Lecture 09

Monday, September 18, 2017 12:12

To do ...

- Quiz 2 this week (**Tues-Fri**)
- HW 6 due **Tues**
- HW 7 due **Thurs**
- WA 1 (FBD only) due **Fri**

Ein Prosit!

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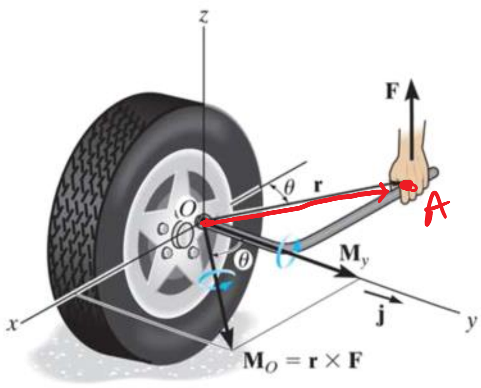
Moment of a force about a specified axis



$$\vec{F} = [0, 0, F]$$

$$\vec{r}_{OA} = [-x, y, 0]$$

$$\vec{M}_O = \vec{r}_{OA} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -x & y & 0 \\ 0 & 0 & F \end{vmatrix} = [yF, xF, 0]$$

$$M_y = \vec{u}_j \cdot \vec{M}_O = \vec{u}_j \cdot (\vec{r}_{OA} \times \vec{F}) = [0,1,0] \cdot [yF, xF, 0] = xF$$

triple scalar product.

scalar magnitude.

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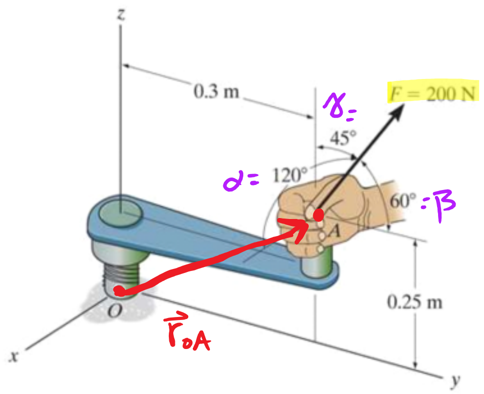

A force is applied to the tool as shown. Find the magnitude of the moment of this force about the x axis of the value.

given: $|\vec{F}|$, α, β, γ, $\vec{u}_{\hat{i}}$

$$\vec{r}_{OA} = \vec{r}_A - \vec{r}_O = [0, 0.3, 0.25]\,m$$

$$\vec{F} = 200[\cos(120), \cos(60), \cos(45)] = [-100, 100, 141.4]\,N$$

$$M_x = \vec{u}_{\hat{x}} \cdot (\vec{r}_{OA} \times \vec{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.3 & 0.25 \\ -100 & 100 & 141.4 \end{vmatrix} = 1(0.3(141.4) - .25(100))\ N \cdot M$$

$$M_x = 17.4\ N \cdot M \quad (CCW)$$

BECAUSE positive.

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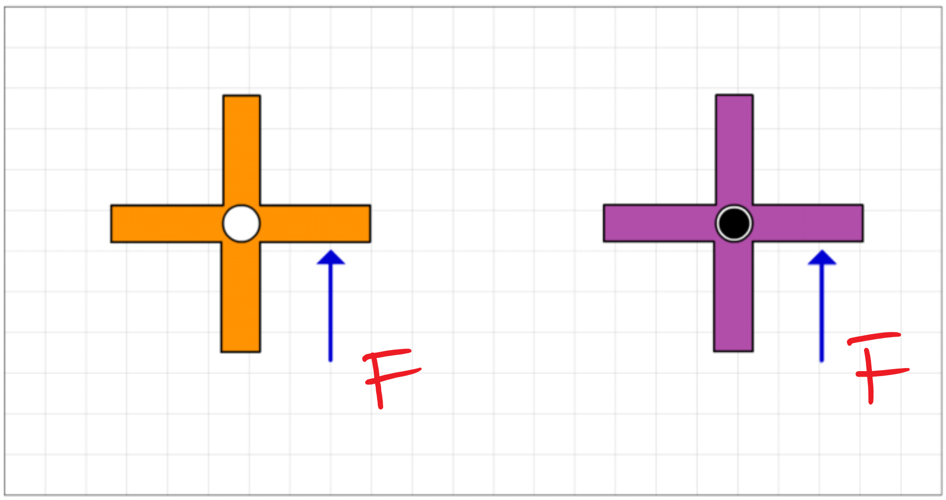
Moment of a couple

Q: What happens?



* translation And Rotation

$\Sigma F \neq 0$

$\Sigma M \neq 0$

* Rotation only

$\Sigma F = 0$

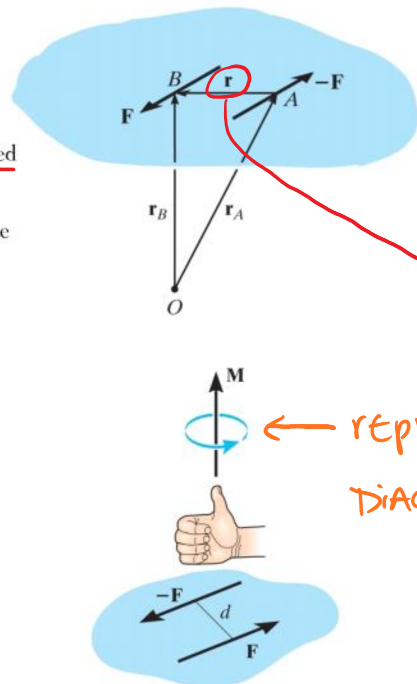
$\Sigma M \neq 0$

Moment of a couple

A **couple** is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d.

Since the resultant force is zero, the only effect of a couple is to produce an actual rotation, or if no movement is possible, there is a tendency of rotation in a specified direction.

The moment produced by a couple is called **couple moment**.



← representation on a diagram.

$$N = \sum(\vec{r} \times \vec{F})$$

$$N_o = \vec{r}_B \times \vec{F} + \vec{r}_A \times -\vec{F}$$

$$N_o = (\vec{r}_B - \vec{r}_A) \times \vec{F}$$

$$N_o = \vec{r} \times \vec{F}$$

* Independent of point of rotation!!

Requirements:

* At least two forces
* lines of action are parallel
* same magnitude : $|\vec{F}_1| = |\vec{F}_2|$
* opposite direction : $\vec{F}_1 = -\vec{F}_2$
* forces separated by perpendicular distance d

Since $\vec{F}_1 = -\vec{F}_2$

the $\sum \vec{F} = \vec{F}_1 - \vec{F}_2 = 0 \Rightarrow$ produces no net force, no translation!

But due to the separation there is PURE ROTATION!

Moment of a couple is a free vector!

↳ only direction and

↳ only direction and magnitude are important, not confined to unique line of action!

Magnitude of a couple moment:

$$M = dF \qquad \text{(scalar)}$$

$$M = \vec{r} \times \vec{F} \qquad \text{(vector analysis)}$$

↑ Any position vector from line of action of $\vec{F}_1$ to the line of action of $\vec{F}_2$!

Direction:

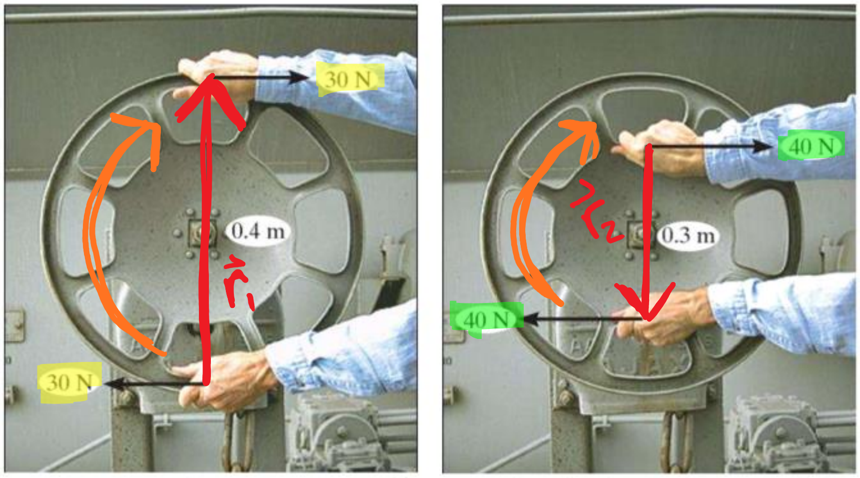
use right hand rule convention,

CCW → positive

CW → negative

Consider the equivalent couples

CW



CW

A torque or moment of 12 N·m is required to rotate the wheel. Why does one of the two grips of the wheel above require less force to rotate the wheel?

$F_1 = 30\ N$

$d_1 = 0.4\ m$

$F_2 = 40\ N$

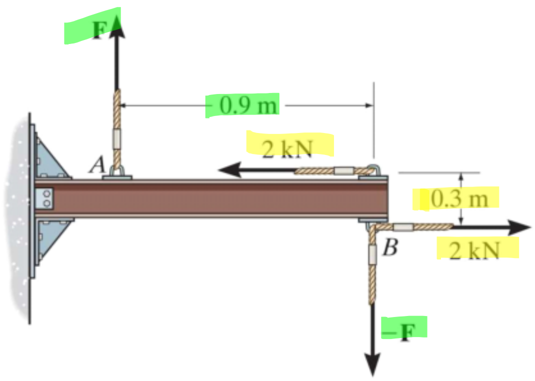
$d_2 = 0.3\ m$

$\vec{M}_1 = (.4)(30) = -12\ N \cdot m$

$\vec{M}_2 = (.3)(40) = -12\ N \cdot m$

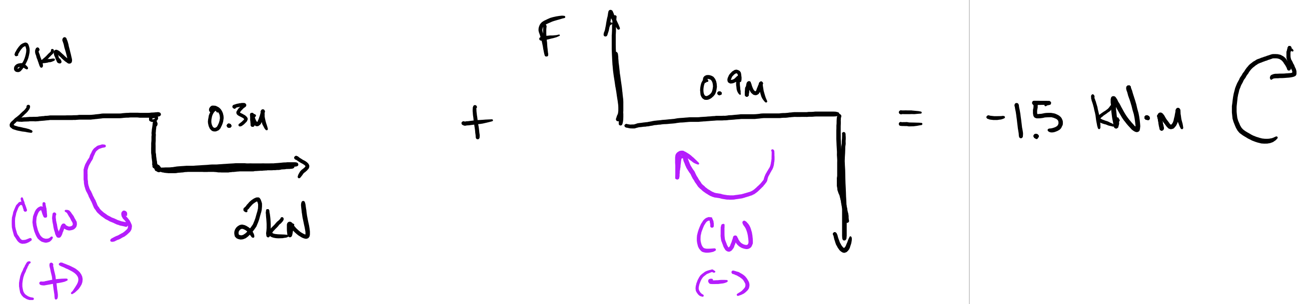
Both CW

*note that $F_2 > F_1$ because $d_1 > d_2$



Two couples act on the beam with the geometry shown. Find the magnitude of F so that the resultant couple moment is 1.5 kN·m clockwise.

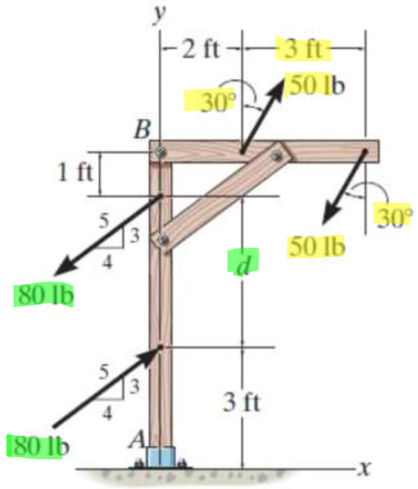
- identify couples, |F|, d



$$\Sigma M_R = (0.3\,m)(2\,kN) - (0.9)F = -1.5\ kN\,m$$

$$0.6\ kN\cdot m + 1.5\ kN\cdot m = 0.9\,F\ m$$

$$F = 2.33\ kN$$

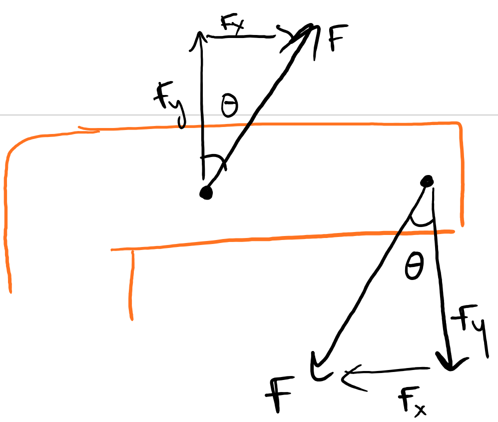


Two couples act on the beam with the geometry shown and d = 4 ft. Find the resultant couple

- identify couples.
- find perpendicular distance OR resolve forces into x & y components.

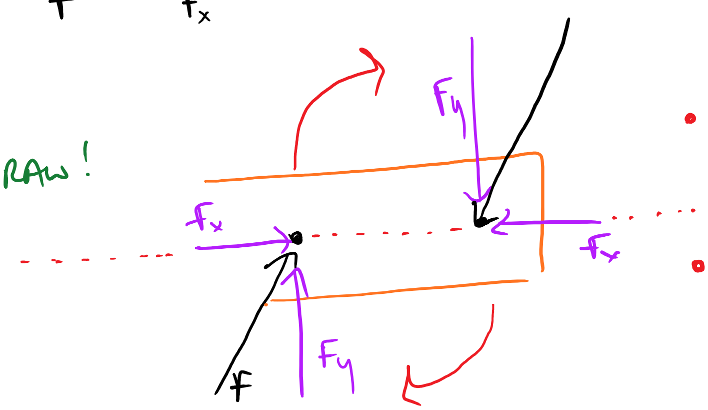
first consider forces on the top beam.

resolve into x & y components



Q: Which components would lead to rotation of the body?

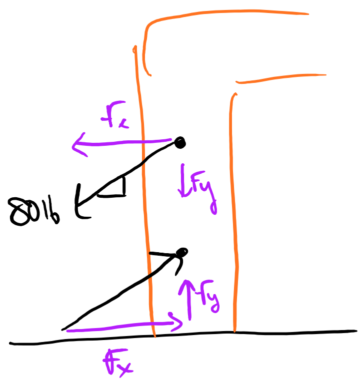
REDRAW!



- Fx would not cause rotation,
- Fy components would rotate in CW direction!

$$\vec{M}_1 = -dF_y = -(3)(50\cos(30))\ \text{ft}\cdot\text{lb}$$

now the second beam.

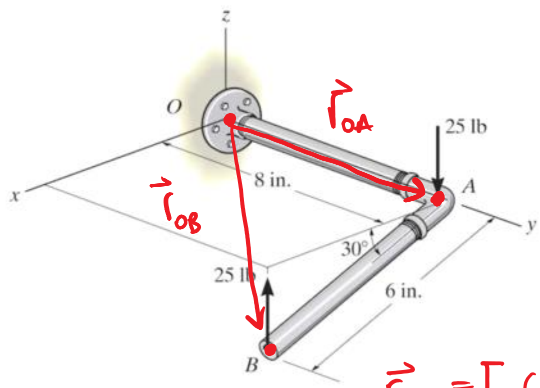


- F_y does not rotate body, only F_x would in CCW direction.

$$\vec{M}_2 = dF_x = (4\text{ft})\left(\frac{4}{5}80\right)\ \text{ft}\cdot\text{lb}$$

$$\vec{M} = \vec{M}_1 + \vec{M}_2 = \cdots = \boxed{126\ \text{ft}\cdot\text{lb}\ \text{CCW}}$$

Determine the couple moment acting on the pipe



$\vec{r}_{OA} = [0, 8, 0]$ in

$\vec{F}_B = [0, 0, -25]$ lb

$\vec{r}_{OB} = [6\cos(30), 8, -6\sin(30)]$ in

$\vec{F}_A = [0, 0, 25]$ lb

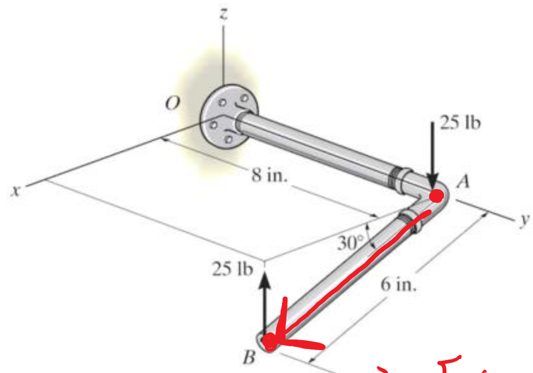
Approach 1:

$$\vec{M} = \vec{r}_{OA} \times \vec{F}_A + \vec{r}_{OB} \times \vec{F}_B$$

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 8 & 0 \\ 0 & 0 & -25 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6\cos(30) & 8 & -6\sin(30) \\ 0 & 0 & 25 \end{vmatrix} = \dots =$$

$$\vec{M} = [0, -25(6\cos(30)), 0] \text{ lb} \cdot \text{in}$$

Determine the couple moment acting on the pipe



$\vec{r}_{A} = [0, 8, 0]$ in

$\vec{r}_{B} = [6\cos(30), 8, -6\sin(30)]$

Approach 2:

use couple moment since

$|\vec{F}_1| = |\vec{F}_2| \quad \vec{F}_1 = -\vec{F}_2$

$\vec{r}_{AB} = \vec{r}_{OB} - \vec{r}_{OA} = [6\cos(30), 0, -6\sin(30)]$ in

$$\vec{M} = \vec{r}_{AB} \times \vec{F}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6\cos(30) & 0 & -6\sin(30) \\ 0 & 0 & 25 \end{vmatrix} = \cdots =$$

$$\vec{M} = [0, -25(6\cos(30)), 0] \text{ lb} \cdot \text{in}$$

* same as previous!

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Moving a force on its line of action (L.O.A)

equivalent to

Moving a force from A to B, when both points are on the vector's line of action, does not change the **external effect**.

→ translation/rotation of body.

→ depends on where and how force is applied to body

* F is a Sliding Vector

→ line of action is defined, produces the same external effect on a body.