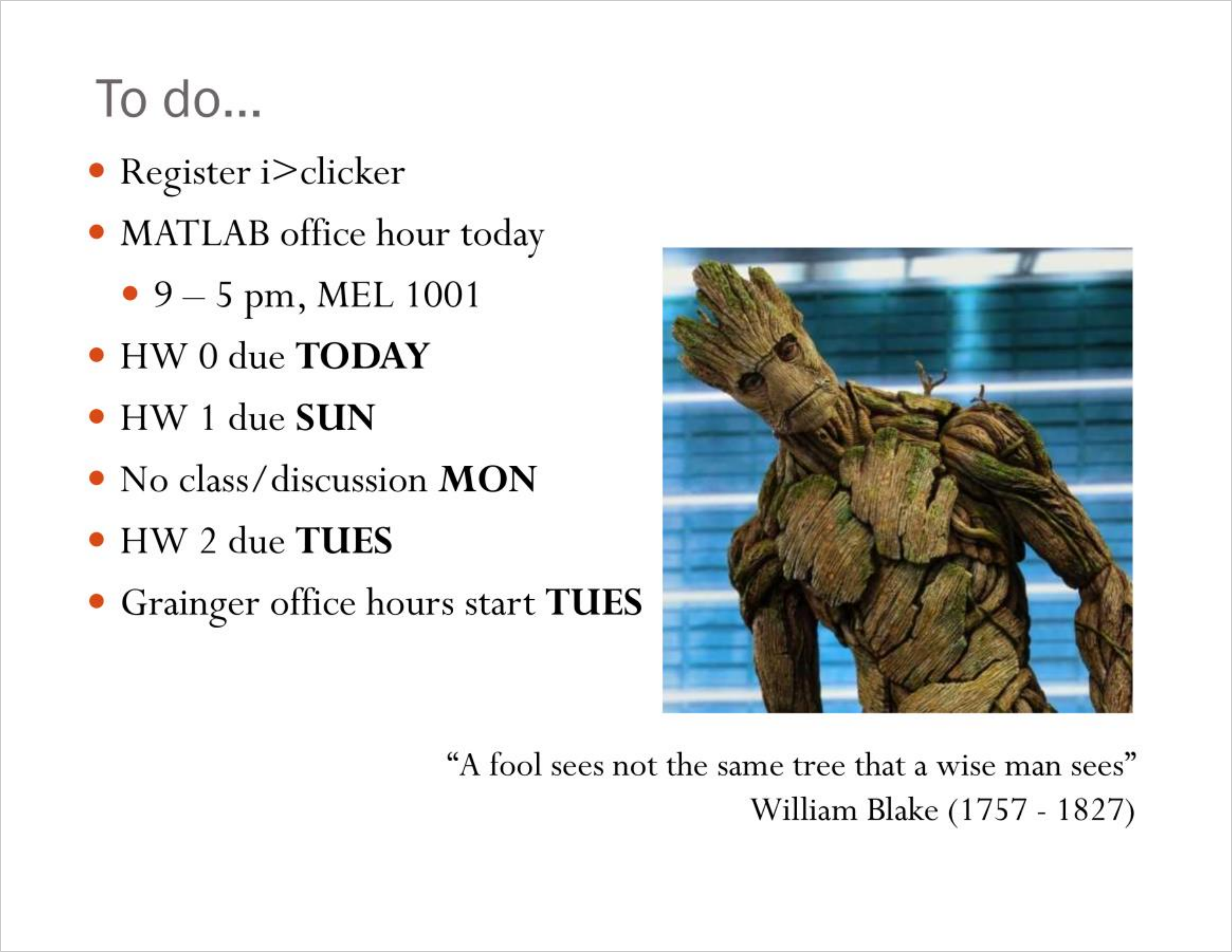
Lecture 03

Friday, September 1, 2017 11:42



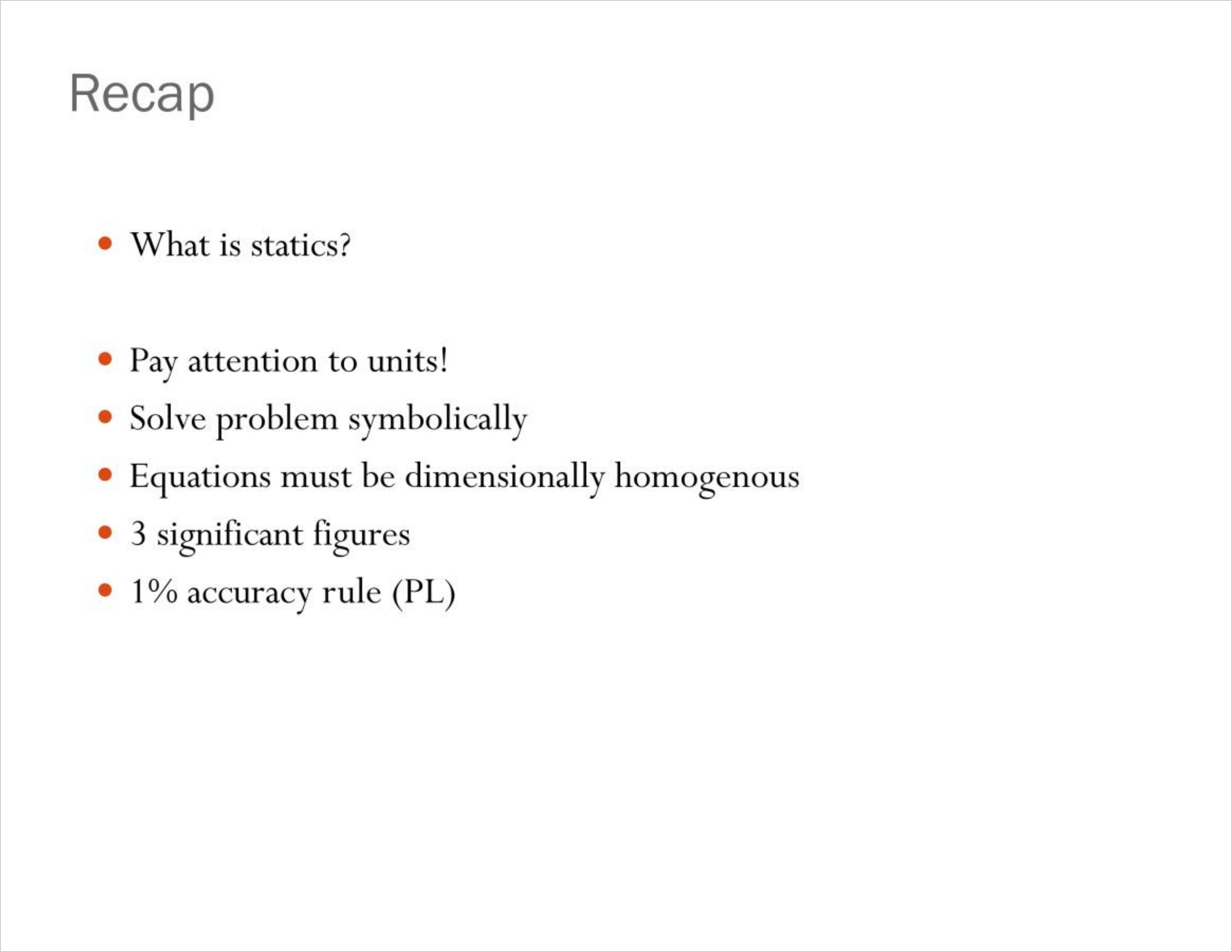
To do…

- Register i>clicker
- MATLAB office hour today
  - 9 – 5 pm, MEL 1001
- HW 0 due **TODAY**
- HW 1 due **SUN**
- No class/discussion **MON**
- HW 2 due **TUES**
- Grainger office hours start **TUES**

> "A fool sees not the same tree that a wise man sees"
> William Blake (1757 - 1827)

Page 2

Friday, September 1, 2017 11:42



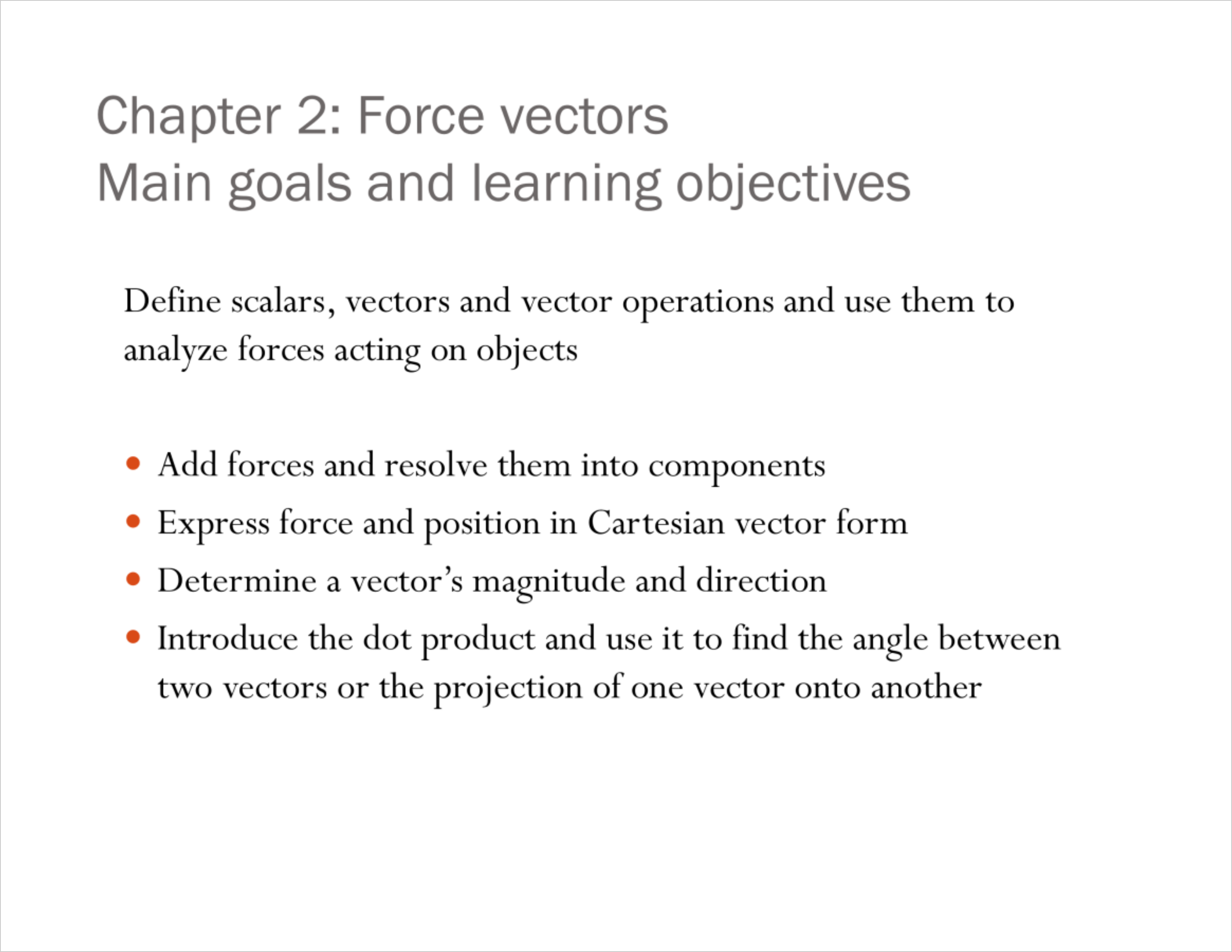
Recap

- What is statics?

- Pay attention to units!
- Solve problem symbolically
- Equations must be dimensionally homogenous
- 3 significant figures
- 1% accuracy rule (PL)

Page 3

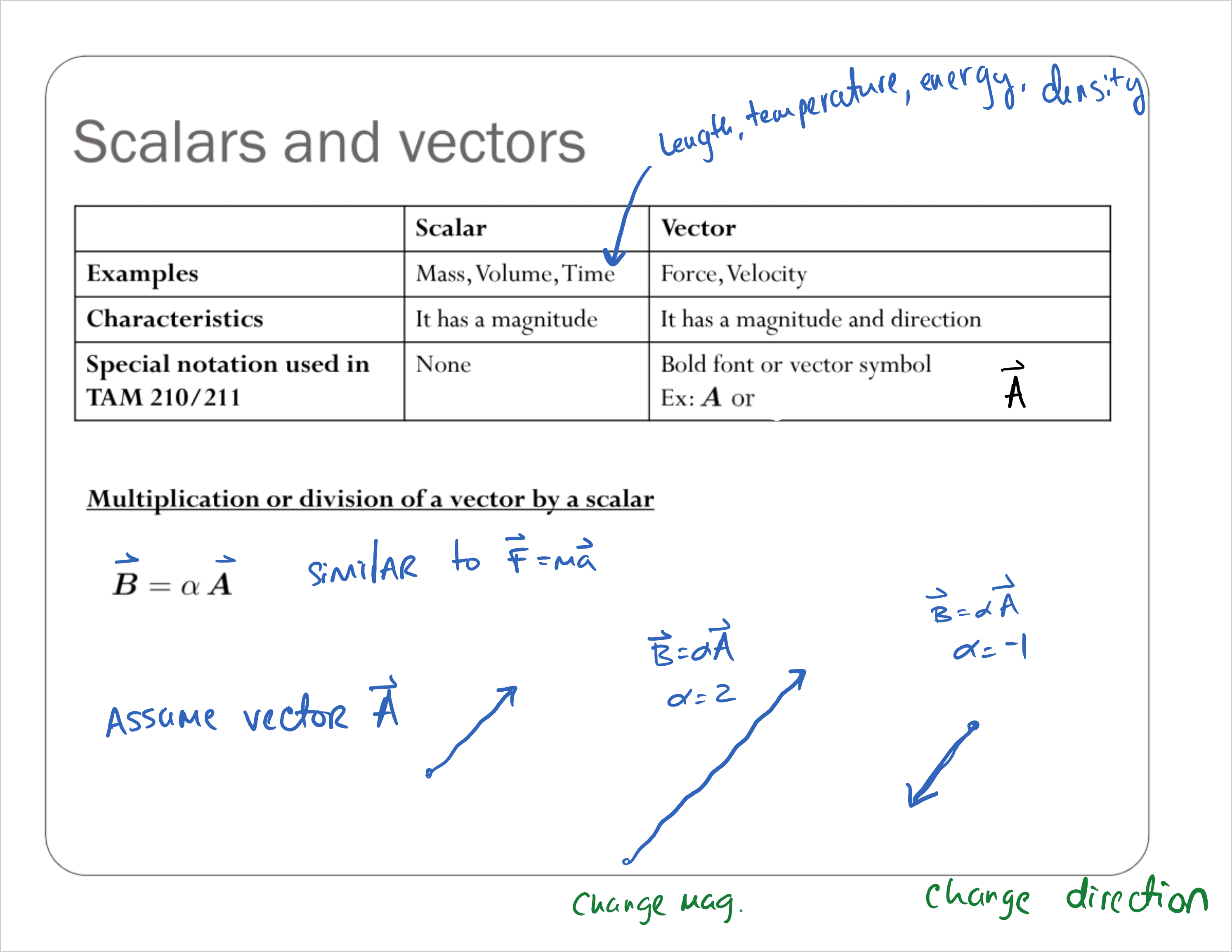
Friday, September 1, 2017 11:42



Chapter 2: Force vectors
Main goals and learning objectives

Define scalars, vectors and vector operations and use them to analyze forces acting on objects

- Add forces and resolve them into components
- Express force and position in Cartesian vector form
- Determine a vector’s magnitude and direction
- Introduce the dot product and use it to find the angle between two vectors or the projection of one vector onto another



Scalars and vectors

Length, temperature, energy, density

	Scalar	**Vector**
Examples	Mass, Volume, Time	Force, Velocity
Characteristics	It has a magnitude	It has a magnitude and direction
Special notation used in TAM 210/211	None	Bold font or vector symbol Ex: ***A*** or $\vec{A}$

Multiplication or division of a vector by a scalar

$\vec{\boldsymbol{B}} = \alpha \vec{\boldsymbol{A}}$ similAR to $\vec{F} = m\vec{a}$

Assume vector $\vec{A}$

$\vec{B} = \alpha \vec{A}$

$\alpha = 2$

$\vec{B} = \alpha \vec{A}$

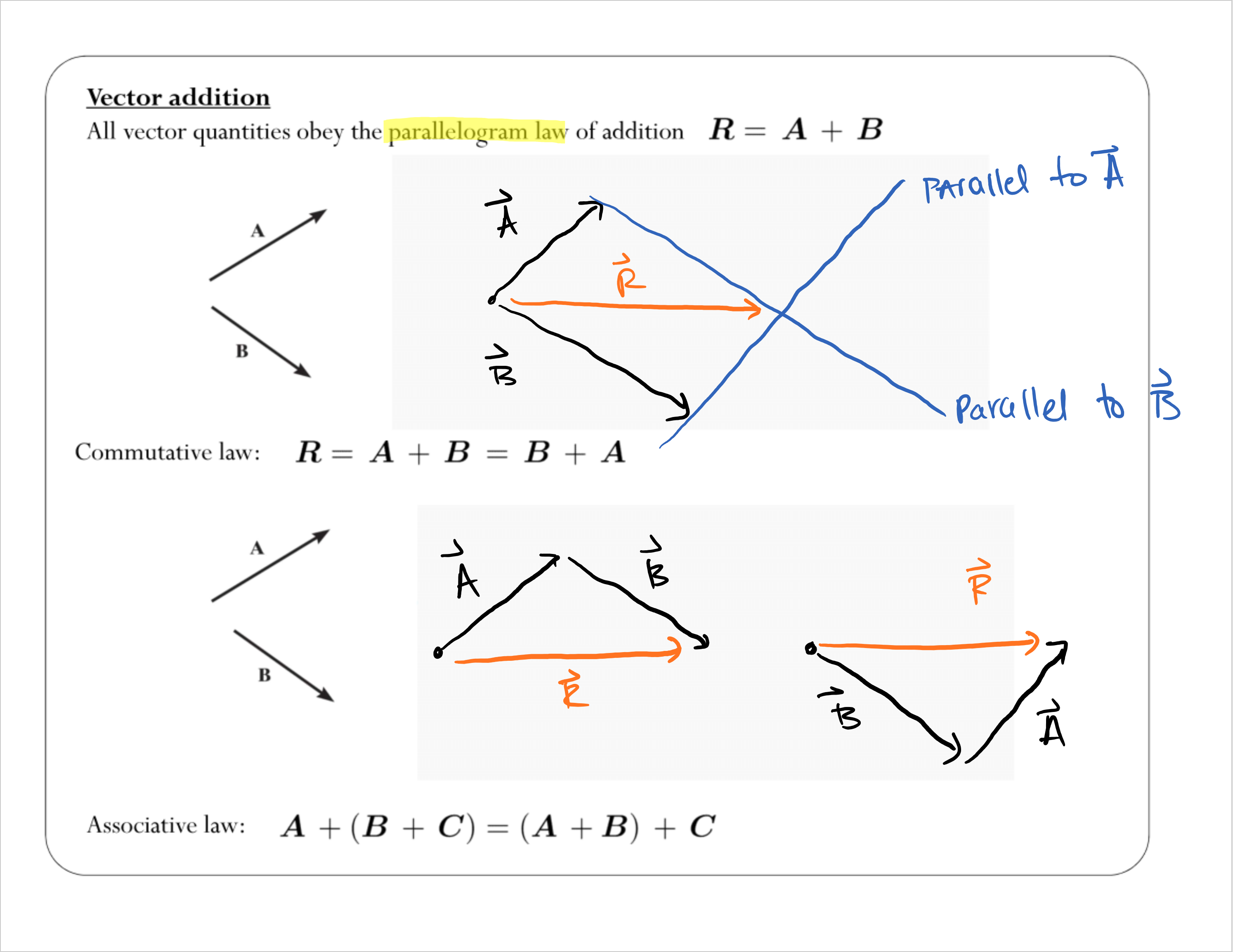
$\alpha = -1$

Change mag.

change direction

Page 5

Wednesday, August 24, 2016 10:49



Vector addition

All vector quantities obey the parallelogram law of addition $\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B}$

Commutative law: $\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B} = \boldsymbol{B} + \boldsymbol{A}$

Associative law: $\boldsymbol{A} + (\boldsymbol{B} + \boldsymbol{C}) = (\boldsymbol{A} + \boldsymbol{B}) + \boldsymbol{C}$

Vector subtraction:

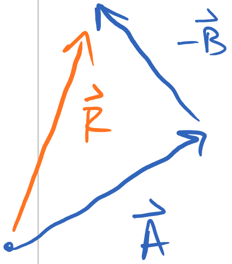
$$\boldsymbol{R} = \boldsymbol{A} - \boldsymbol{B} = \boldsymbol{A} + (-\boldsymbol{B})$$

note! $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$

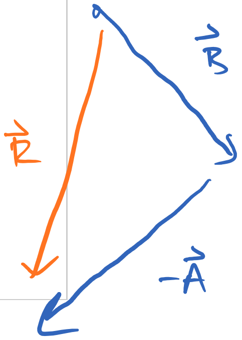
$(-\boldsymbol{B})$ has the same magnitude as $\boldsymbol{B}$ but is in opposite direction.

CONSIDER previous slide

$\vec{A} - \vec{B}$



$\vec{B} - \vec{A}$



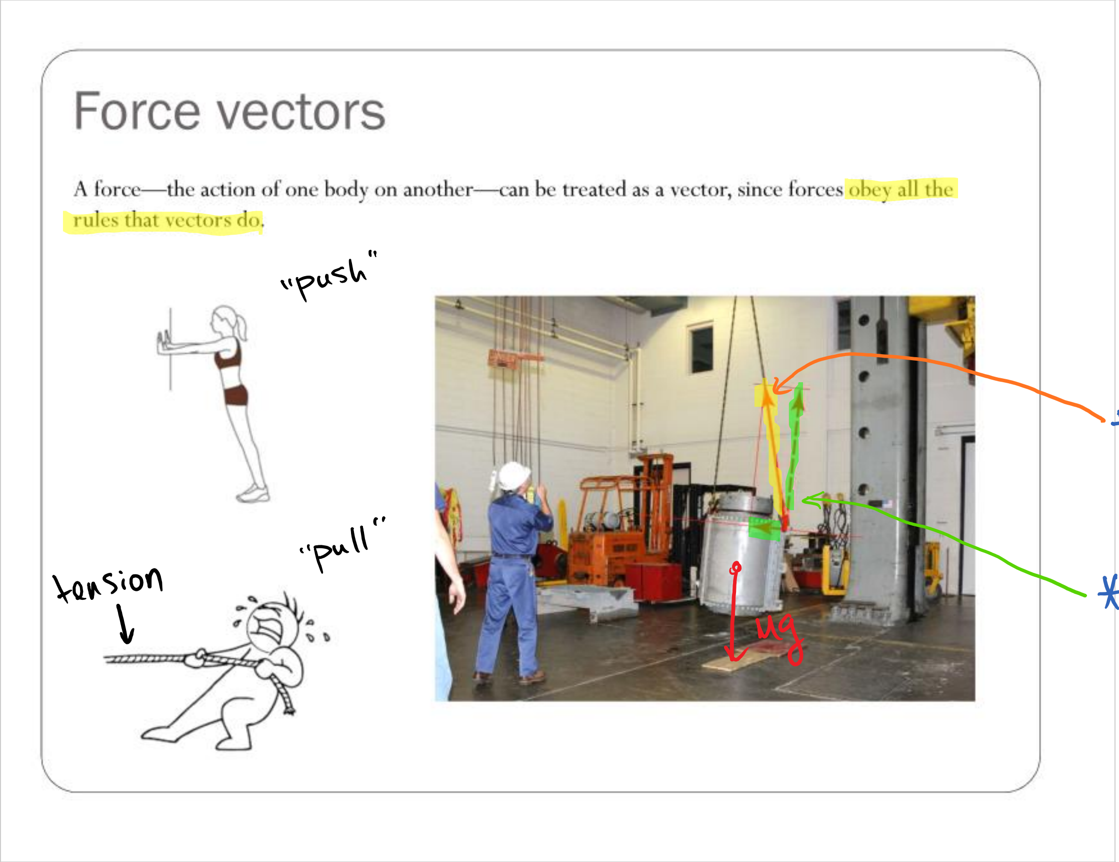
Scalar/Vector multiplication:

$$\alpha(\boldsymbol{A} + \boldsymbol{B}) = \alpha\vec{A} + \alpha\vec{B}$$

$$(\alpha + \beta)\boldsymbol{A} = \alpha\vec{A} + \beta\vec{A}$$

Page 7

Wednesday, August 24, 2016 10:49



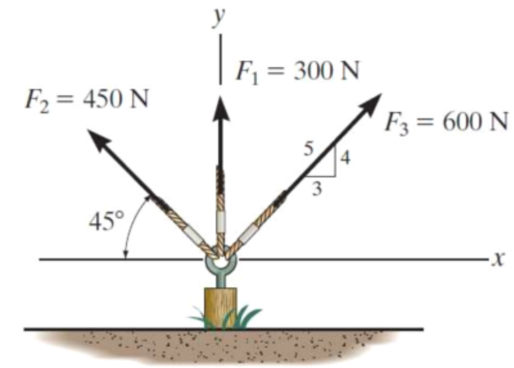
Force vectors

A force—the action of one body on another—can be treated as a vector, since forces obey all the rules that vectors do.

Generally asked to solve two types of problems

- * Determine Resultant force
- * resolve force into Components

Example

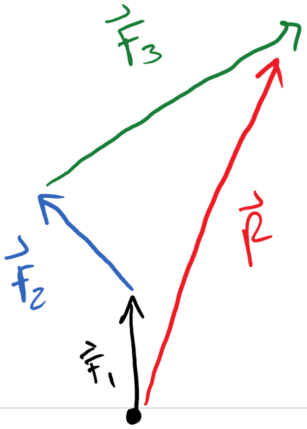


Given: Three concurrent forces acting on a tent post.

Find: The magnitude and angle of the resultant force.

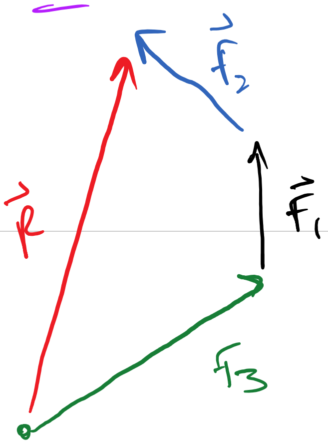
use the parallelogram law:

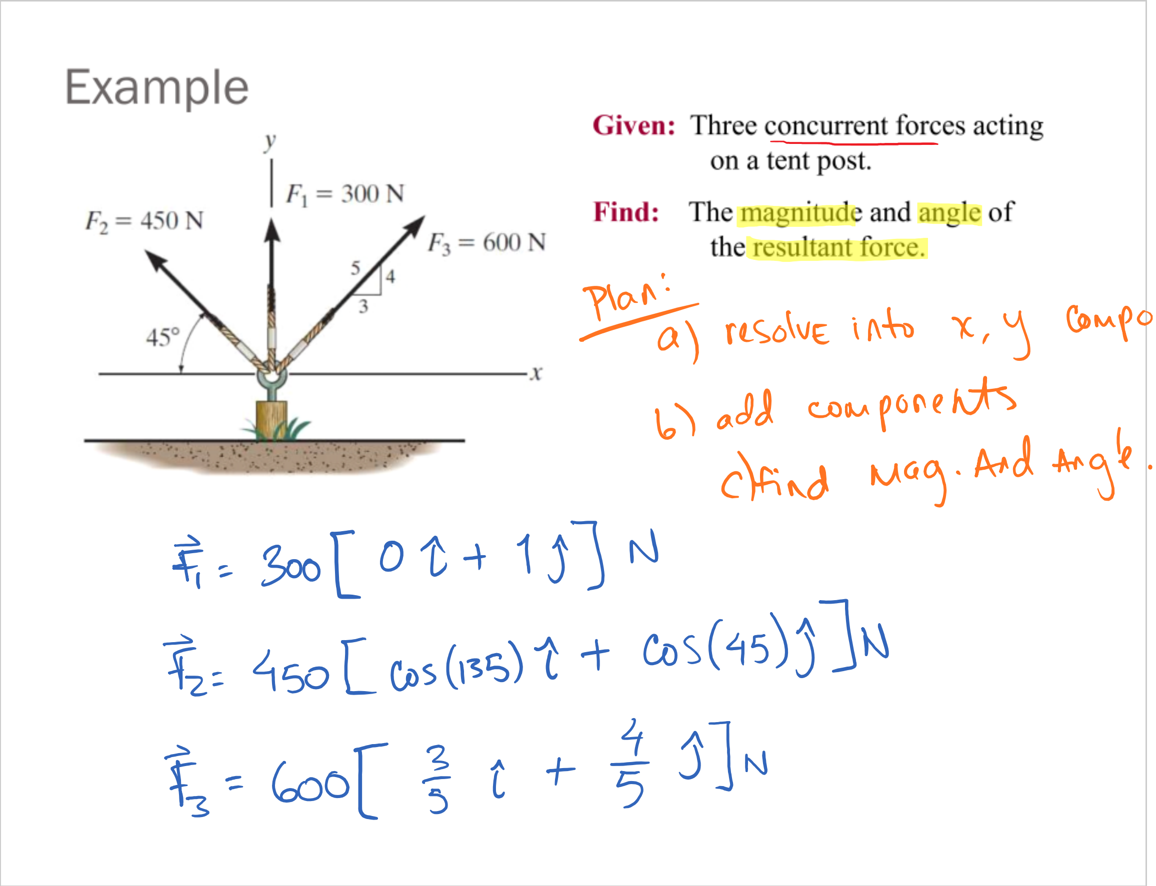
$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$



Q: Does the order of $\vec{F}_1, \vec{F}_2, \vec{F}_3$ matter?

↳ nope! not for Addition. Try other combinations.





Example

Given: Three concurrent forces acting on a tent post.

Find: The magnitude and angle of the resultant force.

Plan:

a) resolve into x, y components.

b) add components

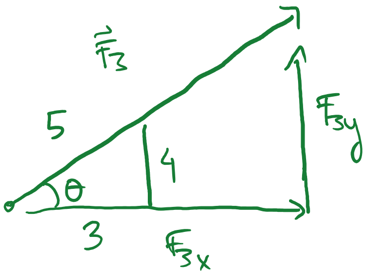
c) find mag. And Angle.

$$\vec{F}_1 = 300\left[0\,\hat{\imath} + 1\,\hat{\jmath}\right] N$$

$$\vec{F}_2 = 450\left[\cos(135)\,\hat{\imath} + \cos(45)\,\hat{\jmath}\right] N$$

$$\vec{F}_3 = 600\left[\frac{3}{5}\,\hat{\imath} + \frac{4}{5}\,\hat{\jmath}\right] N$$

find using similar triangles.



$$\frac{F_{3x}}{F_3} = \frac{3}{5} \rightarrow F_{3x} = \frac{3}{5} F_3$$

b.) $\vec{F}_R = \sum F_x\,\hat{\imath} + \sum F_y\,\hat{\jmath}$

$$\vec{F}_R = \underbrace{(F_{1x} + F_{2x} + F_{3x})}\,\hat{\imath} + \underbrace{(F_{1y} + F_{2y} + F_{3y})}\,\hat{\jmath}$$

$$\vec{F}_R = F_{Rx}\,\hat{\imath} + F_{Ry}\,\hat{\jmath}$$

c.) Magnitude:

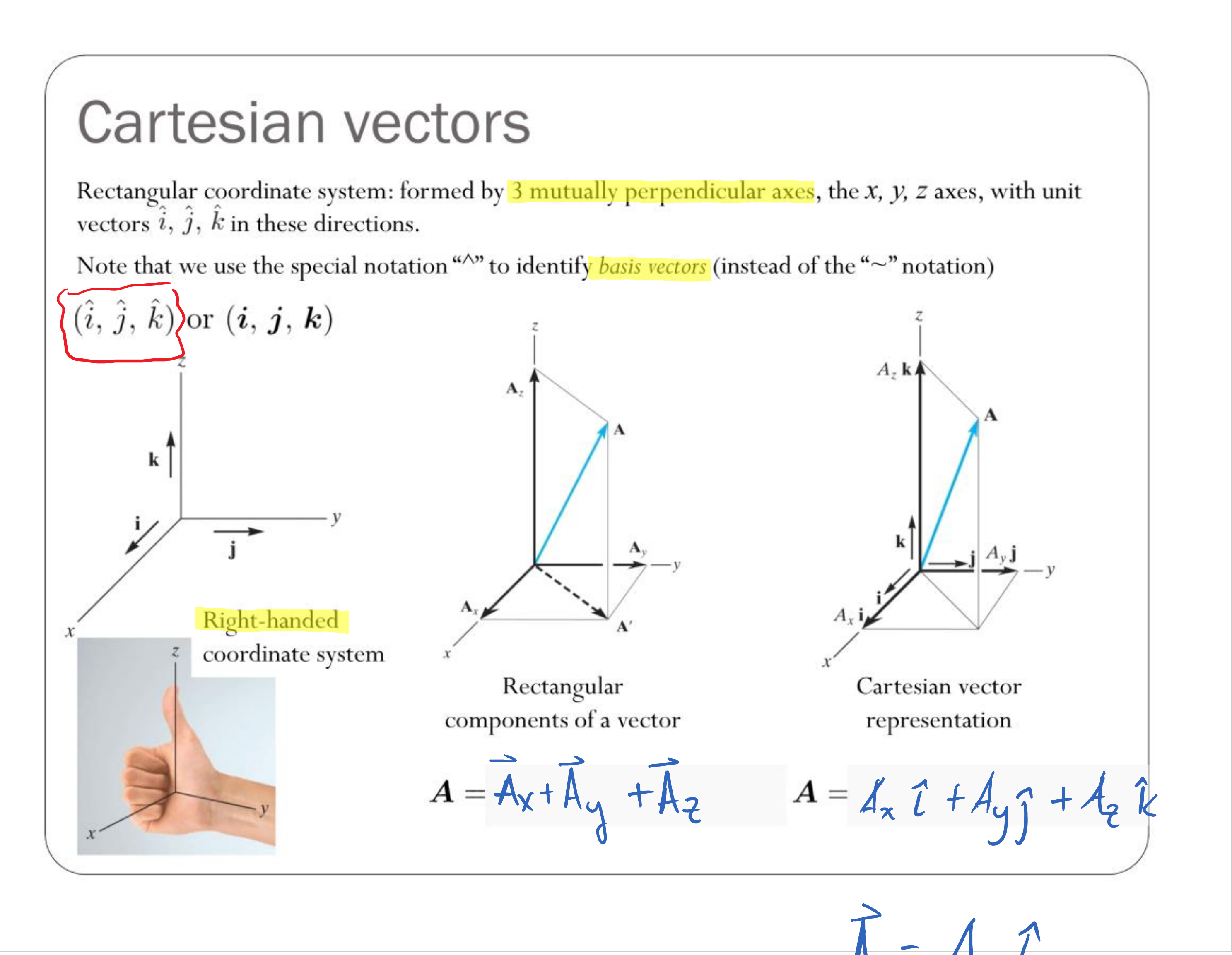
$$|\vec{F}_R| = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

Direction:

$$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right)$$

Page 10

Wednesday, August 24, 2016 10:49



Cartesian vectors

Rectangular coordinate system: formed by 3 mutually perpendicular axes, the x, y, z axes, with unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ in these directions.

Note that we use the special notation "^" to identify *basis vectors* (instead of the "~" notation)

$(\hat{i},\ \hat{j},\ \hat{k})$ or $(\boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k})$

Right-handed coordinate system

Rectangular components of a vector

Cartesian vector representation

$\mathbf{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$

$\mathbf{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

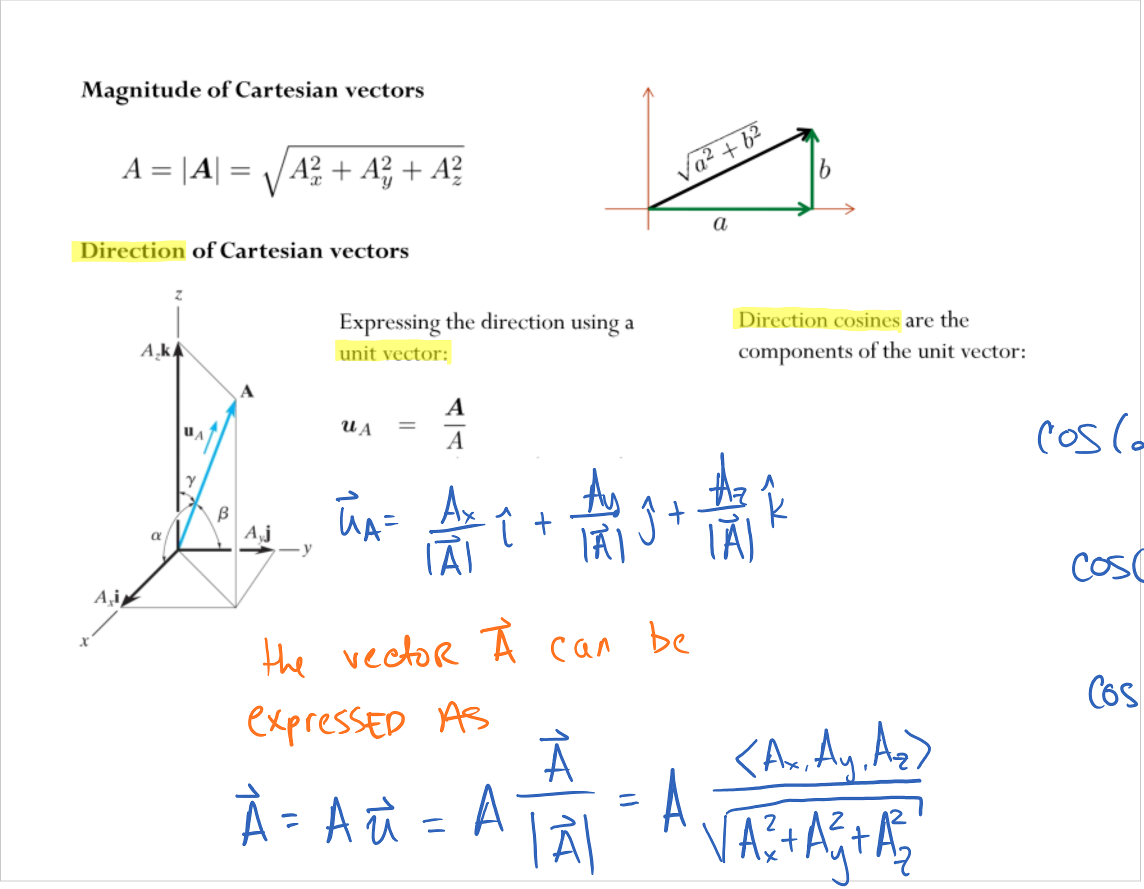
$$\vec{A}_x = A_x \hat{i}$$

scalar component

basis vector

Page 11

Friday, August 26, 2016 10:22



Magnitude of Cartesian vectors

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction of Cartesian vectors

Expressing the direction using a unit vector:

$$\boldsymbol{u}_A = \frac{\mathbf{A}}{A}$$

Direction cosines are the components of the unit vector:

$$\vec{u}_A = \frac{A_x}{|\vec{A}|}\hat{i} + \frac{A_y}{|\vec{A}|}\hat{j} + \frac{A_z}{|\vec{A}|}\hat{k}$$

the vector $\vec{A}$ can be expressed As

$$\vec{A} = A\vec{u} = A\frac{\vec{A}}{|\vec{A}|} = A\frac{\langle A_x, A_y, A_z\rangle}{\sqrt{A_x^2+A_y^2+A_z^2}}$$

$$\cos(\alpha) = \frac{A_x}{|\vec{A}|}$$

$$\cos(\beta) = \frac{A_y}{|\vec{A}|}$$

$$\cos(\gamma) = \frac{A_z}{|\vec{A}|}$$

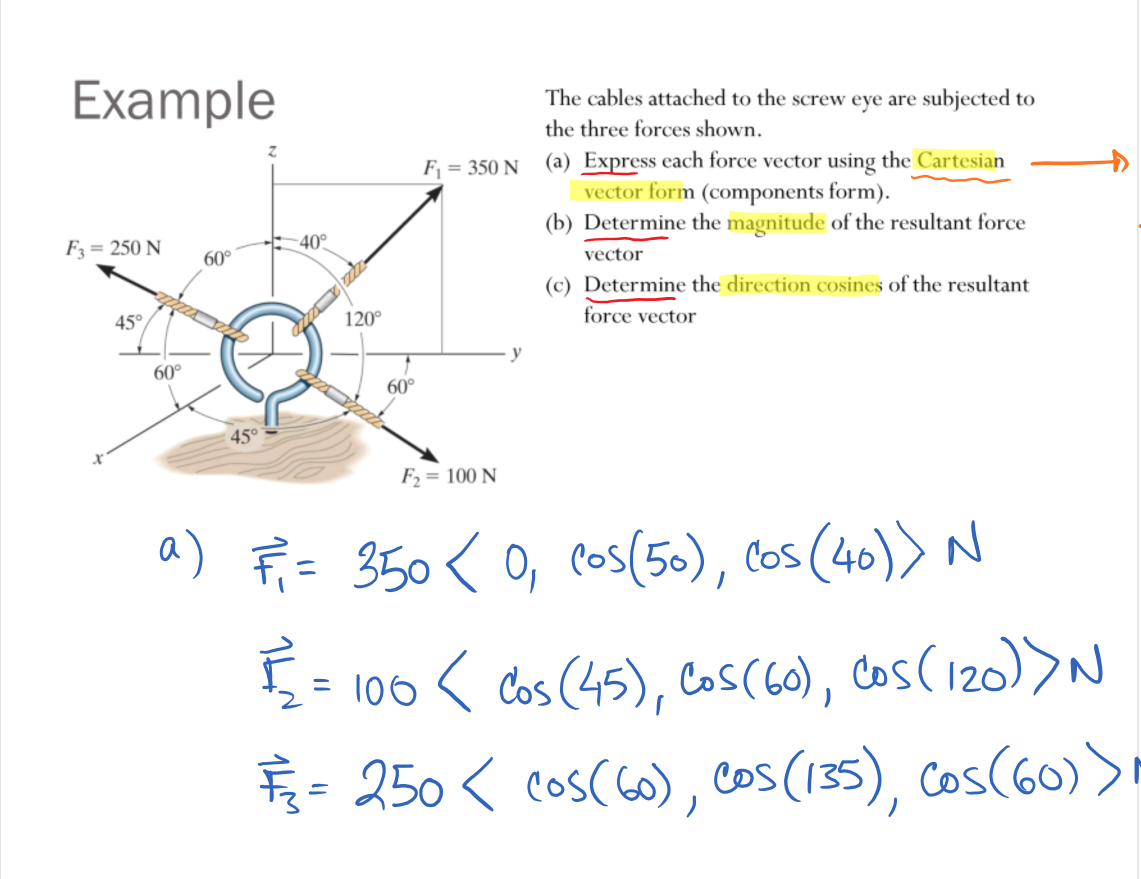
magnitude of the unit vector is

$$|\vec{u}| = 1$$

∴

the vector $\vec{A}$ cAn be expressed As.

$$\vec{A} = A\langle\cos(\alpha), \cos(\beta), \cos(\gamma)\rangle$$



Example

The cables attached to the screw eye are subjected to the three forces shown.

(a) Express each force vector using the Cartesian vector form (components form).

(b) Determine the magnitude of the resultant force vector

(c) Determine the direction cosines of the resultant force vector

$\longrightarrow \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{A} = A\langle \cos\alpha, \cos\beta, \cos\gamma \rangle$

a) $\vec{F}_1 = 350\langle 0, \cos(50), \cos(40)\rangle$ N

$\vec{F}_2 = 100\langle \cos(45), \cos(60), \cos(120)\rangle$ N

$\vec{F}_3 = 250\langle \cos(60), \cos(135), \cos(60)\rangle$ N

b) $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$|\vec{F}_R| = \sqrt{\Sigma F_x^2 + \Sigma F_y^2 + \Sigma F_z^2}$

c) $\cos(\alpha_R) = \dfrac{F_{Rx}}{|\vec{F}_R|}$ $\cos(\beta_R) = \dfrac{F_{Ry}}{|\vec{F}_R|}$ $\cos(\gamma_R) = \dfrac{F_{Rz}}{|\vec{F}_R|}$