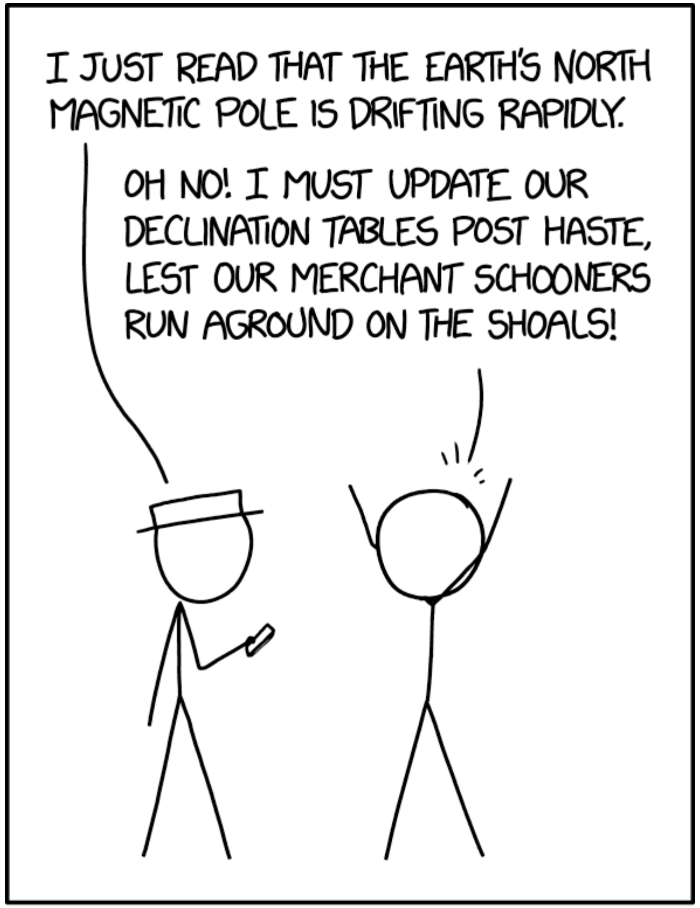
Hall Probe Measurement of Magnetic Fields

Prof. Jeff Filippini

Physics 401

Spring 2020

ILLINOIS



I LIKE WHEN THE EARTH'S MAGNETIC FIELD DOES WEIRD STUFF, BECAUSE IT'S A HUGE, COOL, URGENT-SEEMING SCIENCE THING, BUT THERE'S NOTHING I PERSONALLY NEED TO DO ABOUT IT.

XKCD #2098

But first: What's happening in this course?

The university has moved to online instruction… which is obviously challenging to make effective for a laboratory class! Our current course plan (*subject to change as circumstances require*) is as follows:

- **Lectures**
  - Via Zoom (https://illinois.zoom.us/j/359169027) at usual time, to allow questions. No attendance will be taken. Slides will be posted on the course website as usual.
  - Video posted Monday evening on our course MediaSpace page for students who cannot view live
- **Laboratory activities**
  - Course staff will record and post "highlight reel" videos on MediaSpace to illustrate equipment and give context to measurements. Thanks to Albert Lam for making this week's.
  - Data for analysis will be posted Tuesday, following procedure from write-up and video. \\engr-file-03\PHYINST\APL Courses\PHYCS401\Common\Data_Spring2020\Hall Probe
- **Reports**
  - Due on usual weekly schedule, all students now have Thursday due dates for last three labs
  - Lab notebooks will no longer be graded – all complete reports get full notebook credit
- **Office hours**
  - All staff are available by e-mail and Zoom (by appointment - your usual lab session is a good time)

Key Goals of this Lab

Study the magnetic field distributions created by various systems using the Hall probe and Gauss meter.

- **Making fields**: Calculate the magnetic field distributions created by common magnet configurations
- **Sensing fields**: Understand the applications of the Hall effect to magnetic field sensing
- **Procedure**: Set up various field sources, measure the magnetic field configurations, compare with experimental data

This is a one-week lab

Outline

1. The magnetic field of current loops
2. Helmholtz coils
3. Solenoids
4. Halbach magnet arrays
5. The Hall effect and field sensors
6. Experiment and analysis notes

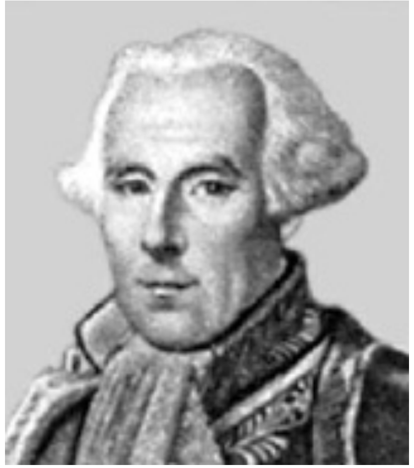
Bonus: SQUID magnetometers

The Biot-Savart Law

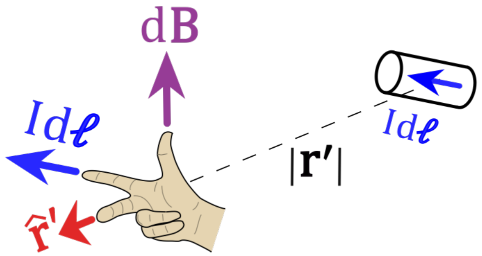
Electric currents generate magnetic fields. How do we compute them?

Jean-Baptiste Biot
(1774-1862)

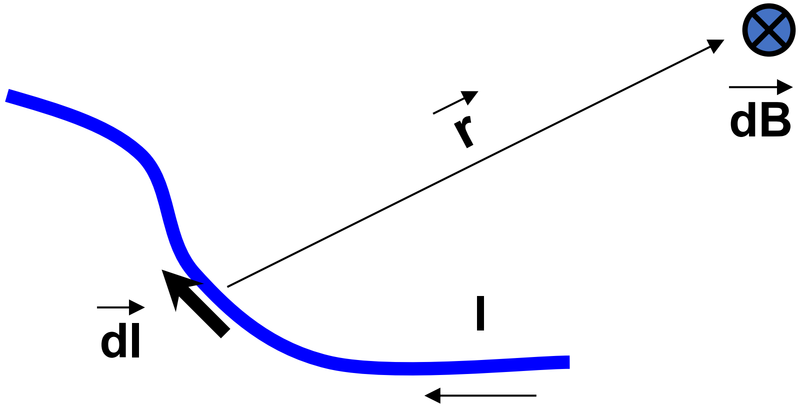
$$d\vec{B} = \frac{\mu_o}{4\pi} \frac{I d\vec{l} \times \vec{r}}{r^3}$$



Félix Savart
(1791-1841)



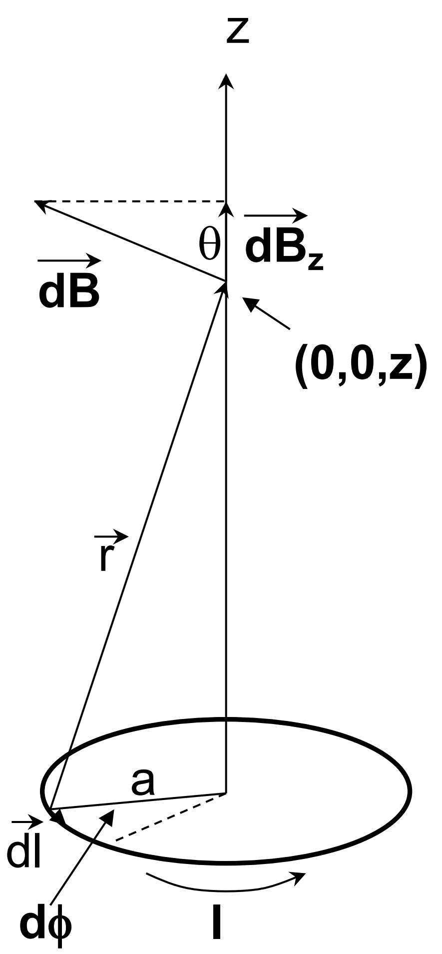
[Wikipedia](#)

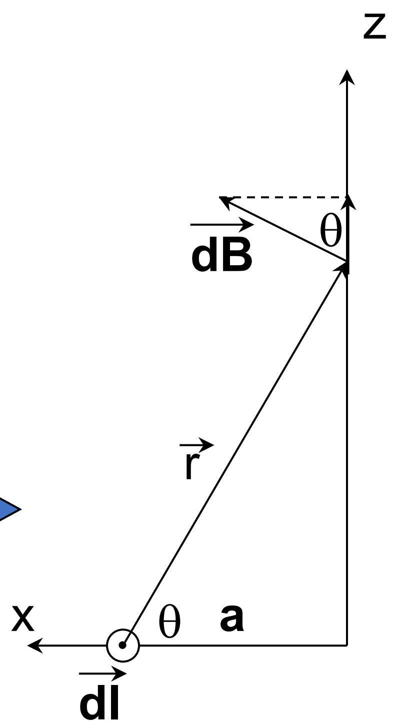


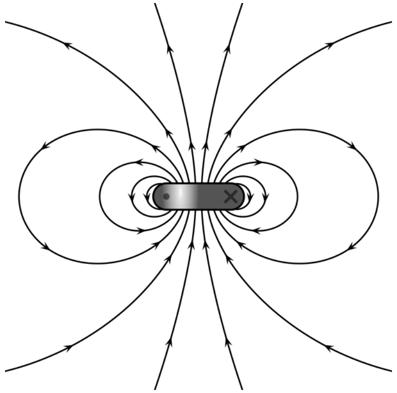
Permeability of free space

$$\mu_0 = 4\pi \times 10^{-7}\ N/A^2$$

Magnetic Field from a Current Loop







For simplicity, consider only the field **on-axis**

Then, by symmetry, we need only consider B_z

$$dB_z = dB\ \cos\theta = dB\ \frac{a}{r}$$

Integrate over sources, i.e. around the loop

$$d\vec{l} = a\ d\vec{\varphi}$$

$$dB_z = \frac{\mu_0}{4\pi}\frac{I\ a^2 d\varphi}{r^3}$$

$$B_z = \int_0^{2\pi}\frac{\mu_0}{4\pi}\frac{I\ a^2}{r^3}d\varphi = \frac{\mu_0 a^2 I}{2r^3}$$

$$\boldsymbol{B_z = \frac{\mu_0 I}{2}\frac{a^2}{(a^2+z^2)^{3/2}}}$$

***Note**: Field decays as $1/z^3$ for z>>a*

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Wikipedia: Magnetic Field

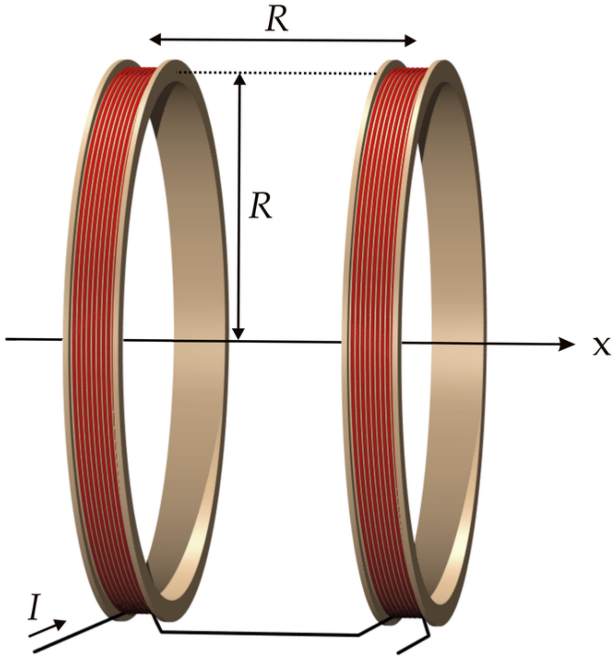
Field Uniformity: Helmholtz Coils

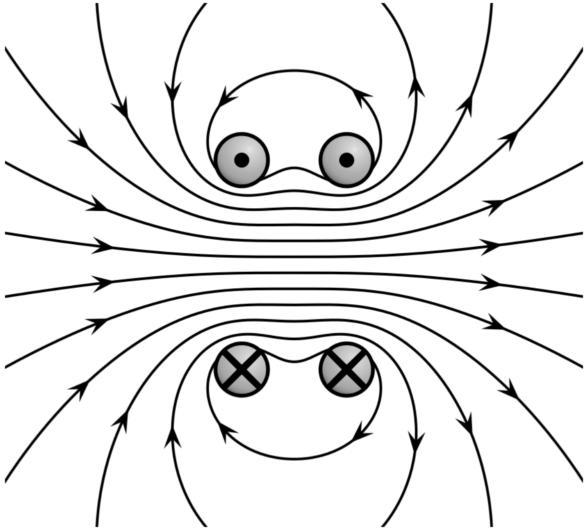
Hermann Ludwig Ferdinand von Helmholtz (1821-1894)

We often want a simple way of generating a **uniform** magnetic field over a significant volume. Examples include:

- Canceling out the Earth's magnetic field around an apparatus
- Measuring magnetic susceptibility of a sample

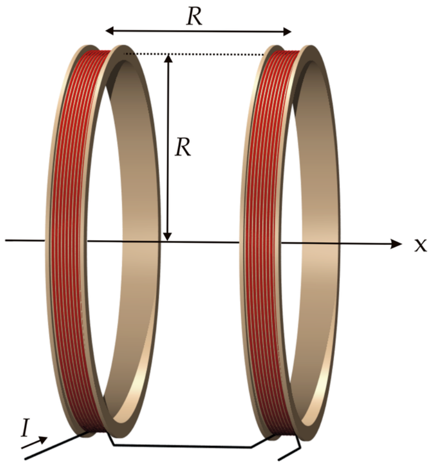
The standard way to do this is through the use of **Helmholtz coils**: a matched pair of (multi-turn) current loops, spaced apart by their radius.





Images [Wikipedia: Helmholtz coil]

Helmholtz Coils: Field Along Axis

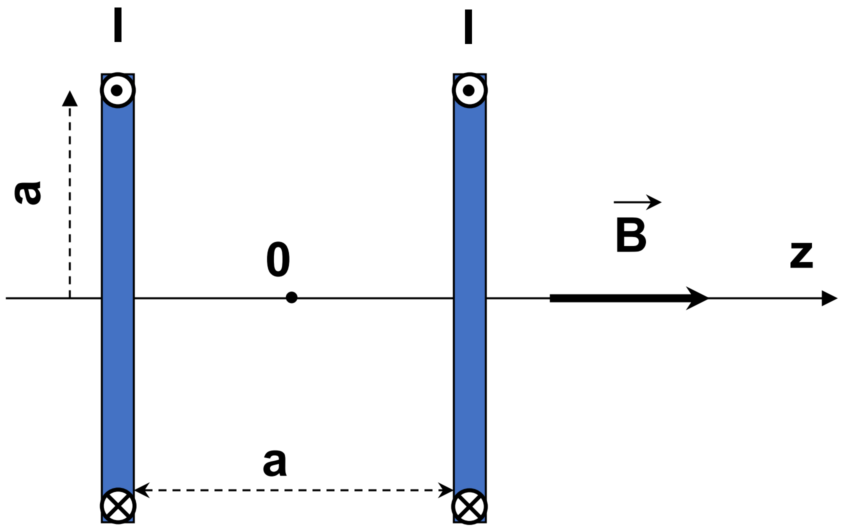


For a single loop of radius a, we have shown:

$$B_z = \frac{\mu_0 I}{2} \frac{a^2}{(a^2 + z^2)^{3/2}}$$

A coil with N loops is equivalent to increasing the current:

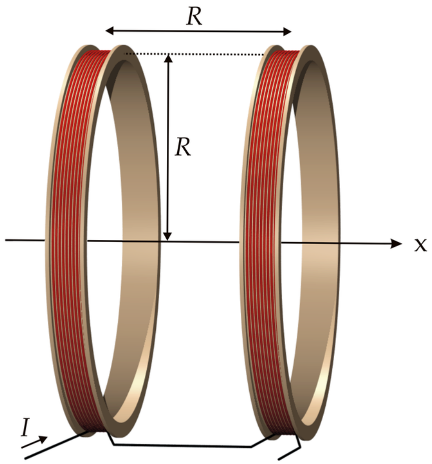
$$B_z = \frac{\mu_0 N I}{2} \frac{a^2}{(a^2 + z^2)^{3/2}}$$



We must also translate the two coils:

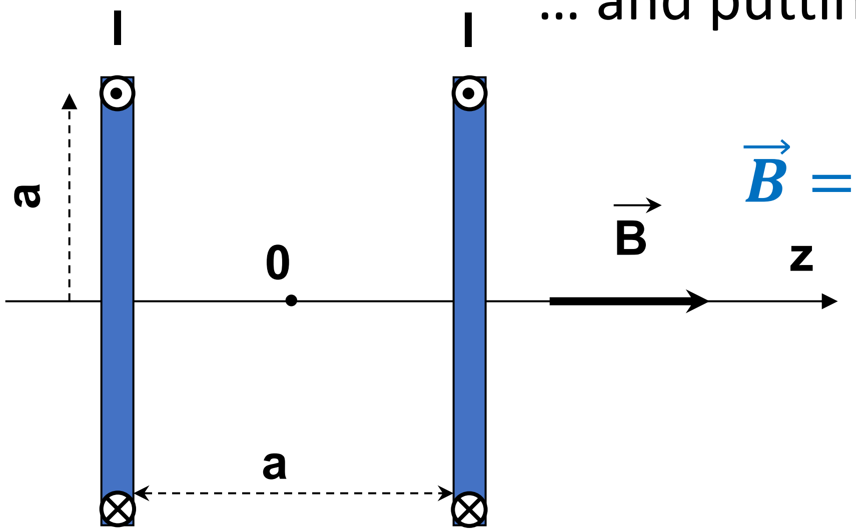
$$z \rightarrow z \pm \frac{a}{2}$$

Helmholtz Coils: Field Along Axis



So for each coil, we have:

$$\vec{B}_{l,r} = \frac{\mu_0 N I}{2} \frac{a^2}{\left(a^2 + \left(z \pm \frac{a}{2}\right)^2\right)^{3/2}} \hat{z}$$

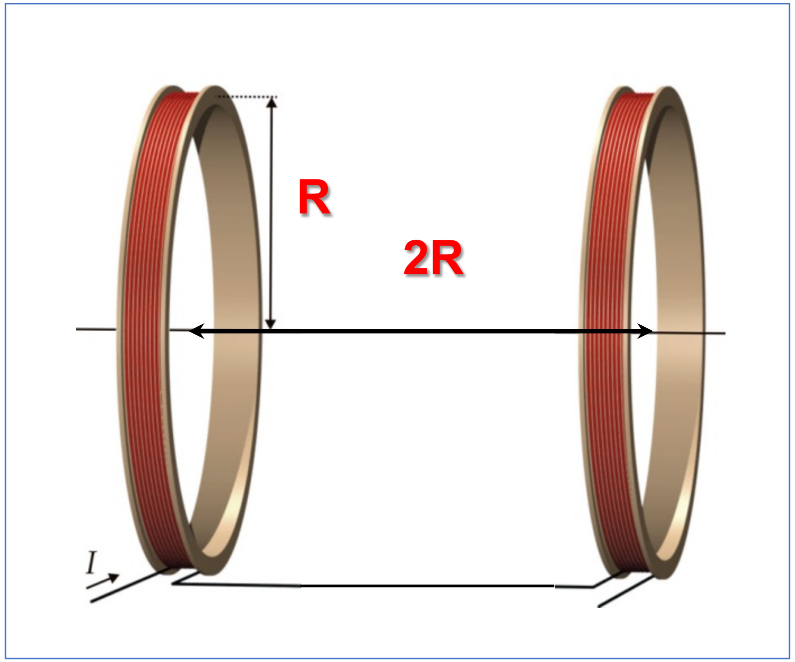


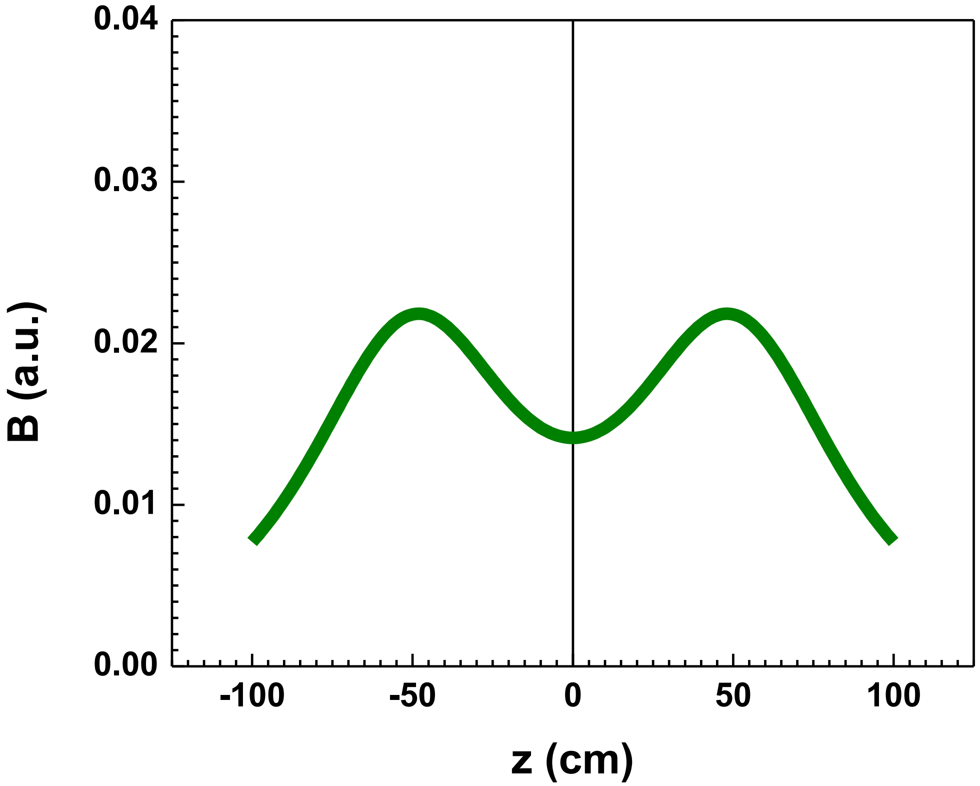
... and putting it all together and rearranging, we have:

$$\vec{B} = \frac{\mu_0 N I}{2a} \left(\frac{1}{\left[1 + \left(\frac{z}{a} + \frac{1}{2}\right)^2\right]^{3/2}} + \frac{1}{\left[1 + \left(\frac{z}{a} - \frac{1}{2}\right)^2\right]^{3/2}} \right) \hat{z}$$

Helmholtz Coils: Distance Between Coils

Case 1: a=2R





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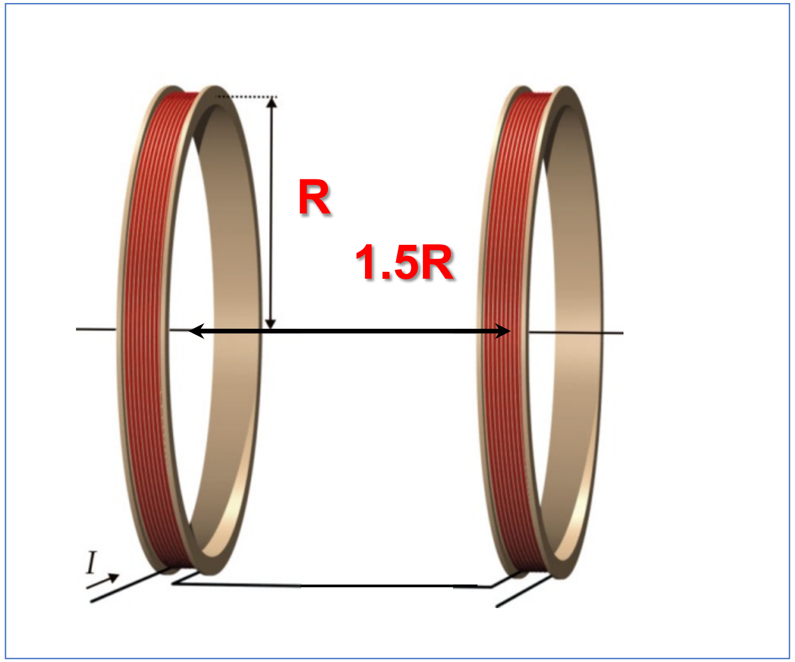
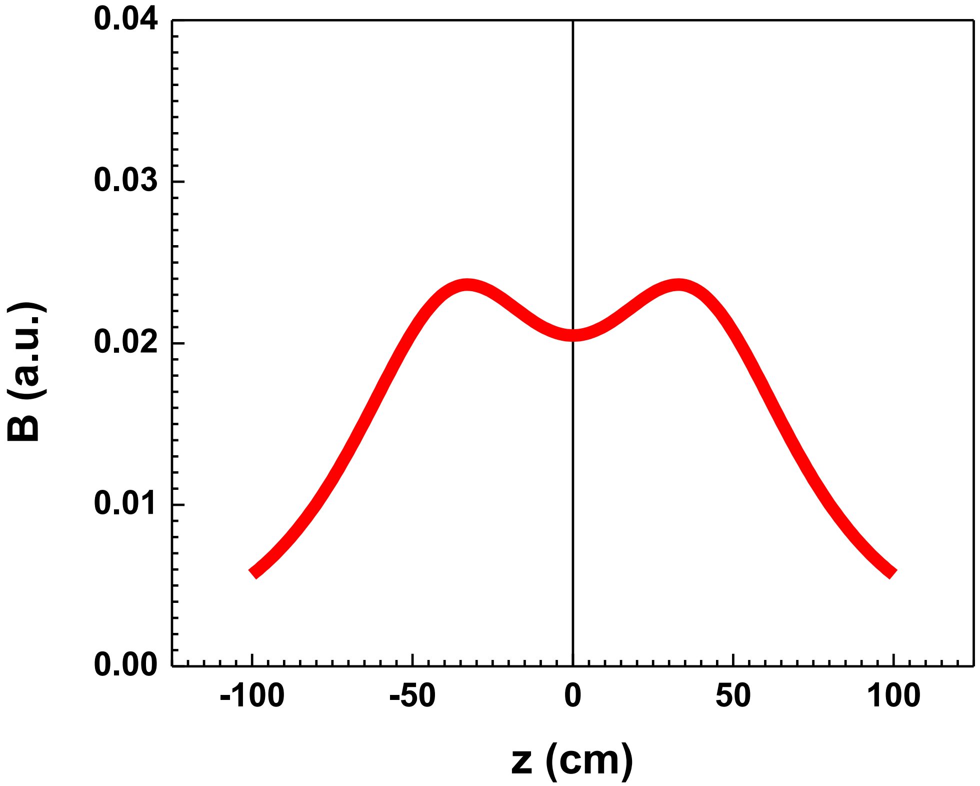
Helmholtz Coils: Distance Between Coils

Case 2: a=1.5R





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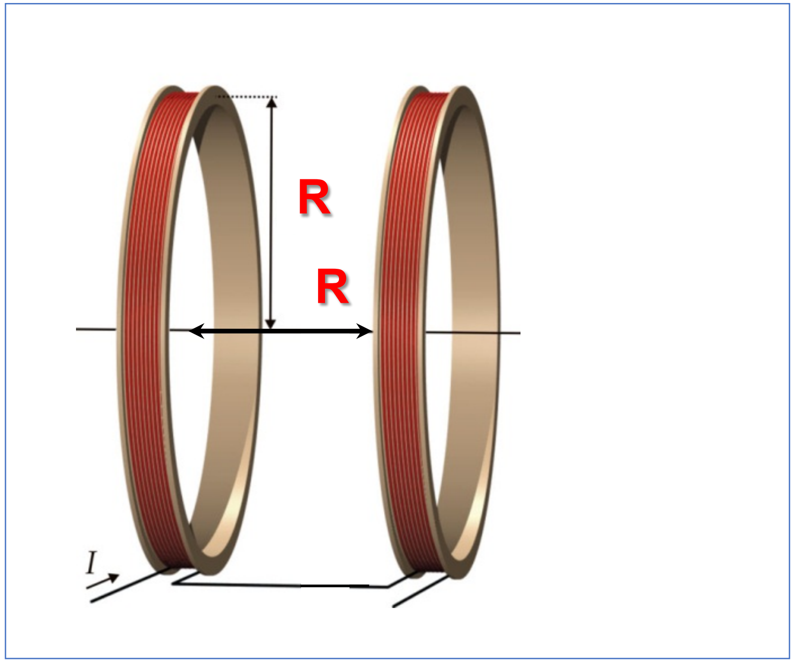
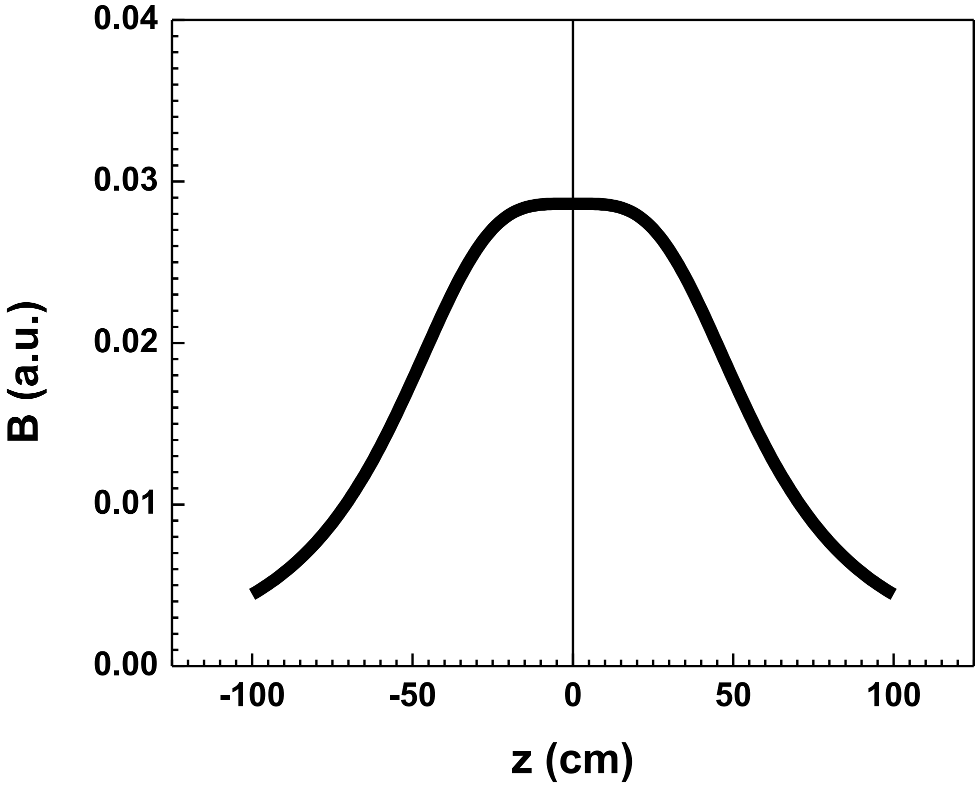
Helmholtz Coils: Distance Between Coils

Case 3: a=R





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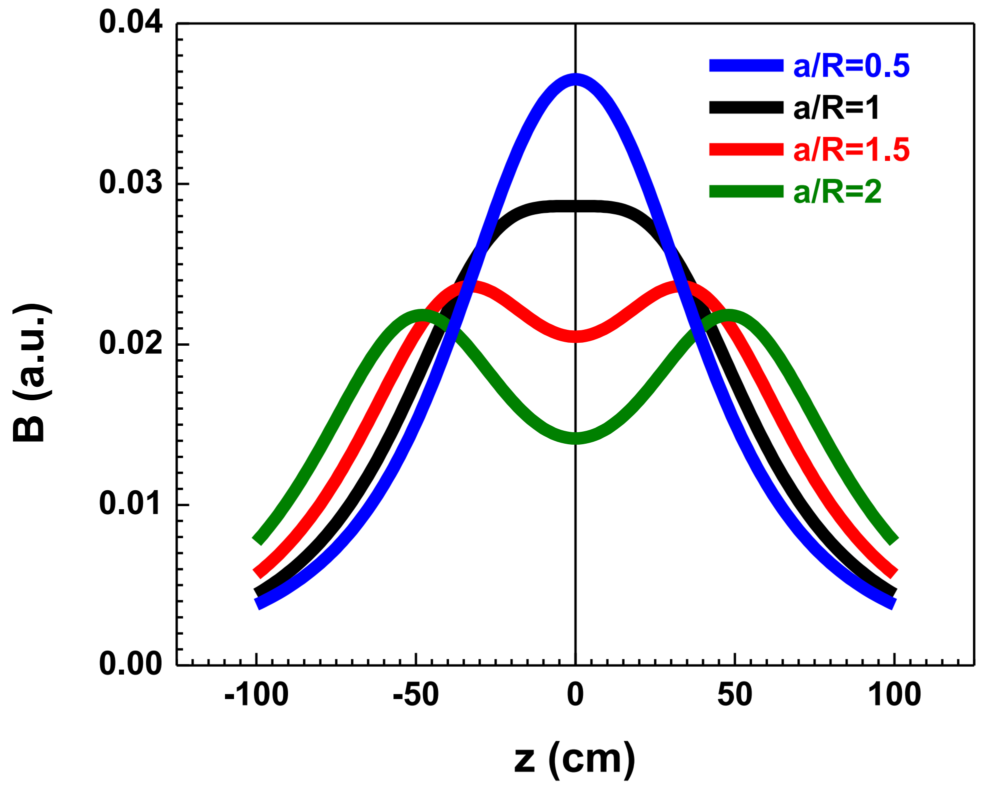
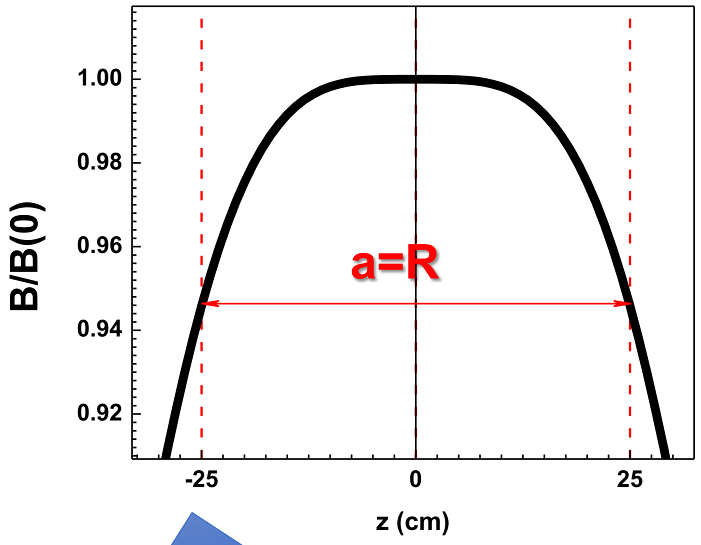
Helmholtz Coils: Distance Between Coils





In the z-range $\left[-\frac{a}{4}, +\frac{a}{4}\right]$, the B-field is uniform to better than 0.5%

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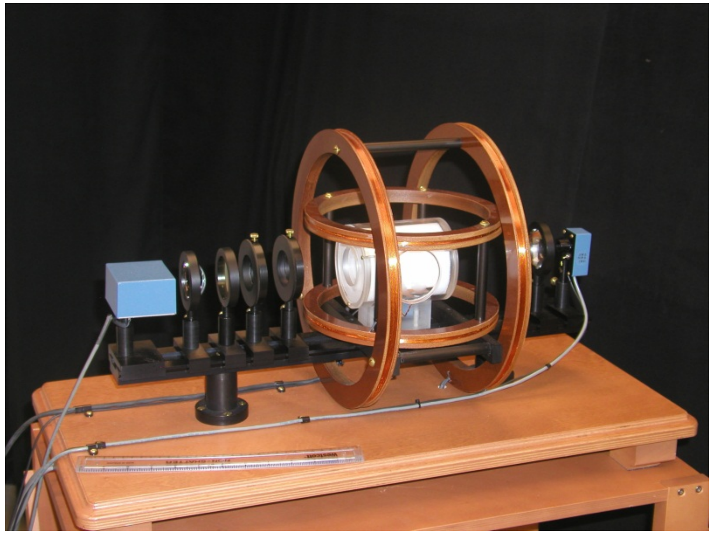
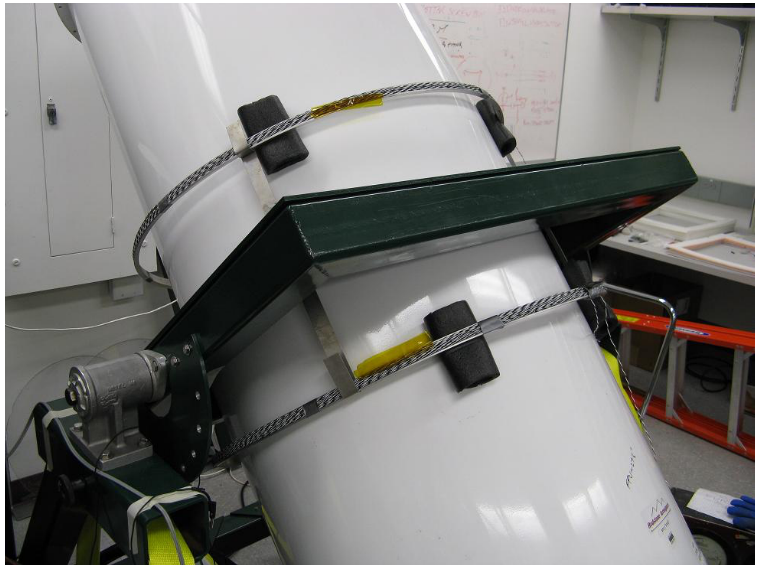
Helmholtz Coils: Summary

Helmholtz coils are a simple way to generate a controlled, uniform field

They're less useful for high fields (e.g. tesla), since it's hard to cram enough turns and current into the necessary geometry

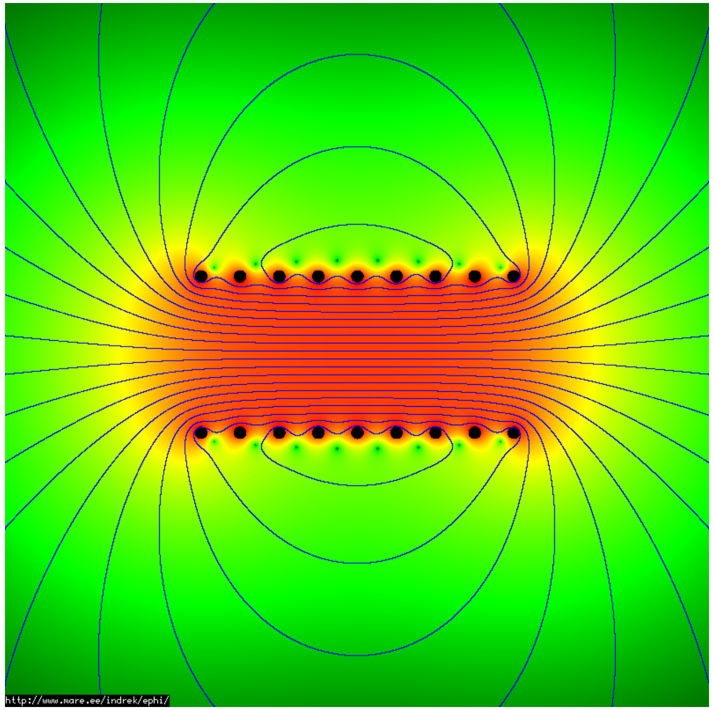


Helmholtz coils in Rb optical pumping experiment. UIUC Physics 403



Helmholtz coil testing of superconducting millimeter-wave camera (JPF, M. Runyan)

Solenoids

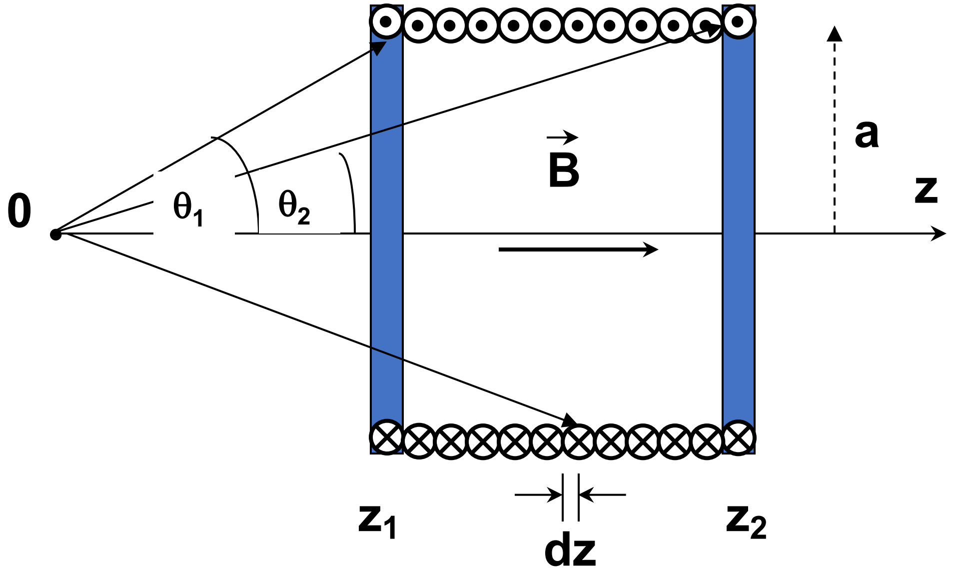


Solenoids are cylindrical coils of wire

We get uniform, high fields in the coil interior

From the exterior the field resembles that of a bar magnet

Solenoid: Field Along Axis



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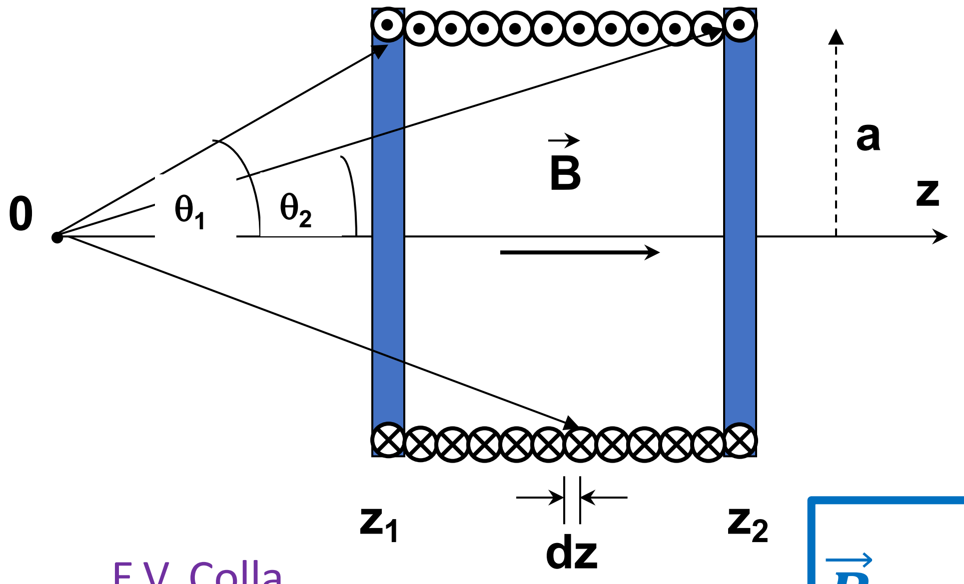
We again start from the field of a single loop, but now we have *n* turns per unit length and a common current *I*.

Then the field contributed by a segment *dz* at distance *z* is given by:

$$d\vec{B} = \left\{ \frac{\mu_0 nI\,dz}{2} \frac{a^2}{(z^2+a^2)^{3/2}} \right\} \hat{z}$$

To get the total field, we need to integrate this from z_1 to z_2.

Solenoid: Field Along Axis



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Segment *dz* at distance *z*, current *I*, *n* turns/length:

$$d\vec{B} = \left\{\frac{\mu_0 nI\,dz}{2}\frac{a^2}{(z^2+a^2)^{3/2}}\right\}\hat{z}$$

Change variables by substituting $z = \frac{a}{\tan\theta}$

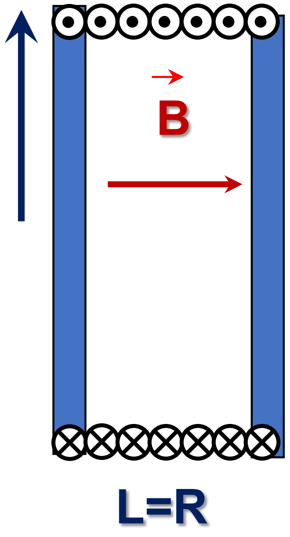
$$\boxed{\vec{B} = -\frac{\mu_0 nI}{2}\hat{z}\int_{\theta_1}^{\theta_2}\sin\theta\,d\theta = \frac{\mu_0 nI}{2}\hat{z}[\cos\theta_1 - \cos\theta_2]}$$

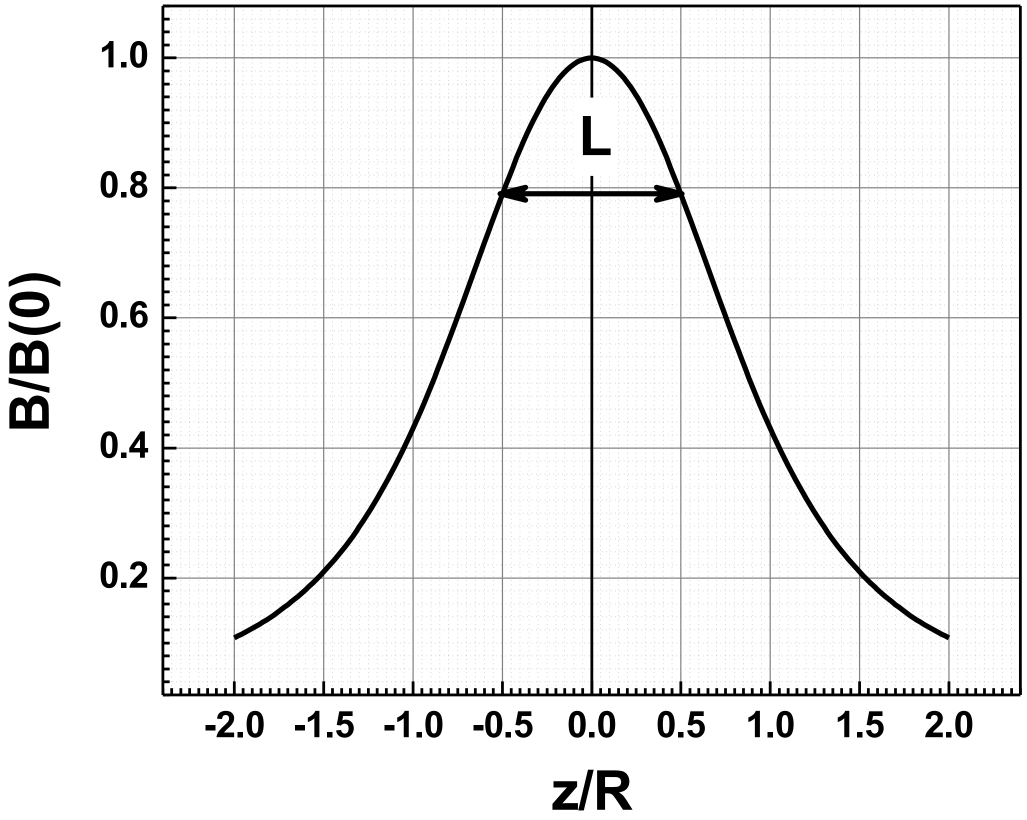
Note that within an **infinite** solenoid, $\theta_{1,2} = \mp\pi$ and so $\vec{B}_\infty = \mu_0 nI\hat{z}$

$$\cos\theta_{1,2} = \frac{z_{1,2}}{\sqrt{z_{1,2}^2 + a^2}}$$

Solenoid: Exploring Field Uniformity

Evaluate along axis at various coil lengths





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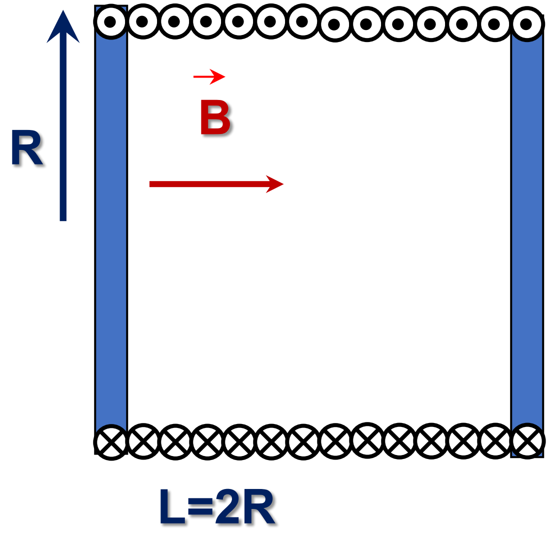
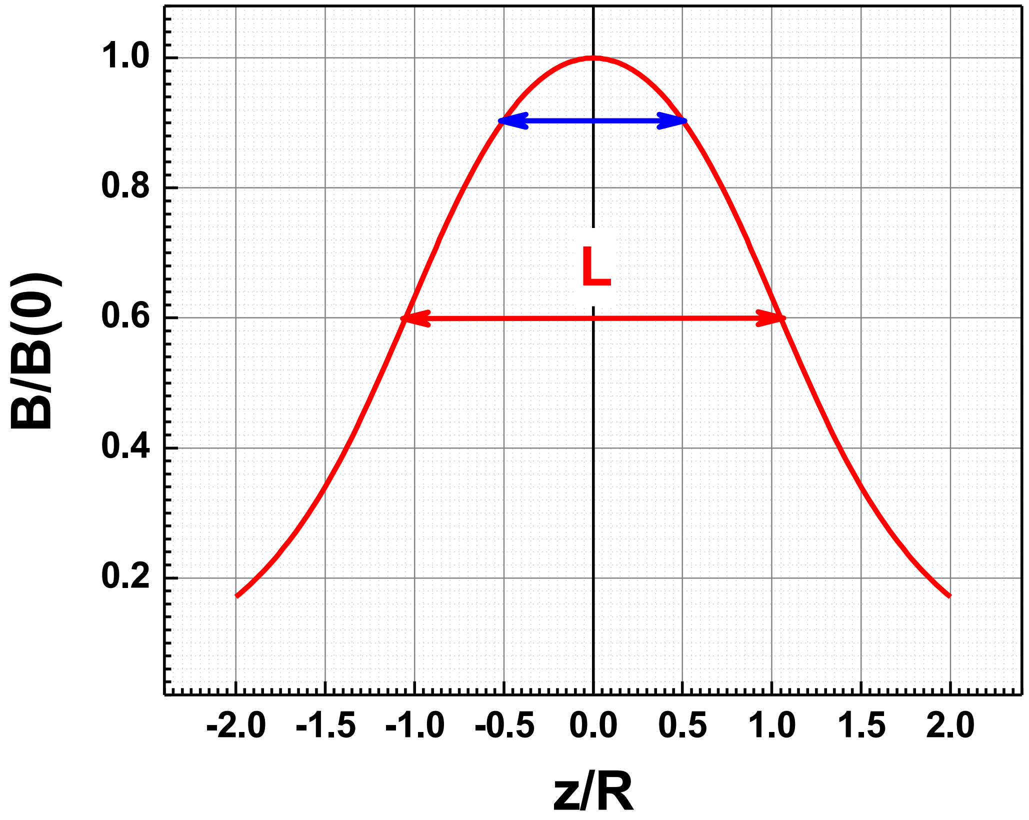
Solenoid: Exploring Field Uniformity

Evaluate along axis at various coil lengths





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Solenoid: Exploring Field Uniformity

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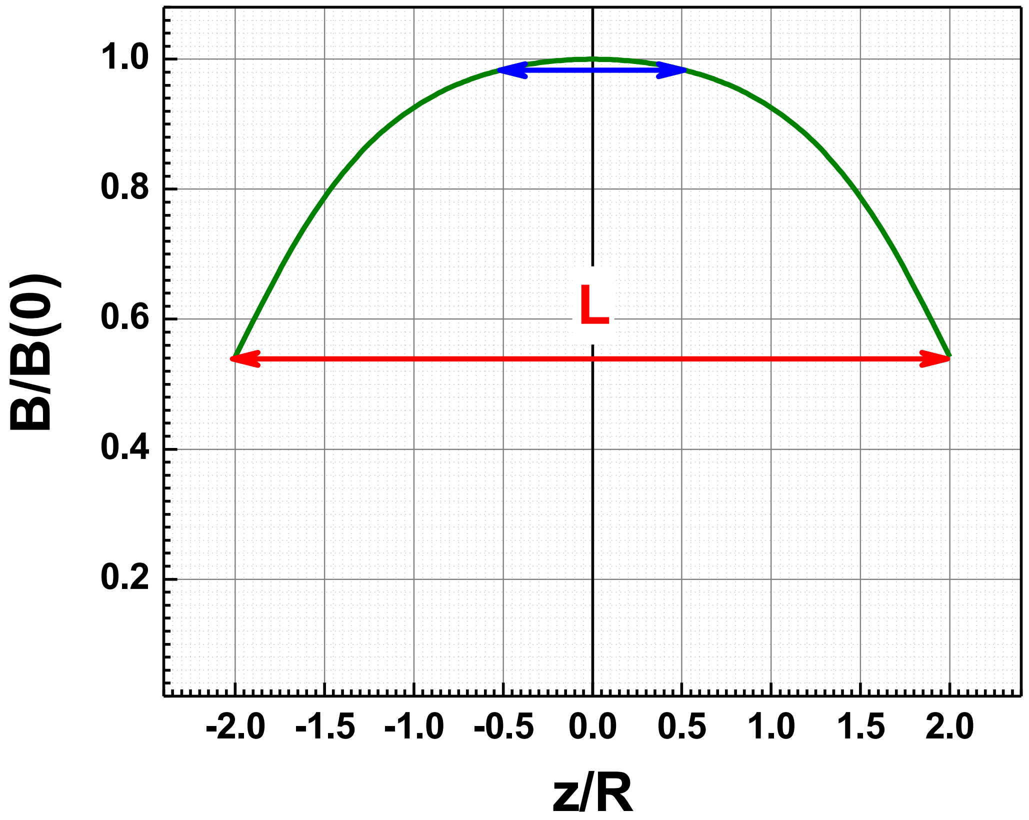

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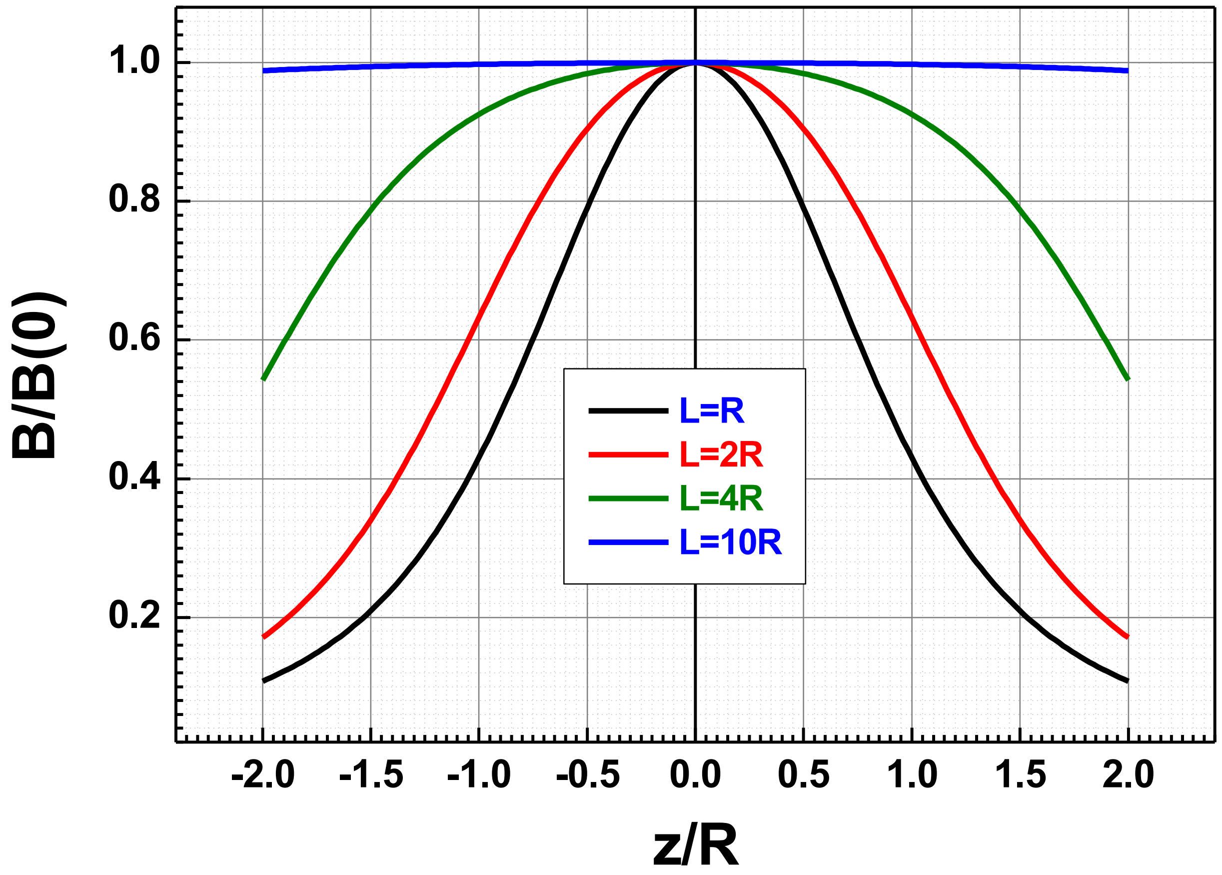
Solenoid: Exploring Field Uniformity



To ensure a uniform field within a solenoid, you need length L>>R

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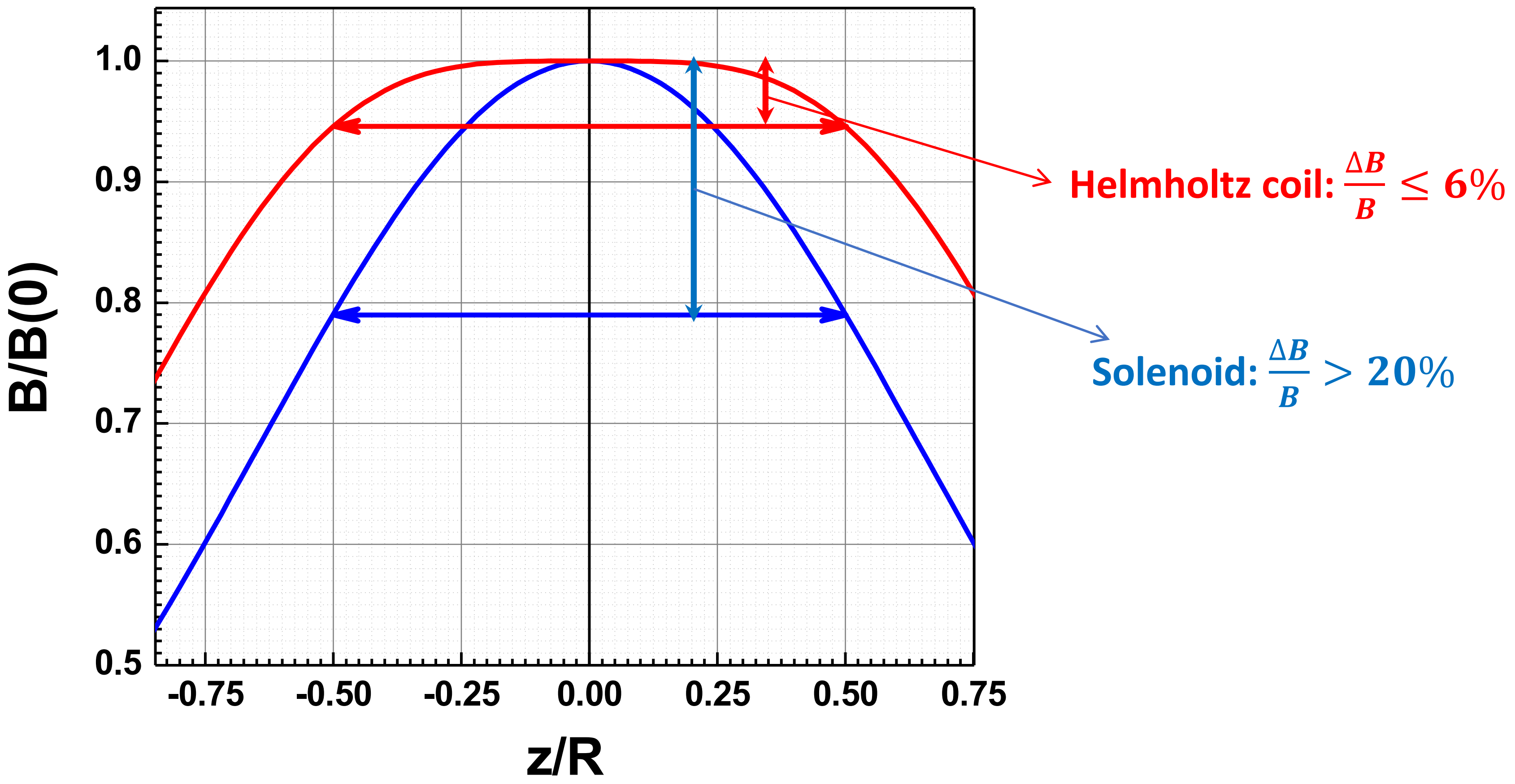
Comparing Solenoids to Helmholtz Coils

L=R



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Ideal Solenoid: Ampere's Law Calculation

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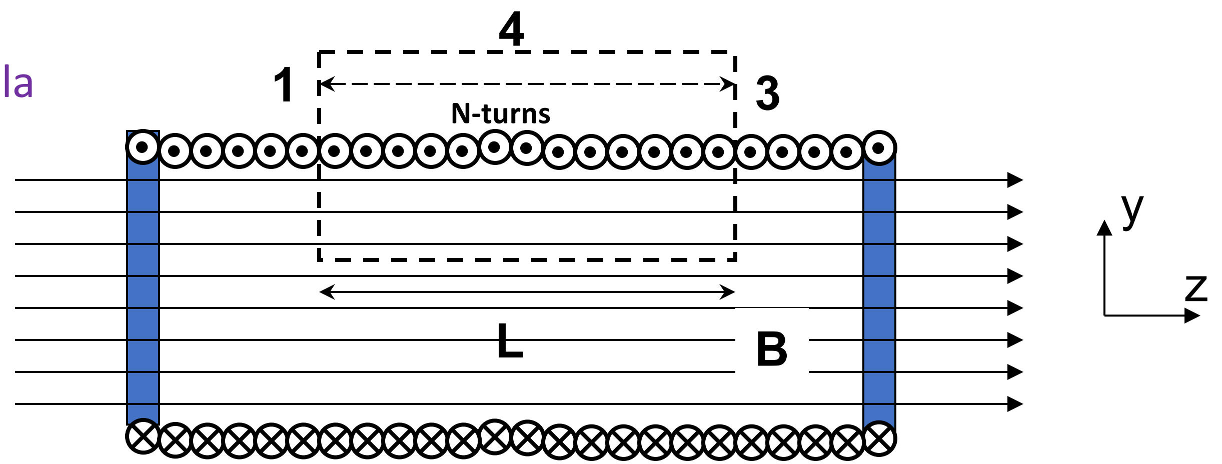


André-Marie Ampère
(1775-1836)

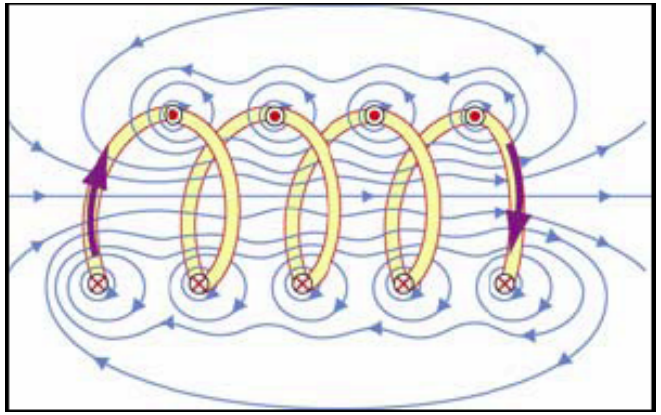
Assume an ideal infinite solenoid (*enforces uniformity along z*) and draw the integration loop above:

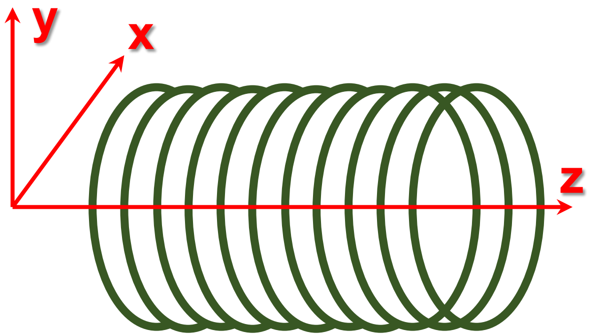
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 NI$$

Inside we must have uniform B=Bz; Outside we must have B=0; Other B components zero throughout

So $0 + LB + 0 + 0 = \mu_0 NI$, and thus: $\boxed{B = \frac{\mu_0 NI}{L} = \mu_0 nI}$

Solenoids: Field Gradients (On-Axis)





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We know that Maxwell's Equations must apply throughout:

$$\vec{\nabla} \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

By symmetry around the z-axis, **on-axis** we must have

$$\frac{\partial B_x}{\partial x} = \frac{\partial B_y}{\partial y}$$

… and thus know that…

$$\frac{\partial B_z}{\partial z} = -2\frac{\partial B_x}{\partial x}$$

Superconducting Solenoids

1 tesla = 10^4 gauss
B_{Earth} = 0.25-0.65 G
This lab: <100G

Ohmic heating limits the field attainable in a conventional solenoid magnet

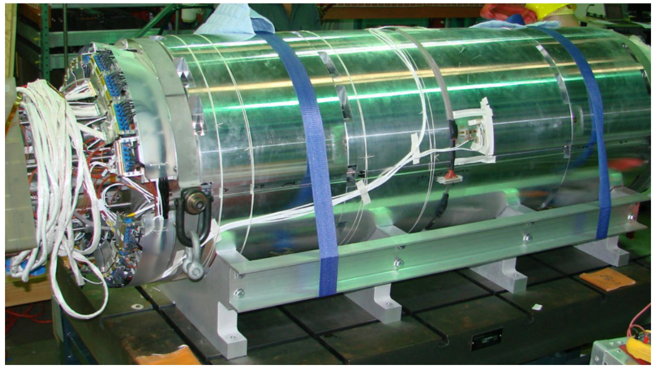
N.B.: ferromagnetic cores help increase field strength up to ~2T, but then saturate

Sustained many-tesla fields possible with solenoids wound from **superconducting wire** (often NbTi) and cooled to **cryogenic** temperatures (typically LHe, ~4.2K)

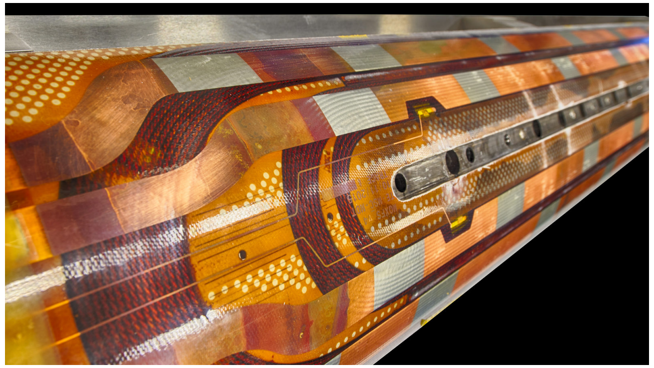
22T LHe-cooled magnet

OXFORD INSTRUMENTS

Siemens Magnetom Aera MRI Scanner
[Wikipedia: Magnetic Resonance Imaging]



HL-LHC Accelerator Magnet
CERN / Fermilab / LBNL
[Symmetry Magazine]



Halbach Array

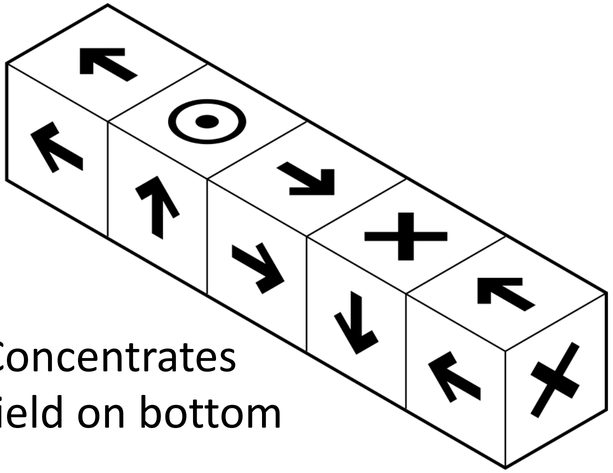
John Mallinson
1932-2015

Klaus Halbach
1924-2000

Discovered by **John Mallinson** (Ampex, Inc.) and independently by **Klaus Halbach** (Lawrence Berkeley National Lab).

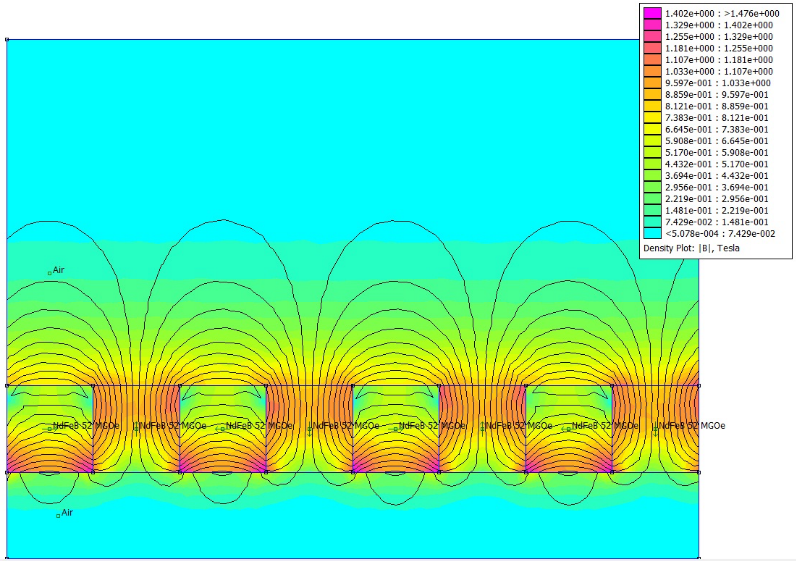
Permanent magnet configuration that concentrates magnetic flux on one side of the array and cancels it on the other.

Used in brushless motors, Maglev trains, particle accelerators; similar to refrigerator magnets (Mallinson!)



Concentrates
field on bottom

Images: Wikipedia



The Hall Effect: Magnetic Sensing

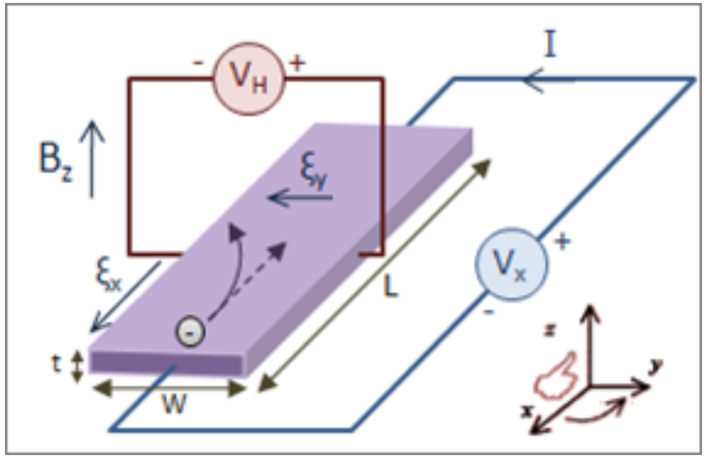
How can we build a compact, robust magnetic field sensor?

Electrons flowing through a conductor are deflected by the Lorentz force from magnetic fields transverse to their motion

This causes electric charge to build up on one side of the conductor (*like charging up a capacitor*), and we detect the resulting electric field as a voltage across the conductor's width

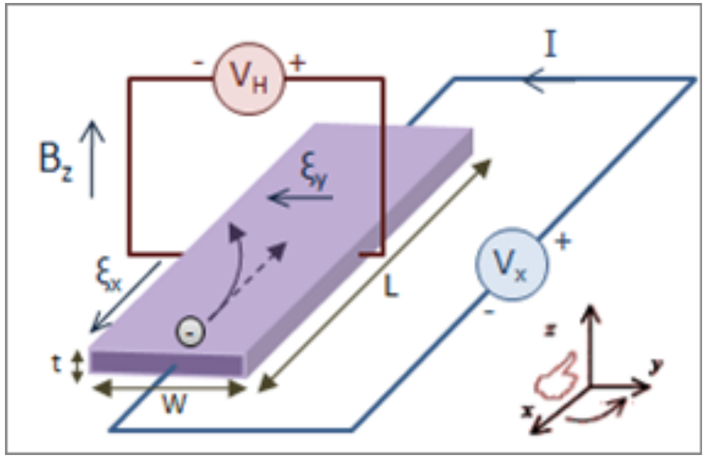
Edwin Herbert Hall (1855-1938)

Modern physics aside
Condensed matter physicists have found analogs in 2D electron systems (Quantum Hall Effect) which show integer (1985 Nobel) and fractional (1998 Nobel) quantization of electrical conductance



Wikipedia: Hall Effect

Hall Effect: Calculation



[Wikipedia: Hall Effect]

The **current** in the +x-direction is $\boldsymbol{I_x = nqAv_x}$, where n is the charge carrier density, q is the carrier charge, $A = wt$ is the bar's cross-sectional area, and v_x is the charge drift velocity (*material-dependent – analogous to terminal velocity*).

With a B-field (+z direction) and a current (+x direction, so electrons move in the –x direction!), the **force** on a carrier q is

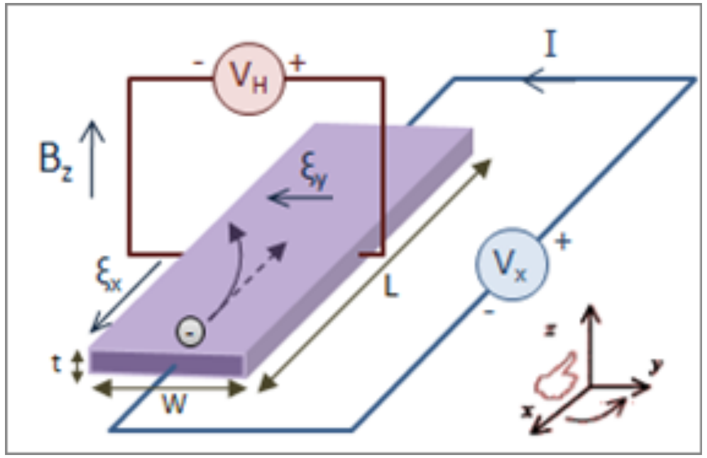
$$\boldsymbol{\vec{F} = q\vec{v} \times \vec{B} = q\left(\frac{I_x}{nqA}\hat{x}\right) \times B_z\hat{z} = -\frac{I_x B_z}{nA}\hat{y}}$$

Carriers will redistribute in the y-direction until the resulting electrostatic force balances this Lorentz force. So then $qE_y\hat{y} - \frac{I_x B_z}{nA}\hat{y} = 0$, and so $E_y = \frac{I_x B_z}{qnA}$.

We measure the equilibrium potential difference across the sample:

$$V_H = -\int_{-\frac{w}{2}}^{+\frac{w}{2}} E_y\, dy = -w\, E_y$$

Hall Effect: Calculation



Wikipedia: Hall Effect

And finally, we have:

$$V_H = \frac{I_x B_z}{q\,n\,t}$$

Defining the Hall coefficient $\boldsymbol{R_H \equiv \frac{1}{n\,q}}$, we can rewrite this as

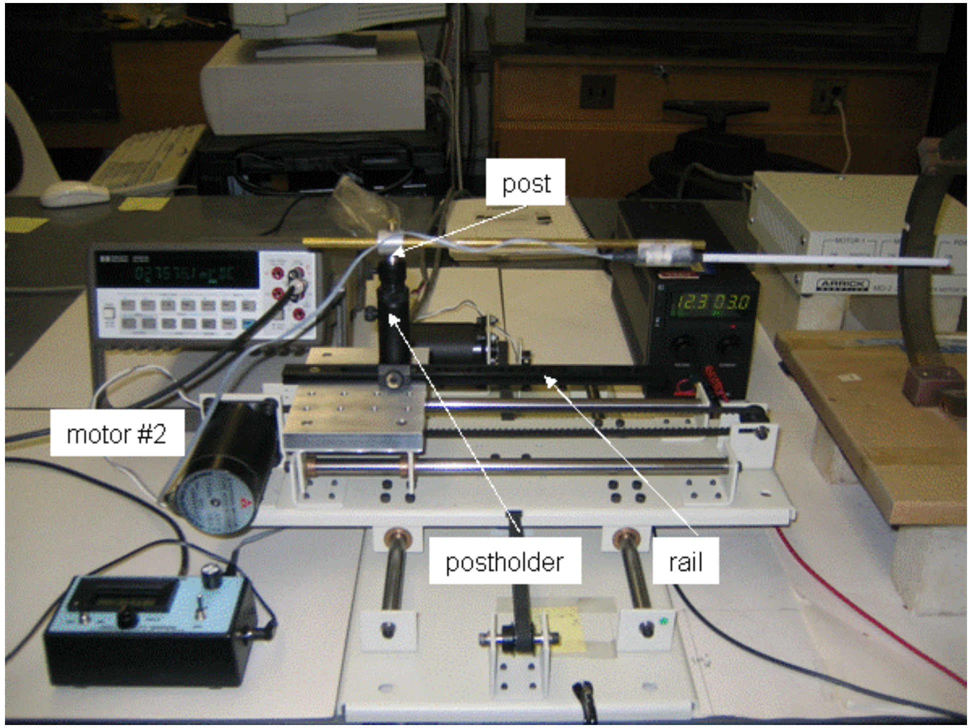
$$\boldsymbol{V_H = R_H \frac{I_x B_z}{t}}$$

The Hall coefficient is an intensive material property tabulated in various reference works

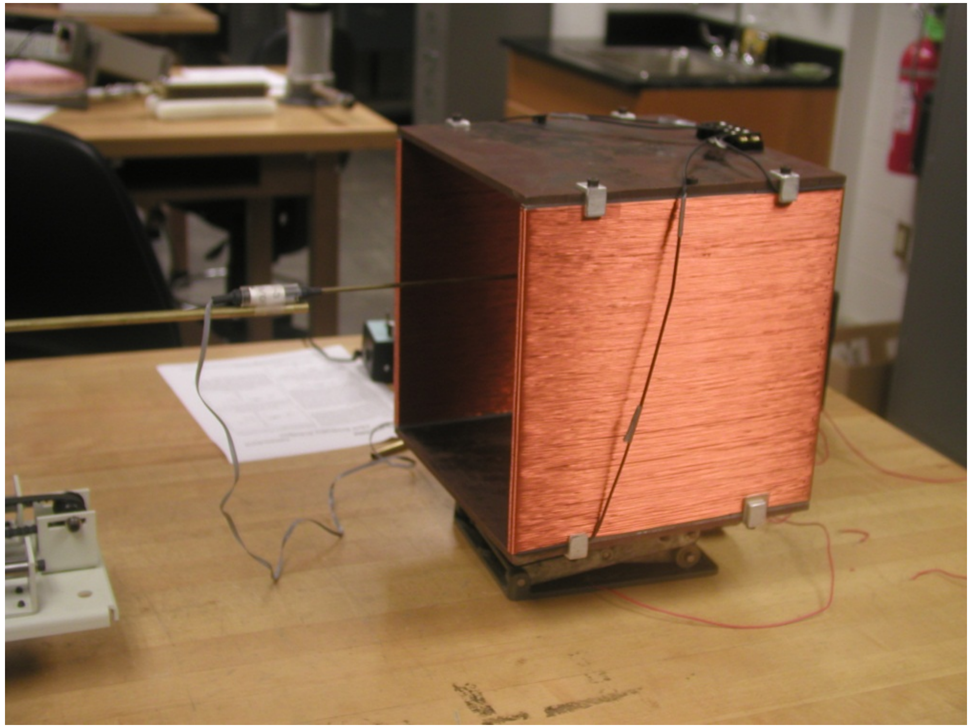
Table 1

Material	R_H (m^3/ C)
Cu	-5.3×10^{-11}
Na	-21.0×10^{-11}
Cr	$+35.0 \times 10^{-11}$
Bi	$-10^{3} \times 10^{-11}$
InAs (approx.)	$-10^{7} \times 10^{-11}$

Laboratory Setup

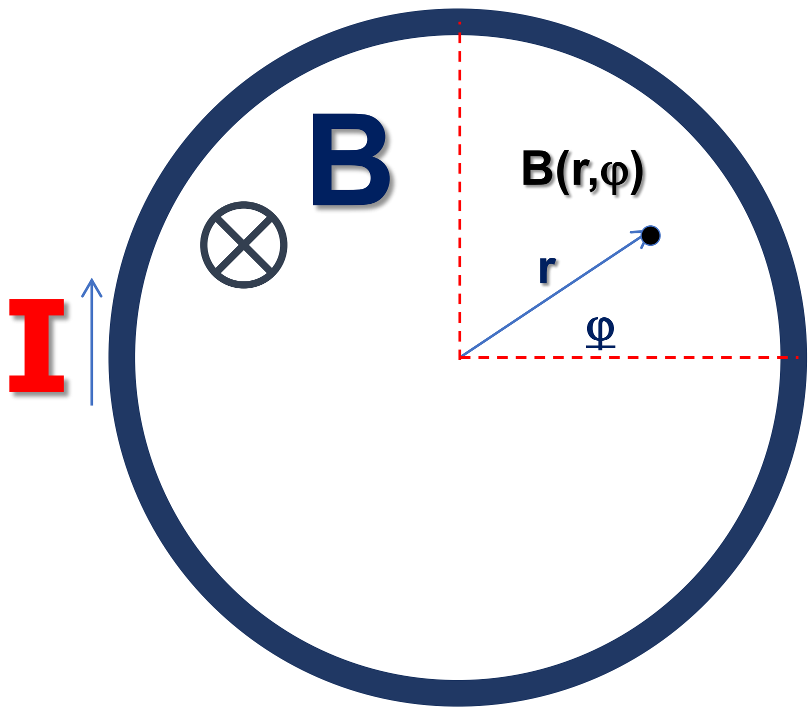


Use a **Hall probe** on an **X-Y scanning stage** measure field profiles



Hall probe scanning the “iron box” magnet

Measurement Tip: Take Advantage of Symmetry!



$B(r,\varphi)=f(r)\neq f(\varphi)$

The magnetic field created by a circular loop (solenoid, Helmholtz coil) depends only on radius r but not on the angle φ

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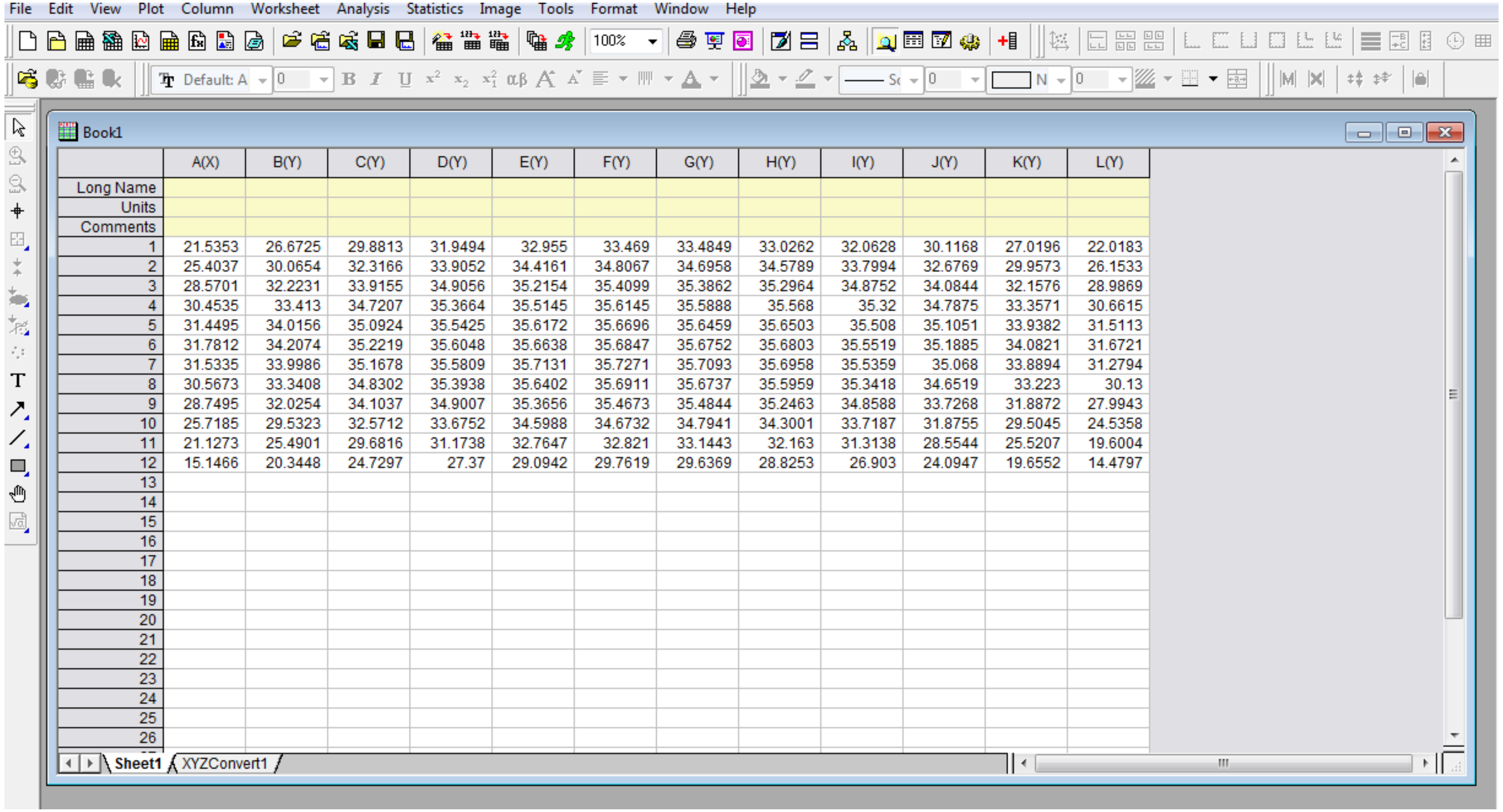
Field Maps: 3D Visualization

Step #1: Plugin your data into the worksheet



	A(X)	B(Y)	C(Y)	D(Y)	E(Y)	F(Y)	G(Y)	H(Y)	I(Y)	J(Y)	K(Y)	L(Y)
Long Name												
Units												
Comments												
1	21.5353	26.6725	29.8813	31.9494	32.955	33.469	33.4849	33.0262	32.0628	30.1168	27.0196	22.0183
2	25.4037	30.0654	32.3166	33.9052	34.4161	34.8067	34.6958	34.5789	33.7994	32.6769	29.9573	26.1533
3	28.5701	32.2231	33.9155	34.9056	35.2154	35.4099	35.3862	35.2964	34.8752	34.0844	32.1576	28.9869
4	30.4535	33.413	34.7207	35.3664	35.5145	35.6145	35.5888	35.568	35.32	34.7875	33.3571	30.6615
5	31.4495	34.0156	35.0924	35.5425	35.6172	35.6696	35.6459	35.6503	35.508	35.1051	33.9382	31.5113
6	31.7812	34.2074	35.2219	35.6048	35.6638	35.6847	35.6752	35.6803	35.5519	35.1885	34.0821	31.6721
7	31.5335	33.9986	35.1678	35.5809	35.7131	35.7271	35.7093	35.6958	35.5359	35.068	33.8894	31.2794
8	30.5673	33.3408	34.8302	35.3938	35.6402	35.6911	35.6737	35.5959	35.3418	34.6519	33.223	30.13
9	28.7495	32.0254	34.1037	34.9007	35.3656	35.4673	35.4844	35.2463	34.8588	33.7268	31.8872	27.9943
10	25.7185	29.5323	32.5712	33.6752	34.5988	34.6732	34.7941	34.3001	33.7187	31.8755	29.5045	24.5358
11	21.1273	25.4901	29.6816	31.1738	32.7647	32.821	33.1443	32.163	31.3138	28.5544	25.5207	19.6004
12	15.1466	20.3448	24.7297	27.37	29.0942	29.7619	29.6369	28.8253	26.903	24.0947	19.6552	14.4797
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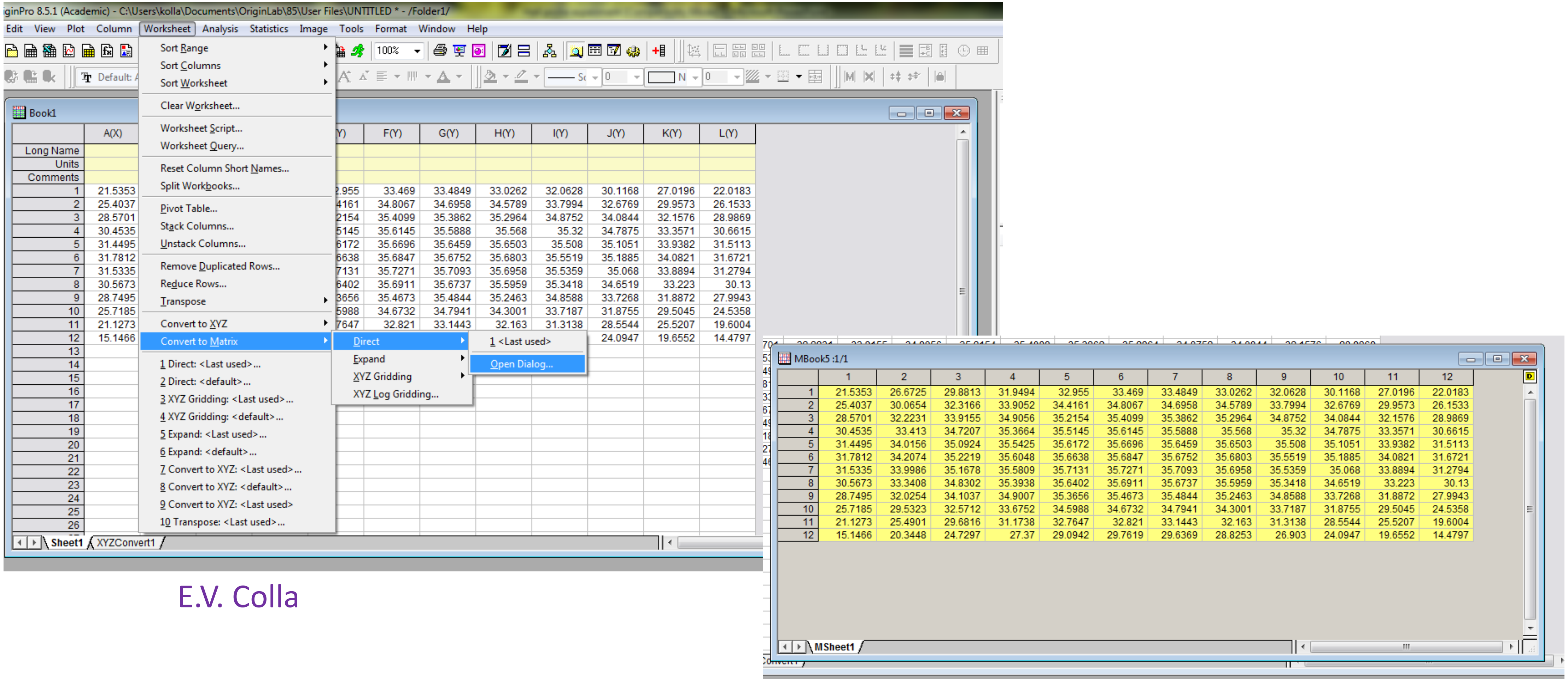
Field Maps: 3D Visualization

Step#2. **Convert data into a matrix**



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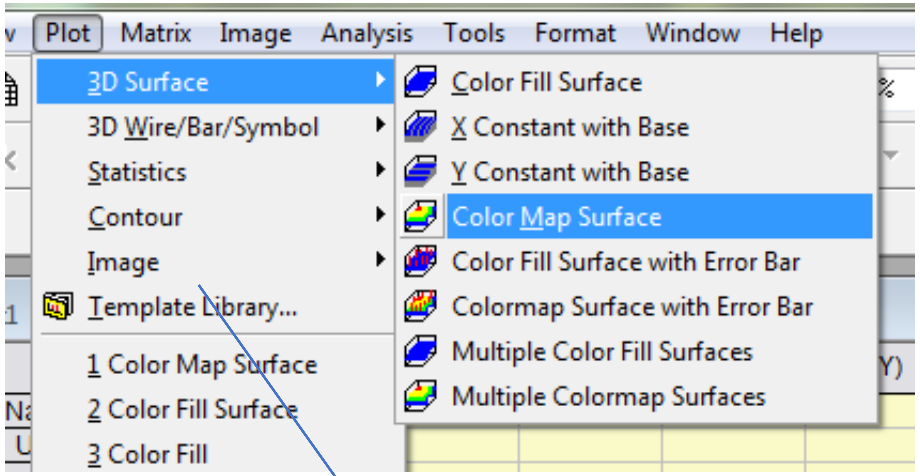
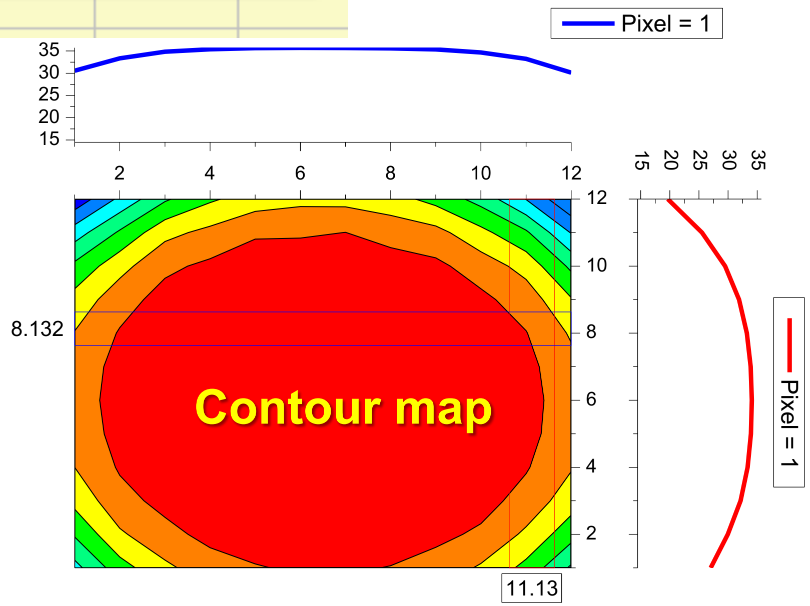
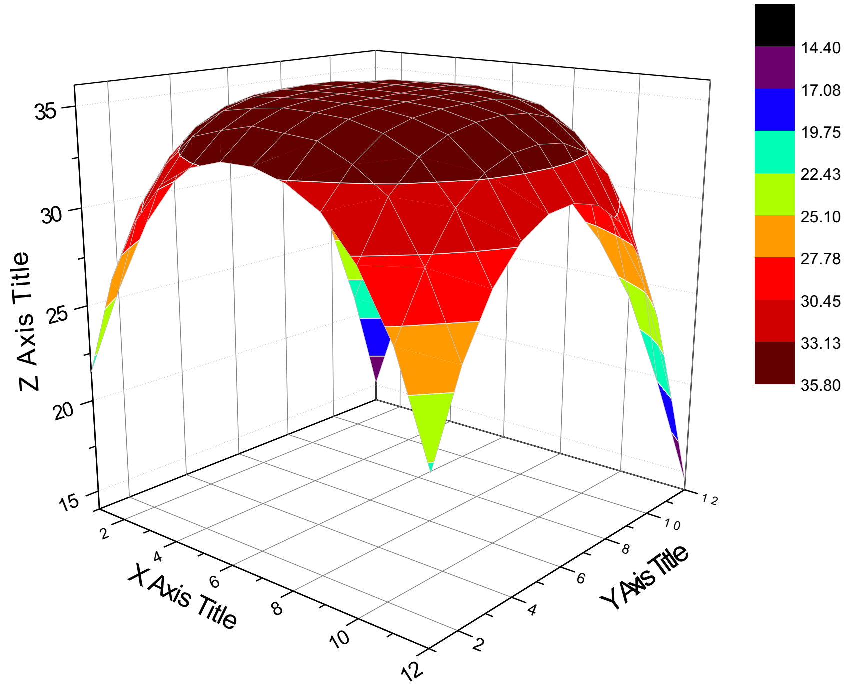
Field Maps: 3D Visualization

Step#3. **Plot the matrix (*A color map is shown here, but you have many options!*)**







Color map surface

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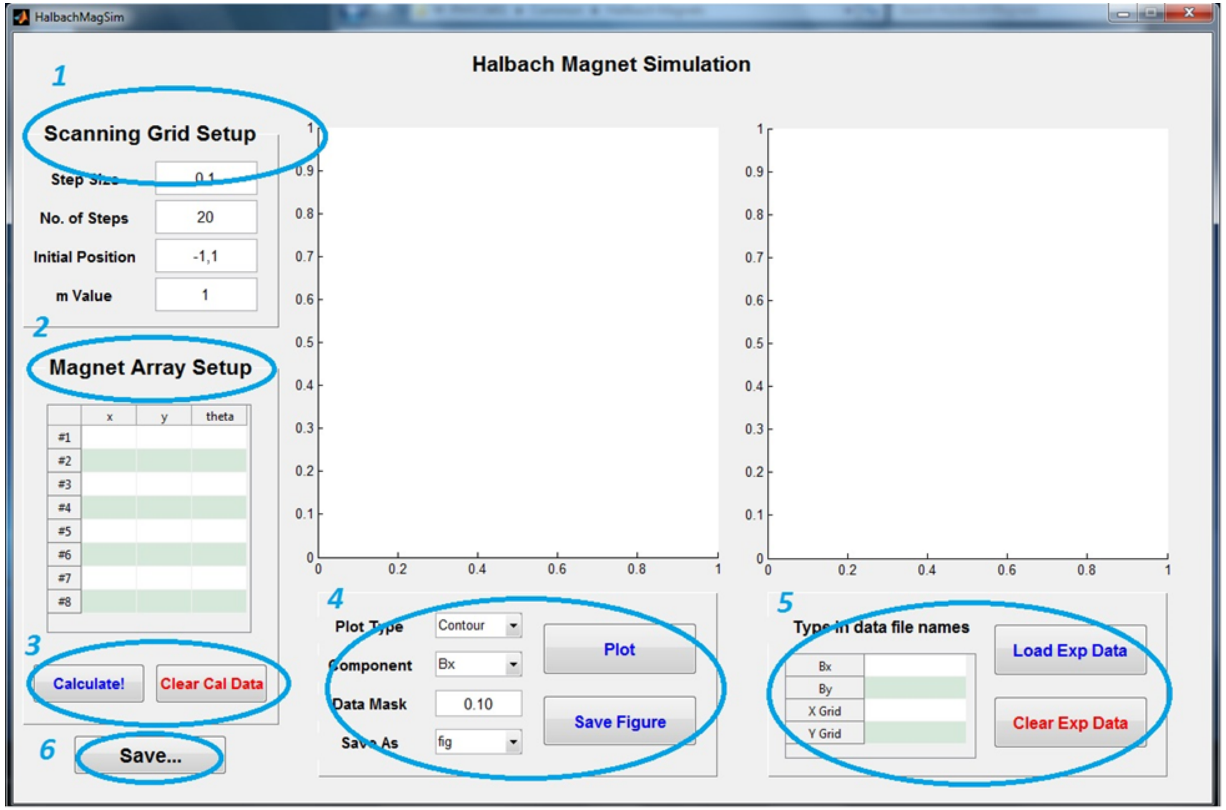
Simulating the Halbach Magnets

\\engr-file-03\PHYINST\APL Courses\PHYCS401\Common\Halbach Magnets



MATLAB code
Courtesy of Longxiang Zhang

E.V. Colla

Documentation:
HalbachSim - The newest and most useful README in the world.docx

Bonus: DC SQUID Magnetometers

How can we build an even more sensitive magnetic sensor?

Some facts about **superconductors**:

1. Their conduction electrons condense into a macroscopic quantum state. This means that we can assign a complex order parameter $\Psi(\vec{r}) = \psi(\vec{r})e^{i\theta(\vec{r})}$ to each point in a superconducting sample.

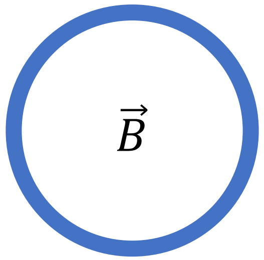
2. The phase of this state is affected by electromagnetic potentials (*for details, learn about gauge theory*).

3. A supercurrent can tunnel through a narrow normal or insulating barrier, allowing a discrete difference in phase (and electric potential) across the gap (*for details, learn about the Josephson effect, e.g. Feynman Lectures vol. 3*)

Bonus: DC SQUID Magnetometers

Taken together, #1 and #2 imply that the magnetic flux through a superconducting loop must be quantized in units of $\Phi_0 = \frac{h}{2e} = 2.07\times10^{-15}\,T\cdot m^2$.

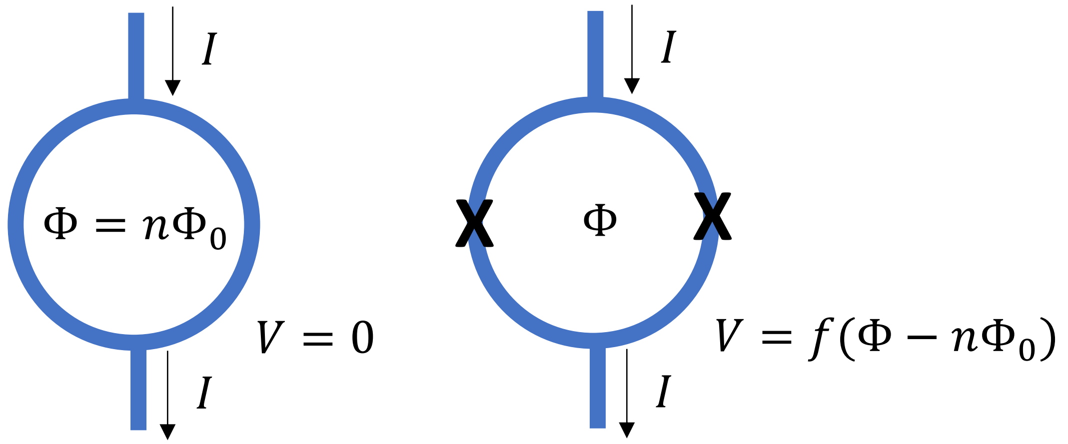
$$\Phi = \int \vec{B}\cdot d\vec{A} = \oint \vec{A}\cdot d\vec{l} = \frac{\hbar}{q}\oint \vec{\nabla}\theta\cdot d\vec{l} = n\Phi_0$$



Phase must change by a multiple of 2π in a walk around the loop, or physical parameters won't be single-valued

A loop is also zero-resistance, so it can carry current without developing a voltage.

But if I break a superconducting loop with tunneling gaps (Josephson junctions), then the flux need not be quantized… and any deviation from $n\Phi_0$ will cause a flux-dependent voltage to appear across the junction(s).



***Oversimplification**: for details, see a superconducting devices course / book!*

Bonus: DC SQUID Magnetometers

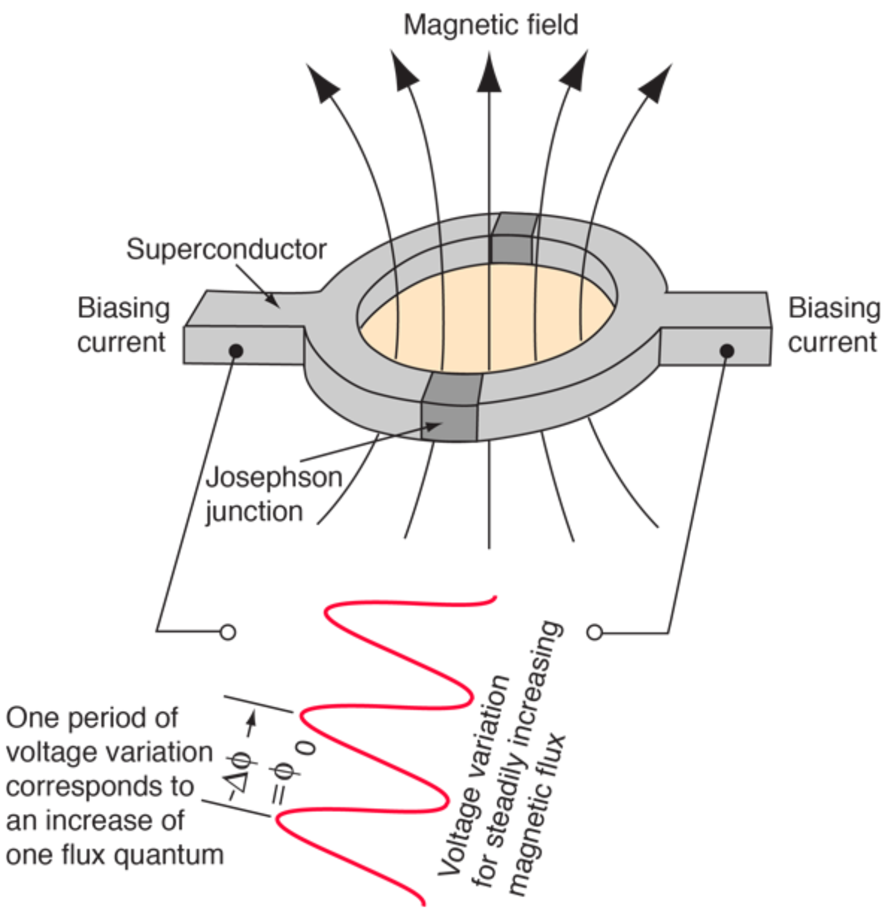


Image: Hyperphysics

DC SQUID: A superconducting loop cut by two (generally matched) Josephson junctions.

When you drive an appropriate current across the pair, the voltage is a sinusoidal function of the flux Φ threading the loop, with period Φ_0.

Many uses!

- Materials characterization
- Biomagnetic fields: brain, heart, stomach, …
- Geomagnetic surveys: oil and mineral prospecting, …
- General relativity measurements: Gravity Probe B
- Superconducting qubits for quantum information
- Readout systems for superconducting Transition-Edge Sensors for cosmic microwave background cameras, dark matter detectors, X-ray spectrometers, …

SQUID research at UIUC: van Harlingen group