

Quantum Zeno effect

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The quantum Zeno effect is the inhibition of transitions between quantum states by frequent measurements of the state. The inhibition arises because the measurement causes a collapse (reduction) of the wave function. If the time between measurements is short enough, the wave function usually collapses back to the initial state. We have observed this effect in an rf transition between two ${}^9\text{Be}^+$ ground-state hyperfine levels. The ions were confined in a Penning trap and laser cooled. Short pulses of light, applied at the same time as the rf field, made the measurements. If an ion was in one state, it scattered a few photons; if it was in the other, it scattered no photons. In the latter case the wave-function collapse was due to a null measurement. Good agreement was found with calculations.

I. INTRODUCTION

The quantum Zeno effect (or paradox) is the inhibition of transitions between quantum states by frequent measurements.¹⁻⁷ Misra and Sudarshan¹ were the first to call the effect by that name, but closely related work was done much earlier.⁸

Consider the decay of an unstable state, such as an unstable particle. An observation that the state has not decayed causes a collapse (reduction) of the wave function to the undecayed state. The probability that the state decays after this collapse grows quadratically with time, for short enough times. Suppose n measurements, spaced in time by T/n , are made. The probability that the state will survive for a time T goes to 1 in the limit $n \rightarrow \infty$. Hence, Misra and Sudarshan argued, a continuously observed state can never decay.¹ This effect is difficult to observe in spontaneous decay because the interval during which the probability grows quadratically is very short compared to the time required to make a measurement. Ghirardi *et al.*³ have shown, by general arguments based on the time-energy uncertainty relations, that the dependence of the lifetime on the frequency of measurements, although present in principle, would be extremely difficult to observe. Deviations from an exponential decay law, expected theoretically for very short and very long times, have not yet been observed experimentally.⁹

The term "quantum Zeno effect" is applied also to the inhibition of *induced* transitions by frequent measurements. This effect can easily be observed experimentally, in contrast to the inhibition of spontaneous transitions. Consider a system made up of two levels, labeled 1 and 2. Assume that the system can be driven from level 1 to level 2 by applying a resonant perturbation for a given length of time. Assume that it is possible to make measurements of the state of the system, which project the system into one of the two levels, and which take a negligible amount of time. If the system is initially in level

1, and we make n equally spaced measurements while the perturbation is applied, the probability of finding the system in level 2 at the end of the period decreases as n increases. Various cases of this type have been examined theoretically.¹⁰⁻¹²

II. THEORY

Cook¹² proposed an experiment on a single, trapped ion to demonstrate the quantum Zeno effect on an induced transition. Trapped ions provide very clean systems for testing calculations of the dynamics of quantum transitions. They can be observed for long periods, free from perturbations and relaxations. Their levels can be manipulated easily with rf and optical fields.

In Cook's proposed experiment, the ion was assumed to have the level structure shown in Fig. 1. Level 1 is the ground state. Level 2 is an excited metastable state. Spontaneous decay from level 2 to level 1 is assumed to be negligible. If the ion is in level 1 at time $\tau = 0$, and a perturbation having the resonance frequency $(E_2 - E_1)/\hbar$ is applied, a coherent superposition state is created. Let P_1 and P_2 be the probabilities for the ion to be in levels 1 and 2. Then $P_2(\tau) = \sin^2(\Omega\tau/2)$

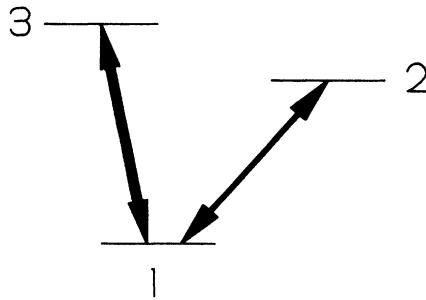


FIG. 1. Energy-level diagram for Cook's proposed demonstration of the quantum Zeno effect.

and $P_1(\tau) = \cos^2(\Omega\tau/2)$, where Ω , the Rabi frequency, is proportional to the amplitude of the applied field. If a measurement of the state of the ion is made after a short time, such that $\Omega\tau \ll 1$, then $P_1(\tau) \approx 1$ and $P_2(\tau) \approx \frac{1}{4}\Omega^2\tau^2 \ll 1$. If, instead, the ion starts out in level 2, the situation is reversed, so that $P_2(\tau) \approx 1$ and $P_1(\tau) \approx \frac{1}{4}\Omega^2\tau^2$.

Assume that level 3 is connected by a strongly allowed transition to level 1 and that it can decay *only* to level 1. The state measurement is carried out by driving the $1 \rightarrow 3$ transition with an optical pulse. This pulse causes a collapse of the wave function. The wave function of the ion is projected by the measurement into level 1 or 2 with probabilities equal to the squares of the wave-function amplitudes for being in level 1 or 2. If the ion is projected into level 1 at the beginning of the pulse, it cycles between level 1 and level 3, and emits a series of photons until the pulse is turned off. If it is projected into level 2, it scatters no photons. The latter case is an example of what Porrati and Puttermann have called a wave-function collapse due to a null measurement.¹³ That is, the *absence* of scattered photons when the optical pulse is applied is enough to cause a collapse of the wave function to level 2. The pulse must be long enough so that an ion in level 1 would have time to scatter a few photons. It does not matter whether a switched-on detector capable of detecting the photons is actually present. The state of the ion is recorded in the electromagnetic field. The measurement (and the subsequent wave-function collapse) takes place after the field has interacted with the ion for a sufficient amount of time.^{12,14} If a measurement finds the ion to be in level 1, the ion returns to level 1 after the end of the measurement, within a time approximately equal to the lifetime of level 3. If a measurement finds the ion to be in level 2, the ion never leaves that level during the measurement. If this measurement is followed immediately by a second one, the result will *almost* always be the same. Thus the optical pulses make nearly ideal, nondestructive measurements. However, since a finite time is required to make a measurement, the wave function can evolve between measurements. Therefore, it is possible for the result of the second measurement to differ from that of the first.

Cook's proposed experiment was to drive the $1 \rightarrow 2$ transition with an on-resonance π pulse (a square pulse of duration $T = \pi/\Omega$) while simultaneously applying a series of short measurement pulses. The duration of a measurement pulse was assumed to be much less than the time between pulses. Suppose the ion is in level 1 at time $\tau = 0$. The π pulse is then applied. Without the measurement pulses, the probability $P_2(T)$ to be in level 2 at $\tau = T$ is 1. Let n measurement pulses be applied at times $\tau = kT/n = k\pi/(n\Omega)$, where $k = 1, \dots, n$. The level populations at the end of the π pulse are easily calculated with the use of the vector representation of a two-level system.¹⁵ The equations simplify if we transform to a coordinate system in which the rotating component of the rf perturbation is stationary. (We ignore the

counter-rotating component.) The system is described by a vector $\mathbf{R} \equiv (R_1, R_2, R_3)$, whose components can be expressed in terms of the density matrix ρ :

$$\begin{aligned} R_1 &\equiv \rho_{12} + \rho_{21}, \\ R_2 &\equiv i(\rho_{12} - \rho_{21}), \\ R_3 &\equiv \rho_{22} - \rho_{11} \equiv P_2 - P_1. \end{aligned} \quad (1)$$

The equation of motion for \mathbf{R} is

$$d\mathbf{R}/dt = \boldsymbol{\omega} \times \mathbf{R}, \quad (2)$$

where $\boldsymbol{\omega} = (\Omega, 0, 0)$. The geometrical interpretation of Eq. (2) is that \mathbf{R} precesses about $\boldsymbol{\omega}$ with fixed magnitude and angular velocity $|\boldsymbol{\omega}| = \Omega$. At $\tau = 0$, $\mathbf{R} = (0, 0, -1)$. Just before the first measurement pulse at $\tau = \pi/(n\Omega)$,

$$\begin{aligned} \mathbf{R} &= [0, \sin(\Omega\tau), -\cos(\Omega\tau)] \\ &= [0, \sin(\pi/n), -\cos(\pi/n)]. \end{aligned} \quad (3)$$

The measurement pulse projects an ion into level 1 or 2. Its effect on the density matrix, which corresponds to an ensemble average, is to set the coherences (ρ_{12} and ρ_{21}) to zero, while leaving the populations (ρ_{22} and ρ_{11}) unchanged. Hence, R_1 and R_2 are set to zero, while R_3 remains unchanged:

$$\mathbf{R} = [0, 0, -\cos(\pi/n)]. \quad (4)$$

Thus, at $\tau = \pi/(n\Omega)$, just after the first measurement pulse, \mathbf{R} is the same as it was at $\tau = 0$, except that its magnitude has been decreased by a factor of $\cos(\pi/n)$. After the second measurement [$\tau = 2\pi/(n\Omega)$], $|\mathbf{R}|$ is decreased by another factor of $\cos(\pi/n)$. This follows from the fact that Eq. (2) is linear with respect to \mathbf{R} . After n measurements, ($\tau = \pi/\Omega$),

$$\mathbf{R}(T) = [0, 0, -\cos^n(\pi/n)]. \quad (5)$$

We use Eq. (1) to express P_2 in terms of R_3 :

$$\begin{aligned} P_2 &= R_3 + P_1 = R_3 + (1 - P_2) \\ &= \frac{1}{2}(1 + R_3). \end{aligned} \quad (6)$$

In deriving Eq. (6), we used the conservation of probability for a closed two-level system: $P_1 + P_2 = 1$. Substituting the value of $R_3(T)$ from Eq. (5) into Eq. (6), we have

$$P_2(T) = \frac{1}{2}[1 - \cos^n(\pi/n)]. \quad (7)$$

It can be shown from Eq. (7) that $P_2(T)$ decreases monotonically toward zero as n goes to infinity. For large n ,¹²

$$P_2(T) \approx \frac{1}{2}[1 - \exp(-\frac{1}{2}\pi^2/n)]. \quad (8)$$

Equation (8) was derived from Eq. (7) by expanding $\cos(\pi/n)$ in a power series and using

$$\lim_{n \rightarrow \infty} (1 - x/n)^n = e^{-x}. \quad (9)$$

III. EXPERIMENT

Our experiment is very similar to that proposed by Cook. Levels 1 and 2 are the $(m_I, m_J) = (\frac{3}{2}, \frac{1}{2})$ and $(\frac{1}{2}, \frac{1}{2})$ hyperfine sublevels in the ground $2s^2 S_{1/2}$ state of ${}^9\text{Be}^+$ (see Fig. 2). These levels are separated by 320.7 MHz at the magnetic field used in the experiment ($B \approx 0.8194$ T). Level 3 is the $(m_I = \frac{3}{2}, m_J = \frac{3}{2})$ sublevel of the $2p^2 P_{3/2}$ state, which decays only to level 1. Spontaneous decay from level 2 to level 1 is negligible.

The experimental apparatus has been described previously.¹⁶⁻¹⁸ About 5000 ${}^9\text{Be}^+$ ions were stored in a cylindrical Penning trap. The pressure in the trap was about 10^{-8} Pa. The storage time of the ions in the trap was several hours. A frequency-doubled cw dye laser generated 313-nm radiation to drive the $1 \rightarrow 3$ transition in order to optically pump, detect, and laser cool the ${}^9\text{Be}^+$ ions. The 313-nm fluorescence from the ions was detected by an imaging photon-counting detector.¹⁶ About 100 000 ${}^{26}\text{Mg}^+$ ions were confined together with the ${}^9\text{Be}^+$ ions. The ${}^{26}\text{Mg}^+$ ions were laser-cooled by 280-nm radiation from a frequency-doubled cw dye laser. The ${}^9\text{Be}^+$ were kept cold ($\lesssim 250$ mK) by long-range Coulomb collisions with the ${}^{26}\text{Mg}^+$ ions even when they were not directly laser-cooled by the 313-nm source.¹⁹

The polarization of the 313-nm beam was perpendicular to the magnetic field. When the 313-nm radiation was nearly resonant with the $1 \rightarrow 3$ transition and no rf field was applied, about $\frac{16}{17}$ of the population was optically pumped to the $(\frac{3}{2}, \frac{1}{2})$ ground-state sublevel (level 1).²⁰⁻²² The remaining population was in the $(\frac{3}{2}, -\frac{1}{2})$ sublevel. When the 313-nm radiation was on continuously, the populations approached the steady state with a time constant of about 1 s.

The measurement sequence for the $1 \rightarrow 2$ transition was as follows: The 313-nm radiation was left on for

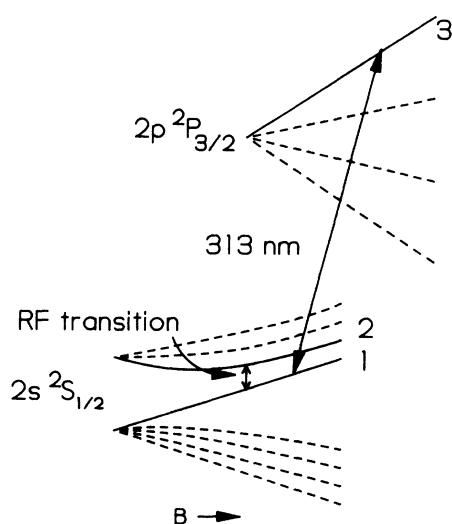


FIG. 2. Diagram of the energy levels of ${}^9\text{Be}^+$ in a magnetic field B . The states labeled 1, 2, and 3 correspond to those in Fig. 1.

about 5 s to prepare most of the ${}^9\text{Be}^+$ ions in level 1 and to empty level 2. The 313-nm radiation was then turned off. The 320.7-MHz rf field was turned on for $T = 256$ ms. Its frequency and amplitude were adjusted to make this an on-resonance π pulse. During the rf pulse, n pulses of length $\tau_p = 2.4$ ms and wavelength 313 nm were applied, where n was 1, 2, 4, 8, 16, 32, or 64. The pulses were long enough to collapse each ion's wave function without causing significant optical pumping. The delay from the beginning of the rf pulse to the beginning of the first 313-nm pulse was $(T/n - 1.3)$ ms. The time between the beginning of one 313-nm pulse and the beginning of the next one was T/n .

After the end of the rf π pulse, the 313-nm radiation was turned on and left on to prepare the state. The number of photons counted in the first 100 ms was recorded. This signal was roughly proportional to the number of ions remaining in level 1. However, background counts, counter deadtime, and optical pumping during the 100 ms cause deviations from this proportionality. In order to calibrate the signal, known level populations were created by applying rf pulses of lengths $\tau = 0, 32, 64, \dots, 544$ ms. The population of level 1 (per ion in the subsystem made up of levels 1 and 2) was then given by $\cos^2(\Omega\tau/2)$. From these data, the precise value of Ω and also the calibration of the signal as a function of the population of level 1 were obtained. The deviations of the calibration points from a smooth curve gave an indication of the measurement errors.

The 313-nm radiation was turned on and off with an electromechanical shutter that had a rise or fall time of about 0.2 ms. The 320.7-MHz rf field was turned on and off with a semiconductor diode switch, which had a switching time of about 150 ns and an on-to-off ratio of about 75 dB. The 280-nm beam was left on continuously. The measurement sequence was the same for the $2 \rightarrow 1$ transition, except that first an rf π pulse, free from 313-nm pulses, transferred the level 1 population to level 2 immediately after the 313-nm radiation was shut off.

The $1 \rightarrow 2$ transition frequency decreases by 22 Hz for a $1-\mu\text{T}$ increase in B . If the transition frequency shifts by more than a small fraction of $1/T \approx 4$ Hz, the observed transition probabilities will deviate significantly from the calculated ones. The center frequency of the $1 \rightarrow 2$ resonance was measured before and after each run. If the frequency shifted by more than about 0.1 Hz, the data from that run were not used. Drift of the magnetic field was the main obstacle to increasing T . Relaxations of the ground-state hyperfine levels are very weak when the 313-nm radiation is not applied. This was shown in studies of the $(m_I, m_J) = (-\frac{1}{2}, \frac{1}{2})$ to $(-\frac{3}{2}, \frac{1}{2})$ transition. The derivative of this transition frequency with respect to B goes to zero at $B \approx 0.8194$ T. At this value of B , weak relaxation processes due, for example, to collisions or to the 280-nm radiation can be studied. This transition showed no sign of population or coherence relaxations for times up to 550 s.¹⁷

The average number of photons scattered by an ion in

TABLE I. Predicted and observed values of the $1 \rightarrow 2$ and $2 \rightarrow 1$ transition probabilities for different values of the number of measurement pulses n . The uncertainties of the observed transition probabilities are about 0.02. The second column shows the transition probabilities that result from a simplified calculation, in which the measurement pulses are assumed to have zero duration and in which optical pumping is neglected.

n	$\frac{1}{2}[1 - \cos^n(\pi/n)]$	1 \rightarrow 2 transition		2 \rightarrow 1 transition	
		Predicted	Observed	Predicted	Observed
1	1.0000	0.995	0.995	0.999	0.998
2	0.5000	0.497	0.500	0.501	0.496
4	0.3750	0.351	0.335	0.365	0.363
8	0.2346	0.201	0.194	0.217	0.209
16	0.1334	0.095	0.103	0.118	0.106
32	0.0716	0.034	0.013	0.073	0.061
64	0.0371	0.006	-0.006	0.080	0.075

level 1 during a single 313-nm pulse of length τ_p is approximately $\tau_p R_c / (\epsilon_d N)$, where R_c is the observed steady-state photon count rate, ϵ_d is the probability of detecting a scattered photon, and N is the total number of ${}^9\text{Be}^+$ ions. The photon detection efficiency ϵ_d was estimated from the solid angle of the lens system and the quantum efficiency of the detector to be about 2×10^{-4} . For typical experimental values $\tau_p = 2.4$ ms, $R_c = 30\,000 \text{ s}^{-1}$, and $N = 5000$, the number of scattered photons per ion per pulse was therefore about 72, more than enough to cause the collapse of the wave function. We emphasize that it is the number of scattered photons which is important, not the number that can be detected by the apparatus. The number of photons detected per ion per pulse is much less than 1. As a further check that the pulses were long enough, some runs were taken with τ_p decreased to 1.4 ms (the shortest that the shutter could make). The results indicated that these pulses were still long enough to collapse the wave functions. These data are not reported here because the pulse shapes were not the same for different pulse repetition rates. This made quantitative interpretation of the data difficult.

With a faster optical shutter, such as an acousto-optic modulator, the 313-nm pulses could be decreased in length and still be long enough to collapse the wave functions. However, the minimum time required for a measurement pulse depends not only on the *average* photon scattering rate, but also on the time required to ensure that every ion will pass through the 313-nm beam. The ${}^9\text{Be}^+$ ions occupied a cylindrical volume with a height of about 1000 μm and a radius of about 350 μm . The 313-nm beam was focused to a radius of about 50 μm . It propagated perpendicular to the axis of the cylinder and intersected the ${}^9\text{Be}^+$ ions near the center of the volume. If the 313-nm beam were expanded radially and directed along the axis, so that it intersected the entire volume, this problem could be avoided.

IV. RESULTS

Table I shows the calculated and observed values of the probabilities of making the $1 \rightarrow 2$ and $2 \rightarrow 1$ transitions

for values of $n = 1, 2, \dots, 64$. The predicted and observed values agree within the measurement error of about 0.02 estimated from the scatter of the signal calibration data. The general decrease of the probabilities with n demonstrates the quantum Zeno effect. Probabilities must take values from 0 to 1. However, with our method of determining the transition probability, random fluctuations in the photon count rate can lead to an *apparent* transition probability which is less than zero or greater than 1. The value of -0.006 for the observed $1 \rightarrow 2$ transition probability for $n = 64$ just means that the number of photons detected was slightly higher than the number expected for a transition probability of 0. Figures 3 and 4 show the probabilities for the $1 \rightarrow 2$ and $2 \rightarrow 1$ transitions, respectively.

The assumptions made in the calculations are as follows: When the rf field is present and the 313-nm radiation is not, the transition between levels 1 and 2 is assumed to proceed without relaxations. During the 313-

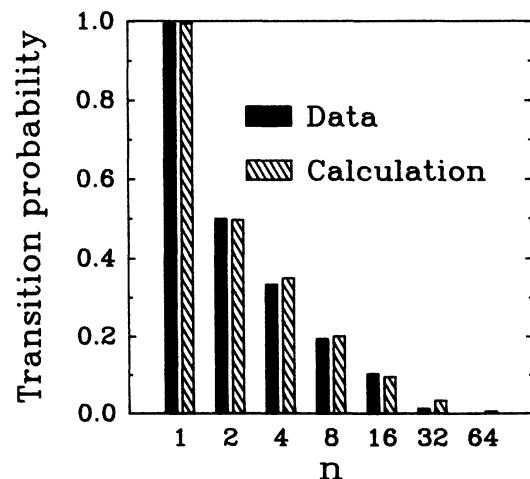


FIG. 3. Graph of the experimental and calculated $1 \rightarrow 2$ transition probabilities as a function of the number of measurement pulses n . The decrease of the transition probabilities with increasing n demonstrates the quantum Zeno effect.

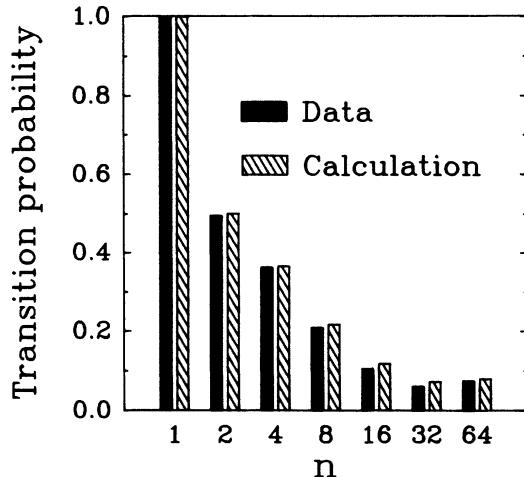


FIG. 4. Graph of the experimental and calculated $2 \rightarrow 1$ transition probabilities as a function of the number of measurement pulses n . The transition probabilities for $n = 32$ and $n = 64$ are higher than the corresponding ones for the $1 \rightarrow 2$ transition because of an optical pumping effect discussed in the text.

nm pulses, the coherence between levels 1 and 2 is assumed to be destroyed so quickly that the rf field can be ignored. The 313-nm radiation causes optical pumping from level 2 to level 1 with a time constant of about 1 s. This rate was measured by using an rf π pulse to prepare the ions in level 2, turning on the 313-nm radiation, and recording the 313-nm fluorescence as a function of time. Optical pumping by the 313-nm pulses can have an observable effect on the measured transition probabilities, especially for large n . This optical pumping causes a decrease in the $1 \rightarrow 2$ transition probability and an increase in the $2 \rightarrow 1$ transition probability. The decrease in the $1 \rightarrow 2$ transition probability is not noticeable in our data, since the transition probability is already small for large n . The increase in the $2 \rightarrow 1$ transition probability is noticeable for $n = 32$ and 64. The predicted transition probabilities shown in Table I and plotted in Figs. 3 and 4 take into account optical pumping, the finite 313-nm pulse durations, and the measured value of Ω . The measured value of Ω differed by less than 3% from the ideal value $\Omega = \pi/T$. For comparison, $\frac{1}{2}[1 - \cos^n(\pi/n)]$, the transition probability calculated if these effects are not included, is also shown in Table I.

For the larger values of n , the sum of the measurement periods, $n\tau_p$, is not negligible compared to T . Therefore, part of the decrease in the transition probability is due merely to the decrease in the time during which rf field can act. For the most extreme case ($n = 64$) $n\tau_p$ is 60% of T . However, even for this case, the decrease in the transition probability is much more than could be accounted for by the decrease in time. For $n = 16$, $n\tau_p$ is only 15% of T , but the transition probability is decreased by about 90%.

V. DISCUSSION

Cook¹² originally proposed an experiment with a single ion. Such an experiment should be feasible, since single ions have been observed in Penning traps^{21,23} and in Paul traps.^{24,25} The ensemble average, which is needed for comparison with calculations of the density matrix, would be obtained by repeating the experiment many times. In the present experiment, each measurement is an average over an ensemble of about 5000 independent ions, since their mutual interactions can be neglected. An experiment might be done with a single Hg^+ ion. Level 1 would be the ground $5d^{10}6s^2 S_{1/2}$ state, level 2 would be the metastable $5d^96s^2 D_{5/2}$ state, and level 3 would be the $5d^{10}6p^2 P_{1/2}$ state. The $1 \rightarrow 2$ transition at 282 nm and the $1 \rightarrow 3$ transition at 194 nm have already been observed in a single Hg^+ ion.²⁵

It might be argued that previous observations of effects such as collisional relaxation of rf transitions already contain the quantum Zeno effect. This is possible, but we know of no experiment in which the effect has been demonstrated simply and unambiguously. Collisions cannot usually be interpreted as quantum measurements. That is, they do not necessarily project a single quantum system, such as an atom, into one state or the other. Rather, in many cases, they perturb the phase of the coherence, represented by R_1 and R_2 in the vector model, for each system. For the ensemble average, R_1 and R_2 are driven to zero, just as in our experiment, but the underlying physics is different. Collisions may also perturb R_3 , which is not desirable for a demonstration of the quantum Zeno effect.

Some experiments that involve continuous measurements have, in a sense, already demonstrated the quantum Zeno effect. Although the measurements are continuous, it takes a finite amount of time to make a measurement with a given degree of uncertainty. Thus a continuous measurement might be regarded as a series of measurements, one after the other. In such experiments, the measurement times are not separated from the free evolution periods, so the analysis is more difficult than for the present experiment. One example of such an experiment is the spin flip resonance of a single, trapped electron.¹⁰ The spin state is detected by coupling the electron to an electronic circuit. As the time required to make a measurement is decreased by increasing the degree of coupling, noise broadens the spin flip resonance. This broadening decreases the rate of transitions induced by a weak microwave field. Another example is the three-level Hg^+ system described previously.²⁵ Radiation resonant with the $5d^{10}6s^2 S_{1/2}$ to $5d^{10}6p^2 P_{1/2}$ ($1 \rightarrow 3$) transition performs a continuous state measurement. Photons are not scattered if the ion is in the $5d^96s^2 D_{5/2}$ state (level 2); otherwise they are. We have observed a decrease of the rate of the $1 \rightarrow 2$ transition, induced by a narrow-band laser, when radiation at the $1 \rightarrow 3$ transition frequency is applied.

Normally, the probability for spontaneous emission of

a photon by an atom grows quadratically only for extremely short times, approximately the inverse of the frequency of the emitted photon. However, it might be possible to increase this time by placing the atom inside a resonant cavity. Jaynes and Cummings²⁶ showed that a two-level atom coupled to a single mode of a resonant, lossless cavity oscillates between the ground and excited states. Suppose an excited atom is placed in a cavity that initially contains no photons. Then the probability for the atom to be in the excited state is $\cos^2(g\tau)$, where g is the coupling constant (the vacuum field Rabi frequency) and τ is the time that the atom has been in the cavity. The probability for the atom to be in the ground state is proportional to τ^2 for τ short compared to g^{-1} .

If the atom is coupled to a single *damped* cavity mode,

the probability to be in the ground state is still proportional to τ^2 , provided that τ is short compared to both g^{-1} and γ_c^{-1} , where γ_c is the dissipation rate of the cavity.²⁷ It might be possible to demonstrate the quantum Zeno effect on the decay of an atom in a cavity. This would not contradict the results of Ghirardi *et al.*,³ since they did not consider such a system.

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