# Lecture 06

Monday, September 11, 2017 11:53

## To do ...

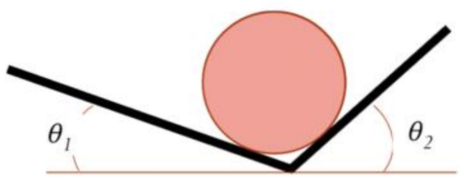
- Quiz 1 – This week! —— Good Luck!
  - CBTF instructions on website
- i>clickers
- HW 4PL due **Tues**
- HW 5ME due **Thurs**

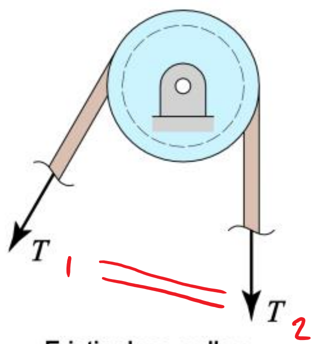
# Recap

- Equilibrium of a particle
- General procedure for analysis
- Free body diagram
- Equation of equilibrium
- Idealizations (pulleys, springs, smooth surfaces)

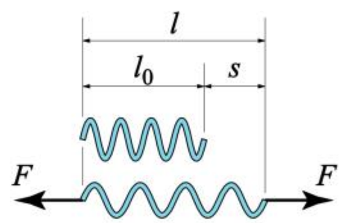
# Idealizations

**Pulleys** are (usually) regarded as frictionless; then the tension in a rope or cord around the pulley is the same on either side.
**Springs** are (usually) regarded as linearly elastic; then the tension is proportional to the *change* in length *s*.




Frictionless pulley




$F = ks = k(l - l_0)$

Linearly elastic spring

* normal force at point of contact perpendicular to surface!

* tension is the same
* pulley - massless frictionless
* cable - rigid, massless

* spring force opposes external force

$s > 0$ stretch

$s < 0$ compress

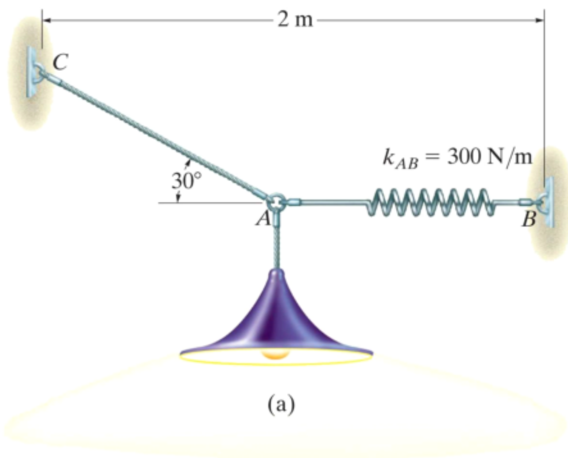
* spring constant

$F = K \Delta s$

force = [?] · length.

dimension of $K \rightarrow \dfrac{\text{force}}{\text{length.}}$

units $\rightarrow \frac{N}{m}$ or $\frac{lb}{ft}$




(a)

Determine the required length of cord AC so that the 8-kg lamp can be suspended in the position shown. The undeformed spring length is 0.4 m and has a stiffness of 300 N/m.

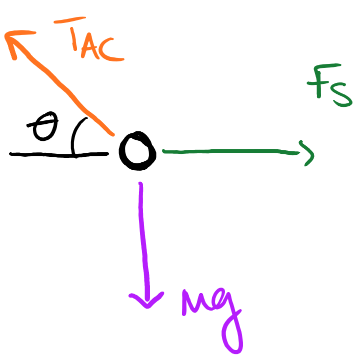
given: $m = 8\ kg$ $\theta = 30°$

$l_o = 0.4 m$

$k = 300\ N/m$

find: $l_{AC}$

Start w/ FBD at A




Equations of equilibrium:

$\Sigma F_x$: $F_s - T_{AC} \cos\theta = 0$ (eq. 1)

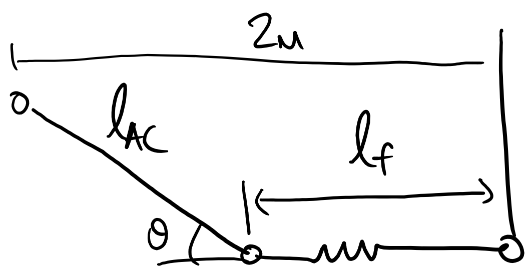
$\Sigma F_y$: $T_{AC} \sin\theta - mg = 0$ (eq. 2) the spring force is $F_s = k\Delta s = k(l_f - l_o)$

$$k(l_f - l_o) = T_{AC}\cos\theta = \left(\frac{Mg}{\sin\theta}\right)\cos\theta$$

(eq. 2)

$$l_f = \left(\frac{Mg}{k}\right)\frac{\cos\theta}{\sin\theta} + l_o = \underline{0.853 \text{ m}}$$

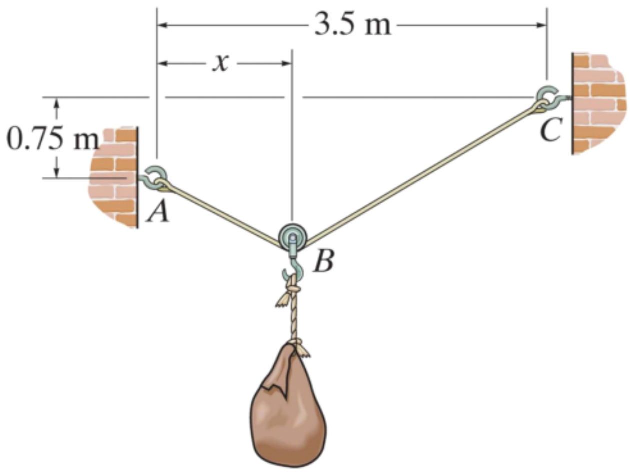
using geometrical constraint:




$$2m = l_f + l_{AC}\cos\theta$$

$$l_{AC} = \frac{2m - l_f}{\cos\theta} = \frac{2 - .853}{\cos\theta} = \boxed{1.32 \text{ m}}$$

Cable ABC has a length of 5 m. Determine the position x and the tension developed in ABC required for equilibrium of the 100-kg sack.

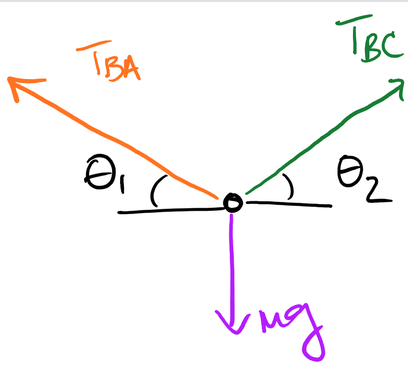



<u>given:</u> $M = 100$ Kg

$l_{ABC} = 5$ M

<u>find:</u> $x$, $T_{ABC}$

<u>FBD At B</u>




<u>Eqns. of equilibrium</u>

$\Sigma F_x$: $T_{BC}\cos\theta_2 - T_{BA}\cos\theta_1 = 0$

$\Sigma F_y$: $T_{BA}\sin\theta_1 + T_{BC}\sin\theta_1 - Mg = 0$

But for a pulley, we know

$$T_{BA} = T_{BC} = T$$

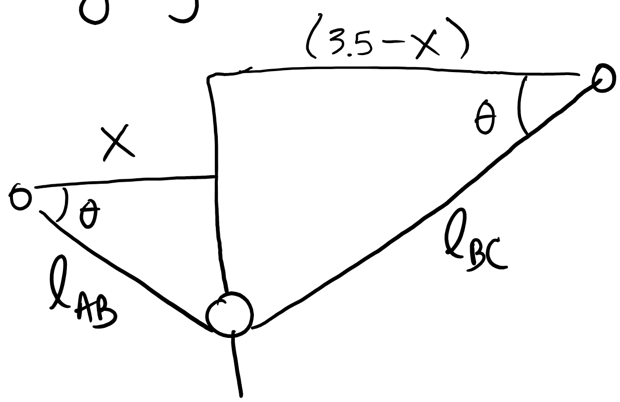
$\therefore$ $T\cos\theta_1 = T\cos\theta_2$ $\therefore \rightarrow$ $\theta_1 = \theta_2$

$\Sigma F_y$: $\quad 2T\sin\theta = mg$

$$\boxed{T = \frac{mg}{2\sin\theta}}$$

* almost there, need to solve for $\theta$

using geometric constraints




total length:

$$5\,m = l_{AB} + l_{BC}$$

Express using $\theta$

$$x = l_{AB}\cos\theta$$

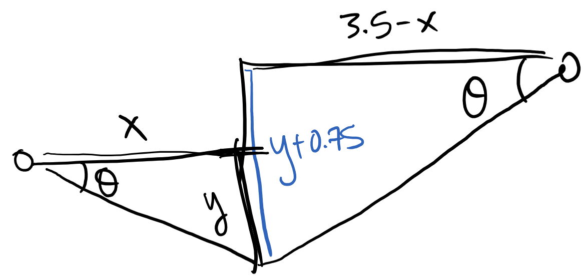
$$(3.5 - x) = l_{BC}\cos\theta$$

$$5\,m = \frac{x}{\cos\theta} + \frac{3.5 - x}{\cos\theta} = \frac{3.5}{\cos\theta}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{3.5}{5}\right) = \boxed{45.57^\circ}$$

* this allows you to solve for T, but still need to find x ...

Again using geometry:




$$x\tan\theta + 0.75 = (3.5-x)\tan\theta$$

$$2x\tan\theta = 3.5\tan\theta - 0.75$$

$$x = \frac{3.5\tan\theta - 0.75}{2\tan\theta} = \boxed{1.38\text{ m}}$$

$$T = \frac{mg}{2\sin\theta} = \boxed{687\text{ N}}$$

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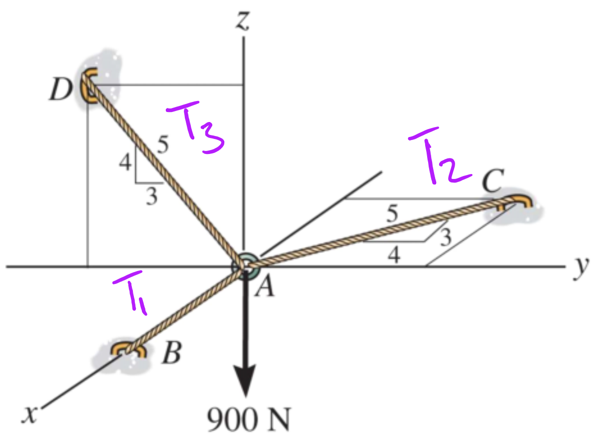
# 3D force systems




Find the tension developed in each cable

given: mag & dir of Applied force

unknown: $T_1$ $T_2$ $T_3$

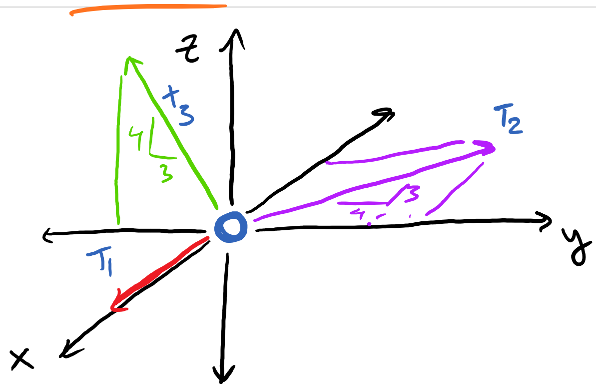
plan: - DRAW FBD of A

- find Direction & magnitude forces

- use equation of equilibrium

$$\Sigma \vec{F} = (\Sigma F_x)\hat{i} + (\Sigma F_y)\hat{j} + (\Sigma F_z)\hat{k}$$

FBD




Determine Direction vectors And use

$$\vec{F}_A = F\,\vec{u}_A$$

$$\vec{u}_1 = [1, 0, 0]$$

$$\vec{u}_2 = [-\tfrac{3}{5}, \tfrac{4}{5}, 0]$$

$$\vec{u}_3 = [0, -\tfrac{3}{5}, \tfrac{4}{5}]$$

| $\vec{F}$ | $\hat{\imath}$ | $\hat{\jmath}$ | $\hat{k}$ |
|---|---|---|---|
| $\vec{T}_1$ | $T_1$ | $0$ | $0$ |
| $\vec{T}_2$ | $-\frac{3}{5}T_2$ | $\frac{4}{5}T_2$ | $0$ |
| $\vec{T}_3$ | $0$ | $-\frac{3}{5}T_3$ | $\frac{4}{5}T_3$ |
| $\vec{P}$ | $0$ | $0$ | $-900$ |
| $\Sigma\vec{F}$ | $\Sigma F_x = 0$ | $\Sigma F_y = 0$ | $\Sigma F_z = 0$ |

$\Sigma F_x$: $\quad T_1 - \frac{3}{5}T_2 = 0$

$\Sigma F_y$: $\quad -\frac{3}{5}T_3 + \frac{4}{5}T_2 = 0$

$\Sigma F_z$: $\quad \frac{4}{5}T_3 - 900 = 0 \quad \Rightarrow \quad T_3 = \frac{5}{4}(900)\,N$

$$\boxed{T_3 = 1125\ N}$$

now using $\Sigma F_y$

$$T_2 = \frac{3}{4}T_3 = \boxed{844\ N}$$

$$T_2 = \frac{3}{4} T_3 = \boxed{844 \text{ N}}$$

finally,

$$T_1 = \frac{3}{5} T_2 = \frac{3}{5}(844) = \boxed{506 \text{ N}}$$