Lecture 12

Monday, September 25, 2017 11:56

To do …

- HW 8 due **Tues**
- HW 9 due **Thurs**

- **Quiz 3 next week, in class, Monday**

- **DRES accommodations for CBTF** – Take to CBTF proctor ASAP
- **DRES accommodations for in class quiz/final** – send a private message to instructors on piazza with PDF of DRES letter ASAP

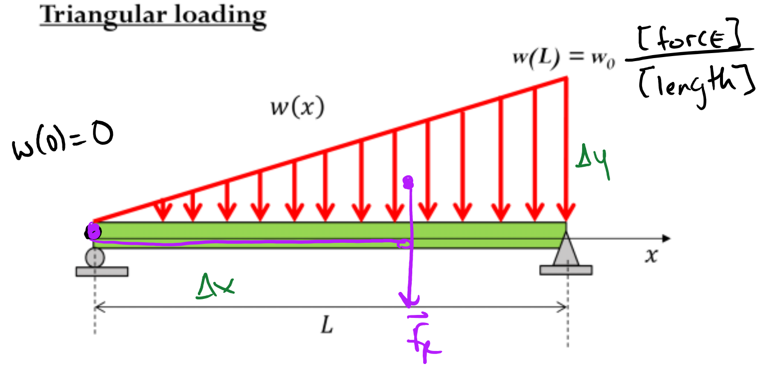
Chapter 4: Force System Resultants

Main goals and learning objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- Provide a method for finding the moment of a force about a specified axis
- Define the moment of a couple
- Method to simplify a force and couple system to an equivalent system
- Indicate how to reduce a simple distributed loading to a resultant force having a specified location

Friday, September 16, 2016 10:55

Triangular loading



1. find $\vec{F}_R = \int_0^L w(x)\,dx$
2. Find $\vec{M}_R = \int_0^L x\,w(x)\,dx$
3. find $\bar{x} = \dfrac{\vec{M}_R}{\vec{F}_R}$

$y(x) = mx + b \leftarrow$ y-intercept

$\uparrow$ slo

$\frac{\text{rise}}{\text{run}} \rightarrow \frac{\Delta y}{\Delta x} = \frac{w_0}{L}$

$\text{Area} = \frac{1}{2}bh = \frac{1}{2}Lw_0 = \vec{F}_R$

$w(x) = \frac{w_0}{L}x$

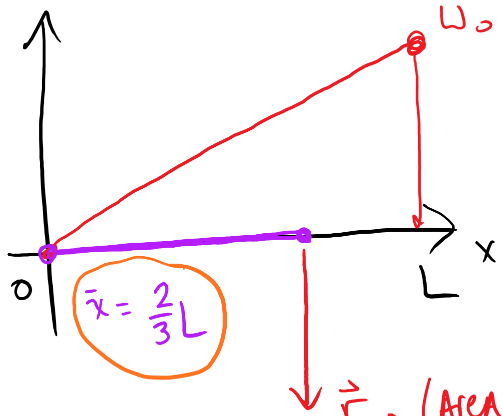
$$\vec{F}_R = \int_0^L w(x)\,dx = \int_0^L \frac{w_0}{L}x\,dx = \frac{w_0}{L}\int_0^L x\,dx = \frac{w_0}{L}\left[\frac{x^2}{2}\Big|_0^L\right]$$

$$\vec{F}_R \quad \frac{1}{2}w_0 L$$ *(AREA of triangle)

$$\vec{M}_R = \int_0^L x\,w(x)\,dx = \int_0^L \frac{w_0}{L}x^2\,dx = \frac{w_0}{L}\int_0^L x^2\,dx = \frac{w_0}{L}\left[\frac{x^3}{3}\Big|_0^L\right]$$

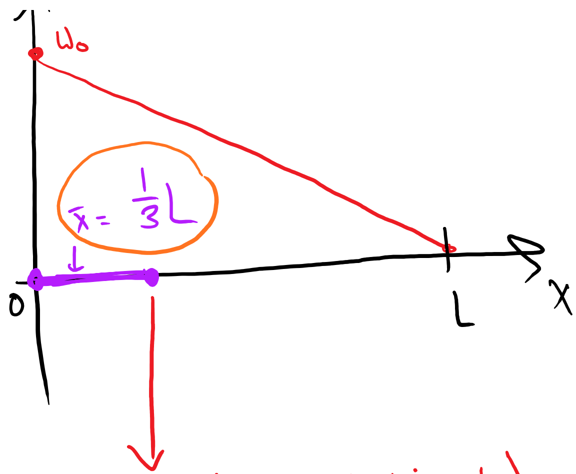
$$\vec{M}_R = \frac{1}{3}w_0 L^2$$

$$\vec{M}_R = \bar{x}\vec{F}_R \Rightarrow \bar{x} = \frac{M_R}{F_R} = \frac{\frac{1}{3}w_0L^2}{\frac{1}{2}w_0L} = \frac{2}{3}L \;//$$



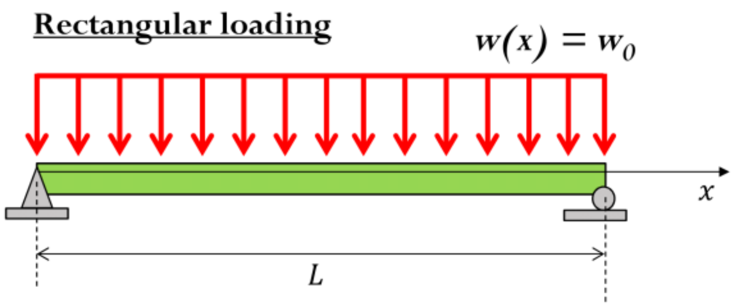
$\vec{F}_R$ - (Area of triangle)

$\frac{1}{2} L w_0$



$\vec{F}_R$ - (Area of triangle)

$\frac{1}{2} L w_0$



1. find $F_R = \int w(x)\,dx$
2. find $M_R = \int x\,dF = \int x\,w(x)\,dx$
3. find $\bar{x} = M_R / F_R$

$$w(x) = mx + b = 0 + w_0 \Rightarrow w(x) = w_0$$

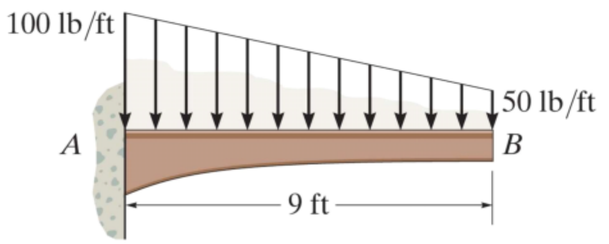
$$\vec{F}_R = \int_0^L w_0\,dx = w_0 x \Big|_0^L = w_0 L$$ * (AREA of RECtAngLE!)

$$\vec{M}_R = \int_0^L x\,dF = \int_0^L w_0 x\,dx = w_0 \frac{x^2}{2}\Big|_0^L = \frac{1}{2} w_0 L^2$$

$$\bar{x} = \frac{M_R}{F_R} = \frac{\frac{1}{2} w_0 L^2}{w_0 L} = \frac{L}{2}$$

Always!

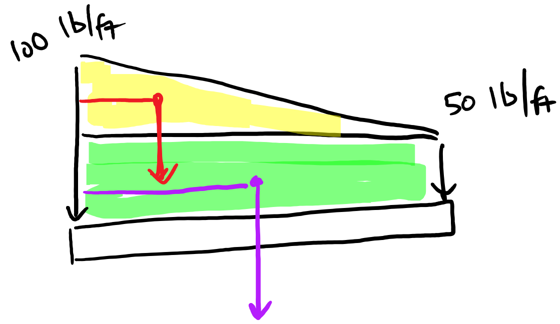
$\vec{F}_R$ – (AREA of RECtAngLE – $w_0 L$)



Determine the magnitude and location of the equivalent resultant of this load.

-DRAW FBD

- Split into simple loads.



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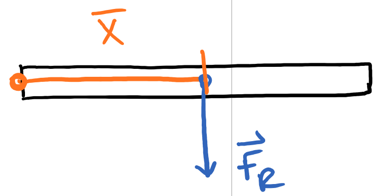


$$\vec{F}_{R_1} = \tfrac{1}{2}(9)(100-50) = \underline{225\text{ lb}}$$

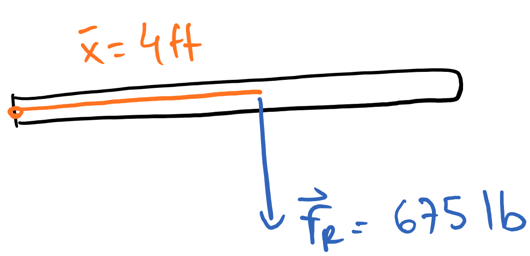
$$\vec{F}_{R_2} = (9)(50) = \underline{450\text{ lb}}$$

$$\vec{F}_R = \vec{F}_{R_1} + \vec{F}_{R_2} = \boxed{675\text{ lb}}$$

Sum moments.

$$(3)(225) + (4.5)(450) = \bar{x}\,(675)$$

$$\boxed{\bar{x} = 4\text{ ft}}$$



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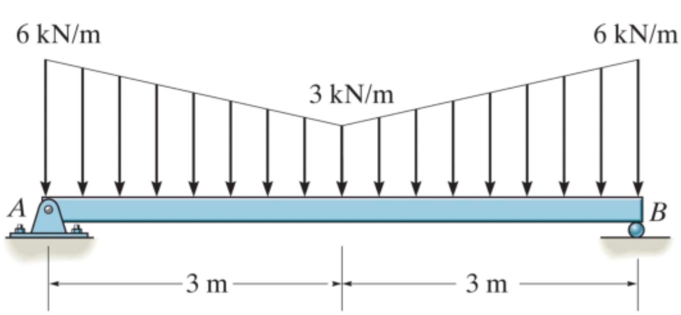
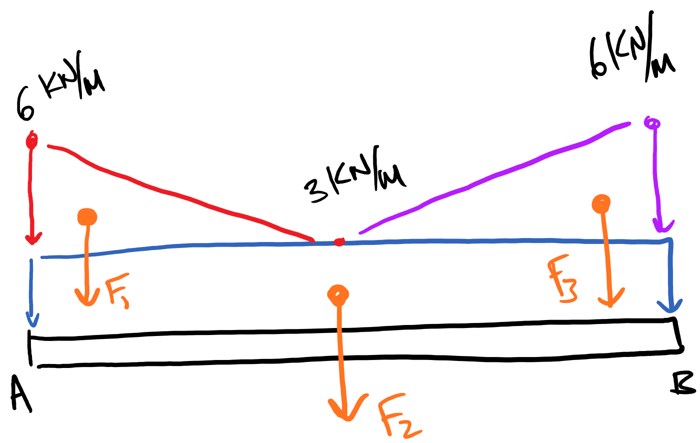


Replace the distributed loading by an equivalent resultant force and couple moment acting at point A.

– DRAW FBD

– split into simple loads.

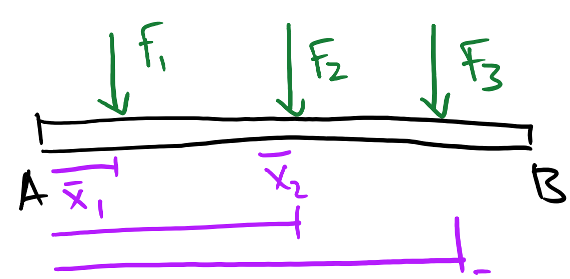


$$\vec{F}_R = \Sigma F = -F_1 - F_2 - F_3$$

$$F_R = -\frac{1}{2}(3_m)\left(3\frac{kN}{m}\right) - (6_m)\left(3\frac{kN}{m}\right) - \frac{1}{2}(3_m)\left(3\frac{kN}{m}\right)$$

$$\boxed{F_R = -27\hat{j}\ \text{kN}}$$

Summing the moments about A:

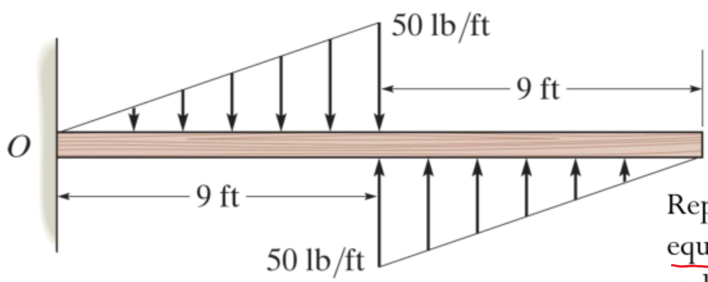


$$\vec{M}_A = -x_1F_1 - x_2F_2 - x_3F_3$$

$$\vec{M}_A = -\left(\tfrac{1}{3}\cdot 3\,m\right)\left(\tfrac{1}{2}(3)(3)\right) - (3\,m)(6\cdot 3) - \left(3 + \tfrac{2}{3}\cdot 3\right)\left(\tfrac{1}{2}\cdot(3)(3)\right)$$

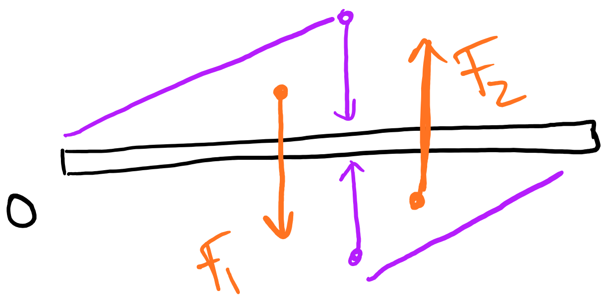
$$\boxed{\vec{M}_A = -81\ \text{kN}\cdot\text{m}\quad (\text{cw})}$$

therefore:



Replace the loading by an equivalent resultant force and couple moment acting at point O.

−DRAW FBD

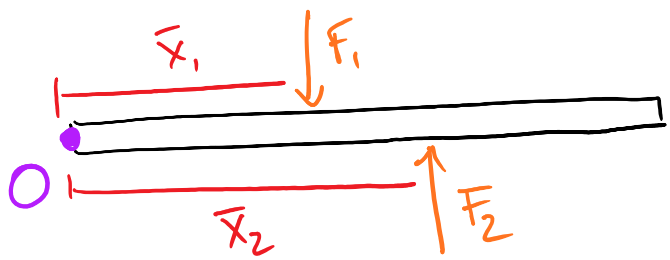


Sum the forces:

$$\vec{F}_R = \sum F_y = -\tfrac{1}{2}(9\,ft)(50\ lb/ft) + \tfrac{1}{2}(9\,ft)(50\ lb/ft)$$

$$\boxed{\vec{F}_R = 0}$$

Sum moments About point O:



$$\vec{M}_O = -\bar{x}_1 F_1 + \bar{x}_2 F_2$$

$$\vec{M}_O = -\left(\tfrac{2}{3}\cdot 9\right)F_1 + \left(9 + \tfrac{1}{3}\cdot 9\right)F_2$$

$$\vec{M}_O = -(6)F_1 + (12)F_2$$

but $|F_1| = |F_2|$; $-\vec{F}_1 = \vec{F}_2$

So it is A couple moment!

$$\vec{M}_O = (6\ \text{ft})\left(\tfrac{1}{2}\cdot 9\ \text{ft}\cdot 50\ \tfrac{\text{lb}}{\text{ft}}\right)$$

$$\boxed{\vec{M}_O = 1350\ \text{lb}\cdot\text{ft} = 1.35\ \text{kip}\cdot\text{ft}\ \ (\text{ccw})}$$

Chapter 5: Equilibrium of rigid bodies

Main goals and learning objectives

- Develop the equations of equilibrium for a rigid body
- Introduce the concept of the free-body diagram for a rigid body
- Solve rigid body equilibrium problems using the equations of equilibrium

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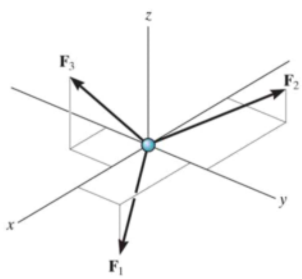
Equilibrium of a Rigid Body



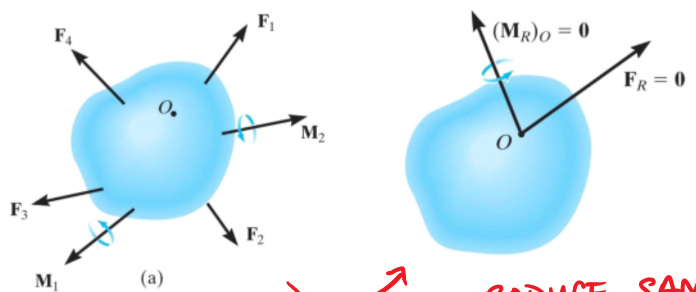
In contrast to the forces on a particle, the forces on a rigid-body are not usually concurrent and may cause rotation of the body.

equilibrium of A particle

$\Sigma F_x = 0$

$\Sigma F_y = 0$

We can reduce the force and couple moment system acting on a body to an equivalent resultant force and a resultant couple moment at an arbitrary point O.



*PRODUCE SAME external effects

Rigid Body Equilibrium

$$\Sigma \vec{F} = \begin{bmatrix} \Sigma F_x \\ \Sigma F_y \\ \Sigma F_z \end{bmatrix} = 0$$

$$\Sigma \vec{M}_o = \Sigma(\vec{r} \times \vec{F}) + \Sigma \vec{M}_c = 0$$

Equilibrium of a Rigid Body

Static equilibrium:

$\vec{F}_R = \Sigma \vec{F} = 0$ — no translation

$\vec{M}_R = \Sigma \vec{M}_O = 0$ — no rotation!

Maintained by reaction forces and moments

- forces from supports/constraints Are exactly Enough to produce zero force And moment

Assumption of rigid body

- Shape/Dimensions unchanged due to Applied loads
- Internal forces Are never Shown
  ↳ cancel out And Do not create external effect on body.

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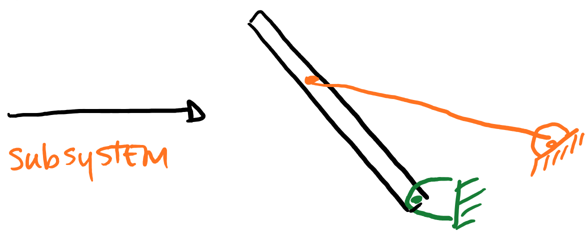
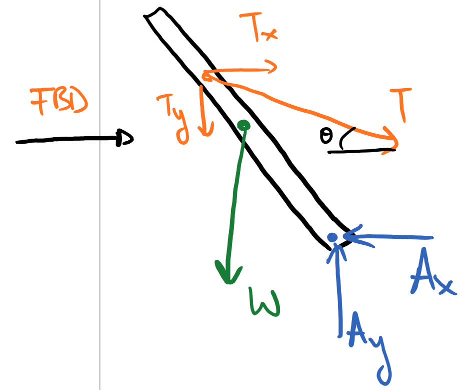
Process of solving rigid body equilibrium problems

1. Create idealized model (modeling and assumptions)





A_x, A_y reaction forces

2. Draw free body diagram showing ALL the external (applied loads and supports)

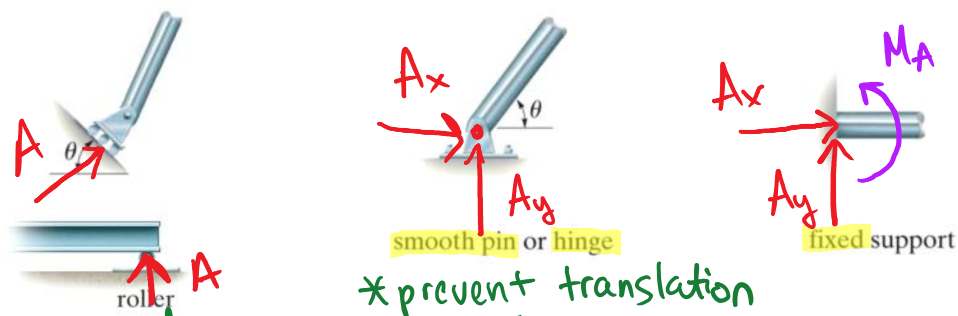
3. Apply equations of equilibrium

*Best way to account for known/unknown forces/couples,

FBD - outlined shape, free of surroundings.

Equilibrium in two-dimensional bodies

Support reactions



* normal force only

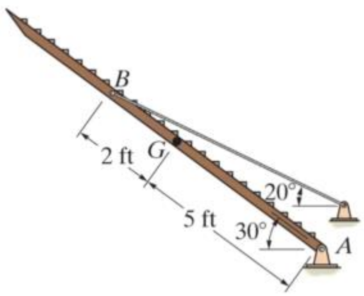
* prevent translation only

* prevent translation And Rotation

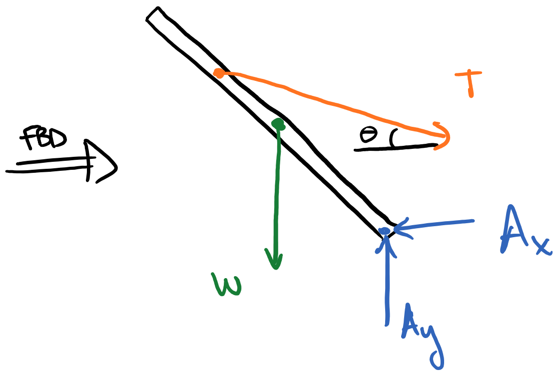
* Support prevents translation by exerting A force in the opposite direction

* Support presents Rotation by exerting A couple Moment in the opposite direction.

The uniform truck ramp has weight 400 lb and is pinned to the body of the truck at each side and held in the position shown by the two side cables. Determine the reaction forces at the pins and the tension in the cables.



idealized model



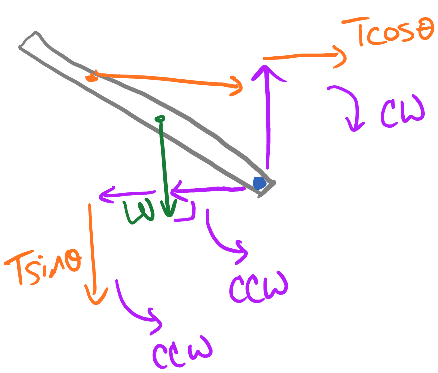
* 3 unknowns: $|T|$, A_x, A_y

Sum of forces:

ΣF_x: $T\cos\theta - A_x = 0 \rightarrow$ (1) $A_x = T\cos\theta$

ΣF_y: $A_y - W - T\sin\theta = 0 \rightarrow$ (2) $A_y = W + T\sin\theta$

Sum of moments:



$$\sum \vec{M}_A = W(5\cos(30)) + (T\sin\theta)(7\cos(30)) - (T\cos\theta)(7\sin(30)) = 0$$

$$T\left(7\cos(30)\sin(20) - 7\sin(30)\cos(20)\right) = 5\cos(30)W$$

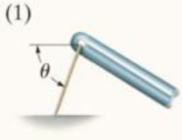
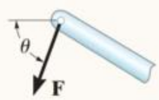
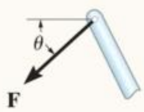
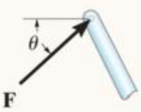
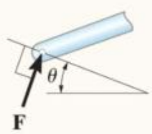
$$\boxed{T = 1425 \text{ lb}}$$

$$A_x = T\cos(20) = \boxed{1339 \text{ lb}}$$

$$A_y = W + T\sin(20) = \boxed{887 \text{ lb}}$$

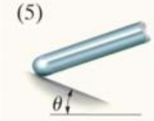
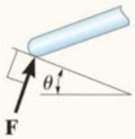
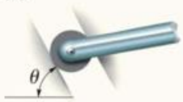
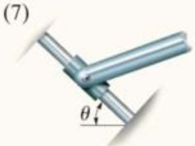
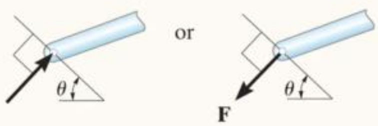
Types of connectors

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

Types of connectors

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(5)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  roller or pin in confined smooth slot		One unknown. The reaction is a force which acts perpendicular to the slot.
(7)  member pin connected to collar on smooth rod		One unknown. The reaction is a force which acts perpendicular to the rod.

continued

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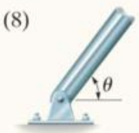
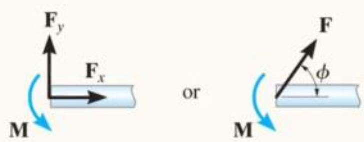
Types of connectors

TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.