Lecture 04

Wednesday, September 6, 2017 12:22

To do ...

- **Quiz 1** sign up now!
  - Tues – Fri of next week (Sept 12 – 15)
  - "Practice" quiz available

- HW3 due **Thurs**
- HW4 PL due **Tues**
- **Quiz 2** (Sept 19 – 22)
- Written assignment coming soon

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Recap

- A force can be treated as a vector since forces obey all the rules that vectors do
- Vector representations
  - Rectangular components
  - Cartesian components
  - Unit vector
  - Directional cosines
- Position vectors

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

$$= A_x \hat{i} + A_y \hat{j} + A_z \hat{k} = [A_x \; A_y \; A_z]$$

$$\vec{u}_A = \frac{\vec{A}}{|\vec{A}|}$$

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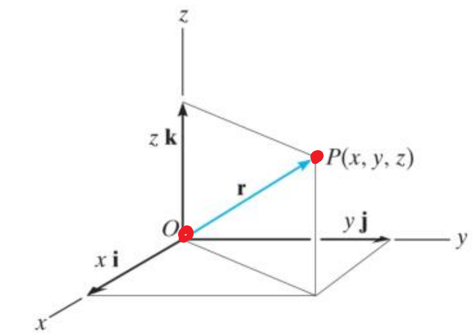
Position vectors



A position vector $\boldsymbol{r}$ is defined as a fixed vector which locates a point in space relative to another point. For example,

$$\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$$

expresses the position of point $P(x,y,z)$ with respect to the origin O.

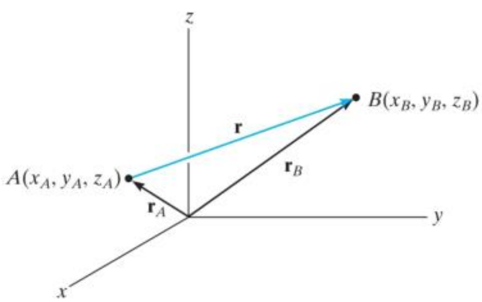
The position vector $\boldsymbol{r}$ of point $\boldsymbol{B}$ with respect to point $\boldsymbol{A}$ is obtained from

$$\vec{r}_A + \vec{r} = \vec{r}_B$$

Hence,

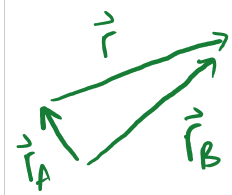
$$\vec{r} = \vec{r}_B - \vec{r}_A$$

$$\vec{r} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$$



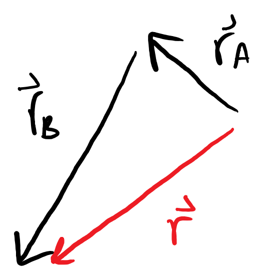
Thus, the $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ components of the positon vector $\boldsymbol{r}$ may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).

vector Addition:

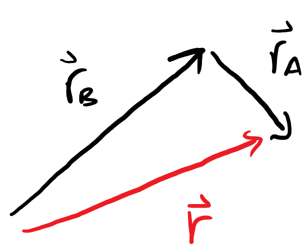


Check:

$\vec{r}_A - \vec{r}_B$



$\vec{r}_B - \vec{r}_A$



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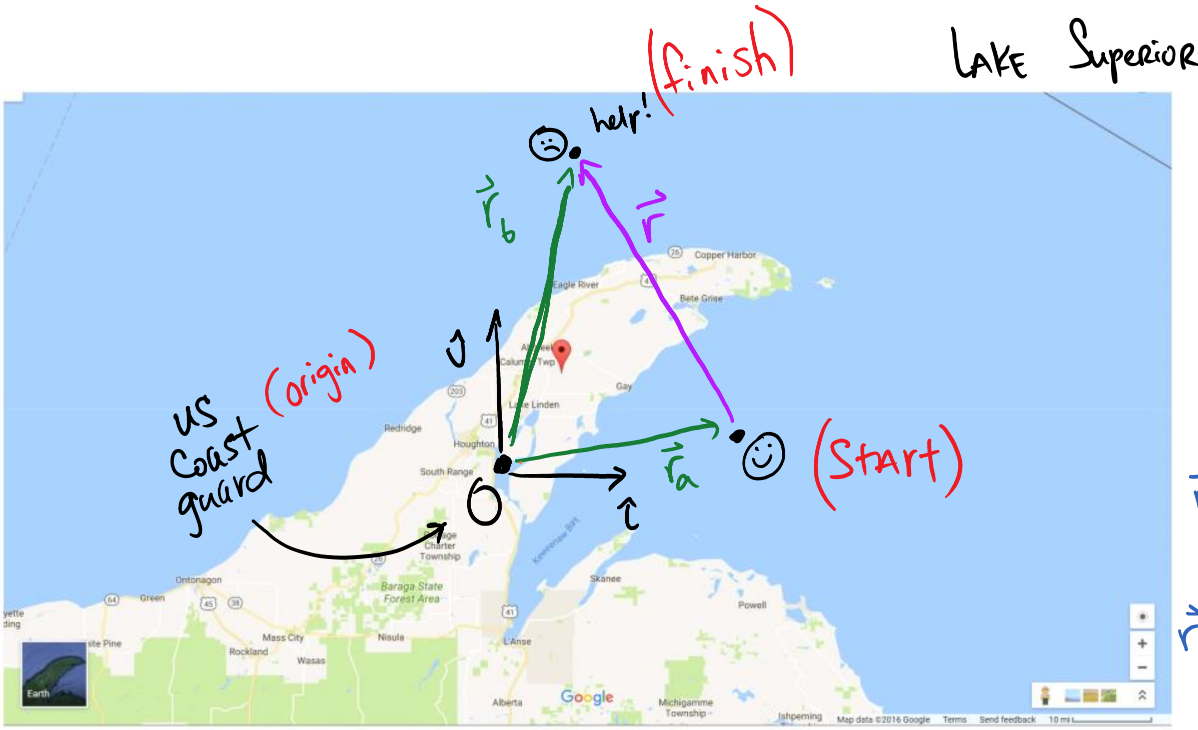

vector $\vec{r}$ with respect to to origin O

$$\vec{r} = \vec{r}_b - \vec{r}_a$$

$\vec{r}$ = end point − start point

* note: As you can see, using the triangle method

$$\vec{r}_a + \vec{r} = \vec{r}_b$$

but we know $\vec{r}_a$ and $\vec{r}_b$, not $\vec{r}$ so we solve and get

$$\vec{r} = \vec{r}_b - \vec{r}_a$$

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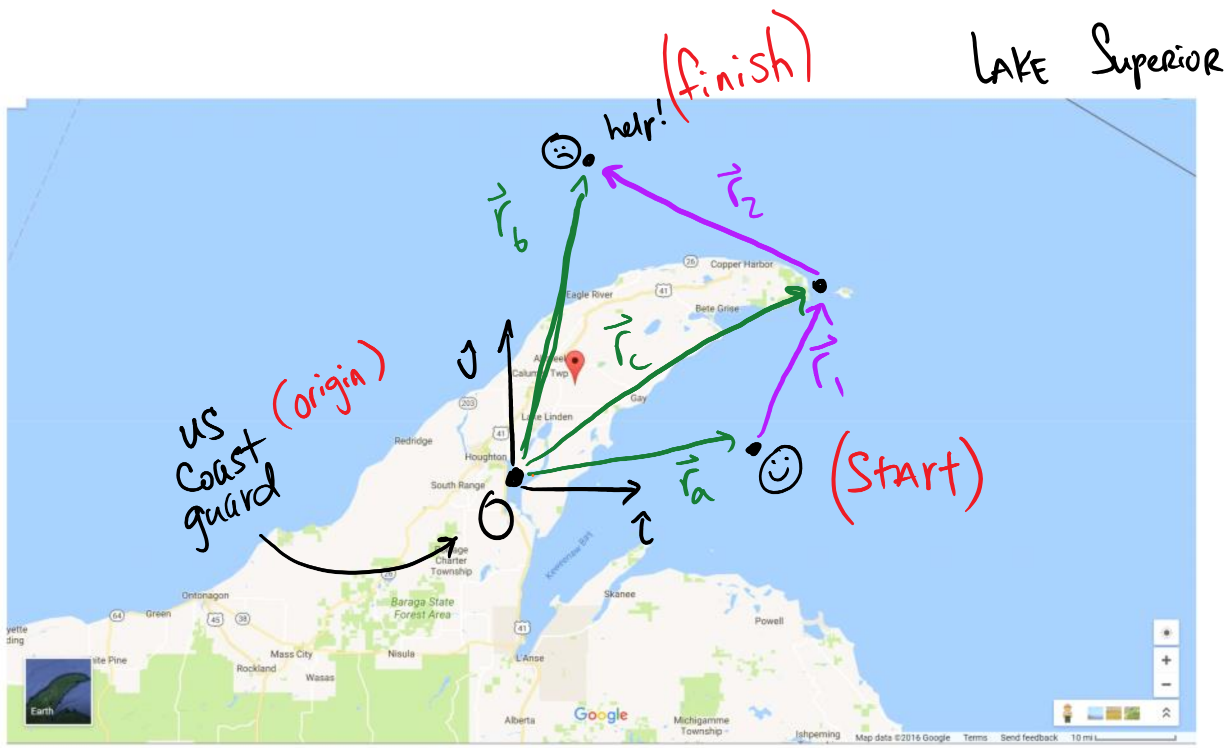
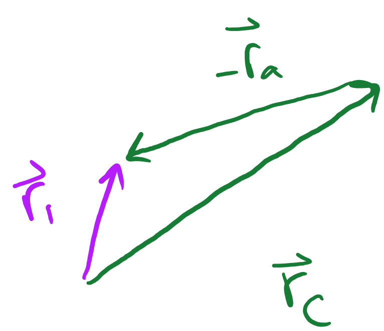


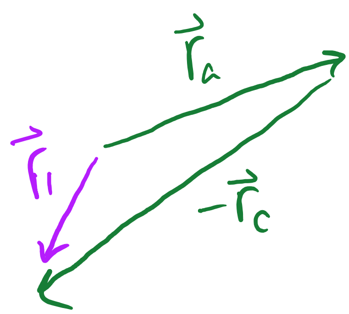
$$\vec{r}_1 = \vec{r}_c - \vec{r}_a$$

$$\vec{r}_2 = \vec{r}_b - \vec{r}_c$$

*note: if you switch the order, you get A different vector! for example

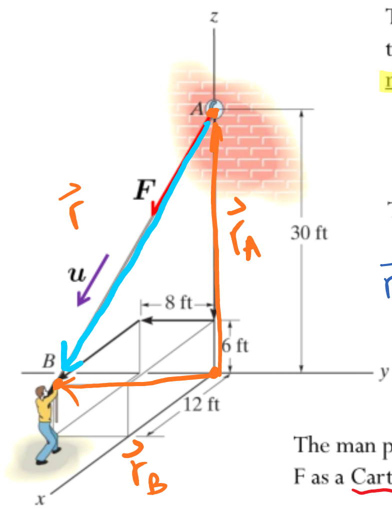
$\vec{r}_1 = \vec{r}_c - \vec{r}_a$ vs. $\vec{r}_1 = \vec{r}_a - \vec{r}_c$





not the same!

Force vector directed along a line



The force vector $\boldsymbol{F}$ acting a long the rope can be defined by the unit vector $\boldsymbol{u}$ (defined the direction of the rope) and the magnitude of the force.

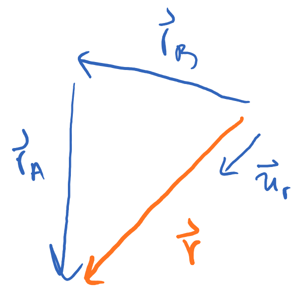
$$\vec{F} = F\,\vec{u}$$

magnitude ↗ ↖ direction

The unit vector $\vec{u}$ is specified by the position vector:

$$\vec{r} = \vec{r}_B - \vec{r}_A = (x_B - x_A)\hat{\imath} + (y_B - y_A)\hat{\jmath} + (z_B - z_A)\hat{k}$$

$$\vec{u}_r = \frac{\vec{r}}{|\vec{r}|} \quad \rightarrow \text{direction, unit less}$$



The man pulls on the cord with a force of 70 lb. Represent the force F as a Cartesian vector.

$$\vec{r}_{AB} = (12-0)\hat{\imath} + (-8-0)\hat{\jmath} + (6-30)\hat{k}$$

$$\vec{r}_{AB} = (12\hat{\imath} - 8\hat{\jmath} - 24\hat{k})\ \text{ft}$$

magnitude (length of the rope)

$$|\vec{r}_{AB}| = \sqrt{12^2 + 8^2 + 24^2} = 28\ \text{ft}$$

the unit vector is

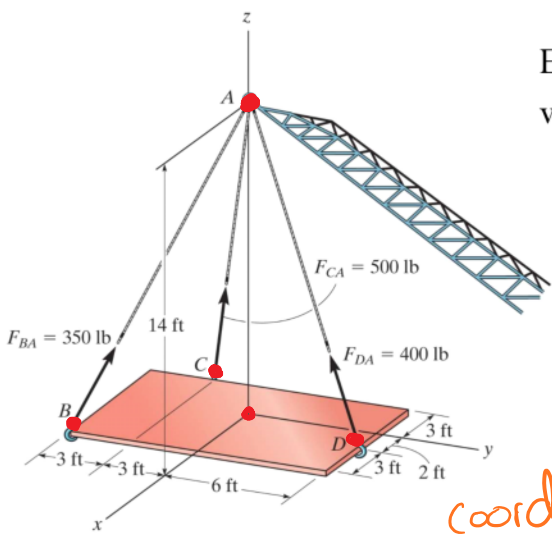
$$\vec{u}_{AB} = \frac{\vec{r}}{|\vec{r}|} = \frac{12}{28}\hat{\imath} - \frac{8}{28}\hat{\jmath} - \frac{24}{28}\hat{k}$$

$$\vec{F}_{AB} = F\,\vec{u}_{AB} = \left[70\left(\frac{12}{28}\right)\hat{\imath} - 70\left(\frac{8}{28}\right)\hat{\jmath} - 70\,\frac{24}{28}\hat{k}\right]\ \text{N}$$

Quick check:

$$\vec{F} = F\,\vec{u} = F\,\frac{\vec{r}}{|\vec{r}|} = F\,\frac{r_x}{|\vec{r}|}\hat{\imath} + F\,\frac{r_y}{|\vec{r}|}\hat{\jmath} + F\,\frac{r_z}{|\vec{r}|}$$

$$|\vec{F}| = \sqrt{F^2\frac{r_x^2}{|\vec{r}|^2} + F^2\frac{r_y^2}{|\vec{r}|^2} + F^2\frac{r_z^2}{|\vec{r}|^2}} = \frac{F}{|\vec{r}|}\cdot\sqrt{r_x^2 + r_y^2 + r_z^2} = \frac{F}{|\vec{r}|}\cdot|\vec{r}| = F$$



Express each force as a Cartesian vector

$$\vec{F}_{BA} = 350 \cdot \frac{\vec{r}_{BA}}{|\vec{r}_{BA}|} \text{ lb}$$

coords.

$A = (0, 0, 14)$ ft

$C = (-3, -3, 0)$ ft

$B = (5, -6, 0)$ ft

$D = (2, 6, 0)$ ft

position vectors:

$\vec{r}_{BA} = \vec{r}_A - \vec{r}_B = [-5, 6, 14]$ ft $\qquad |\vec{r}_{BA}| = 16.031$ ft

$\vec{r}_{CA} = \vec{r}_A - \vec{r}_C = [3, 3, 14]$ ft $\qquad |\vec{r}_{CA}| = 14.629$ ft

$\vec{r}_{DA} = \vec{r}_A - \vec{r}_D = [-2, -6, 14]$ ft $\qquad |\vec{r}_{DA}| = 15.362$ ft

$$\vec{F}_{BA} = 350 \left(\frac{\vec{r}_{BA}}{|\vec{r}_{BA}|} \right) \text{lb} = 350 \, \frac{[-5, 6, 14]}{16.031} \text{ lb}$$

$[3, 3, 14]$

$$\vec{F}_{CA} = 500\left(\frac{\vec{r}_{CA}}{|\vec{r}_{CA}|}\right)\text{lb} = 500\ \frac{[3,3,14]}{14.629}\ \text{lb}$$

$$\vec{F}_{DA} = 400\left(\frac{\vec{r}_{DA}}{|\vec{r}_{DA}|}\right)\text{lb} = 400\ \frac{[-2,-6,14]}{15.362}\ \text{lb}$$

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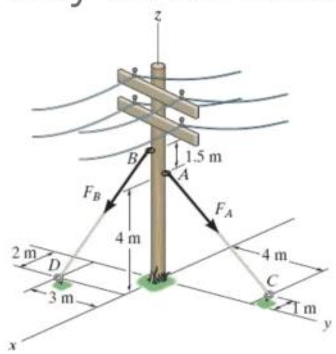
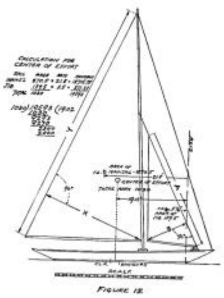
Why do we care?



*what seems to be the problem officer... :(



*Build And design stuff that works!

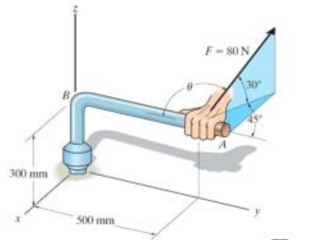
Force vector directed along a line

Don't look up!

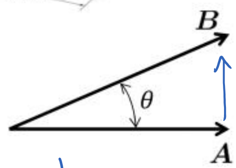
Dot (or scalar) product

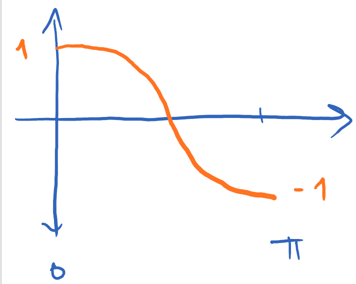
The dot product of vectors **A** and **B** is defined as such

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$



$$\cos\theta = \left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$$





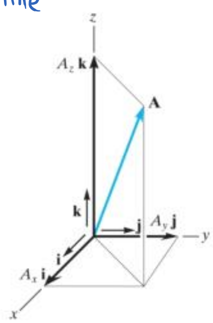
* find angle between two vectors
* find components of a vector parallel/perp to a line

Very Important!

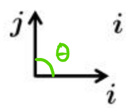
* $\vec{A}\cdot\vec{B}$ is a scalar
* the units are A·B
  ie. $\vec{F_1}\cdot\vec{F_2} \Rightarrow N \cdot N$

Cartesian vector formulation:

$$\boldsymbol{A}\cdot\boldsymbol{B} = A_xB_x + A_yB_y + A_zB_z$$

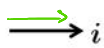


Note that:



$\boldsymbol{i}\cdot\boldsymbol{j} = 0$

$\theta = 90 \therefore \cos(90) = 0$



$\boldsymbol{i}\cdot\boldsymbol{i} = 1$

$\theta = 0 \therefore \cos 0 = 1$

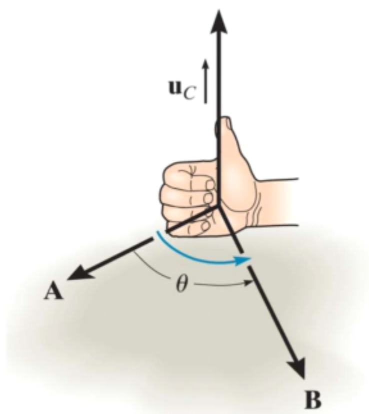
$$\vec{A}\cdot\vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})\cdot(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$= A_xB\,\underbrace{\hat{\imath}\cdot\hat{\imath}}_{\hat{\imath}\cdot\hat{\imath} = 1} + A_xB_y\,\underbrace{\hat{\imath}\cdot\hat{\jmath}}_{\hat{\imath}\cdot\hat{\jmath} = 0} + A_xB_z\,\underbrace{\hat{\imath}\cdot\hat{k}}_{\hat{\imath}\cdot\hat{k} = 0} + \cdots$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Cross (or vector) product

The cross product of vectors **A** and **B** yields the vector **C**, which is written



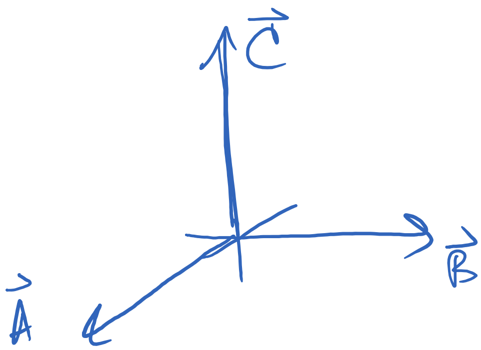
$$\boxed{\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}}$$

The magnitude of vector **C** is given by:

$$C = |C| = |\vec{A}||\vec{B}| \sin\theta$$

The vector **C** is perpendicular to the plane containing **A** and **B** (specified by the **right-hand rule**). Hence,

$$\boldsymbol{C} = AB\sin\theta \ \vec{u}_c$$

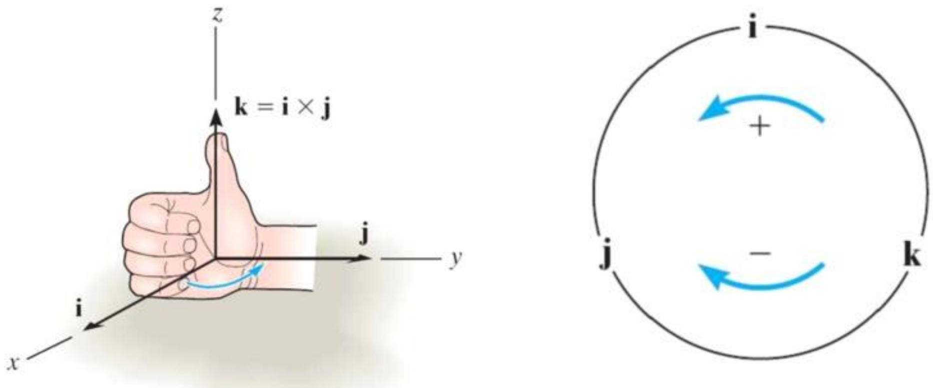


what is $\vec{B} \times \vec{A} = ?$

$\vec{A} \times -\vec{C} = ?$

Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $i \times i = 0$



Considering the cross product in Cartesian coordinates

$$\boldsymbol{A} \times \boldsymbol{B} = (A_x \boldsymbol{i} + A_y \boldsymbol{j} + A_z \boldsymbol{k}) \times (B_x \boldsymbol{i} + B_y \boldsymbol{j} + B_z \boldsymbol{k})$$

$$\vec{A} \times \vec{B} = A_x B_x \underbrace{(\hat{\imath} \times \hat{\imath})}_{0} + A_x B_y \underbrace{(\hat{\imath} \times \hat{\jmath})}_{\hat{k}} + A_x B_z \underbrace{(\hat{\imath} \times \hat{k})}_{-\hat{\jmath}} + \cdots$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{\imath} - (A_x B_z - A_z B_x)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$$

* VECTOR quantity!

Cross (or vector) product

Also, the cross product can be written as a determinant.

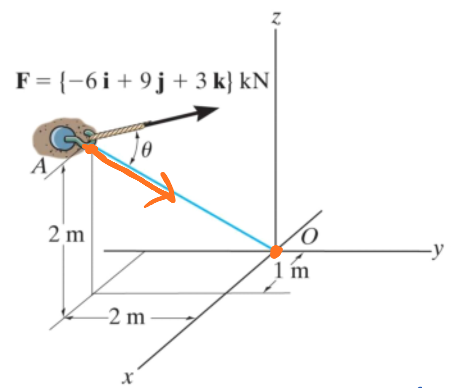
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be determined using 2 × 2 determinants.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} - \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{\imath} - (A_x B_z - A_z B_x)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$$

Example



Given: The force acting on the hook at point A. ; (x, y, z) w.r.t. origin.

Find: The angle between the force vector and the line AO, and the magnitude of the projection of the force along the line AO.

Plan:

1. find $\vec{r}_{AO}$
2. find θ
3. find F_{AO} with dot product

1. $$\vec{r}_{AO} = (x_o - x_A)\hat{\imath} + (y_o - y_A)\hat{\jmath} + (z_o - z_A)\hat{k}$$

$$\vec{r}_{AO} = (0-1)\hat{\imath} + (0-(-2))\hat{\jmath} + (0-2)\hat{k} = -1\hat{\imath} + 2\hat{\jmath} - 2\hat{k} \text{ m}$$

$$|\vec{r}_{AO}| = \sqrt{1^2 + 2^2 + 2^2} = 3\text{m}$$

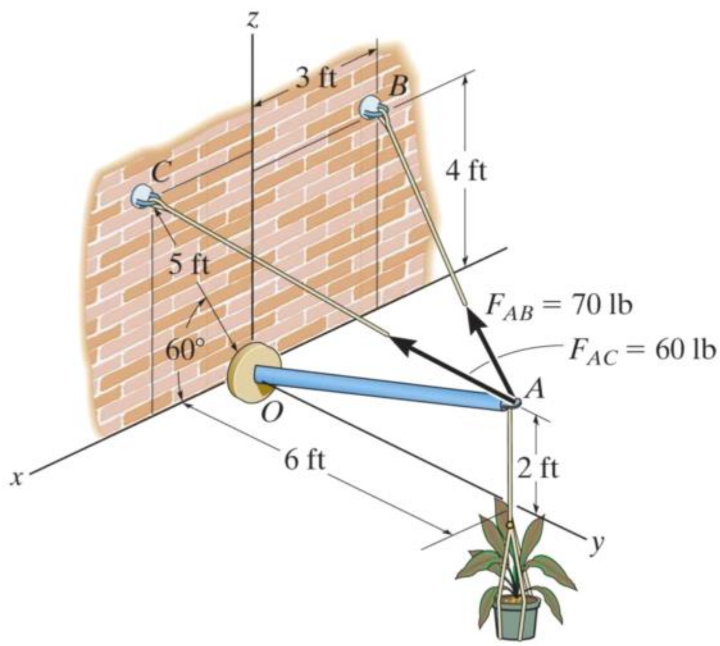
2. $$\theta = \cos^{-1}\left(\frac{\vec{F} \cdot \vec{r}_{AO}}{|\vec{F}||\vec{r}_{AO}|}\right) = \cos^{-1}\left(\frac{(-6\hat{\imath} + 9\hat{\jmath} + 3\hat{k})\text{kN} \cdot (-1\hat{\imath} + 2\hat{\jmath} - 2\hat{k})\text{m}}{|\vec{F}|\,|\vec{r}_{AO}|}\right)$$

$$\theta = \cos^{-1}\left(\frac{18 \text{ kN}\cdot\text{m}}{|\vec{F}||\vec{r}_{AO}|}\right) = \boxed{\dots}$$

3. $$\vec{F}_{AO} = \vec{F}\cos\theta$$

OR

$$F_{AO} = \vec{F} \cdot \vec{u}_{AO} = \vec{F} \cdot \frac{\vec{r}_{AO}}{|\vec{r}_{AO}|} = \frac{F_x r_x + F_y r_y + F_z r_z}{|\vec{r}_{AO}|}$$



Determine the projected component of the force vector F_{AC} along the axis of strut AO. Express your result as a Cartesian vector