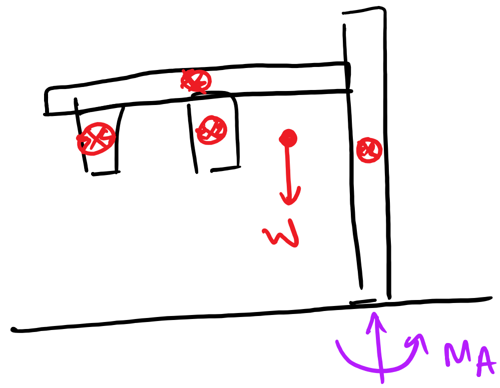
vs

trucks



high speeds & sharp turns!

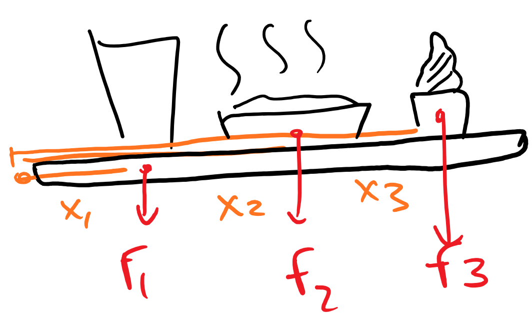
the street light

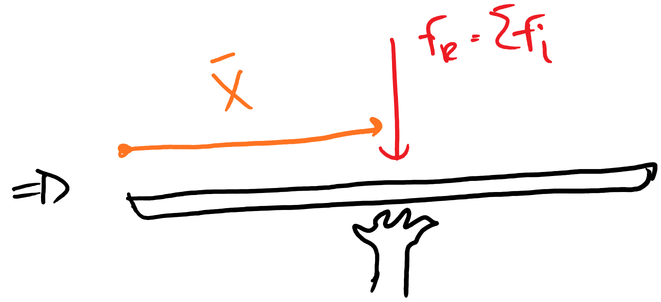


Reactions?

tray of food

Hold at both ends OR balance w/ one hand!





place hand under $\vec{F}_R$ ∴ no tipping!

$$\bar{x} F_R = \sum x_i F_i$$

$$\bar{x} F_R = \underbrace{\sum x_i f_i}_{\text{moments!}}$$

tipping.

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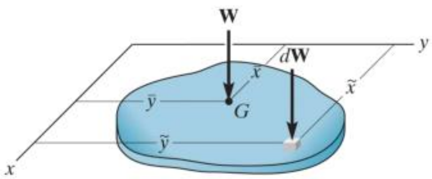
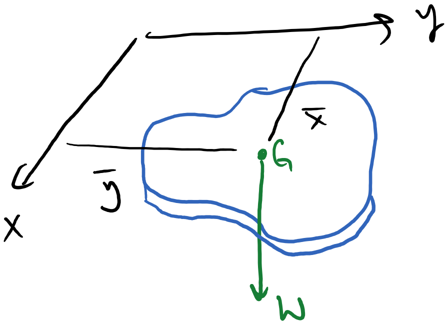
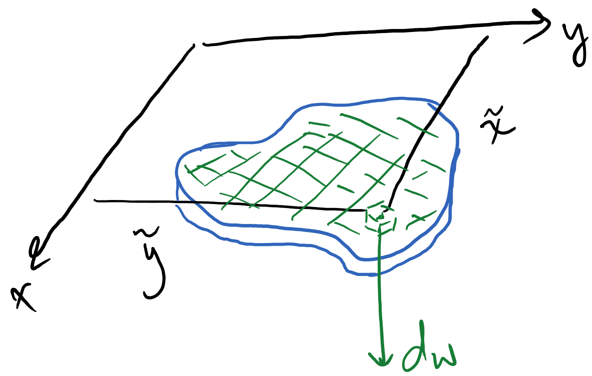
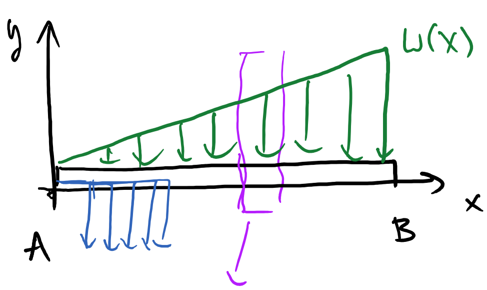
Center of gravity

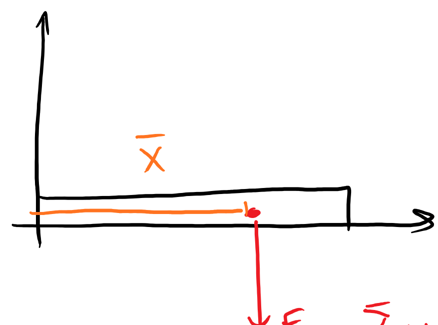




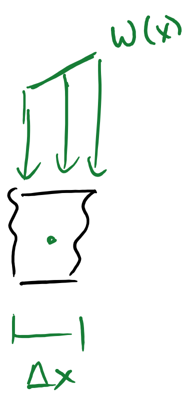


Analogous to distributed loads on beams.





$F_R = \sum w(x) dx = \int w(x)\,dx$



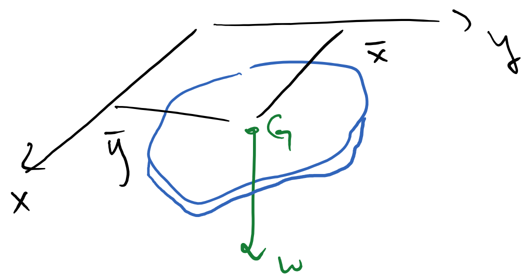
Sum moments about A.

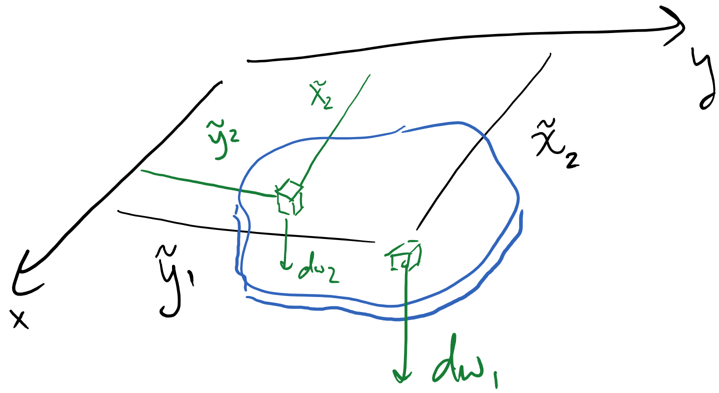
$$\Sigma M_A = \sum x_i\, w(x)\,\Delta x = \int x\, w(x)\,dx = \bar{x} F_R$$

$$\bar{x} = \int x\, w(x)\,dx$$

Δx

$$\bar{x} = \frac{\int x\, w(x)\, dx}{\int w(x)\, dx}$$





Sum the moments About x And y AXES.

$\sum M_y$: $\quad \bar{x}\, W = \tilde{x}_1 dw_1 + \tilde{x}_2 dw_2 + \cdots + \tilde{x}_n dw_n = \sum_i^n \tilde{x}_i\, dw_i$

$\sum M_x$: $\quad \bar{y}\, W = \tilde{y}_1 dw_1 + \tilde{y}_2 dw_2 + \cdots + \tilde{y}_n dw_n = \sum_i^n \tilde{y}_i\, dw_i$

in the limit:

$$\left.\begin{aligned} \bar{x} \int dw &= \int \tilde{x}\, dw \\ \bar{y} \int dw &= \int \tilde{y}\, dw \end{aligned}\right\} \text{Center of Gravity}$$

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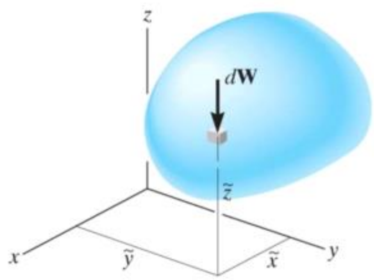
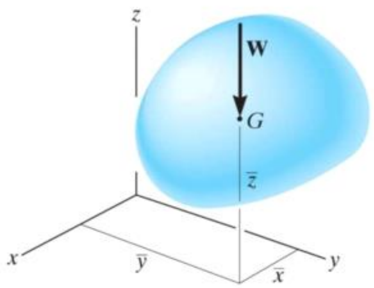
Center of gravity



A body is composed of an infinite number of particles, and so if the body is located within a gravitational field, then each of these particles will have a weight dW.



The **center of gravity (CG)** is a point, often shown as G, which locates the resultant weight of a system of particles or a solid body.

From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G.

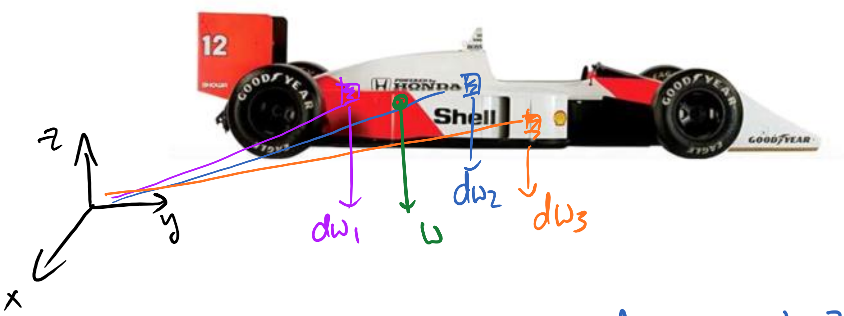
If dW is located at point $(\tilde{x}, \tilde{y}, \tilde{z})$ then

$$\bar{x}\,W = \int \tilde{x}\,\mathrm{dW}$$

$$\bar{y}\,W = \int \tilde{y}\,\mathrm{dW}$$

$$\bar{z}\,W = \int \tilde{z}\,\mathrm{dW}$$

Center of gravity



Sum the moments about $x, y, \, \& \, z$

$$\vec{r}\, W = \int_V \vec{r}\, dw \Rightarrow \begin{Bmatrix} \vec{\bar{x}} \\ \vec{\bar{y}} \\ \vec{\bar{z}} \end{Bmatrix} = \frac{\int_V \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} dw}{\int dw}$$

[center of Gravity]

CONSIDER A constant gravitational field

$$W = mg \longrightarrow dw = g\,dm$$

$$\bar{x} = \frac{\int \tilde{x}\, dw}{\int dw} = \frac{\int \tilde{x}\, g\, dm}{\int g\, dm} = \frac{\int \tilde{x}\, dm}{\int dm}$$

[center of mass]

CONSIDER the density of A MATERIAL.

$$M = \rho V \qquad \rho \rightarrow \text{density.}$$

$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm}$$

if $\rho =$ constant (i.e. homogeneous MATERIAL!) then $\bar{x} = \dfrac{\int \tilde{x} \rho\, dv}{\int \rho\, dv} = \dfrac{\int \tilde{x}\, dV}{\int dV}$

[center of volume]

$\Downarrow$

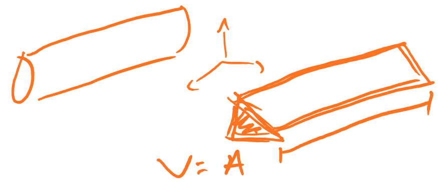
[... 3D body]

$\Downarrow$

[Centroid of A 3D body]

CONSIDER A SHAPE with symmetry

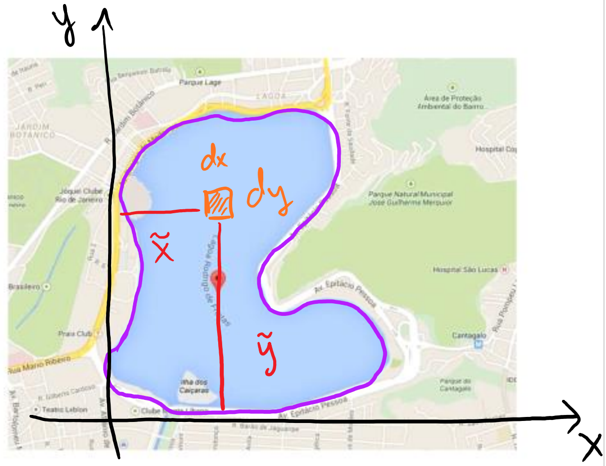
$$\bar{x} = \frac{\int \tilde{x}\, dV}{\int dV} = \frac{\int \tilde{x}\, t\, dA}{\int t\, dA} = \frac{\int \tilde{x}\, dA}{\int dA}$$



[Center of Area]

[Centroid of A 2D body.]

Center of Area



Consider the lagoon,

Start with dW, is g constant? → $dW = g\,dM$

what about dM, is ρ constant? → $dM = \rho\,dV$

what about dV, is the depth constant? no!

$$\bar{x} = \frac{\int \tilde{x}\,dv}{\int dv}$$ but if yes then $$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA}$$

Procedure for Analysis

- Select coordinate system, axes, differential element
  - dL line, dV volume
  - dA area
- express differential element in terms of coordinates describing the curve.

- experess $\tilde{x}, \tilde{y}$ in terms "

- Substitute $\tilde{x}, \tilde{y}$, And dL, dA into $\bar{x} = \dfrac{\int \tilde{x}\, dA}{\int dA}$

- express the function in terms of the SAme variable'.

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Center of Mass	Center of Volume	Center of Area
$\bar{x} = \dfrac{\int \tilde{x}\, dm}{\int dm}$	$\bar{x} = \dfrac{\int \tilde{x}\, dV}{\int dV}$	$\bar{x} = \dfrac{\int \tilde{x}\, dA}{\int dA}$
$\bar{y} = \dfrac{\int \tilde{y}\, dm}{\int dm}$	$\bar{y} = \dfrac{\int \tilde{y}\, dV}{\int dV}$	$\bar{y} = \dfrac{\int \tilde{y}\, dA}{\int dA}$
$\bar{z} = \dfrac{\int \tilde{z}\, dm}{\int dm}$	$\bar{z} = \dfrac{\int \tilde{z}\, dV}{\int dV}$	$\bar{z} = \dfrac{\int \tilde{z}\, dA}{\int dA}$

$\tilde{x}, \tilde{y}, \tilde{z}$ $\longrightarrow$ centroid of differential element

dv, dA, or dL

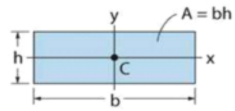
Centroid

The centroid, C, is a point defining the geometric center of an object.

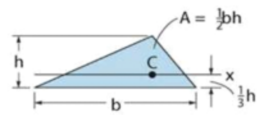
The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogeneous (density or specific weight is constant throughout the body).

If an object has an axis of symmetry, then the centroid of object lies on that axis.

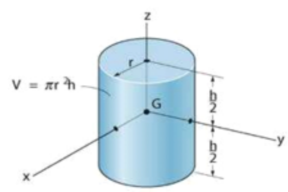
In some cases, the centroid may not be located on the object.



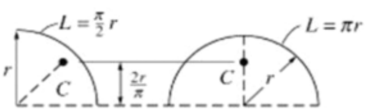
Rectangular area



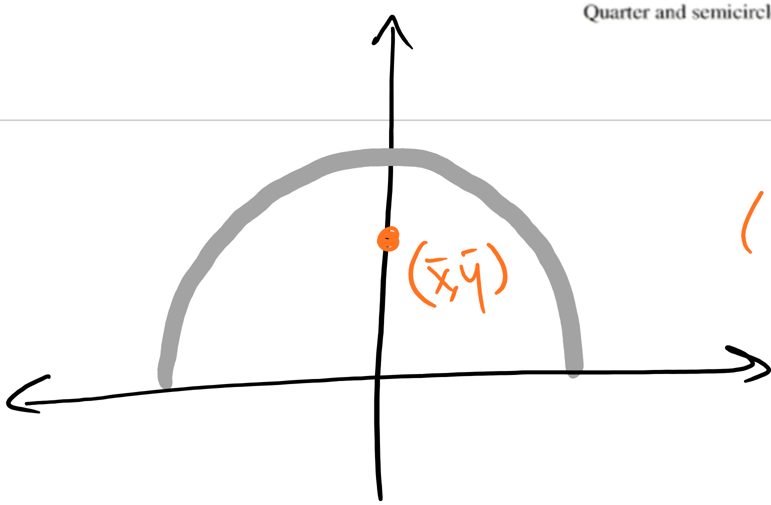
Triangular area



Cylinder

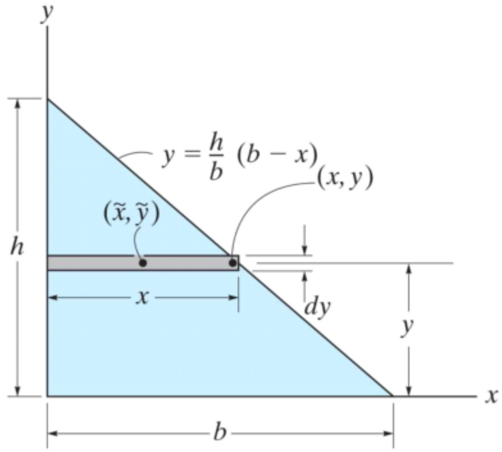


Quarter and semicircle arcs



$$(\bar{x},\bar{y}) = \left(0, \frac{\int \tilde{y}\,dL}{\int dL}\right)$$

symmetry!



Determine the distance $\overline{y}$ measured from the x axis to the centroid of the area of the triangle.

Consider the rectangular differential element:

$$dA = x\,dy$$

the centroid of each differential element is:

$$(\tilde{x}, \tilde{y}) = \left(\frac{x}{2}, y\right)$$

the area is bounded by the curve:

$$y = \frac{h}{b}(b - x) \quad \text{or} \quad x = \frac{b}{h}(h - y)$$

now apply the equation for $\overline{y}$:

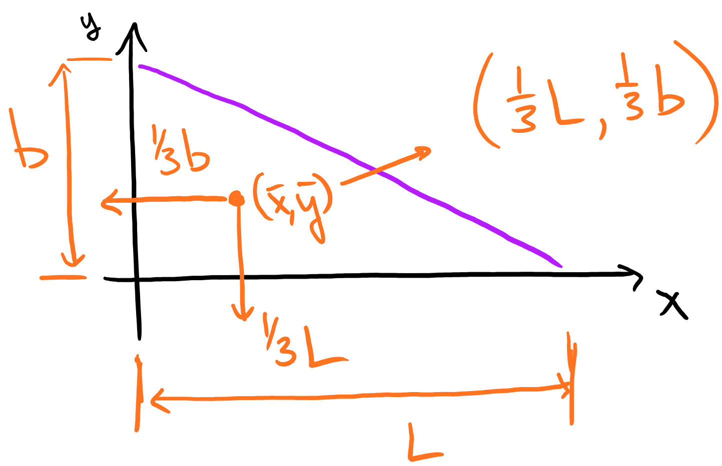
$$\overline{y} = \frac{\int \tilde{y}\,dA}{\int dA} = \frac{\int y\,x\,dy}{\int x\,dy} = \underbrace{\frac{\int_0^h y\left(\frac{b}{h}(h-y)\right)dy}{\int_0^h \frac{b}{h}(h-y)\,dy}}_{\text{expressed as}}$$

single variable y

$$\bar{y} = \frac{\frac{b}{h}\int_0^h (yh - y^2)\,dy}{\frac{b}{h}\int_0^h h - y\,dy} = \frac{\frac{hy^2}{2} - \frac{y^3}{3}\Big|_0^h}{hy - \frac{y^2}{2}\Big|_0^h}$$

$$\bar{y} = \frac{\frac{h^3}{2} - \frac{h^3}{3}}{h^2 - \frac{h^2}{2}} = \frac{\frac{h^3}{6}}{\frac{h^2}{2}} = \frac{h^3}{6}\cdot\frac{2}{h^2} = \boxed{\frac{h}{3}}$$

* this should look familiar...



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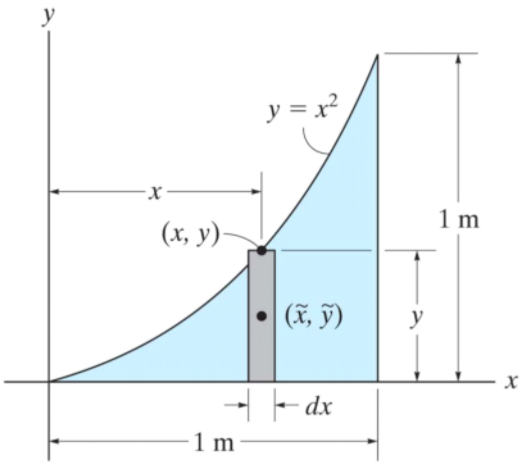


Locate the centroid of the area.

the differential element:

$$dA = y\,dx$$

with centroid:

$$(\tilde{x}, \tilde{y}) = \left(x, \frac{y}{2}\right)$$

bounded by the curve:

$$y = x^2$$

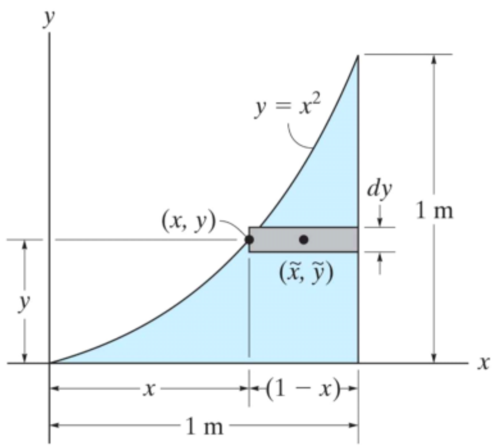
the eqns. for the centroid are:

$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA} = \frac{\int x\,y\,dx}{\int y\,dx} = \underbrace{\frac{\int_0^1 x^3\,dx}{\int_0^1 x^2\,dx}}_{\text{single variable } x} = \cdots$$

$$\bar{x} = \frac{\left.\frac{x^4}{4}\right|_0^1}{\left.\frac{x^3}{3}\right|_0^1} = \frac{3}{4}\,\text{m}$$

$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA} = \frac{\int \frac{y}{2}\, y\, dx}{\int y\, dx} = \frac{\frac{1}{2}\int_0^1 x^4\, dx}{\underbrace{\int_0^1 x^2\, dx}_{\text{single variable } x}} = \cdots$$

$$\bar{y} = \frac{\frac{1}{2} \cdot \frac{x^5}{5}\Big|_0^1}{\frac{x^3}{3}\Big|_0^1} = \frac{3}{10}\, m$$



Locate the centroid of the area.

the differential element:

$$dA = (1-x)\,dy$$

with centroid

$$(\tilde{x}, \tilde{y}) = \left(x + \frac{1-x}{2},\ y\right) = \left(\tfrac{1}{2}(x+1),\ y\right)$$

bounded by:

$$y = x^2 \implies x = \sqrt{y}$$

the eqns for the centroid:

$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA} = \frac{\int \frac{1}{2}(x+1)(1-x)\,dy}{\int (1-x)\,dy} = \frac{\frac{1}{2}\int (1-x^2)\,dy}{\int 1-x\,dy}$$

$$\bar{x} = \underbrace{\frac{\frac{1}{2}\int_0^1 (1-y)\,dy}{\int_0^1 (1-y^{1/2})\,dy}}_{\text{single variable } y} = \cdots$$

$$\bar{x} = \frac{1}{2}\left(y - \frac{y^2}{2}\right)\Big|_0^1 = \frac{3}{\;}\ \text{m}$$

$$\frac{}{\frac{2}{3}y^{3/2}\Big|_0^1}$$

$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA} = \frac{\int y(1-x)\,dy}{\int (1-x)\,dy} = \frac{\int_0^1 y - y^{3/2}\,dy}{\underbrace{\int_0^1 1 - y^{1/2}\,dy}_{\text{single variable } y}} = \ldots$$

$$\bar{y} = \frac{\frac{y}{2} - \frac{2}{5}y^{5/2}\Big|_0^1}{\frac{2}{3}y^{3/2}\Big|_0^1} = \frac{3}{10}\text{ m}$$