Lecture 08

Friday, September 15, 2017 11:42

To do ...

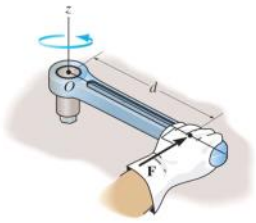
- Quiz 1 – last day!
- Quiz 2 – next week – sign up now!
  - Tues – Fri (9/19-9/22)

- HW 6 due **Tues**
- HW 7 due **Thurs**

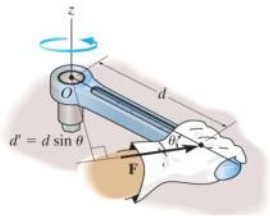
- Written Assignment due **Fri (9/22)**
  - Separate white or engineering paper
  - Upload a SINGLE PDF file

Recap

- Moment of a force
  - Scalar representation



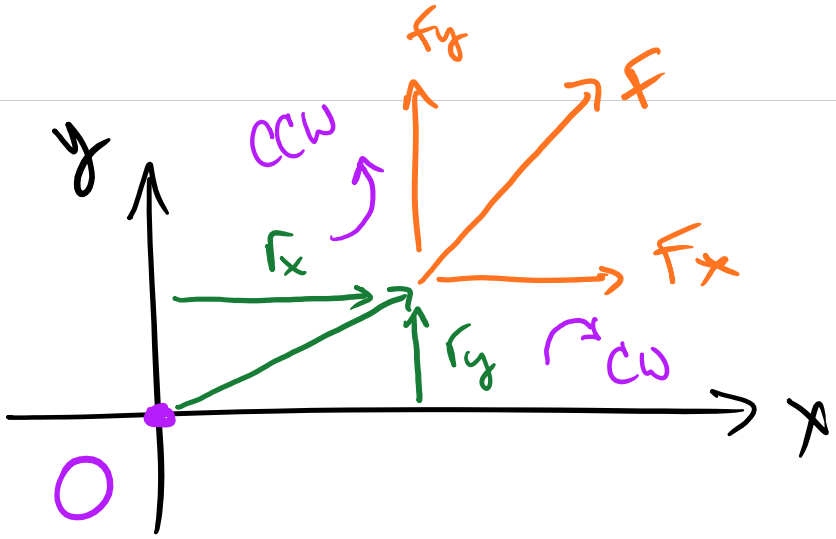
  - Vector representation



$\vec{M} = dF \rightarrow$ dimensions: force · length.

direction:

"+" ccw ↺ "−" cw ↻

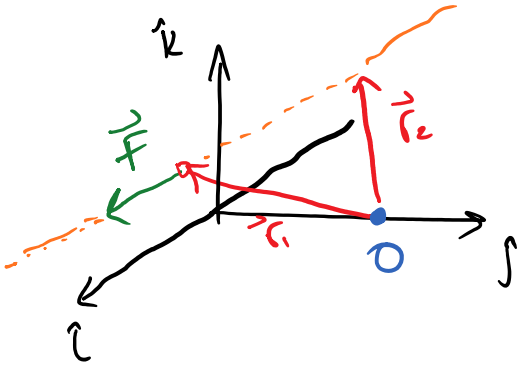


$$\vec{M} = rF = r_x F_y - r_y F_x$$

Moment of a force – vector formulation

The **moment of a force about a point** provides a measure of the **tendency for rotation** (sometimes called a torque).

The moment of a force **F** about point **O**, or actually about the moment axis passing through O and perpendicular to the plane containing **O** and **F**, can be expressed using the cross (vector) product, namely:



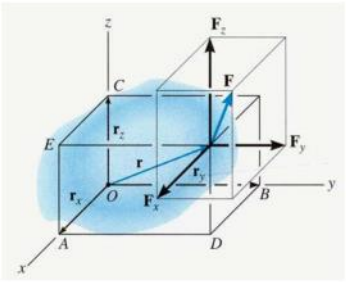
$\vec{M}_o = \vec{r} \times \vec{F}$

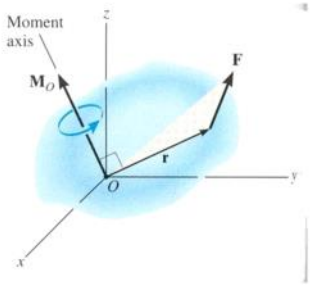
↑ position vector from O to any point along line of action of $\vec{F}$

Scalar magnitude → $|\vec{M}| = |\vec{r}||\vec{F}|\sin\theta$

vector → $\vec{M} = |\vec{r}||\vec{F}|\sin\theta \ (\vec{u}_r \times \vec{u}_F)$

Moment of a force – vector formulation





$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

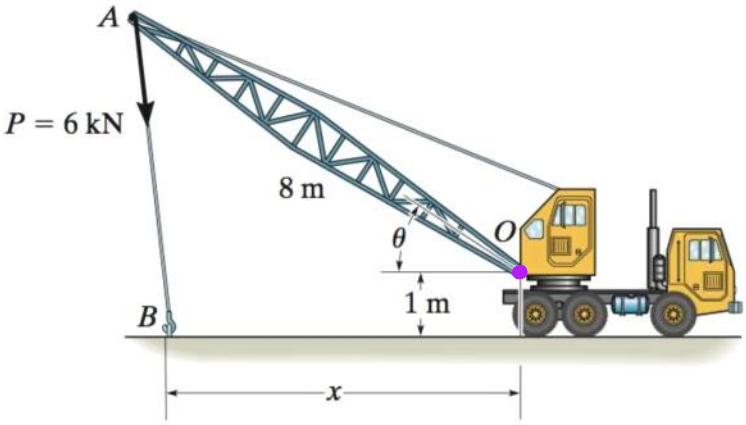
$$\vec{M} = (r_y F_z - r_z F_y)\hat{\imath} - (r_x F_z - r_z F_x)\hat{\jmath} + (r_x F_y - r_y F_x)\hat{k}$$

Components cause Rotation About x-axis

Rotation About y-Axis

Rotation about z-Axis

Example – Vector Formulation



Given: The angle $\theta = 30°$ and $x = 10$ m.

Find: The moment by **P** about point O.

the moment about O is

$$\vec{M}_o = \vec{r}_{OA} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_{OAx} & r_{OAy} & 0 \\ P_x & P_y & 0 \end{vmatrix}$$

$$\vec{M}_o = (r_{OAx} P_y - r_{OAy} P_x)\hat{k} \quad kN_M$$

* need to find $\vec{r}_{OA}$ And $\vec{P} = |\vec{P}| \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$

Using the problem geometry, we have:

$$A = (-8\cos\theta, 8\sin\theta, 0)$$

$$B = (-10, -1, 0)$$

Therefore,

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = \left[-10 + 8\cos\theta,\ -1 - 8\sin\theta,\ 0\right]$$

$$\vec{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{\left[-10 + 8\cos\theta,\ -1 - 8\sin\theta\right]}{\sqrt{(\cdots)^2 + (\cdots)^2}}$$

$$\vec{r}_{OA} = \left[-8\cos\theta,\ 8\sin\theta\right]$$

plug in to:

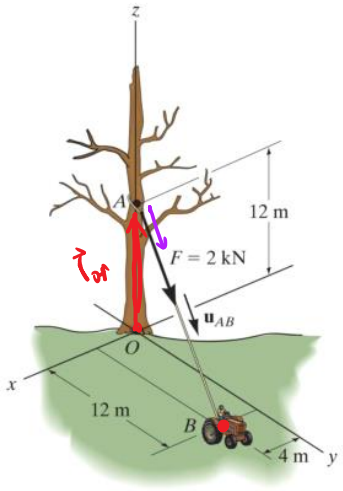
$$\vec{M} = \vec{r}_{OA} \times \vec{P} = |\vec{P}|\ \vec{r}_{OA} \times \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = |\vec{P}| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8\cos\theta & 8\sin\theta & 0 \\ u_{ABx} & u_{ABy} & 0 \end{vmatrix}$$

* now try the problem using

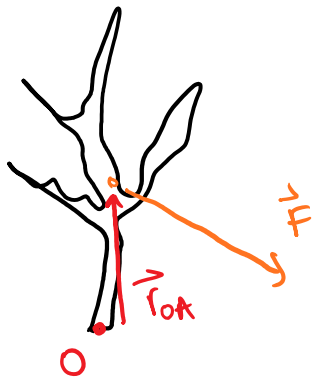
$$\vec{M}_o = \vec{r}_{OB} \times \vec{P}$$

and see if you get the same ANSWER!

Determine the moment produced by the force **F** about point **O**.



DRAW the FBD.



$\vec{r}_A = [0, 0, 12]$ m

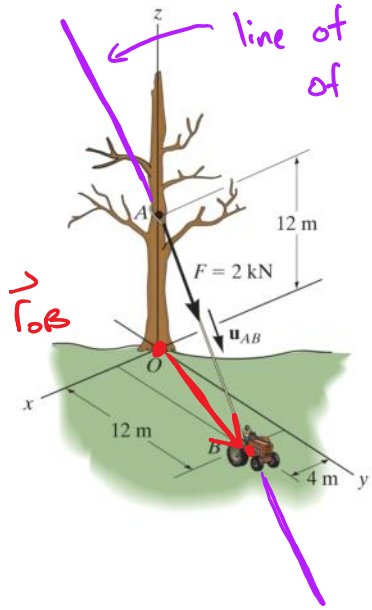
$\vec{r}_B = [4, 12, 0]$ m

$$\vec{F} = F\,\vec{u}_{AB} = F\,\frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = F\,\frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_{AB}|} = 2\,\frac{[4, 12, -12]}{\sqrt{4^2+12^2+12^2}}\ \text{kN}$$

$$\vec{M}_O = \vec{r}_{OA} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 12 \\ F_x & F_y & F_z \end{vmatrix} = (-12F_y)\hat{i} - (-12F_x)\hat{j} + 0\hat{k}$$

$$\vec{M}_O = [-12F_y,\ 12F_x,\ 0]\ \text{kN}\cdot\text{m}$$

Determine the moment produced by the force **F** about point **O**.

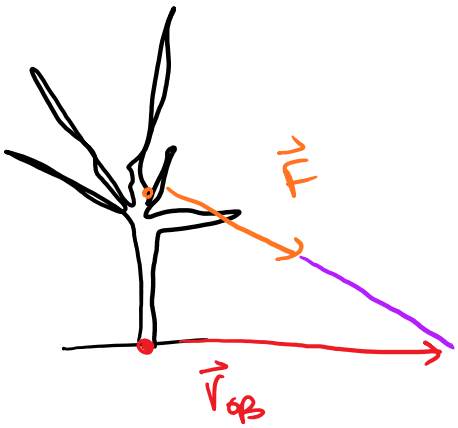


line of action of $\vec{F}$

$r_A = [0, 0, 12]$ M

$r_B = [4, 12, 0]$ M

CONSIDER the SAME PROBLEM with A different MOMENT ARM:



$\vec{F} = f\,\vec{u}_{AB} = f\,\dfrac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = 2\,\dfrac{[4, 12, -12]}{\sqrt{4^2 + 12^2 + 12^2}}$ kN

$\vec{r}_{OB} = \vec{r}_B - \vec{r}_O = [4, 12, 0]$ M

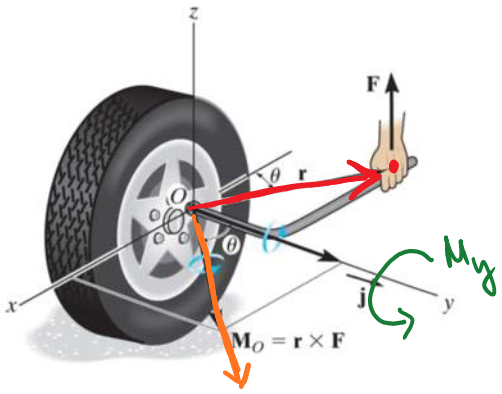
$$\vec{M}_o = \vec{r}_{OB} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 12 & 0 \\ f_x & f_y & f_z \end{vmatrix} =$$

$$\vec{M}_o = [12 f_z,\ -4 f_z,\ 4(f_y - 3 f_x)] \text{ kN} \cdot \text{M}$$

✍ Check to see if the

SAME ANSWER AS previous.

Moment of a force about a specified axis



consider the force and moment Arm:

$\vec{F} = [0, 0, F]$

$\vec{r} = [-x, y, 0]$

$$\vec{M}_o = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -x & y & 0 \\ 0 & 0 & F \end{vmatrix} = (yF)\hat{i} - (-xF)\hat{j} + 0\hat{k}$$

$\vec{M}_o = [yF, xF, 0]$ N·M

* Rotation About both x And y Axis!

to determine magnitude Along specific axis, use:

$M_y = \vec{M}_o \cdot \vec{u}_j = [yF, xF, 0] \cdot [0, 1, 0] =$ **xF N·M**

scalar magnitude