Useful Formulae and Integrals

$$\int_0^\infty x^n e^{-x} dx = n! \qquad \text{(n = positive integer)}$$

$$\int_0^\infty e^{-\beta x^2} dx = \frac{1}{2} (\pi/\beta)^{1/2}$$

$$\int_0^\infty x\, e^{-\beta x^2} dx = \frac{1}{2\beta}$$

$$\int_0^\infty x^2 e^{-\beta x^2} dx = \frac{1}{4} (\pi/\beta^3)^{1/2}$$

$$\int_0^\infty x^3 e^{-\beta x^2} dx = \frac{1}{2\beta^2}$$

Stirling's approximation:

$$\log_e (n!) \simeq n \log_e n - n \qquad (n >> 1)$$

$$n! = \sqrt{2\pi\, n}\, n^n \exp\left(-n + \frac{1}{12n} + \circ\left(\frac{1}{n^2}\right)\right)$$

$$(a+b)^c = \sum_{r=0}^{c} a^r\, b^{(c-r)} \frac{c!}{r!(c-r)!}$$

Spherical Harmonics Y_ℓ^m

$$Y_0^0 = \frac{1}{\sqrt{4\pi}}$$

$$Y_1^1(\theta,\phi) = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_1^0(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^{-1}(\theta,\phi) = \sqrt{\frac{3}{8\pi}}\sin\theta\, e^{-i\phi}$$

$$Y_2^2(\theta,\phi) = \sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_2^1(\theta,\phi) = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^0(\theta,\phi) = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$$

$$Y_2^{-1}(\theta,\phi) = \sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{-i\phi}$$

$$Y_2^{-2}(\theta,\phi) = \sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{-2i\phi}$$

Legendre Polynomials $P_\ell(z)$

$$P_o(z) = 1$$

$$P_1(z) = z$$

$$P_2(z) = \tfrac{1}{2}(3z^2 - 1)$$

$$P_3(z) = \tfrac{1}{2}(5z^3 - 3z)$$

$$Y_\ell^o(\theta,\phi) = \sqrt{\frac{2\ell+1}{4\pi}}\, P_\ell(\cos\theta)$$

The Bessel and Neumann functions $Z_n(x) = \begin{cases} J_n(x) \\ Y_n(x) \end{cases}$ are solutions of Bessel's equation

$$\frac{d^2Z_n(x)}{dx^2} + \frac{1}{x}\frac{dZ_n(x)}{dx} + \left(1 - \frac{n^2}{x^2}\right) Z_n(x) = 0$$

$e = 4.80 \times 10^{-10}$ esu $= 1.60 \times 10^{-19}$ Coulomb

$c = 3.00 \times 10^{10}$ cm/sec $= 3.00 \times 10^{8}$ m/sec

$\hbar = 1.05 \times 10^{-27}$ erg sec $= 1.05 \times 10^{-34}$ J sec

$m_e = 9.11 \times 10^{-28}$ g $= 9.11 \times 10^{-31}$ kg

$m_p = 1.67 \times 10^{-24}$ g $= 1.67 \times 10^{-27}$ kg

$N_o = 6.02 \times 10^{23}$ particles/mole

$k_B = 1.38 \times 10^{-23}$ J K^{-1} $= 1.38 \times 10^{-16}$ ergK^{-1}

$a_o = 0.529 \times 10^{-8}$ cm

$\varepsilon_o \cong \dfrac{1}{4\pi\times 9\times 10^9}\ \dfrac{\text{Farad}}{\text{m}}$

$\mu_o = 4\pi \times 10^{-7}\ \dfrac{\text{Tesla}\cdot\text{m}}{\text{Ampere}}$

VECTOR OPERATIONS IN CYLINDRICAL AND SPHERICAL COORDINATES

CYLINDRICAL COORDINATES

Coordinates (r, φ, z) Unit vectors $(\vec{i}_1, \vec{i}_2, \vec{i}_3)$

Gradient

$$\nabla f = \vec{i}_1 \frac{\partial f}{\partial r} + \vec{i}_2 \frac{1}{r}\frac{\partial f}{\partial \varphi} + \vec{i}_3 \frac{\partial f}{\partial z}$$

Curl

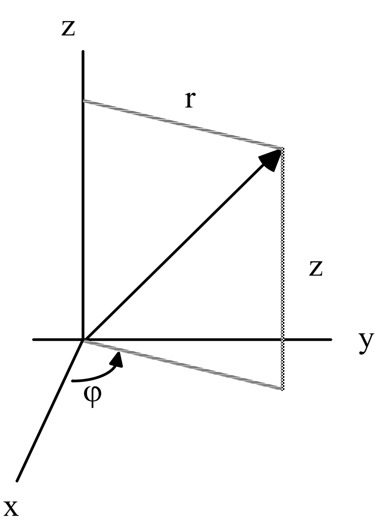
$$\nabla \times \vec{A} = \vec{i}_1\left(\frac{1}{r}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right) + \vec{i}_2\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \vec{i}_3\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi) - \frac{1}{r}\frac{\partial A_r}{\partial \varphi}\right)$$

Divergence

$$\nabla \bullet \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(r\,A_r) + \frac{1}{r}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

Laplacian

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$



VECTOR OPERATIONS IN CYLINDRICAL AND SPHERICAL COORDINATES

SPHERICAL COORDINATES

Coordinates (r, θ, φ) Unit vectors $(\vec{i}_1, \vec{i}_2, \vec{i}_3)$

Gradient
$$\nabla f = \vec{i}_1 \frac{\partial f}{\partial r} + \vec{i}_2 \frac{1}{r}\frac{\partial f}{\partial \theta} + \vec{i}_3 \frac{1}{r \sin\theta}\frac{\partial f}{\partial \varphi}$$

Curl
$$\nabla \times \vec{A} = \vec{i}_1 \frac{1}{r \sin\theta}\left(\frac{\partial}{\partial \theta}(\sin\theta\, A_\varphi) - \frac{\partial A_\theta}{\partial \varphi}\right) + \vec{i}_2 \left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{1}{r}\frac{\partial}{\partial r}(r\, A_\varphi)\right)$$
$$+ \vec{i}_3 \left(\frac{1}{r}\frac{\partial}{\partial r}(r\, A_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right)$$

Divergence
$$\nabla \bullet \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\, A_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

Laplacian
$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$$
$$= \frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$$