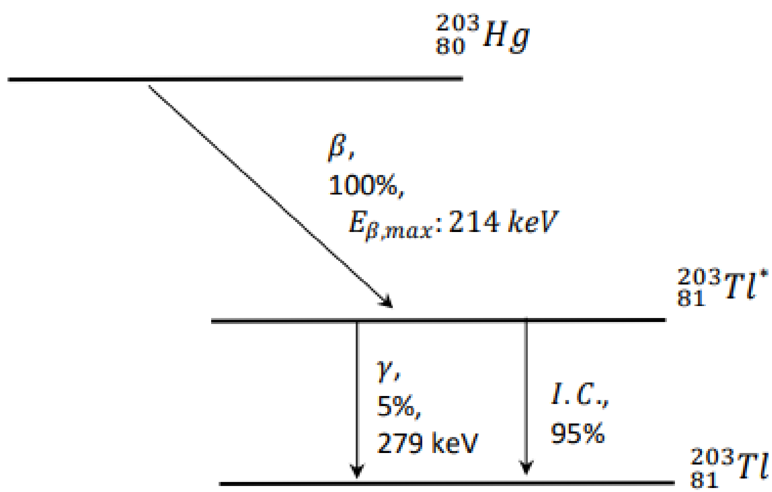
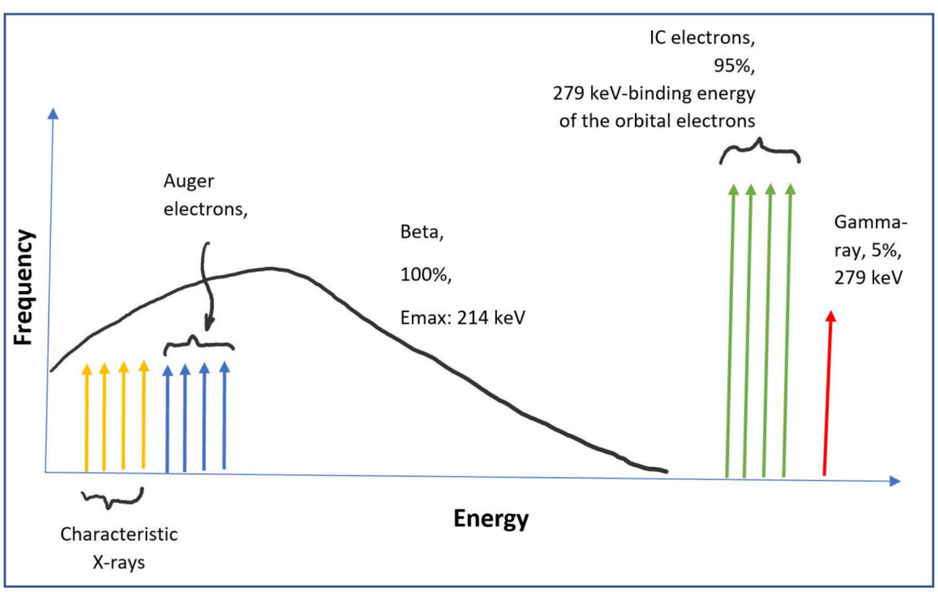
NPRE 441 **Quiz 1 Solutions**

2/14/23

1. (25 points total) Plot the energy spectrum for all types of radiation emitted by Hg-203. 5 points assigned each for beta particles, conversion electrons, gamma rays, auger electrons, and characteristics X-rays.

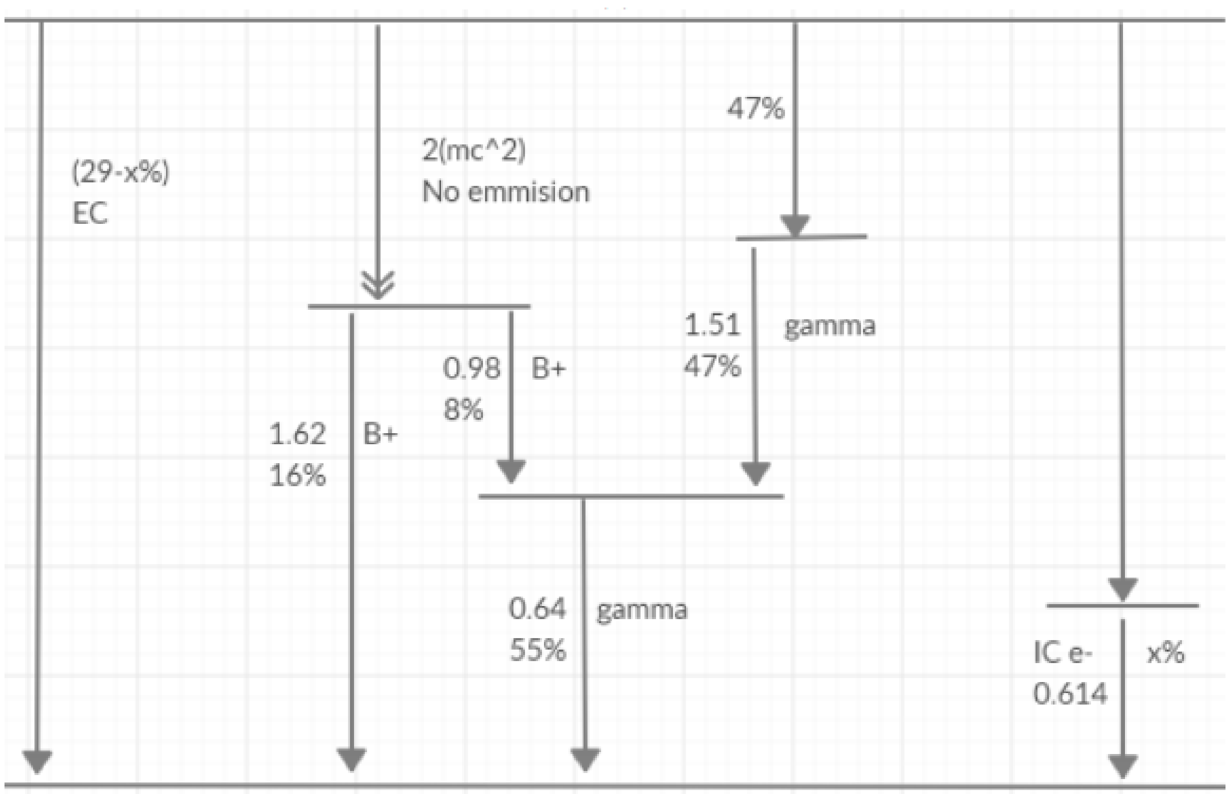
$$ {}^{203}_{80}Hg \rightarrow {}^{203}_{81}Tl + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu} $$





2. (25 points total) Nuclide A decays into B by β^+ emission (24 %) or by electron capture (76 %). The major radiations, energies (MeV), and frequencies per disintegration are:
β^+: 1.62 (16 %), 0.98 (8 %)
γ: 1.51 (47 %), 0.64 (55 %), 0.511 (48 %)
Daughter x-rays
e^-: .0614

(a) Draw the nuclear decay scheme, labeling types of decay, percentages, and energies. 15 points assigned for properly drawing the decay diagram.



(b) What leads to the emission of daughter x-rays?
The emission of daughter or characteristic x-rays is due to electronic rearrangements by outer electrons filling vacant orbits. 10 points assigned for answering (b) sufficiently.

3. (25 points total) Starting with a 40 mg sample of Ra-226.

$$\begin{aligned}
\lambda_{Ra} &= \frac{\ln 2}{t_{\frac{1}{2}Ra}} \\
&\simeq 1.377 \times 10^{-11} \text{ s} \\
\lambda_{Ra} &\simeq 2.100 \times 10^{-6} \text{ s}
\end{aligned}$$

$$\begin{aligned}
N_{Ra}(0) &= \mathrm{m}_{Ra} \cdot \frac{N_A}{\mathrm{MM}_{Ra}} \\
N_A &\simeq 6.02 \times 10^{23} \ \frac{\text{atoms}}{\text{mol}} \\
\mathrm{MM}_{Ra} &\simeq 226 \ \frac{\text{grams}}{\text{mol}} \\
N_{Ra}(0) &= 1.066 \times 10^{20}
\end{aligned}$$

$$N_{Ra}(t) = N_{Ra}(0) \cdot e^{-\lambda_{Ra} t}$$

See Problem 4

$$\begin{aligned}
N_{Rn} &= \frac{N_{Ra}(0)\lambda_{Ra}}{\lambda_{Rn} - \lambda_{Ra}} \cdot (e^{-\lambda_{Ra} t} - e^{-\lambda_{Rn} t}) \\
Q_{Ra} &= \lambda_{Ra} \cdot N_{Ra} \\
Q_{Rn} &= \lambda_{Rn} \cdot N_{Rn}
\end{aligned}$$

$$\begin{aligned}
Q_{Rn} &= \lambda_{Rn} \cdot N_{Rn} \\
&= \lambda_{Rn} \cdot \frac{N_{Ra}(0)\lambda_{Ra}}{\lambda_{Rn} - \lambda_{Ra}} \cdot (e^{-\lambda_{Ra} t} - e^{-\lambda_{Rn} t})
\end{aligned}$$

(a) (5 points) How long will it take for 10mCi of Rn-222 to build up,

$$\begin{aligned}
10 \text{ mCi} &= 3.7 \times 10^{8} \text{ Bq} \\
3.7 \times 10^{8} &= Q_{Rn} \\
&= \lambda_{Rn} \cdot \frac{N_{Ra}(0)\lambda_{Ra}}{\lambda_{Rn} - \lambda_{Ra}} \cdot (e^{-\lambda_{Ra} t} - e^{-\lambda_{Rn} t}) \\
e^{-\lambda_{Ra} t} - e^{-\lambda_{Rn} t} &= .2523 \\
\lambda_{Ra} t &<<< \lambda_{Rn} t \\
e^{-\lambda_{Ra} t} &\simeq 1 \\
t &= -\frac{\ln(1 - .2523)}{\lambda_{Rn}} \\
&\simeq 1.384 \times 10^{5} \text{ s} \\
&\simeq 38.45 \text{ hours}
\end{aligned}$$

$$\simeq 1.602 \text{ days}$$

(b) (15 points between (b) and (c)) What will be the activity of Rn-222 after 2 years?

$$\begin{aligned} Q_{Rn} &= \lambda_{Rn} \cdot \frac{N_{Ra}(0)\lambda_{Ra}}{\lambda_{Rn} - \lambda_{Ra}} \cdot (e^{-\lambda_{Ra}t} - e^{-\lambda_{Rn}t}) \\ &= 1.465^9 \text{ Bq} \\ &= .0396 \text{ Ci} \end{aligned}$$

(c) What will be the activity of Rn-222 after 1000 years?

$$\begin{aligned} Q_{Rn} &= \lambda_{Rn} \cdot \frac{N_{Ra}(0)\lambda_{Ra}}{\lambda_{Rn} - \lambda_{Ra}} \cdot (e^{-\lambda_{Ra}t} - e^{-\lambda_{Rn}t}) \\ &= 9.510^8 \text{ Bq} \\ &= .0257 \text{ Ci} \end{aligned}$$

(d) (5 points) What is the ratio of the specific activity of Rn-222 to that of Ra-226?

$$\begin{aligned} \text{Specific Activity} &= \frac{\text{activity}}{\text{mass}} \\ \text{Ratio} &= \frac{\lambda_{Rn}}{\text{MM}_{Rn}} \cdot \frac{\text{MM}_{Ra}}{\lambda_{Ra}} \\ &= 155314 \end{aligned}$$

4. (25 points total) Consider a serial decay scheme involving 4 isotopes, A $\leftarrow$ B $\leftarrow$ C $\leftarrow$ D, with D being a stable isotope. Assuming that at $t = 0$, there is only isotope A with abundance N_0, derive the equations for activity of isotopes A, B, and C as functions of time.
   5 points assigned for Q_A
   10 points assigned for Q_B
   10 points assigned for Q_C

$$\begin{aligned}
\frac{dN_A}{dt} &= -\lambda_A N_A \\
\frac{dN_B}{dt} &= \lambda_A N_A - \lambda_B N_B \\
\frac{dN_C}{dt} &= \lambda_B N_B - \lambda_C N_C \\
\frac{dN_D}{dt} &= \lambda_C N_C \\
N_A(t=0) &= N_0
\end{aligned}$$

$$\begin{aligned}
Q_A &= \lambda_A N_A \\
Q_B &= \lambda_B N_B \\
Q_C &= \lambda_C N_C \\
Q_D &= 0
\end{aligned}$$

$$\begin{aligned}
\frac{dN_A}{dt} &= -\lambda_A N_A \\
N_A' + \lambda_A N_A &= 0 \\
N_A e^{\lambda_A t} &= N_0 \\
N_A &= N_0 \cdot e^{-\lambda_A t}
\end{aligned}$$

$$\begin{aligned}
\frac{dN_B}{dt} &= \lambda_A N_A - \lambda_B N_B \\
N_B' + \lambda_B N_B &= \lambda_A N_0 \cdot e^{-\lambda_A t} \\
\frac{d}{dt}(N_B e^{\lambda_B t}) &= e^{\lambda_B t} \cdot \lambda_A N_0 \cdot e^{-\lambda_A t} \\
N_B e^{\lambda_B t} &= \int e^{\lambda_B t} \cdot \lambda_A N_0 \cdot e^{-\lambda_A t} dt \\
&= \lambda_A N_0 \int e^{(\lambda_B - \lambda_A)t} dt \\
&= \frac{\lambda_A N_0}{\lambda_B - \lambda_A} e^{(\lambda_B - \lambda_A)t} + C_1 \\
N_B(t=0) &= 0
\end{aligned}$$

$$
\begin{aligned}
0 &= \frac{\lambda_A N_0}{\lambda_B - \lambda_A} + C_1 \\
C_1 &= -\frac{\lambda_A N_0}{\lambda_B - \lambda_A} \\
N_B e^{\lambda_B t} &= \frac{\lambda_A N_0}{\lambda_B - \lambda_A} e^{(\lambda_B - \lambda_A)t} - \frac{\lambda_A N_0}{\lambda_B - \lambda_A} \\
N_B &= \frac{\lambda_A N_0}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t})
\end{aligned}
$$

$$
\begin{aligned}
\frac{dN_C}{dt} &= \lambda_B N_B - \lambda_C N_C \\
N'_C + \lambda_C N_C &= \lambda_B \frac{\lambda_A N_0}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) \\
\frac{d}{dt}(N_C e^{\lambda_C t}) &= \frac{\lambda_A \lambda_B N_0}{\lambda_B - \lambda_A} \left(e^{(\lambda_C - \lambda_A)t} - e^{(\lambda_C - \lambda_B)t}\right) \\
N_C e^{\lambda_C t} &= \int \frac{\lambda_A \lambda_B N_0}{\lambda_B - \lambda_A} \left(e^{(\lambda_C - \lambda_A)t} - e^{(\lambda_C - \lambda_B)t}\right) dt \\
&= \frac{\lambda_A \lambda_B N_0}{\lambda_B - \lambda_A} \left(\int (e^{(\lambda_C - \lambda_A)t} dt - \int e^{(\lambda_C - \lambda_B)t}) dt\right) \\
&= \frac{\lambda_A \lambda_B N_0}{\lambda_B - \lambda_A} \left(\frac{e^{(\lambda_C - \lambda_A)t}}{\lambda_C - \lambda_A} - \frac{e^{(\lambda_C - \lambda_B)t}}{\lambda_C - \lambda_B}\right) + C_2 \\
N_C(t = 0) &= 0 \\
0 &= \frac{\lambda_A \lambda_B N_0}{\lambda_B - \lambda_A} \left(\frac{1}{\lambda_C - \lambda_A} - \frac{1}{\lambda_C - \lambda_B}\right) + C_2 \\
C_2 &= -\frac{\lambda_A \lambda_B N_0}{\lambda_B - \lambda_A} \left(\frac{1}{\lambda_C - \lambda_A} - \frac{1}{\lambda_C - \lambda_B}\right) \\
N_C e^{\lambda_C t} &= \frac{\lambda_A \lambda_B N_0}{\lambda_B - \lambda_A} \left(\frac{e^{(\lambda_C - \lambda_A)t}}{\lambda_C - \lambda_A} - \frac{e^{(\lambda_C - \lambda_B)t}}{\lambda_C - \lambda_B}\right) - \frac{\lambda_A \lambda_B N_0}{\lambda_B - \lambda_A} \left(\frac{1}{\lambda_C - \lambda_A} - \frac{1}{\lambda_C - \lambda_B}\right) \\
N_C &= \frac{\lambda_A \lambda_B N_0}{\lambda_B - \lambda_A} \left[\left(\frac{e^{-\lambda_A t}}{\lambda_C - \lambda_A} - \frac{e^{-\lambda_B t}}{\lambda_C - \lambda_B}\right) - \left(\frac{1}{\lambda_C - \lambda_A} - \frac{1}{\lambda_C - \lambda_B}\right) e^{-\lambda_C t}\right]
\end{aligned}
$$