Unification of Electromagnetic and Weak interaction

We start by considering the simple case of the first family of the lepton sector. From the phenomenology of charged-current interaction, only left-handed leptons participate in CC interaction. The gauge symmetry is assumed to be $SU(2)_L$

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \; ; \quad \psi_R = e_R$$

where ν_L, e_L form the weak isospin doublet and $\psi_R = e_R$ is a weak isospin singlet. Also, assume no ν_R (only left-handed ν exists)

The Lagrangian is $\mathcal{L}_0 = \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R$

using $e = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$, then $\mathcal{L}_0 = \bar{e}\, i\gamma^\mu \partial_\mu e + \bar{\nu}_L i\gamma^\mu \partial_\mu \nu_L$

Comparing with $\mathcal{L}_0$ for $SU(3)$, we find that there is no mass term for $\mathcal{L}_0$ in $SU(2)_L$. This is due to the fact that mass terms like $\bar{\psi}_L m \psi_L$ or $\bar{\psi}_R m \psi_R$ vanish. (note that $\psi_L = \frac{1-\gamma^5}{2}\psi$ and $\bar{\psi}_L m \psi_L = \bar{\psi}(\frac{1+\gamma^5}{2}) m (\frac{1-\gamma^5}{2})\psi = 0$).

A mass term like $\bar{\psi}_R m \psi_L$ is not invariant under $SU(2)_L$ transformation (since ψ_L is a doublet and ψ_R a singlet under $SU(2)_L$), and cannot exist either.

We conclude that $SU(2)_L$ cannot be the whole story, since both ν and e are massless, which is contradictory to reality.

The $SU(2)_L$ transformation is given by $U = e^{-ig w_i t}$

$t_{iL} = \frac{1}{2} \tau_i$, when it operates on the ψ_L doublet
$t_{iR} = 0$, when it operates on the ψ_R singlet

We note that $\mathcal{L}_0$ is also invariant under $U(1)$ transformation :

$$U(w) = e^{-ig' w F}$$

where g' is the coupling constant, F is the generator

What is F referring to ? Is it the charge Q ?
Since $\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ contains members of different electric charge, one concludes ~~that F cannot be Q~~ if one wishes to unify $SU(2)_L$ and $U(1)$.
One can generalize 'electric charge' to 'hypercharge' where

$$Y = 2(Q - T_3)$$ (T_3 is the 3rd component of weak isospin)

Particle	T	T_3	Q	Y
ν_e	$\frac{1}{2}$	$+\frac{1}{2}$	0	-1
e_L	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
e_R	0	0	-1	-2

We see that $\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$ both have the same hypercharge $Y = -1$.

Under $SU(2)_L \times U(1)_Y$, we have

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I + \mathcal{L}_G$$

where

$$\mathcal{L}_0 + \mathcal{L}_I = \bar{\psi}_L i \gamma^\mu D_\mu^L \psi_L + \bar{\psi}_R i \gamma^\mu D_\mu^R \psi_R$$

Again, $\psi_L = \begin{pmatrix} \nu_e \\ e_L^- \end{pmatrix}$, $\psi_R = e_R^-$

$$D_\mu^L \psi_L = (\partial_\mu + i g A_{i\mu} \frac{\tau_i}{2} + i g' B_\mu \frac{Y_L}{2}) \psi_L$$

$$D_\mu^R \psi_R = (\partial_\mu + i g' B_\mu \frac{Y_R}{2}) \psi_R$$

We have

3 gauge fields $A_{i\mu}$ for $SU(2)_L$

1 gauge field B_μ for $U(1)_Y$

$$\mathcal{L}_G = -\frac{1}{4} W_{\mu\nu}^i W_i^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

where

$$W_{\mu\nu}^i = \partial_\mu A_\nu^i - \partial_\nu A_\mu^i - g \epsilon_{ijk} A_\mu^j A_\nu^k$$ (note for SU(2) $f_{ijk} = \epsilon_{ijk}$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$A_\mu^{i\prime} = A_\mu^i + \partial_\mu \omega^i - g f_{ijk} A_{j\mu} \omega_k$$

Two outstanding problems:

1) e^- is massless

2) gauge bosons for $SU(2)_L$ are massless

How to solve these two problems?

To solve these problems, we introduce two complex scalar fields, forming an $SU(2)$ doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

The notation ϕ^+ suggests it is charged. A complex field contains ~~[illegible]~~ two charge-conjugated fields. One can consider ϕ^+ as a (very rough) analog of K^+, K^- scalar meson pair. Similarly, ϕ^0 is analogous to $K^0, \bar{K}^0$ meson pair.

The number of components for ϕ is 4, which is the same as the number of gauge bosons in $SU(2) \times U(1)$.

As we will see later, the $SU(2)$-doublet nature of ϕ can be used to generate a 'Yukawa'-coupling term in the Lagrangian relevant for the masses for electron (and neutrino).

The introduction of the scalar field ϕ leads to a new term in the Lagrangian:

$$\mathcal{L}_S = (D_\mu \phi)^+ (D^\mu \phi) - V(\phi)$$

where $D_\mu \phi = (\partial_\mu + i g A_\mu + i g' B_\mu \frac{Y_H}{2}) \phi$ is to make ϕ invariant under $SU(2)_L$ and $U(1)_Y$.

The potential term $V(\phi)$ experienced by ϕ is given as

$$V = \mu^2 \phi^+ \phi + \lambda (\phi^+ \phi)^2$$

(Note that $\mathcal{L}_0^{K-G} = \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$ for spin-0 particle

The introduction of the scalar doublet fields allows a new coupling between the scalar field and the fermion field. The gauge-invariant 'Yukawa' term (The name 'Yukawa' is referring to the nucleon-nucleon-pion coupling in analogy to the fermion-fermion-scalar coupling case here)

$$\mathcal{L}_Y = -C_e\left[\bar{\psi}_R(\phi^\dagger \psi_L) + (\bar{\psi}_L \phi)\psi_R\right]$$

$\mathcal{L}_Y$ is invariant under $SU(2)_L$, since ϕ and ψ_L are both $SU(2)$ doublets.

To ensure $U(1)$ invariance for $\mathcal{L}_Y$, where $U(w) = e^{-iwfY}$ for $U(1)$ transformation acting on $\mathcal{L}_Y$, we obtain $Y_H - Y_L + Y_R = 0$ or $Y_H = (-1) - (-2) = 1$, where Y_H is the hypercharge of the scalar doublet (Higgs).

Since $Q = T_3 + Y/2$, we conclude that ϕ^+ has $Q = +1$ and ϕ^0 has $Q = 0$, which justifies the notations for ϕ^+ and ϕ^0.

It is important to note that $\mathcal{L}_Y$ now connects e_R and e_L ($\bar{e}_R e_L$ and $\bar{e}_L e_R$), hence electron can now be massive, due to the presence of the Scalar Higgs field.

We still need to make the gauge boson massive (for W^+, W^-, and Z^0), while the photon is massless. The solution involves the concept of "spontaneous symmetry Breaking" for the scalar fields under $SU(2)_L$ and $U(1)_Y$.

Spontaneous symmetry Breaking (SSB)

1) Global $U(1)$ symmetry

Consider a complex scalar field singlet ϕ:

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

$$\mathcal{L} = (\partial_\mu \phi^+)(\partial^\mu \phi) - V(\phi)$$

where the potential term has the form

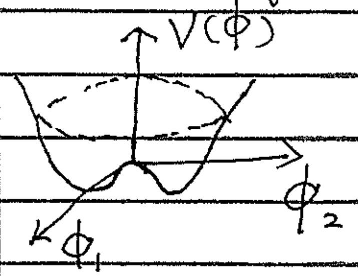
$$V(\phi) = \mu^2 \phi^+ \phi + \frac{1}{4}\lambda(\phi^+\phi)^2$$

The inclusion of the $(\phi^+\phi)^2$ term allows the presence of Spontaneous Symmetry Breaking, as we will see.

$\mathcal{L}$ is clearly invariant under global $U(1)$ transformation

$$\phi \rightarrow \phi' = e^{i\alpha G}\phi$$

~~Plotting $V(\phi)$ as a function of ϕ_1 and ϕ_2 for~~ the case of $\mu^2 < 0$, $\lambda > 0$, we have



$U(1)$ invariance implies symmetry under rotation along an axis perpendicular to ϕ_1, ϕ_2. However, the minima of $V(\phi)$ occur at locations where $\phi_1^2 + \phi_2^2 = \frac{-4\mu^2}{\lambda} \equiv v^2$

From $\phi(x) = \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x))$, we change to polar coordinates :

$$\phi_1(x) = \rho(x)\cos(-\theta(x)/v)$$

$$\phi_2(x) = \rho(x)\sin(-\theta(x)/v)$$

then,

$$\phi(x) = (\rho(x)/\sqrt{2})\exp[-i\theta(x)/v]$$

Selecting $\rho = v = \sqrt{\frac{-4\mu^2}{\lambda}}$ and $\theta = 0$ as the physical vacuum, and expand ϕ around this vacuum we have

$$\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))\exp[-i\theta(x)/v]$$

$$(\phi_1(x), \phi_2(x)) \Longrightarrow (h(x), \theta(x))$$

The $U(1)$ symmetry (rotation along the vertical axis) is now spontaneously broken.

In terms of $h(x)$ and $\theta(x)$, $\mathcal{L}$ becomes

$$\mathcal{L} = \frac{1}{2}\partial_\mu h\,\partial^\mu h + \mu^2 h^2 + \frac{1}{2}\partial_\mu\theta\,\partial^\mu\theta + \frac{\mu^4}{\lambda} + \cdots$$

Hence, $h(x)$ excitation has mass of $\sqrt{2}(-\mu^2)^{1/2}$

$\theta(x)$ excitation is massless (no quadratic term in $\theta(x)$) This simply reflects the zero curvature along the θ direction around the vacuum. We find :

When $\mu^2 > 0$ (no SSB), we have $[(\mu^2)^{1/2}, (\mu^2)^{1/2}]$ as the masses

when $\mu^2 < 0$ (SSB), we have $[\sqrt{2}(-\mu^2)^{1/2}, 0]$ as the masses

The result of the SSB is that one of the scalar fields, $\theta(x)$, is now massless. This is a consequence of the Goldstone Theorem, which states:

"Each Spontaneously broken symmetry introduces a massless particle (called Goldstone Boson)"

2) Global SU(2) SSB

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \end{pmatrix}$$

$$\mathcal{L} = (\partial_\mu \phi^+)(\partial^\mu \phi) - \mu^2 \phi^+ \phi - \frac{\lambda}{4}(\phi^+ \phi)^2$$

ϕ contains 4 degrees of freedom, each with the same mass, μ (when $\mu^2 > 0$)

$\mathcal{L}$ is invariant under

$SU(2)$: $\phi' = \exp(-i\vec{\alpha}\cdot\vec{\tau}/2)\phi$

$U(1)$: $\phi' = \exp(-i\alpha)\phi$

(There are 4 conserved charges (symmetries), 3 from SU(2) and 1 from U(1))

when $\mu^2 > 0$, vacuum occurs at $\phi = 0$

when $\mu^2 < 0$, minimum occurs at

$\phi^\dagger \phi = -2\mu^2/\lambda \equiv v^2/2$

or $\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = v^2$

Choose the vacuum such that

$$\phi = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \quad ; \quad \text{i.e.} \quad \phi_1 = \phi_2 = \phi_4 = 0, \quad \phi_3 = v$$

Excitation around the vacuum can again be expressed in terms of radial ($H(x)$) and axial components ($\vec{\theta}(x)$):

$$\phi(x) = \exp\left[-i\left(\vec{\theta}(x)\cdot\vec{\tau}/2\right)/v\right] \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+H(x)) \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu H \partial^\mu H - \mu^2 H^2 + \cdots\cdots$$

The masses for $\vec{\theta}(x)$ and $H(x)$ excitations are $(0, 0, 0, \sqrt{2}|\mu|)$, while the original masses before SSB are (μ, μ, μ, μ) (when $\mu^2 > 0$)

Why are there only three massless Goldstone bosons after SSB, and not 4 Goldstone bosons (there were 4 symmetries, one from $U(1)$ and three from $SU(2)$) ?

The answer is that the chosen vacuum might not break all possible symmetries. Hence, the number of massless Goldstone bosons might be less than the maximal number.

In particular, the infinitesimal transformation $\delta\phi = -i\varepsilon'(1+\tau_3)\phi$, applied to the SSB vacuum state, gives $\delta\phi = 0$. Hence the $(\underset{U(1)}{1} + \underset{SU(2)}{\tau_3})$ symmetry is still preserved by the vacuum.

where are these massless spin-0 Goldstone bosons?
To address this question, we consider the case for local SU(n) transformation.

3) Local U(1) SSB

For local SU(n) symmetry, we need to add the terms from the gauge fields. For local U(1):

$$\mathcal{L} = [(\partial^\mu + iq A^\mu)\phi^\dagger][(\partial_\mu + iq A_\mu)\phi] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\lambda(\phi^\dagger\phi)^2 + \mu^2(\phi^\dagger\phi)$$

(we change the sign for μ^2, so that $\mu^2 > 0$ for SSB)

Adding the massless gauge field A^μ, we now have four field degrees of freedom (2 from the complex scalar field ϕ, and 2 from massless vector gauge field A^μ)

Repeating the steps taken for the global U(1), we can expand $\phi(x)$ around the SSB vacuum:

$$\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))\exp[-i\,\theta(x)/v]$$

The gauge freedom in local U(1) implies invariance under

$$\phi'(x) = e^{-i\alpha(x)}\phi(x)$$
$$A'^\mu(x) = A^\mu(x) + \frac{1}{q}\partial^\mu\alpha(x)$$

The gauge freedom in $\phi(x)$ and $A^\mu(x)$ allows us to choose

$$\alpha(x) = -\theta(x)/v$$

hence, $\phi'(x) = e^{-i\alpha(x)}\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))$

Expressing $\mathcal{L}$ in terms of $\phi'(x)$ (as $\mathcal{L}$ is invariance under $\phi \to \phi'$ transformation), we have

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) + \underbrace{\frac{1}{2} q^2 v^2 A_\mu A^\mu}_{\text{massive gauge boson}} + \frac{1}{2}\partial_\mu h \partial^\mu h \underbrace{- \mu^2 h^2}_{\text{massive scalar particle}} + \cdots$$

Hence, the gauge boson becomes massive (at the expense of the 'disappearance' of the Goldstone boson $\theta(x)$)

From $\mathcal{L}$, we see that the mass of the scalar field is $\sqrt{2}\mu$, and the mass of the gauge boson become qv.

We see that before the SSB, the initial field degrees of freedom contain $\underbrace{2}_{\text{for the}}$ scaler field, and another 2 for the massless gauge boson (only transverse fields). After the SSB, we have 1 for the scaler field (the Goldstone boson has disappeared), and 3 for the massive gauge boson (longitudinal excitation is also possible)

Now, we return to $SU(2)_L \times U(1)_Y$, adding the scalar field (and spontaneously broken symmetry), and the gauge freedom for ϕ ($\phi \to \phi'$), we have, dropping the ' symbol in ϕ:

$$\mathcal{L}_S = (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$
(from the scalar field)

$$\mathcal{L}_G = -\frac{1}{4} W^i_{\mu\nu} W_i^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
(from the gauge fields)

$$\mathcal{L}_\ell = \overline{\Psi}_L i\gamma^\mu D_\mu^L \Psi_L + \overline{\Psi}_R i\gamma^\mu D_\mu^R \Psi_R$$
(from the fermion field)

$$\mathcal{L}_{eY} = -C_e \left[\overline{\Psi}_R (\phi^\dagger \Psi_L) + (\overline{\Psi}_L \phi) \Psi_R \right]$$
(from the Yukawa coupling)

1) We first find the implications of the scalar field term, $\mathcal{L}_S$:

$$D_\mu \phi = \left(\partial_\mu + i g A_\mu + i g' B_\mu \frac{Y_H}{2}\right) \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}$$

where $A_\mu = \frac{1}{2} \tau_i A_{i\mu} = \frac{1}{2} (\tau_1 A_{1\mu} + \tau_2 A_{2\mu} + \tau_3 A_{3\mu})$

$$= \frac{1}{\sqrt{2}} (\tau_+ W_\mu + \tau_- W_\mu^\dagger) + \frac{1}{2} \tau_3 A_{3\mu}$$

$\tau_\pm = \frac{1}{2}(\tau_1 \pm i \tau_2)$ are the raising/lowering operators

$W_\mu = \frac{1}{\sqrt{2}}(A_{1\mu} - i A_{2\mu})$ and $W_\mu^\dagger = \frac{1}{\sqrt{2}}(A_{1\mu} + i A_{2\mu})$

Then,

$$(D_\mu \phi)^\dagger (D_\mu \phi) = \frac{1}{4} g^2 v^2 W_\mu^\dagger W^\mu + \frac{1}{8} v^2 (g A_{3\mu} - g' B_\mu)^2 + \cdots$$

$$\Downarrow$$

$$M_W^2 W_\mu^\dagger W^\mu$$

Hence, the first term shows that $\boxed{M_W = \frac{1}{2} g v}$

For the neutral gauge bosons, A_3 and B, the cross term in $(g A_{3\mu} - g' B_\mu)^2$ shows that A_3 and B are mixed. The mass eigenstates, A and Z, are given as

$$A_\mu = \sin\theta_W A_{3\mu} + \cos\theta_W B_\mu$$
$$Z_\mu = \cos\theta_W A_{3\mu} - \sin\theta_W B_\mu$$

(θ_W is the 'Weinberg' angle)

then,

$$\frac{1}{8} v^2 (g A_{3\mu} - g' B_\mu)^2$$
$$= \frac{1}{8} v^2 \{ A_\mu^2 (g \sin\theta_W - g' \cos\theta_W)^2 + Z_\mu^2 (g\cos\theta_W + g'\sin\theta_W)^2$$
$$+ 2 A_\mu Z^\mu (g \sin\theta_W - g'\cos\theta_W)(g\cos\theta_W + g'\sin\theta_W)$$

For A to be the massless photon, the coeffient in front of A_μ^2 must vanish $\Longrightarrow g\sin\theta_W - g'\cos\theta_W = 0$

or,

$$g\sin\theta_W = g'\cos\theta_W, \quad \boxed{g'/g = \tan\theta_W},$$
$$\cos\theta_W = g/(g^2+g'^2)^{1/2}, \quad \sin\theta_W = g'/(g^2+g'^2)^{1/2}$$

Therefore, the relative $U(1)_Y$, $SU(2)_L$ coupling strength, g'/g, is given by $\tan\theta_W$.

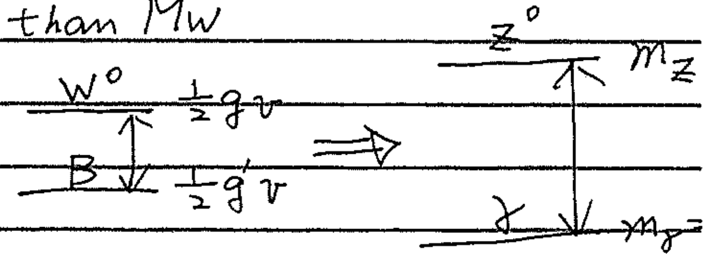
From the coefficient of Z_μ^2, we see that the mass

$$M_Z = \frac{1}{2} v (g\cos\theta_W + g'\sin\theta_W) = \frac{1}{2} g v \frac{1}{\cos\theta_W}$$

or $\boxed{M_W = \cos\theta_W M_Z}$

Hence, M_Z is heavier than M_W

Graphically, we have



The potential term in $\mathcal{L}_S$

$$V(\phi) = \frac{1}{4}\mu^2 v^2 + \mu^2 H^2 + \lambda\left(vH^3 + \frac{1}{4}H^4\right)$$

Hence $\boxed{M_H = \sqrt{-2\mu^2}}$

2) We next examine the $\mathcal{L}_{eY}$ Yukawa term

$$\mathcal{L}_{eY} = -C_e\left[\bar\Psi_R(\phi^\dagger\Psi_L) + (\bar\Psi_L\phi)\Psi_R\right]$$

$$\phi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}, \quad \Psi_R = e_R, \quad \Psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e = e_L + e_R$$

$$\mathcal{L}_{eY} = -\frac{C_e}{\sqrt{2}}\left(\underbrace{v\bar e e}_{e\text{ mass term}} + \underbrace{H\bar e e}_{e-H\text{ coupling}}\right), \quad \bar e e = \bar e_L e_R + \bar e_R e_L$$

We find that $\boxed{m_e = \frac{C_e v}{\sqrt{2}}}$

Also, the coupling of e to Higgs scalar particle is proportional to C_e, which is proportional to m_e. Hence, the coupling strength between e and H is proportional to electron's mass

3) We now look at the $\mathcal{L}_\ell = \mathcal{L}_0 + \mathcal{L}_I$ lepton term

$$\mathcal{L}_\ell = \bar\Psi_L i\gamma^\mu D_\mu^L \Psi_L + \bar\Psi_R i\gamma^\mu D_\mu^R \Psi_R$$

where

$$D_\mu^L\Psi_L = \left(\partial_\mu + ig A_{i\mu}\frac{L_i}{2} + ig' B_\mu \frac{Y_L}{2}\right)\Psi_L$$

$$D_\mu^R\Psi_R = \left(\partial_\mu + ig' B_\mu \frac{Y_R}{2}\right)\Psi_R$$

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \psi_R = e_R, \quad e = e_L + e_R$$

Hence,

$$\mathcal{L}_L = \bar{\nu}_L i \gamma^\mu \partial_\mu \nu_L + \bar{e} i \gamma^\mu \partial_\mu e \quad \leftarrow \text{Kinetic terms}$$
$$+ \left(-\frac{g}{\sqrt{2}}\right)(J^{\mu +} W_\mu + J^\mu W_\mu^+) \quad \leftarrow \text{charged current term}$$
$$+ (-g \sin\theta_W)\, j_\mu^{em} A^\mu \quad \leftarrow \text{EM current term}$$
$$+ \left(-\frac{g}{\cos\theta_W}\right) j_\mu^Z Z^\mu \quad \leftarrow \text{neutral current term}$$

The kinetic term reads $\bar{\nu}_L i \gamma^\mu \partial_\mu \nu_L + \bar{e}_L i \gamma^\mu \partial_\mu e_L + \bar{e}_R i \gamma^\mu \partial_\mu e_R$

The charged current term contains the charged currents

$$J_\mu = j_\mu^1 - i j_\mu^2 = \bar{\psi}_L \gamma_\mu \left(\frac{\tau_1}{2}\right) \psi_L - i \bar{\psi}_L \left(\frac{\tau_2}{2}\right) \psi_L$$
$$= \bar{\psi}_L \gamma_\mu \tau_- \psi_L = \bar{e}_L \gamma_\mu \nu_L = \frac{1}{2} \bar{e} \gamma_\mu (1 - \gamma^5) \nu$$
$$J_\mu^+ = \bar{\nu}_L \gamma_\mu e_L = \frac{1}{2} \bar{\nu} \gamma_\mu (1 - \gamma^5) e$$

(which gives the V–A form for charged curren

Again, $W_\mu = \frac{1}{\sqrt{2}} (A_{1\mu} - i A_{2\mu})$

$$W_\mu^+ = \frac{1}{\sqrt{2}} (A_{1\mu} + i A_{2\mu})$$

are the W^+, W^- gauge fields coupled to charged curren

The EM current term contains

$$j_\mu^{em} = \bar{e} \gamma^\mu e = \bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R$$

and the EM coupling $\boxed{e = g \sin\theta_W}$

or $e = g' \cos\theta_W$

showing comparable ~~compar~~ coupling strengths among e, g, and g'

Finally, for the neutral current term we have

$$j_\mu^Z = \bar{\psi}_L \gamma_\mu Z_L \psi_L + \bar{\psi}_R \gamma_\mu Z_R \psi_R$$

where $Z_L = T_{3L} - Q \sin^2\theta_W$

$Z_R = - Q \sin^2\theta_W$

Before SSB we have 5 parameters :

g, g' $SU(2)_L$ and $U(1)_Y$ Coupling constants

λ, μ^2 for the scalar field potential

C_e for the Yukawa Coupling

After SSB we have 5 parameters related to the above 5 parameters

$$\tan\theta_W = g'/g$$
$$e = g \sin\theta_W$$
$$m_e = \frac{1}{\sqrt{2}} C_e v = \frac{1}{\sqrt{2}} C_e \sqrt{-\mu^2/\lambda}$$
$$M_W = \frac{1}{2} g v = \frac{1}{2} g \sqrt{-\mu^2/\lambda}$$
$$M_H = \sqrt{-2\mu^2}$$

Since we now know the values of θ_W, e, m_e, M_W, M_H, we can deduce the values of g, g', λ, μ^2, C_e.