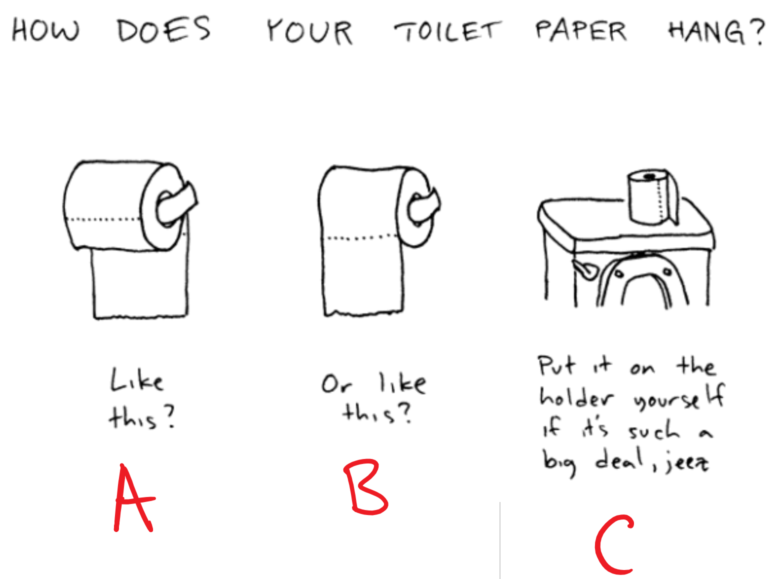
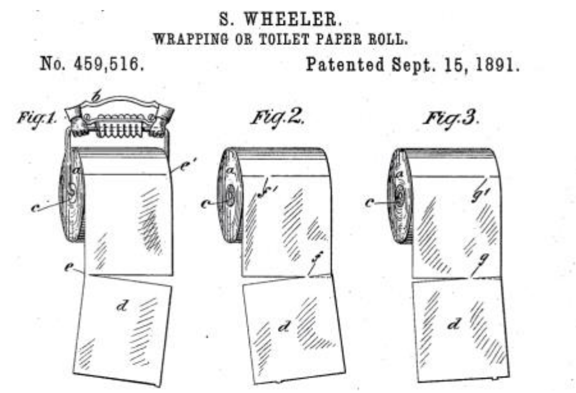
Lecture 11

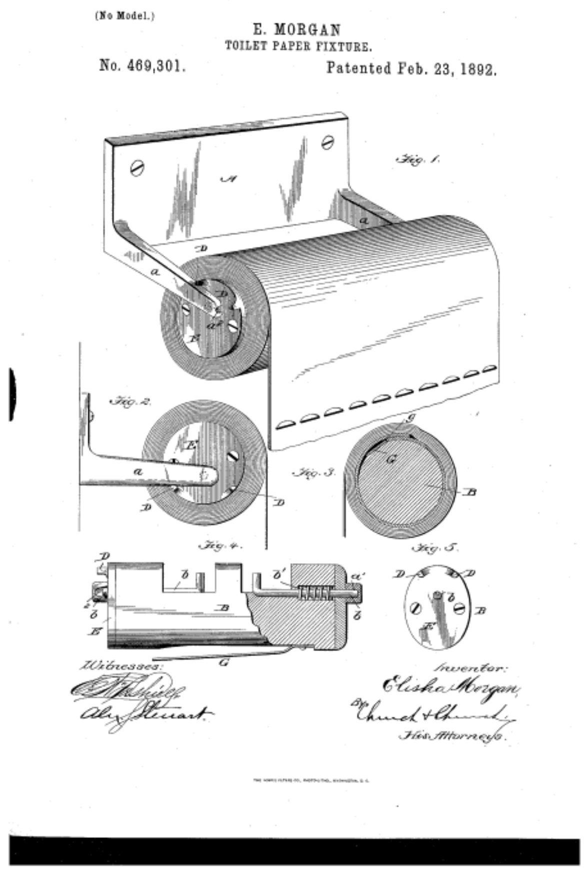
Friday, September 22, 2017 11:41

To do ...

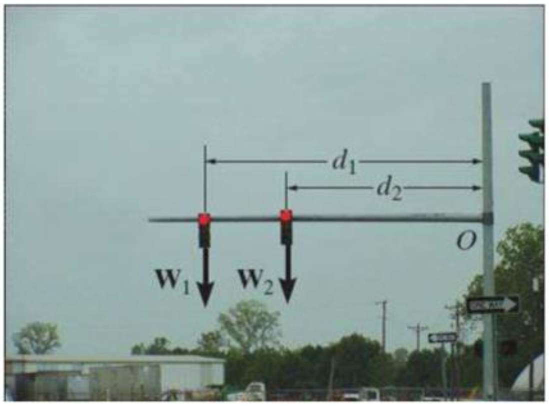
- Quiz 2 this week (**ends on today!**)
- WA 1 **due TODAY @ 11:59 pm**
- HW 8 due **Tues**
- HW 9 due **Thurs**
- **In class quiz 3 MONDAY, Oct 2**
- **Thank you for your feedback!!**
- **Happy Autumn Equinox!!**

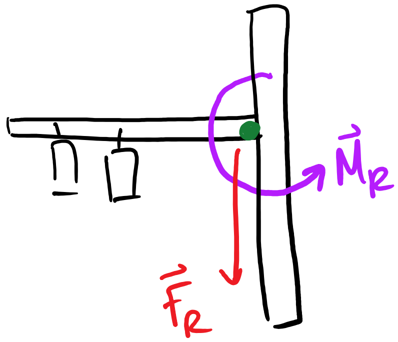


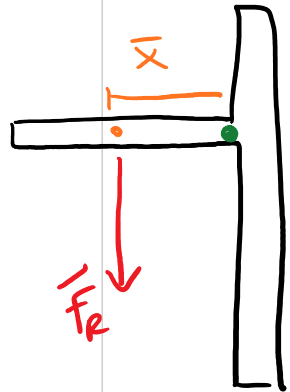

S. WHEELER.
WRAPPING OR TOILET PAPER ROLL.
No. 459,516.
Patented Sept. 15, 1891.
Fig.1.
Fig.2.
Fig.3.


(No Model.)
E. MORGAN
TOILET PAPER FIXTURE.
No. 469,301.
Patented Feb. 23, 1892.
Witnesses:
Inventor:
Elisha Morgan
His Attorneys.

What is the equivalent system?



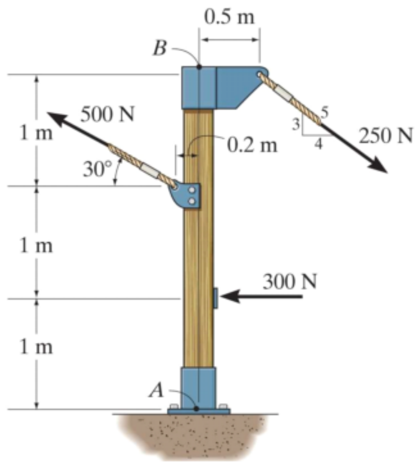




note that $\vec{F}_R \perp \vec{M}_R$

$$\bar{x} = \frac{\vec{M}_R}{\vec{F}_R}$$

same external effects



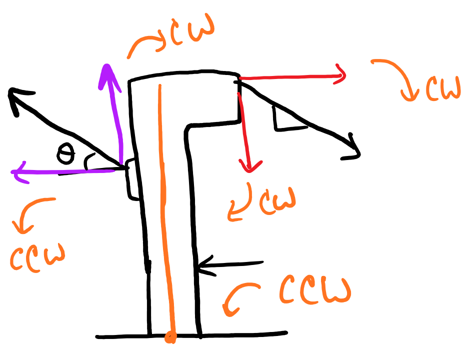
Replace the force system acting on the post by a resultant force and resultant moment about point A, and specify where its line of action intersects the post AB measured from point A.

- sum forces
- find and sum moments

$$\Sigma F_x: \quad \frac{4}{5}250 - 300 - 500\cos(30) = -533\text{ N}$$

$$\Sigma F_y: \quad 500\sin(30) - \frac{3}{5}250 = 100\text{ N}$$

$$|\vec{F}_R| = \sqrt{F_x^2 + F_y^2} = \underline{542\text{ N}} \qquad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \underline{10.6°}$$

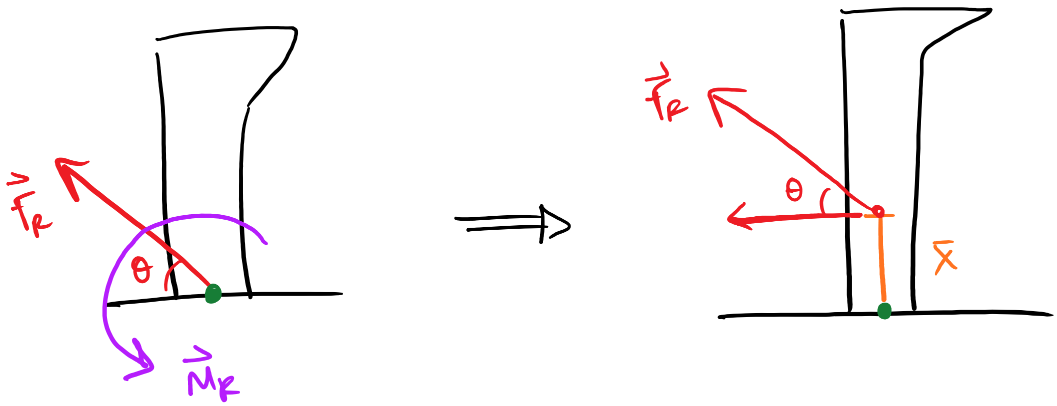


$$\vec{M}_R = \Sigma M = (1)(300) + (2)(500\cos(30)) - (0.2)(500\sin(30))$$
$$- (0.5)\left(\frac{3}{5}250\right) - (3)\left(\frac{4}{5}250\right) = \bar{x}\,\underline{\vec{F}_{R_x}}$$

only x-component creates

$$\underline{\bar{x} = 0.827\text{ m}}$$

moment.



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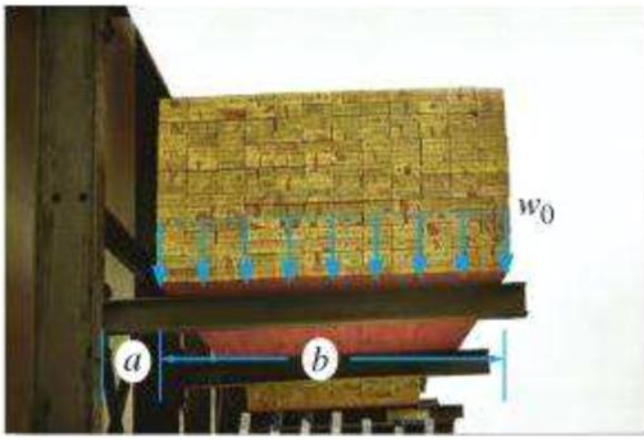
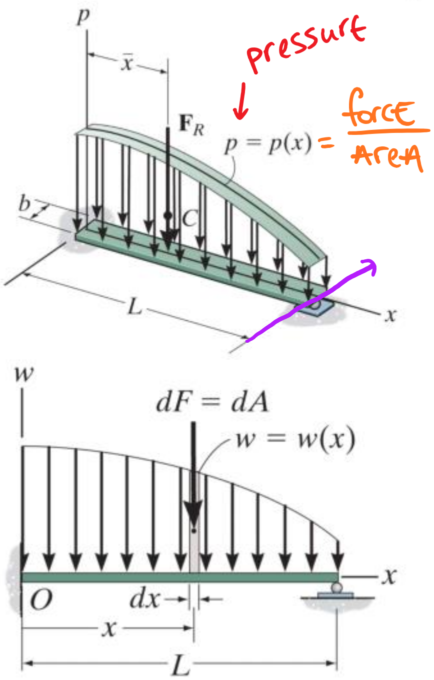
Reduction of a simple distributed load



vs. force applied at single point.

Reduction of a simple distributed load



In structural analysis, we often are presented with a **distributed load** $w(x)$ (force/unit length) and we need to find the equivalent loading F.

Example of such forces are winds, fluids, or the weight of items on the body's surface.

$$w(x) = p(x) \cdot b = \frac{N}{m^2} \cdot m = \frac{N}{m} = \frac{\text{force}}{\text{length}}$$

$$w(x) = \frac{dF}{dx}$$

Replace coplanar parallel force system with a single equivalent resultant force

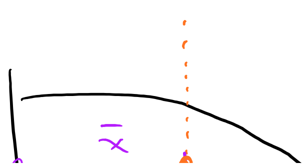
$$M_i = x\, dF(x)$$

$$\vec{M}_R = \sum M_i = \sum x\, dF(x)$$

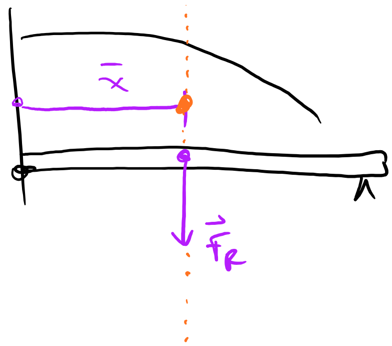
$$\vec{F}_R = \sum dF = \int_0^L dF = \int_0^L w(x)\,dx = \text{Area under the curve.}$$

$$\vec{M}_R = \sum x\, dF(x) = \int_0^L x\, dF(x) = \int_0^L x\, w(x)\,dx = \text{Resultant moment.}$$

Where is the resultant force located?



$$\vec{M}_R = \bar{x}\, \vec{F}_R$$



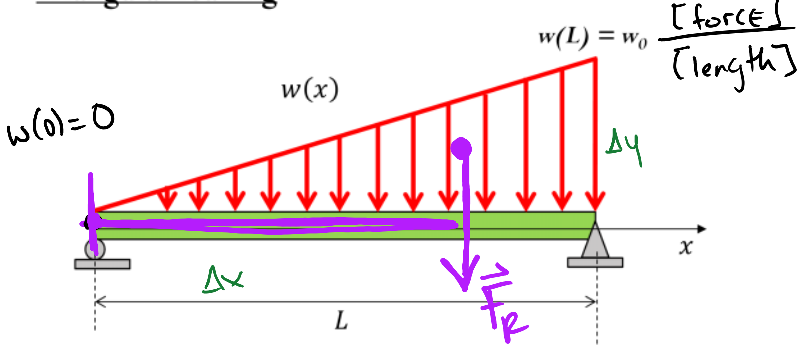
line of action passes through centroid!

$$M_R = \bar{x} F_R$$

$$\int_0^L x\,\omega(x)\,dx = \bar{x} \int_0^L \omega(x)\,dx$$

$$\bar{x} = \frac{\int_0^L x\,\omega(x)\,dx}{\int_0^L \omega(x)\,dx} =$$ geometric center, centroid

Triangular loading



1. find $\vec{F}_R = \int_0^L w(x)\,dx$
2. Find $\vec{M}_R = \int_0^L x\,w(x)\,dx$
3. find $\bar{x} = \frac{\vec{M}_R}{\vec{F}_R}$

$y(x) = mx + b$ ← y-intercept
↑
slo

rise/run → $\frac{\Delta y}{\Delta x} = \frac{w_0}{L}$

$\text{AREA} = \frac{1}{2}bh = \frac{1}{2}Lw_0 = \vec{F}_R$

$$w(x) = \frac{w_0}{L}x$$

$$\vec{F}_R = \int_0^L w(x)\,dx = \int_0^L \frac{w_0}{L}x\,dx = \frac{w_0}{L}\int_0^L x\,dx = \frac{w_0}{L}\left[\frac{x^2}{2}\Big|_0^L\right]$$

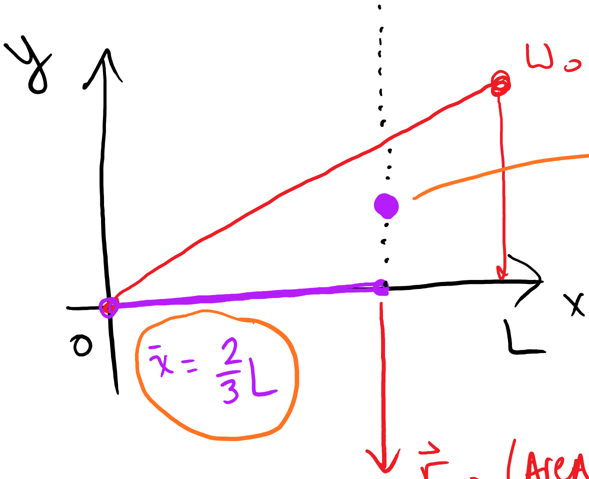
$\vec{F}_R \quad \frac{1}{2}w_0L$ * (AREA of triangle)

$$\vec{M}_R = \int_0^L x\,w(x)\,dx = \int_0^L \frac{w_0}{L}x^2\,dx = \frac{w_0}{L}\int_0^L x^2\,dx = \frac{w_0}{L}\left[\frac{x^3}{3}\Big|_0^L\right]$$

$$\vec{M}_R = \frac{1}{3}w_0L^2$$

$$\vec{M}_R = \bar{x}\,\vec{F}_R \Rightarrow \bar{x} = \frac{M_R}{F_R} = \frac{\frac{1}{3}w_0L^2}{\frac{1}{2}w_0L} = \frac{2}{3}L$$

$M_R = \bar{x}\ T_R$ T_R $\frac{1}{2} w_o L$ //



Geometric center

$(\bar{x}, \bar{y})$

* line of Action passes through $(\bar{x}, \bar{y})$

$\vec{F}_R$ - (Area of triangle)

$\vec{F}_R = \frac{1}{2} L w_o$ (magnitude)

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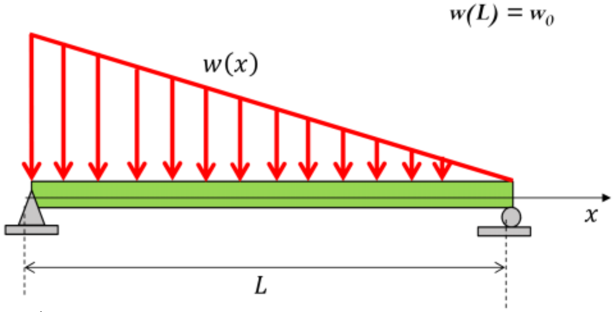
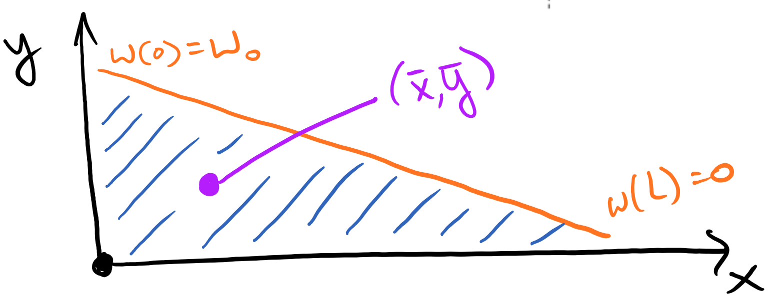
Triangular loading



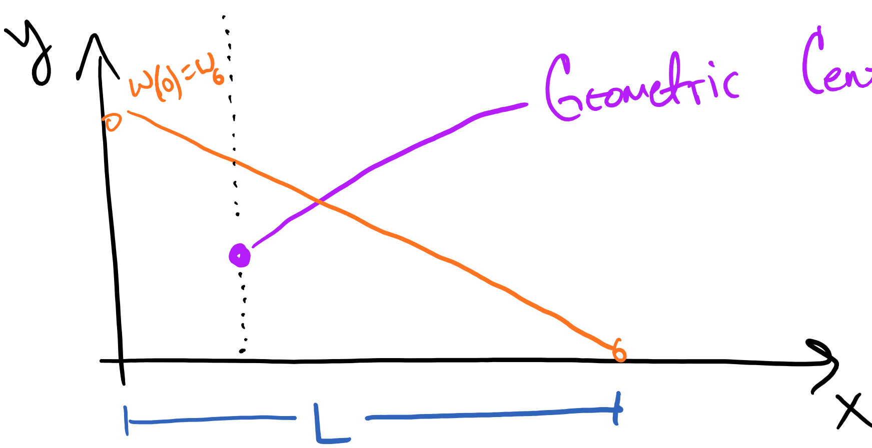
- find $\vec{F}_R$
- find $\bar{x}$



for triangular loading

* magnitude

$$\vec{F}_R = (\text{Area of triangle}) = \frac{1}{2} L W_0$$



Geometric Center - $(\bar{x},\bar{y})$

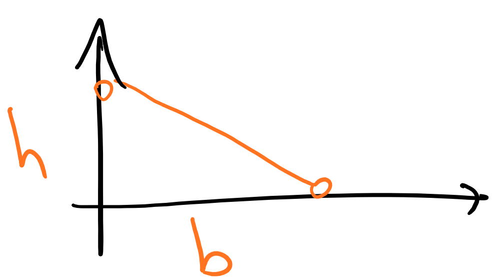
the line of Action passes through $(\bar{x},\bar{y})$, for a triangle in this configuration,

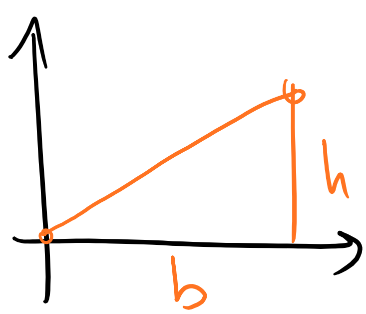
$$\boxed{\bar{x} = \frac{1}{3} L}$$

to summarize:

* Magnitude is equal to the area under the curve $w(x)$

↳ triangle is $F_R = \int w(x)\,dx = \frac{1}{2} b h$

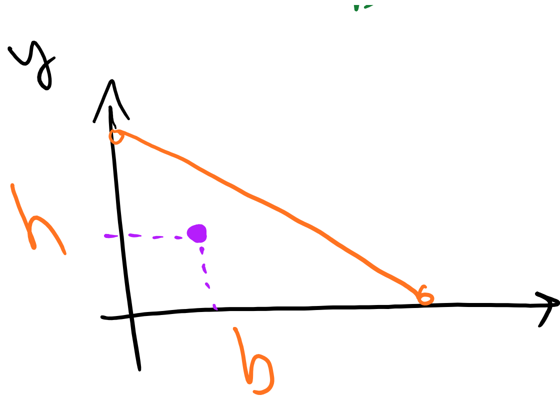


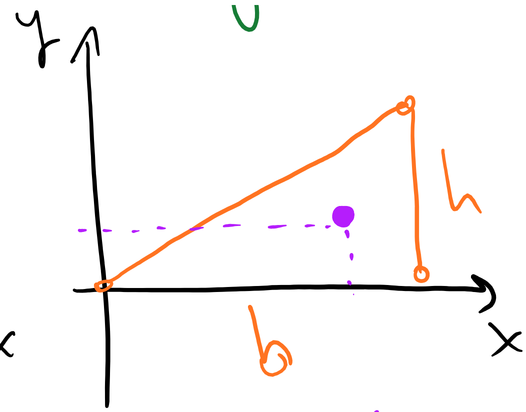


$$\boxed{F_R = \frac{1}{2} b h}$$

* the force acts at the geometric center

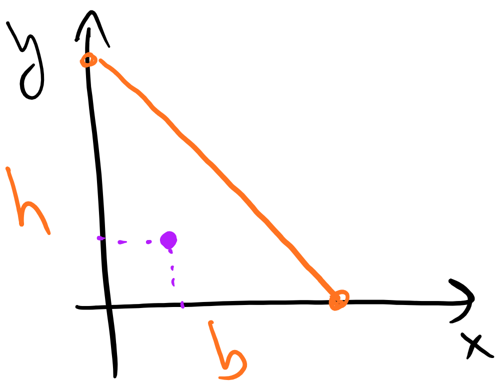
↳ for a triangle...

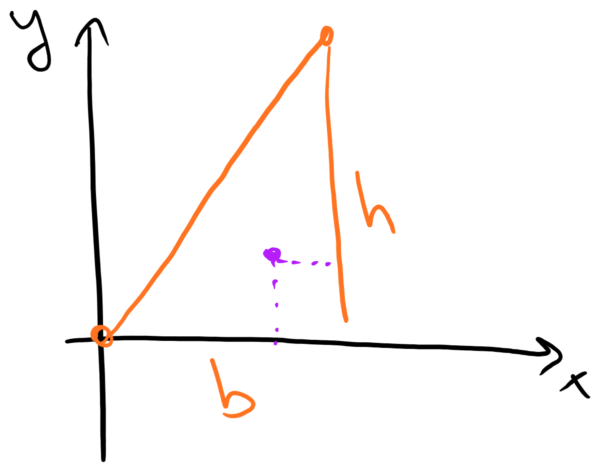




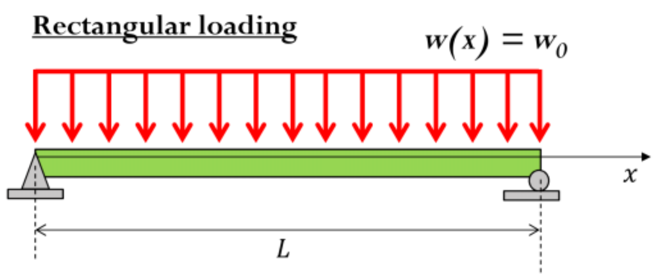
$(\bar{x}, \bar{y}) = \left(\frac{1}{3}b, \frac{1}{3}h\right)$ $\qquad$ $(\bar{x}, \bar{y}) = \left(\frac{2}{3}b, \frac{1}{3}h\right)$

Q: What about this one?





$(\bar{x}, \bar{y}) = \ ?$ $\qquad$ $(\bar{x}, \bar{y}) = \ ?$



1. find $F_R = \int w(x)dx$
2. find $M_R = \int x\,dF = \int x\,w(x)\,dx$
3. find $\bar{x} = M_R / F_R$

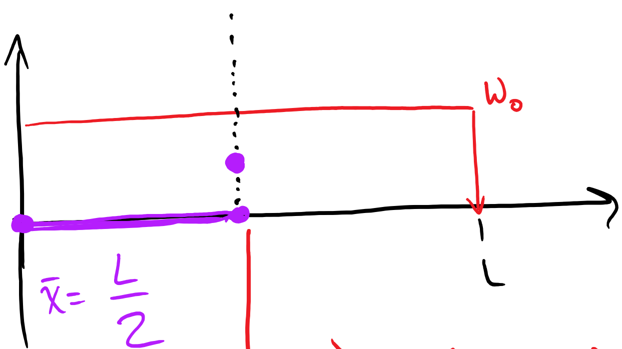
$$w(x) = mx + b = 0 + w_0 \Rightarrow w(x) = w_0$$

$$\vec{F}_R = \int_0^L w_0\,dx = w_0 x \Big|_0^L = w_0 L$$ * (AREA of Rectangle!)

$$\vec{M}_R = \int_0^L x\,dF = \int_0^L w_0 x\,dx = w_0 \frac{x^2}{2}\Big|_0^L = \frac{1}{2} w_0 L^2$$

$$\bar{x} = \frac{M_R}{F_R} = \frac{\frac{1}{2} w_0 L^2}{w_0 L} = \frac{L}{2}$$

Always!



Geometric center $-(\bar{x}, \bar{y})$

$\vec{F}_R$ - (AREA of Rectangle - $w_0 L$)