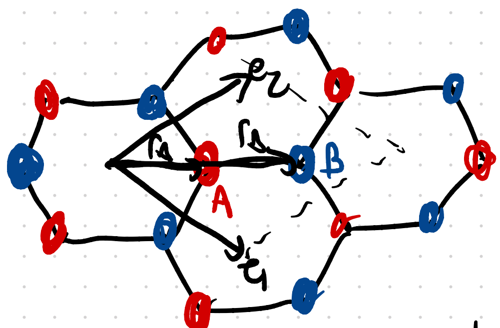
Lecture 24

Fractional Chern Insulator Workshop @ ICMT 4/18, 4/19

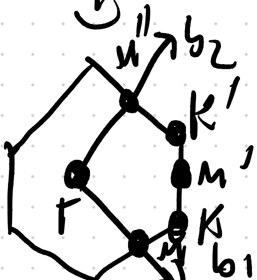
$p6/mmm1'$



$$\vec{r}_A = \frac{1}{3}(\vec{e}_1 + \vec{e}_2)$$

$$\vec{r}_B = \frac{2}{3}(e_1 + e_2)$$

Reciprocal lattice



TRIMS: Γ, M, M', M''

$$K = \frac{2}{3}\vec{b}_1 + \frac{1}{3}\vec{b}_2 \qquad K' = \frac{2}{3}\vec{b}_2 + \frac{1}{3}b_1$$

Tight binding basis functions: $|\tau\,\sigma\,\vec{R}\rangle$ transform like p_z orbitals

τ: A, B

σ: ↑, ↓, electron spin

$\vec{R}$: unit cell index

$$\langle \tau\sigma R|\vec{x}|\tau\sigma R\rangle = \vec{R} + \vec{r}_\tau$$

Band representation:

$$I(R + \vec{r}_A) = -R - \vec{r}_A$$
$$= -R + \vec{r}_B - \vec{e}_1 - \vec{e}_2$$

Inversion symmetry $\quad u_I|A\sigma\vec{R}\rangle = |B\sigma\, {-R}-e_1-e_2\rangle(-1)$

(-1) from p_z transformation

$$u_I|B\sigma R\rangle = |A\sigma\, {-R}-e_1-e_2\rangle(-1)$$

$$B(I) = (-1)\,\tau_x \otimes \sigma_0$$

$$\begin{array}{c|cc} & A & B \\ \hline A & 0 & 1 \\ B & 1 & 0 \end{array}$$

spin rotation

$$U_{C_6}|A\sigma R\rangle = |B\sigma' C_6 R - e_1\rangle \left(e^{-\frac{i\pi}{6}\sigma_z}\right)_{\sigma'\sigma} e^{-2\pi i/6}$$

$$U_{C_6}|B\sigma R\rangle = |A\sigma' C_6 R - e_1 + e_2\rangle \left(e^{-\frac{i\pi}{6}\sigma_z}\right)_{\sigma'\sigma} e^{-2\pi i/6}$$

rotation on e_z

$$B(C_6) = e^{-2\pi i/6}\, \tau_x \otimes e^{-\frac{i\pi}{6}\sigma_z}$$

$$B(C_3) = B(C_6)^2 = e^{-2\pi i/3}\, \tau_0 \otimes e^{-\frac{i\pi}{3}\sigma_z}$$

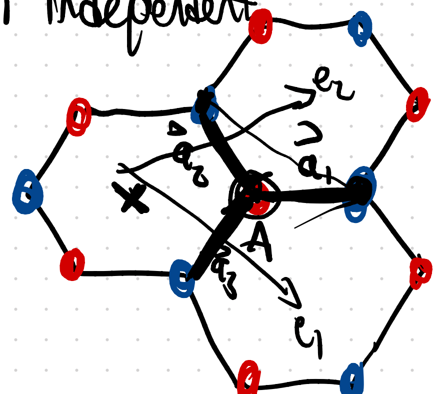
$$B(C_2) = B(C_6)^3 = e^{-i\pi}\, \tau_x \otimes e^{-i\pi/2\,\sigma_z} = i\tau_x \otimes \sigma_z$$

TRS: $T|\tau\sigma\vec{R}\rangle = |\tau\sigma'\vec{R}\rangle(-i\sigma_y)_{\sigma'\sigma}$

$$\underline{B(T) = -i\tau_0\otimes\sigma_y}\,\mathcal{K}$$

Nearest-Neighbor tight-binding model

Spin independent



$$\vec{a}_1 = \vec{r}_B - \vec{r}_A = \frac{1}{3}(e_1+e_2)$$

$$\vec{a}_2 = C_3\vec{a}_1 = \frac{1}{3}(\vec{e}_2 - 2\vec{e}_1) = a_1 - e_1$$

$$\vec{a}_3 = C_3\vec{a}_2 = \vec{a}_1 - \vec{e}_2 = \frac{1}{3}(e_1 - 2\vec{e}_2)$$

$$t = \langle B\sigma\vec{R}|H|A\sigma\vec{R}\rangle$$

3fold rotation symmetry $\{C_3|\vec{e}_1\}$ ← 120° rotation about $\vec{r}_A$

$$t = \langle B\sigma 0|H|A\sigma 0\rangle$$
$$= \langle B\sigma -e_1|H|A\sigma 0\rangle$$
$$= \langle B\sigma -\vec{e}_2|H|A\sigma 0\rangle$$

TRS: $t = t^*$

Inversion symmetry: $\langle A\sigma 0|H|B\sigma 0\rangle = \langle B\sigma 0|H|A\sigma 0\rangle$
$= t$

Momentum space Hamiltonian

$$h^{\tau\tau'}_{\sigma\sigma'}(\vec{k}) = \sum_R e^{-ik\cdot(R+r_\tau-r_{\tau'})}\langle \tau\sigma R|H|\tau'\sigma'\vec{0}\rangle$$

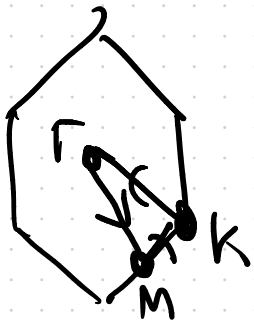
$$= \delta_{\sigma\sigma'}\begin{pmatrix} 0 & t\sum_{i=1}^{3} e^{ik\cdot a_i} \\ t\sum_{i=1}^{3} e^{-ik\cdot\vec{a}_i} & 0 \end{pmatrix}^{B}$$

$$\Rightarrow h(k) = \begin{pmatrix} 0 & q(k) \\ q^*(k) & 0 \end{pmatrix}\otimes\sigma_0 \qquad q(k) = t\sum_i e^{ik\cdot a_i}$$

$$k = k_1\vec{b}_1 + k_2\vec{b}_2$$

$$q(k) = t\left(e^{\frac{2\pi i}{3}(k_1+k_2)} + e^{\frac{2\pi i}{3}(k_2-2k_1)} + e^{\frac{2\pi i}{3}(k_1-2k_2)} \right)$$

$$\varepsilon_{\pm\sigma}(k) = \pm |q(k)|$$



$$\Gamma : (0,0)$$

$$q(\Gamma) = 3t$$

$$h(\Gamma) = 3t\,\tau_x \otimes \sigma_0$$

$$\varepsilon_{\pm\sigma}(k) = \pm 3t$$

$$M : \left(k_1 = \tfrac{1}{2}, k_2 = 0\right)$$

$$q(M) = t\left(e^{\frac{i\pi}{3}} + e^{-\frac{2\pi i}{3}} + e^{-\frac{i\pi}{3}}\right)$$

$$= t\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$$

$$h(M) = t\begin{pmatrix} 0 & \frac{1}{2}+\frac{i\sqrt{3}}{2} \\ \frac{1}{2}-\frac{i\sqrt{3}}{2} & 0 \end{pmatrix}\sigma_0$$

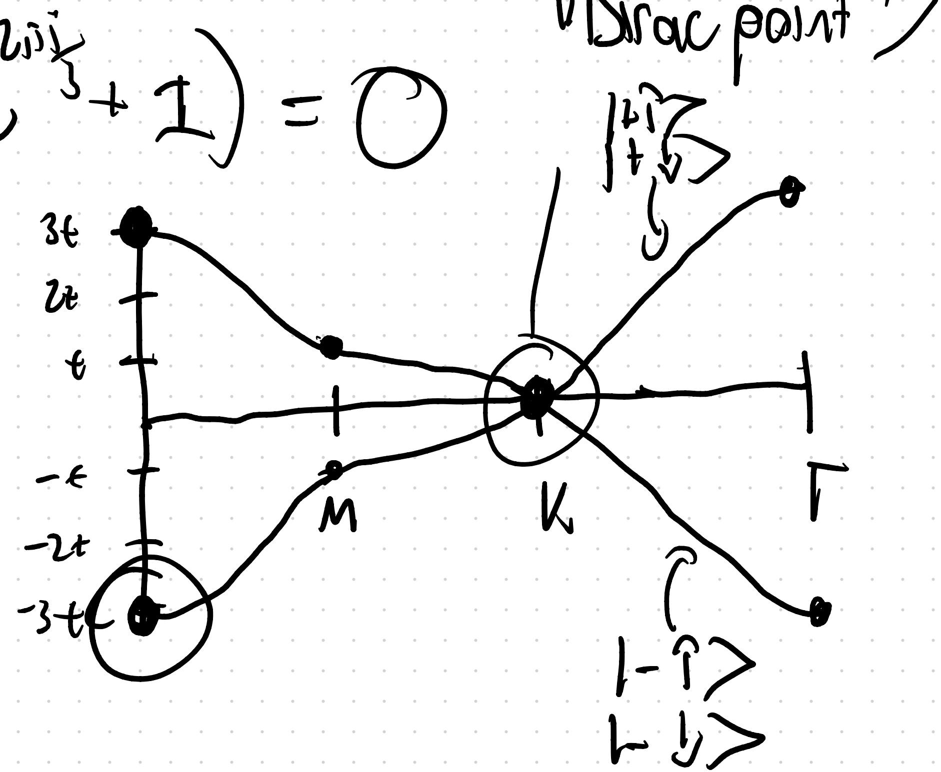
$$\varepsilon_{\pm\sigma}(M) = \pm t$$

$K := \left(k_1 = \frac{2}{3}, k_2 = \frac{1}{3}\right)$

$q(K) = t\left(e^{\frac{2\pi i}{3}} + e^{\frac{-2\pi i}{3}} + 1\right) = 0$

$h(K) = 0$

$\varepsilon_{\pm\theta}(K) = 0$



Inversion & TRS & Wilson loops

Embedding matrix $V(\vec{b}) = \begin{pmatrix} e^{ib\cdot r_A} & \\ & e^{ib\cdot r_B} \end{pmatrix}$

$$h(k+\vec{b}) = V^{\dagger}(b)\,h(k)\,V(b)$$

$$B(I) = -\tau_x \otimes \sigma_0$$

$\boxed{t>0}$

$$B^{\dagger}(I)\,h(k)\,B(I) = \tau_x \begin{pmatrix} 0 & q(k) \\ q^*(k) & 0 \end{pmatrix} \tau_x \sigma_0$$

$$= \tau_x \left(\mathrm{Re}\,q\,\tau_x - \mathrm{Im}\,q\,\tau_y \right) \tau_x \sigma_0$$

$$= \begin{pmatrix} 0 & q^*(k) \\ q(k) & 0 \end{pmatrix} = h(-k)$$

TRS $\quad -i\tau_0\sigma_y \mathcal{K}\, h(k) (-i\tau_0\sigma_y \mathcal{K})^{-1} = h^*(k) = h(-k)$

Γ: $\quad h(\Gamma) = h(0) = 3t\, \tau_x \otimes \sigma_0 = -3t\, B(I)$

$[B(I), h(\Gamma)] = 0$ $\qquad$ $1 - \sigma k = 0 \Rightarrow$ have inversion eigenvalues $+1$

M: $k = \frac{1}{2} b_1$

$$B(I)\, h(\tfrac{1}{2} b_1) B^\dagger(I) = h(-\tfrac{1}{2} b_1)$$
$$= h(\tfrac{1}{2}\vec{b}_1 - \vec{b}_1)$$
$$= V(b_1) h(\tfrac{1}{2} b_1) V^\dagger(b_1)$$

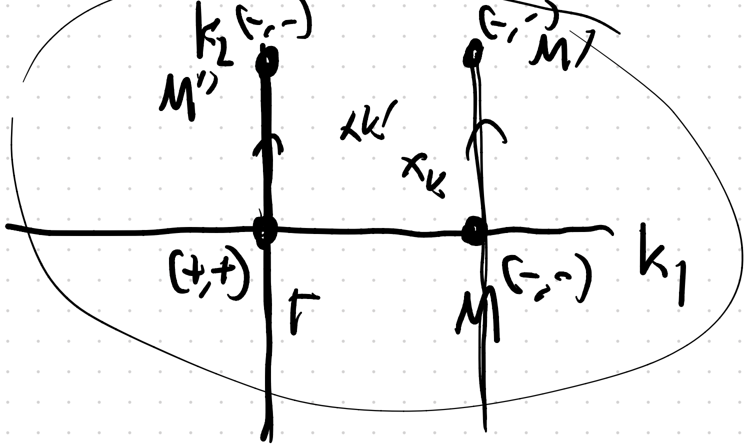
Little group representative matrix $V^\dagger(b_1)B(I)$

$$[V^\dagger(b_1)B(I), h(\tfrac{1}{2}b_1)] = 0$$

$$V^\dagger(b_1)B(I) = \begin{pmatrix} e^{-ib_1\cdot r_A} & 0 \\ 0 & e^{-ib_1\cdot r_B} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \sigma_0$$

$$= \begin{pmatrix} 0 & -e^{-2\pi i/3} \\ -e^{2\pi i/3} & 0 \end{pmatrix}\sigma_0 = \begin{pmatrix} 0 & e^{i\pi/3} \\ e^{-i\pi/3} & 0 \end{pmatrix}\sigma_0 = \frac{h(\frac{1}{2}b_1)}{t}$$

$|-, \sigma, k=M_1\rangle$ have inversion eigenvalue -1



$$W^{\sigma\sigma'}_{1\leftarrow 0}(k_1) = U^{\sigma\,\dagger}_{-,(k_1,k_2=1)} \cdot \prod_{k_2}^{1\leftarrow 0} P(k_1,k_2)\cdot U^{\sigma'}_{-(k_1,0)}$$

When $k_1 = 0$ or $\frac{1}{2} \leftarrow k_1^*$

$$W^{\sigma\sigma'}_{1\leftarrow 0}(k_1^*) = B^{--}_{(k_1^*,0)}(I)\, W^{\dagger}_{\frac{1}{2}\leftarrow 0}(k_1^*)\, B^{--}_{(k_1^*,\frac{1}{2})}(I)\, W_{\frac{1}{2}\leftarrow 0}(k_1^*)$$

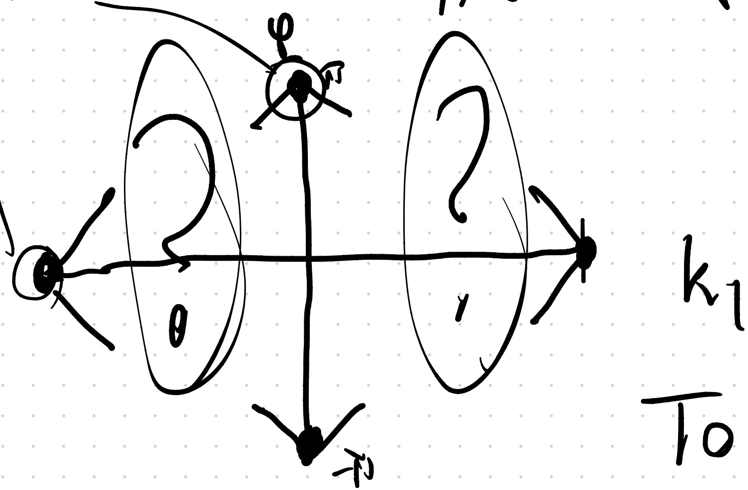
Proportional to σ_0

$$= B^{-}_{(k_1^+,0)}(I)\, B^{-}_{(k_1^+,\pi)}(I) = \begin{cases} -\sigma_0, & k_1 = 0 \\ +\sigma_0, & k_1 = \frac{1}{2} \end{cases}$$

$e^{\pm i\pi}$

e^{i0}

Protected by TRS



To say more about topology, need to open a gap @ K & K'