Interactions of Neutrons with Matter

Reading Material:

- Chapter 5 in <<Introduction to Health Physics>>, Third edition, by Cember.
- Chapter 9 in <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.
- Chapter 2 in <<Radiation Detection and Measurements>>, Third Edition, by G. F. Knoll.

Introduction

☞The neutrons was discovered by Chadwick in 1932.

☞Nuclear fission, induced by the capture of a slow neutron in ^{235}U was discovered by Hahn and Strassman in 1939.

☞The fact that several neutrons emitted when fission takes place suggested that a self-sustaining chain reaction might be possible.

☞Under Fermi's direction, the world's first man-made nuclear reactor went critical on Decembe 2, 1942.

Neutron occupies a central position in the modern world of atoms and radiation.

Neutron Sources – Nuclear Reactor

- Fission reactions of ^{235}U and ^{239}Pu emit neutrons.
- These neutrons possess energies ranging from a few keV to more than 10MeV.
- Neutrons from reactor are usually degraded in energy, having passing through parts of the reactor and coolant as well as structural materials.

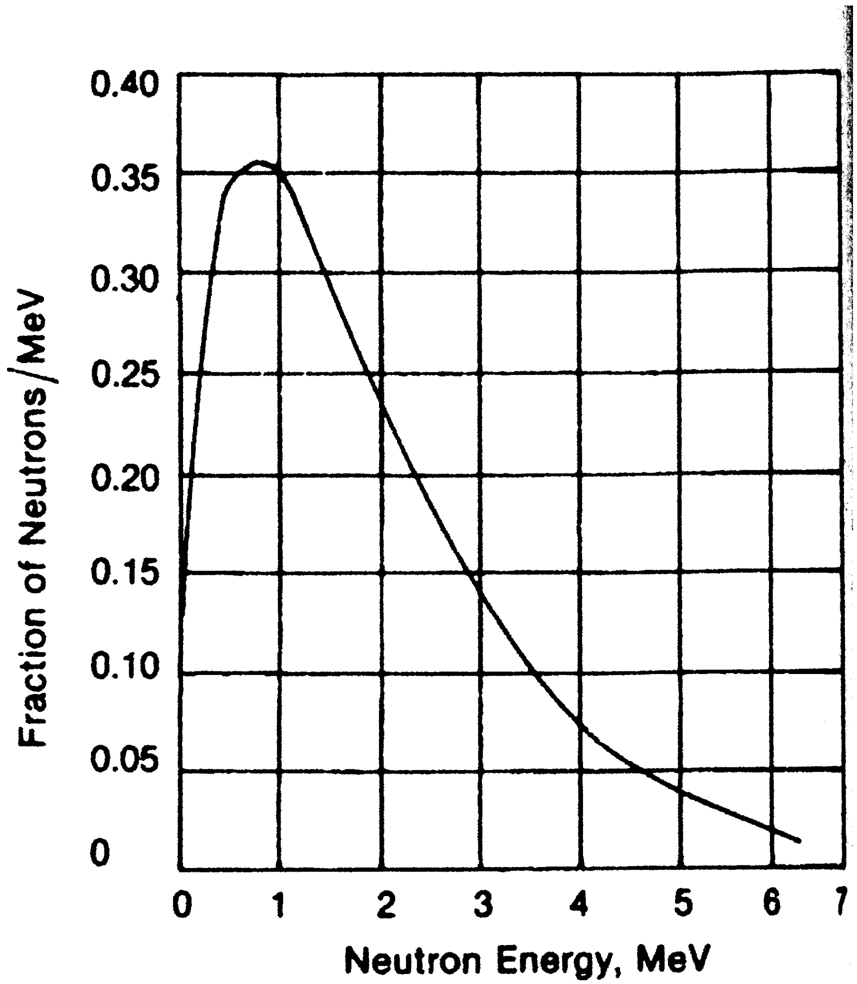


FIGURE 5.19. Energy distribution of fission neutrons. The most probable energy is 0.7 MeV and the average energy is 2 MeV.

Figure from Page 150, Introduction to Health Physics

Interactions of Neutrons with Matters

Attenuation of neutrons:

- ☞ Neutrons are uncharged and can travel appreciable distance in matter without interaction.
- ☞ Under conditions of "good geometry", a narrow beam of monoenergetic neutrons is attenuated exponentially.

Interactions of Neutrons with Matters

Neutrons can interact with an atomic nuclei through

- **Elastic scattering**: the total kinetic energy is conserved – the energy loss by the neutron is equal to the kinetic energy of the recoil nucleus.

- **Inelastic scattering**: the nucleus absorbs some energy internally and is left to an excited state.

- **(Thermal) neutron capture**: the neutron is captured or absorbed by a nucleus, leading to a reaction such as (n,p), (n,2n), (n,α) or (n,γ). The reaction changes the atomic number and/or atomic mass number of the struck nucleus.

Interactions of Neutrons with Matters

☞Hydrogen nucleus does not have excited state. Only elastic scattering and neutron capture is possible.

☞The neutron cross section for carbon has considerable structures resultant from the combined effects of elastic, inelastic and capture process.

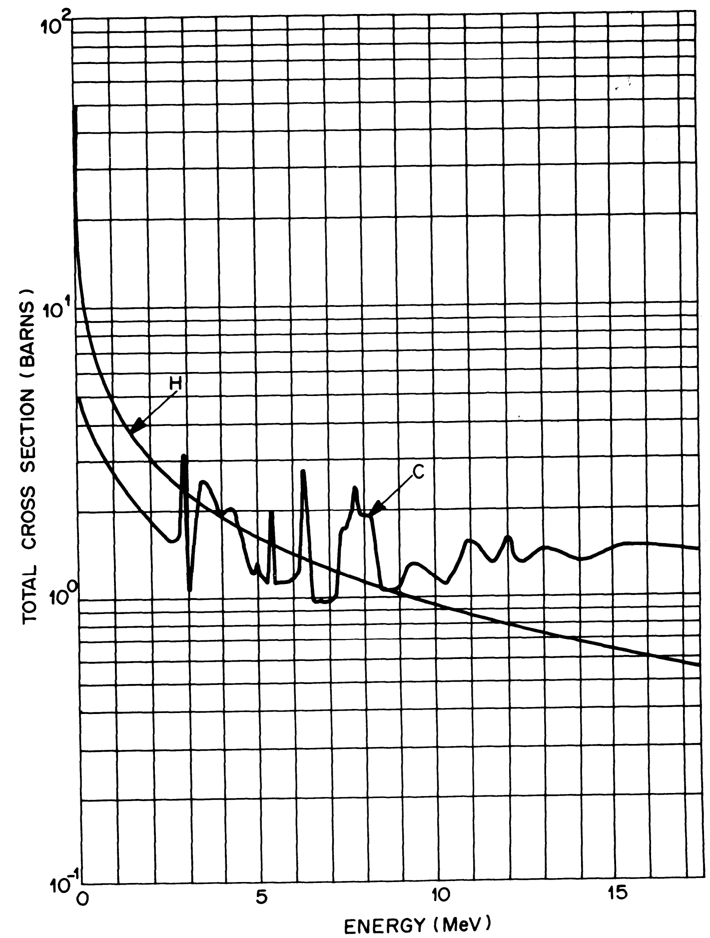


FIGURE 9.2. Total cross sections for neutrons with hydrogen and carbon as functions of energy.

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p213

Elastic Scattering of Neutrons

Elastic scattering is the most important process for slowing down fast neutrons. Due to the rapid increase in the probability of neutron capture, the neutrons, once slowed down, will eventually be captured by target nuclei.

Here we will discuss several aspects of neutron scattering in matter:

☞ Maximum energy transfer.

☞ Angular distribution of scattered neutrons.

☞ Energy distribution of scattered neutrons.

☞ Average logarithm energy decrement of a neutron in multiple scattering.

Elastic Scattering of Neutrons

Kinematics of neutron scattering:

- ☞ Energy transfer as a function of scattering angle.
- ☞ Angular distribution of scattered neutrons.
- ☞ Energy spectrum of scattered neutrons.
- ☞ Average logarithm energy decrement of a neutron in multiple scattering.

Elastic Scattering of Neutrons

The **elastic scattering** plays an important role in neutron energy measurements. For example, a proton-neutron telescope illustrated below can be used to accurately measure the spectrum of neutrons in a collimated beam.

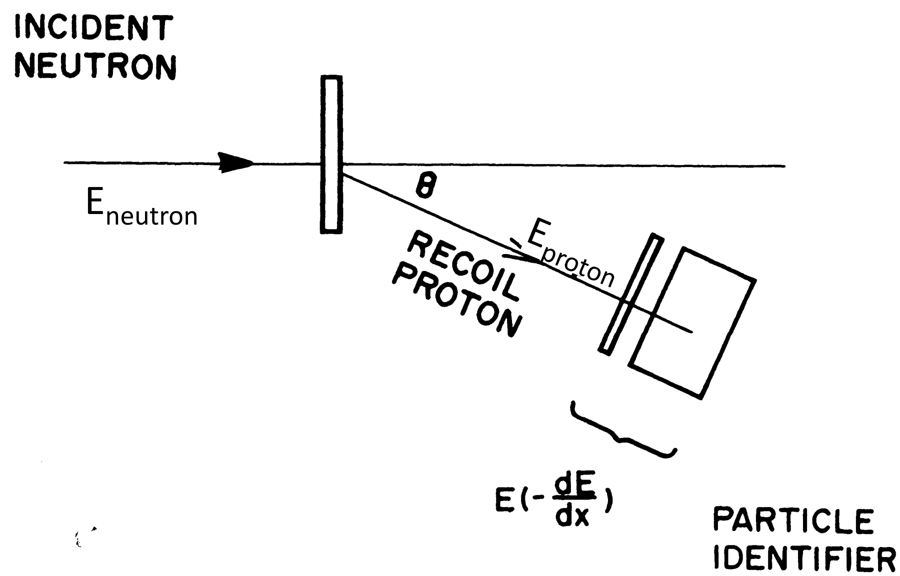


FIGURE 10.36. Arrangement of proton-recoil telescope for measuring spectrum neutron beam.

$$E_{proton} = E_{neutron} \cos^2 \theta$$

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p281

Elastic Scattering of Neutrons

The **maximum energy** that a neutron of mass M and kinetic energy E_n can transfer to a nucleus of mass m in a single elastic collision given by

$$E_{\max} = E_n \frac{4Mm}{(M+m)^2}$$

TABLE 9.4. Maximum Fraction of Energy Lost, Q_{max}/E_n from Eq. (9.3), by Neutron in Single Elastic Collision with Various Nuclei

Nucleus	Q_{max}/E_n
$^{1}_{1}H$	1.000
$^{2}_{1}H$	0.889
$^{4}_{2}He$	0.640
$^{9}_{4}Be$	0.360
$^{12}_{6}C$	0.284
$^{16}_{8}O$	0.221
$^{56}_{26}Fe$	0.069
$^{118}_{50}Sn$	0.033
$^{238}_{92}U$	0.017

Elastic Scattering of Neutrons

- The **elastic scattering** is the dominating mechanism whereby fast neutrons deliver dose to tissue.

- The recoil nuclei are essentially ionizing particles traveling in media and losing their energy through ionization and excitation.

- As we will see later, over 85% of the “first-collision” dose in soft tissue (composed of H, C, O and N) arises from n-p scattering for neutron energy below 10MeV.

Elastic Scattering of Neutrons

Kinematics of neutron scattering:

☞ Energy transfer as a function of scattering angle.

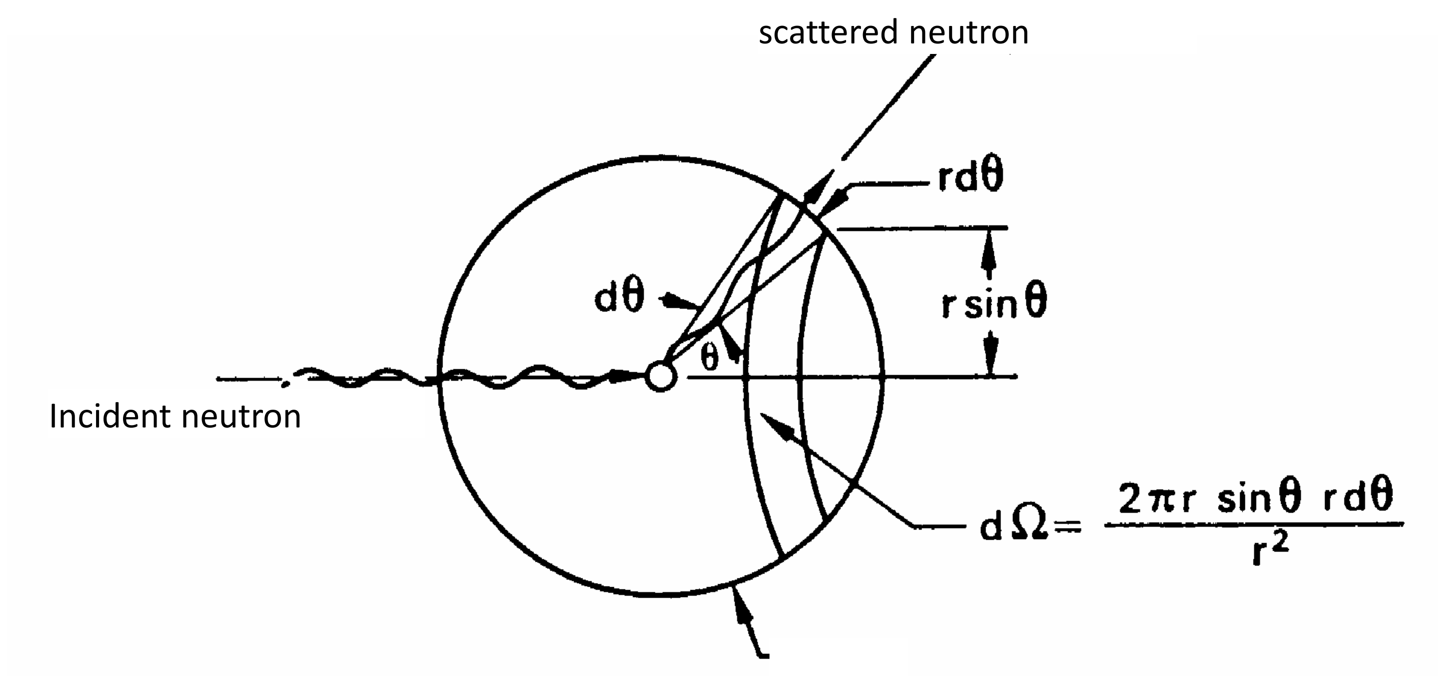
☞ Angular distribution of scattered neutrons.

☞ Energy spectrum of scattered neutrons.

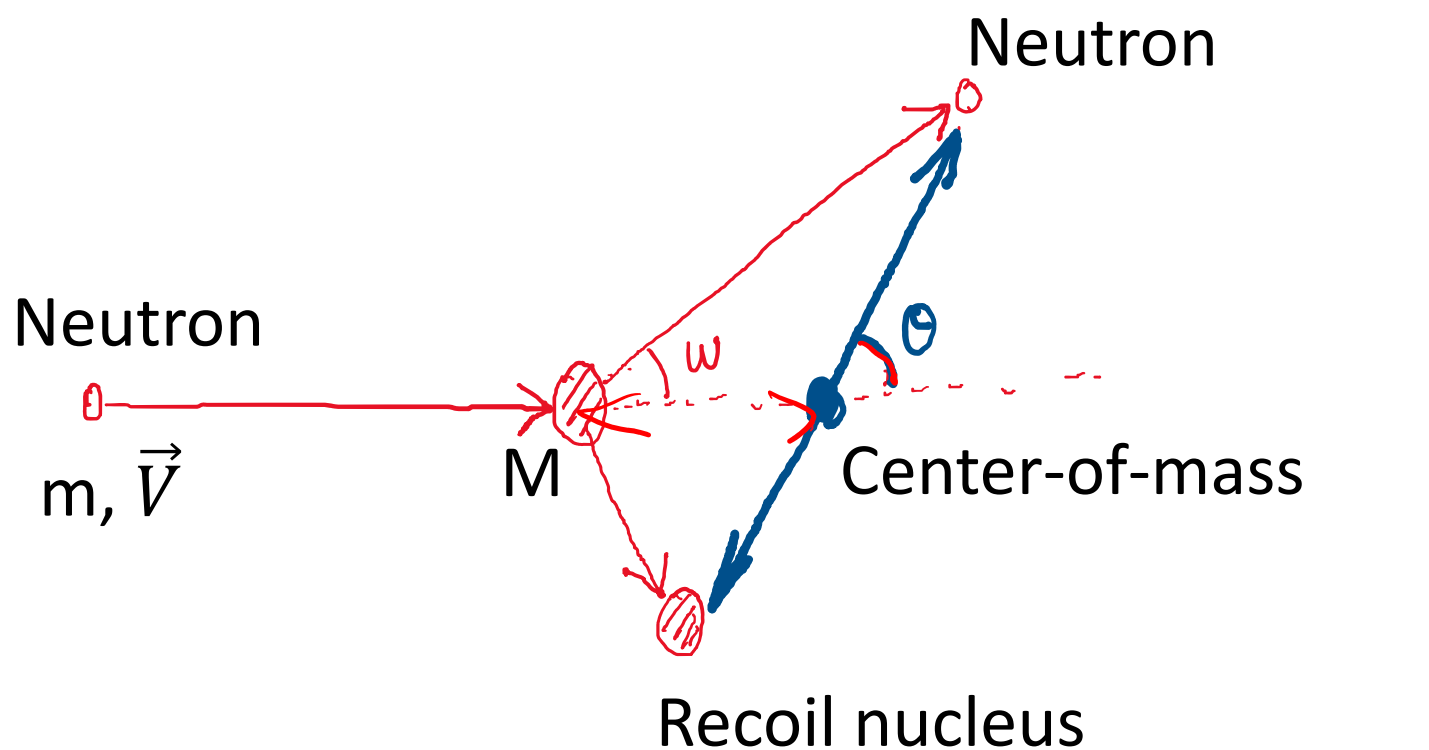
☞ Average logarithm energy decrement of a neutron in multiple scattering.

Angular Distributions of the Scattered Neutrons

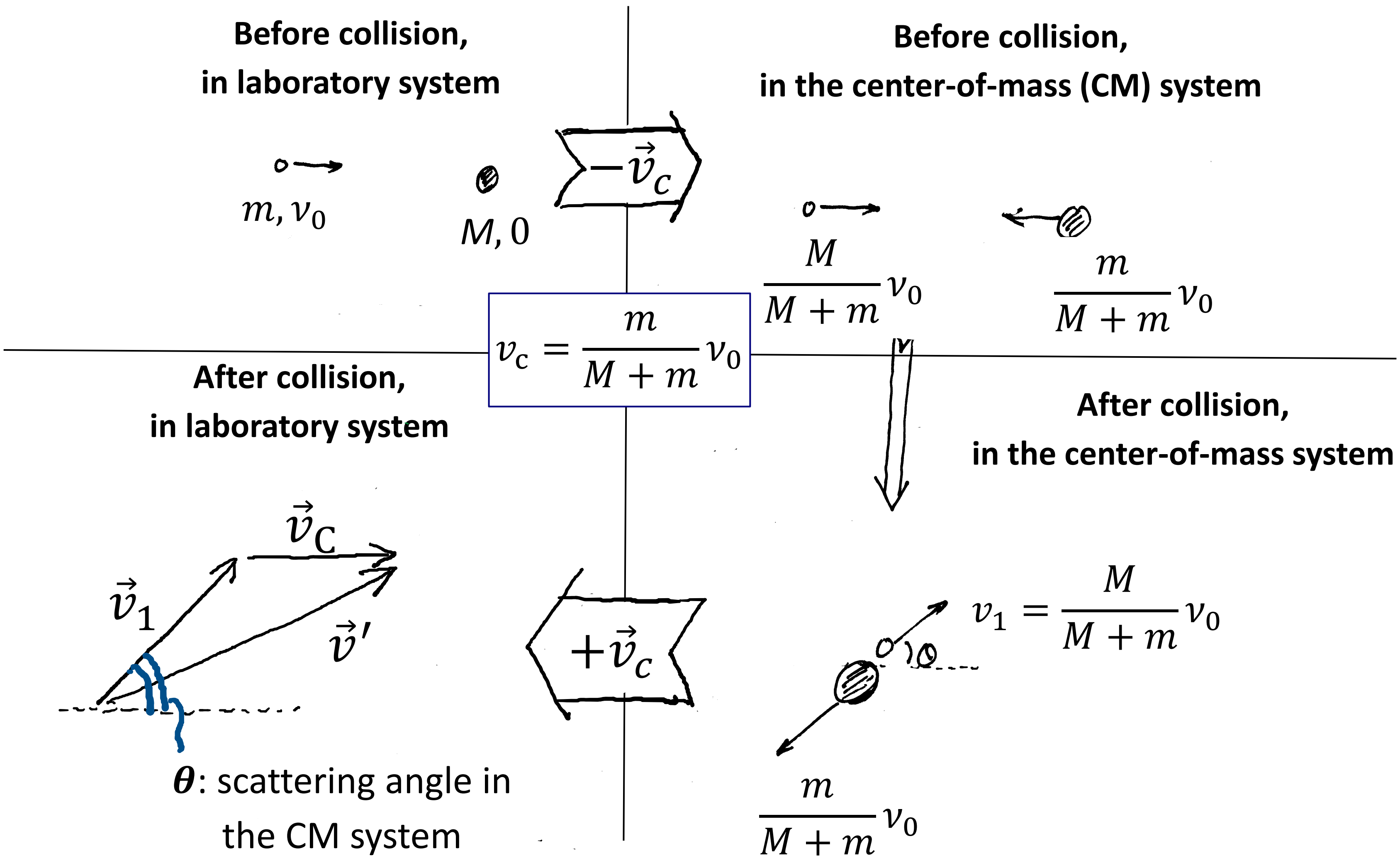
☞ For neutron energies up to 10MeV, it is experimentally observed that the scattering of neutrons is **isotropic** in the **center-of-mass coordinate system**. The neutron and the recoil nuclei are scattered with equal probability in any direction in this 3-D coordinate system.



Angular Distributions of the Scattered Neutrons



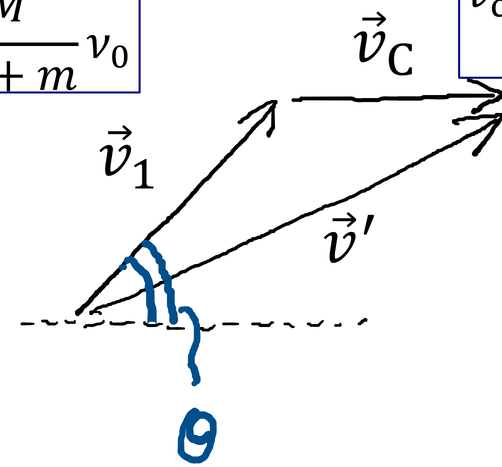
Angular Distributions of the Scattered Neutrons



Angular Distributions of the Scattered Neutrons

$$\boxed{v_1 = \frac{M}{M+m} v_0}$$

$$\boxed{v_c = \frac{m}{M+m} v_0}$$



The speed of the scattered neutron within the laboratory frame is

$$\vec{v}' = \vec{v}_1 + \vec{v}_c.$$

Therefore

$$v'^2 = v_1^2 + v_c^2 - 2v_1 v_c \cos(\pi - \theta)$$
$$= v_1^2 + v_C^2 + 2v_1 v_C \cos\theta.$$

The kinetic energy of the scattered neutron, E', is

$$E' = \frac{1}{2} m v'^2,$$

and

$$\frac{E'}{E_0} = \frac{1}{v_0^2}\left[\left(\frac{M}{M+m} v_0\right)^2 + \left(\frac{m}{M+m} v_0\right)^2 + 2\frac{M}{M+m}\frac{m}{M+m} v_0^2 \cos\theta\right].$$

Therefore,

$$\boxed{\frac{E'}{E_0} = \frac{M^2+m^2+2Mm\cos\theta}{(M+m)^2}}.$$

Energy Spectrum of the Scattered Neutrons

Since the scattering in the CM system is isotropic, the probability of the scattered neutron falling into an angular interval $[\theta, \theta + \mathrm{d}\theta]$ is

$$p(\theta) \cdot \mathrm{d}\theta = [2\pi\, r^2 \sin\theta\, \mathrm{d}\theta]/(4\pi r^2) = \frac{1}{2}\sin\theta \cdot \mathrm{d}\theta.$$

The probability of the outgoing neutron carrying a kinetic energy falling into a given window $[E', E' + \mathrm{d}E']$ is given by

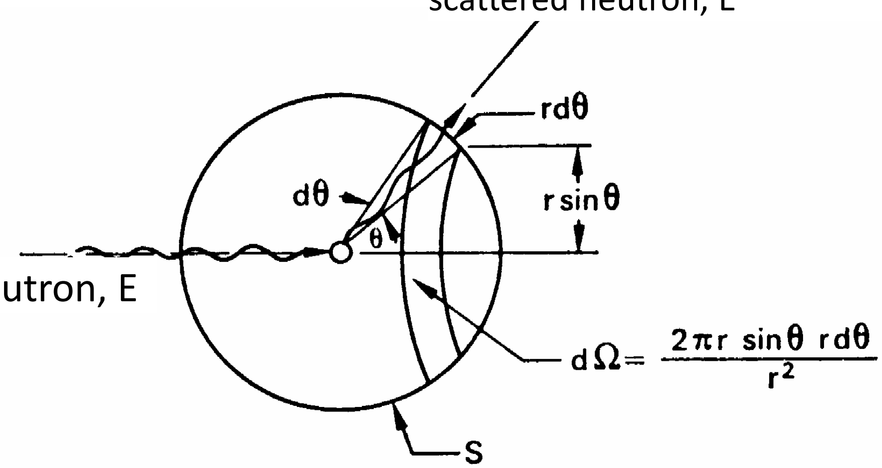
$$p(E')\,\mathrm{d}E' = -p(\theta) \cdot \mathrm{d}\theta$$

$$= -\frac{1}{2}\sin\theta \cdot \mathrm{d}\theta = -\frac{1}{2}\sin\theta \cdot \left(\frac{dE'}{d\theta}\right)^{-1} \cdot \mathrm{d}E'$$

$$= -\frac{1}{2}\sin\theta \left[E_0 \frac{2Mm}{(\mathrm{M}+m)^2}(-\sin\theta)\right]^{-1} \cdot \mathrm{d}E'$$

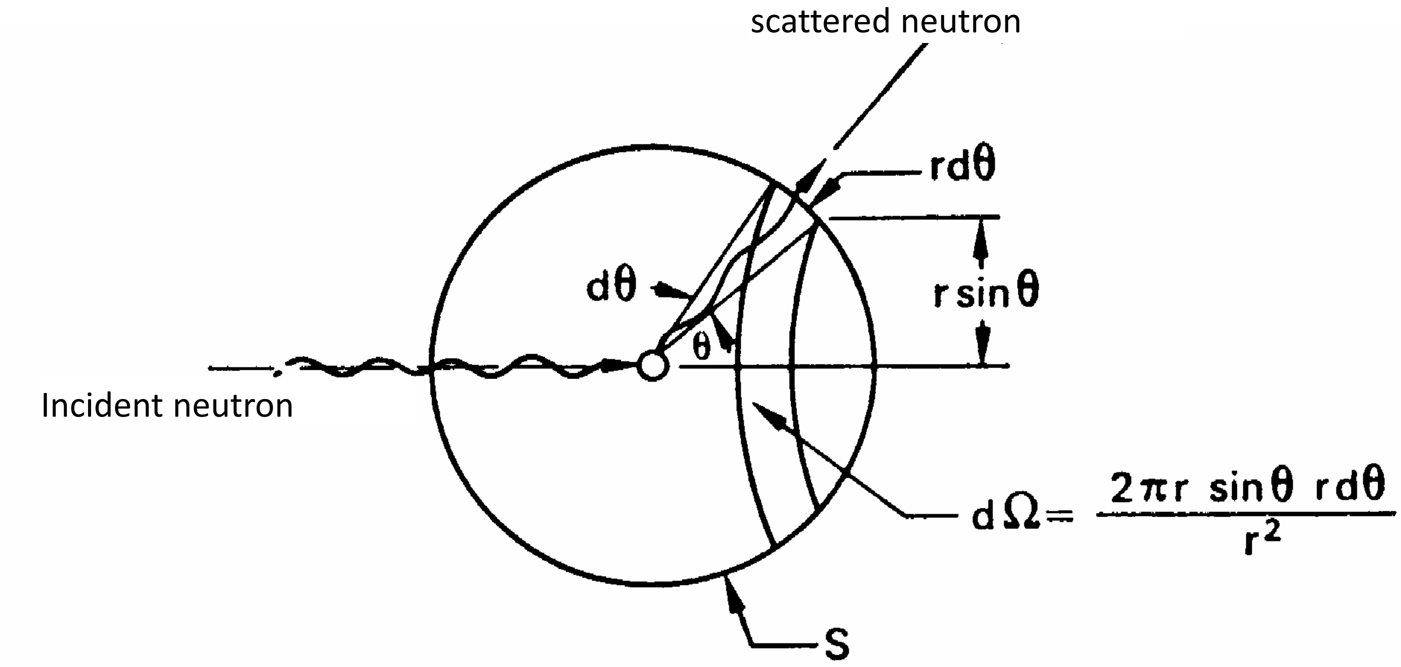
$$= \frac{1}{2}\sin\theta \cdot \frac{(M+m)^2}{2Mm\sin\theta} \cdot \frac{1}{E_0} \cdot \mathrm{d}E'$$

$$= \frac{(M+m)^2}{4Mm} \cdot \frac{1}{E_0} \cdot \mathrm{d}E'$$



$$\boxed{p(E')\,\mathrm{d}E' = \frac{(M+m)^2}{4Mm} \cdot \frac{1}{E_0} \cdot \mathrm{d}E'}$$

Angular Distributions of the Scattered Neutrons



Since the scattering of the neutron is isotropic in the CM system, the probability of the scattered neutron going into an angular interval dθ can be written as

$$p(\theta)\cdot d\theta = [2\pi \sin\theta \cdot d\theta]/4\pi = \frac{1}{2}\sin\theta\cdot d\theta$$

Energy Spectrum of the Scattered Neutrons

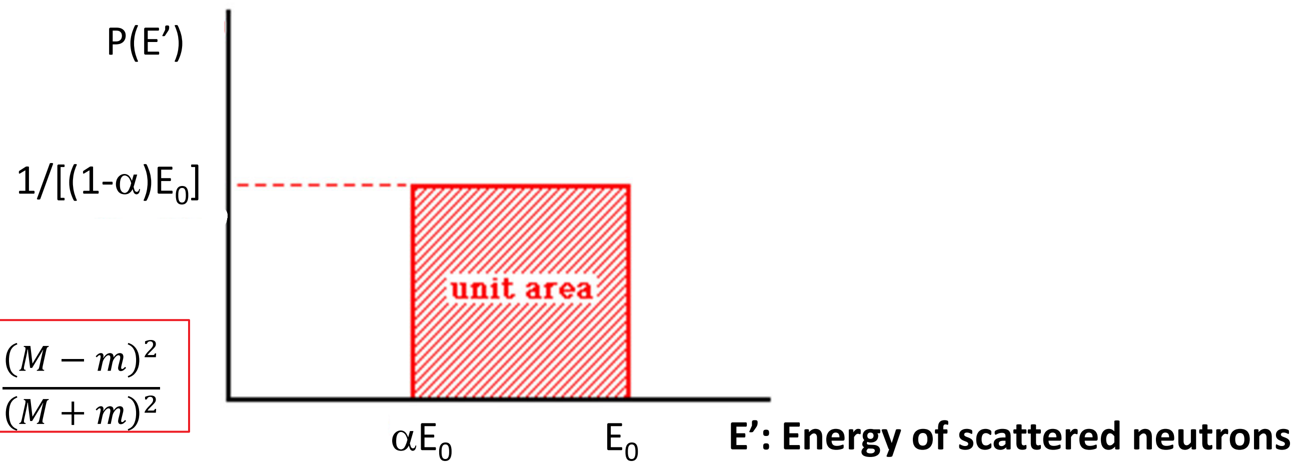
The probability of the outgoing neutron carrying kinetic energy falling into a given window $[E', E' + \mathrm{d}E']$ is given by

$$p(E')\,\mathrm{d}E' = \frac{(M+m)^2}{4Mm} \cdot \frac{1}{E_0} \cdot \mathrm{d}E' = \frac{1}{1-\alpha} \cdot \frac{1}{E_0} \cdot \mathrm{d}E' \,,$$

where

$$\alpha = \frac{(M-m)^2}{(M+m)^2} \,.$$

Energy Spectrum of Scattered Neutrons



$$\alpha = \frac{(M-m)^2}{(M+m)^2}$$

The fraction of energy carried by the scattered neutron is

$$\frac{E'}{E_0} = \frac{M^2 + m^2 + 2Mm\cos\theta}{(M+m)^2}$$

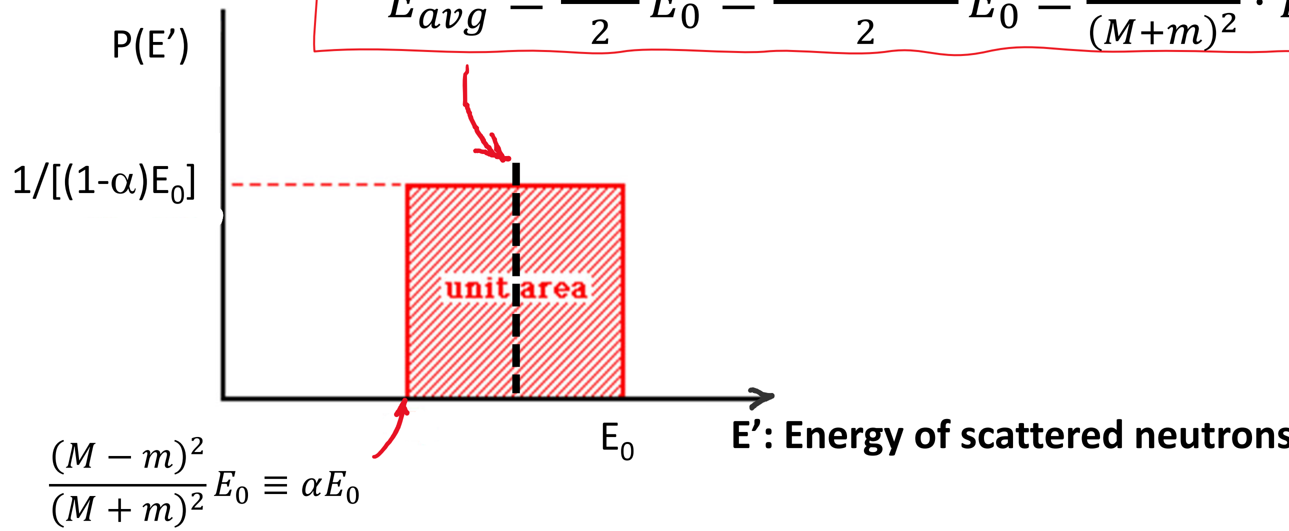
The distribution of the energy of the scattered neutrons is given by

$$p(E') = \frac{1}{1-\alpha}\frac{1}{E_0}, E' \subset [\alpha E_0, E_0].$$

Energy Spectrum of Scattered Neutrons

Average energy carried by the scattered neutron:

$$E'_{avg} = \frac{1+\alpha}{2} E_0 = \frac{1+\frac{(M-m)^2}{(M+m)^2}}{2} E_0 = \frac{M^2+m^2}{(M+m)^2} \cdot E_0$$



αE$_0$

Average energy transferred to the recoil nucleus:

$$E_{avg\_energy\_loss} = E_0 - E'_{avg} = \frac{2Mm}{(M+m)^2} \cdot E_0$$

Elastic Scattering of Neutrons

Kinematics of neutron scattering:

☞ Energy transfer as a function of scattering angle.

☞ Angular distribution of scattered neutrons.

☞ Energy spectrum of scattered neutrons.

☞ Average logarithm energy decrement of a neutron in multiple scattering.

Average Logarithmic Energy Decrement

For fast neutrons undergo successive collisions in the absorber. The average decrease per collision in the logarithm of the neutron energy (the **average logarithmic energy decrement**) remains constant:

$$\xi = \overline{\Delta \ln E} = \overline{\ln E_0 - \ln E} = \overline{\ln \frac{E_0}{E}} = -\overline{\ln \frac{E}{E_0}}$$

E — for the scattered neutron

and

$$\xi = 1 + \frac{\alpha \ln \alpha}{1 - \alpha}$$

where

$$\alpha = [(M - m)/(M + m)]^2$$

The **average logarithmic energy decrement**) is independent of the neutron energy and is a function only of the mass of the scattering nuclei.

Average Logarithmic Energy Decrement of Scattered Neutrons

The average logarithmic energy decrement per collision is defined as

$$\xi = \overline{\Delta \ln E} = \overline{\ln E_0 - \ln E} = -\overline{\ln \frac{E}{E_0}}$$

$$= \int_{\alpha \cdot E_0}^{E_0} -\ln\left(\frac{E}{E_0}\right) \cdot p(E) \cdot dE$$

$$= -p(E) \cdot \left(E \ln \frac{E}{E_0} - E\right)\Big|_{\alpha E_0}^{E_0}$$

$$= -\frac{(M+m)^2}{4Mm} \frac{1}{E_0}\left[\left(E_0 \ln \frac{E_0}{E_0} - E_0\right) - \left(\alpha E_0 \ln \alpha - \alpha E_0\right)\right]$$

$$= \frac{1}{1-\alpha}\left(1 + \alpha \ln \alpha - \alpha\right)$$

$$= \frac{1}{1-\alpha}\left(1 - \alpha + \alpha \ln \alpha\right) = 1 + \frac{\alpha \ln \alpha}{1-\alpha}$$

where

$$\alpha = \left(\frac{M-m}{M+m}\right)^2.$$

Average Logarithmic Energy Decrement

Since

$$\overline{\ln \frac{E}{E_0}} = -\xi,$$

$$\frac{E}{E_0} = e^{-\xi},$$

The median fraction of the incident neutron's energy that is transferred to the nucleus during a collision is

$$f = 1 - \frac{\overline{E}}{E_0} = 1 - e^{-\xi}$$

Average Logarithmic Energy Decrement

If the slowing-down medium contains n kinds of nuclides, each of microscopic scattering cross section ξ σ_{si} , and the **average logarithmic energy decrement** is

$$\xi = \frac{\sum_{i=1}^{n} \sigma_{si} N_i \xi_i}{\sum_{i=1}^{n} \sigma_{si} N_i}$$

Fast- and Thermal-Diffusion Lengths

Fast-diffusion length: the average straight-line distance covered by fast neutrons traveling in a given medium.

Thermal-diffusion length: the average distance covered by thermalized neutrons before it is absorbed. It is measured by the thickness of a slowing down medium that attenuates the beam of thermal neutrons by a factor of e. Thus, the attenuation of a beam of thermal neutrons by a substance of thickness t (cm), whose thermal diffusion length is L (cm) is given by

$$n = n_0 e^{-t/L}$$

TABLE 5.6. Fast and Thermal Diffusion Lenghts of Selected Materials

Substance	Fast Diffusion Length, cm	Thermal Diffusion Length, cm
H_2O	5.75	2.88
D_2O	11	171
Be	9.9	24
C (graphite)	17.3	50

Interaction of Slow Neutrons (E<0.5eV)

- ☞ Significant interactions include ***elastic scattering*** and ***neutron induced nuclear reactions***.
- ☞ Due to the low neutron energy, very little energy can be transferred by elastic scattering.
- ☞ The more significant effect of elastic scattering is to ***slow down*** slow neutrons and turn them into ***thermal neutrons*** (average E<0.025eV at room temperature).
- ☞ Neutron absorption followed by the immediate emission of a gamma ray photon and other particles.

Interaction of Slow Neutrons (E<0.5eV)

- The most important interactions between slow neutrons and absorbing materials are ***neutron-induced reactions***, such as (n,γ), (n,α), (n,p) and (n, fission) etc. These interactions lead to more prominent signatures for neutron detection.

$$\text{neutron} + \text{target nucleus} \Rightarrow \begin{cases} \text{recoil nucleus} \\ \text{protons} \\ \text{alpha particles} \\ \text{fission fragments} \end{cases}$$

Neutron Induced Reactions

$$ {}_0^1n + {}_1^1H \rightarrow {}_1^2H + {}_0^0\gamma $$

- Neutron absorption followed by the immediate emission of a gamma ray photon.
- Since the thermal neutron has negligible energy by comparison, the gamma photon has the energy Q=2.22MeV released by the reaction, which represents the binding energy of the deuteron.
- The capture cross section per atom is 0.33barn.
- When tissue is exposed to thermal neutrons, this reaction provides a source of gamma rays that delivers dose to the tissue.

Neutron Induced Reactions

$$ {}_{0}^{1}n + {}_{2}^{3}He \rightarrow {}_{1}^{3}H + {}_{1}^{1}p $$

- Cross section for thermal neutron is 5330 barns.
- Q=765keV.
- Commonly used in proportional counters for fast neutron detection.

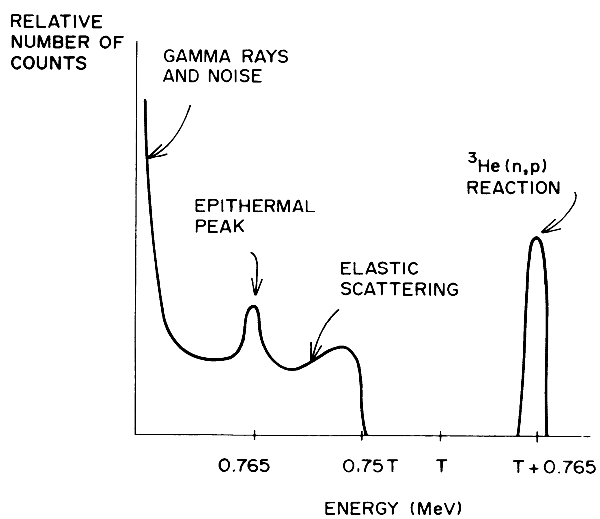


FIGURE 10.35. Pulse-height spectrum from ^{3}He proportional counter for monoenergetic neutrons of energy T.

Neutron Induced Reactions

$$ {}_{0}^{1}n + {}_{3}^{6}Li \rightarrow {}_{2}^{4}He + {}_{1}^{3}H $$

- Cross section for thermal neutron is 940 barns.
- Q=4.78MeV.
- Widely used for thermal neutron detection.

  Neutron sensitive LiI scintillator can be made or Li can be added to other scintillator to register neutrons.

  ^{6}Li is 7.42% abundant and Li enriched in the isotope ^{6}Li is available.

Neutron Induced Reactions

$$ {}_{0}^{1}n + {}_{5}^{10}B \rightarrow {}_{3}^{7}Li + {}_{2}^{4}He $$

- ☞ Cross section for thermal neutron is 3840 barns.
- ☞ Q=2.31MeV when the daughter nucleus is in an excited state (93%) and 2.79MeV when the Li nucleus is in ground state (7%).
- ☞ Widely used for thermal neutron detection.

  BF_3 is a gas that can be used directly in a neutron counter.

  Boron is also used as a liner inside the tubes of proportional counters for neutron detection.

Neutron Induced Reactions

$$ {}_0^1n + {}_7^{14}N \rightarrow {}_6^{14}C + {}_1^1p $$

- Cross section for thermal neutron is 1.70 barns.
- Q=0.626MeV.
- Since the range of the proton and the ^{14}C nucleus are relatively small, their energy is deposited locally at the site where the neutron was captured.
- Capture by hydrogen and nitrogen are the only two processes through which neutron deliver a significant does to soft tissue.

Neutron Induced Reactions

$$ {}^{1}_{0}n + {}^{23}_{11}Na \rightarrow {}^{24}_{11}Na + {}^{0}_{0}\gamma $$

- Cross section for thermal neutron is 0.534 barns.
- Q=0.626MeV.
- ^{24}Na undergo radioactive decay with the emission of two gamma rays, having energies of 2.75MeV and 1.37MeV per disintegration with a half-life of 15 hours.
- Since ^{23}Na is a normal constituent of blood, activation of blood sodium can be sued as a dosimetry tool when persons are exposed to relatively high does of neutrons, for example, in a criticality accident.

Energetics of Threshold Reactions

- Consider the following reaction

$$ {}_{0}^{1}n + {}_{16}^{32}S \rightarrow {}_{15}^{32}P + {}_{1}^{1}p $$

- The neutrons must have an energy of above a certain threshold to enable this reaction.
- These reactions are called **endothermic reactions**, in which energy is converted into mass and therefore Q<0.

Energetics of Threshold Reactions

Example :

Calculate the threshold energy for the reaction $^{32}S(n,p)^{32}P$

Solution:

The reaction

$$^{1}_{0}n + ^{32}_{16}S \rightarrow ^{1}_{1}H + ^{32}_{15}P$$

The atomic mass difference is given by

$$\Delta = A_{^1H} + A_{^{32}S} - A_n - A_{^{32}P}$$

$$= -0.9276 \text{ MeV}$$

The threshold energy is

$$E_{th} = -Q\left(1 + \frac{M_1}{M_3 + M_4 - M_1}\right) = 0.9276\left(1 + \frac{1}{1+32-1}\right)$$

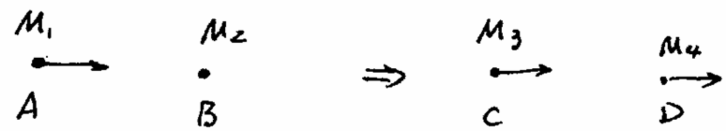
$$= 0.957 \text{ MeV}.$$

Energetics of Threshold Reactions

Nuclide	Natural Abundance (%)	Mass Difference $\Delta = M - A$ (MeV) (at. mass – at. mass No.)	Type of Decay	Half-Life	Major Radiations, Energies (MeV), and Frequency per Disintegration (%)
$^{22}_{11}$Na	—	−5.182	β^+ 89.8% EC 10.2%	2.602 y	β^+ : 0.545 max (avg 0.215) γ : 0.511 (180%, $\gamma^\pm$), 1.275 (100%), Ne X rays
$^{23}_{11}$Na	100.	−9.528	—	—	—
$^{24}_{11}$Na	—	−8.418	β^-	15.00 h	β^- : 1.390 max (avg 0.554) γ : 1.369 (100%), 2.754 (100%)
$^{24}_{12}$Mg	78.60	−13.933	—	—	—
$^{26}_{12}$Mg	11.3	−16.214	—	—	—
$^{26}_{13}$Al	—	−12.211	β^+ 81.8% EC 18.2%	7.16×10^5 y	β^+ : 1.174 max (avg 0.544) γ : 0.511 (164%, $\gamma^\pm$), 1.130 (2.5%), 1.809 (100%), Mg X rays
$^{26m}_{13}$Al	—	−11.982	β^+	6.4 s	β^+ : 3.21 max γ : 0.511 (200%, $\gamma^\pm$)
$^{32}_{15}$P	—	−24.303	β^-	14.29 d	β^- : 1.710 max (avg 0.695) No γ
$^{32}_{16}$S	95.0	−26.013	—	—	—
$^{35}_{16}$S	—	−28.847	β^-	87.44 d	β^- : 0.167 max (avg 0.0488) No γ
$^{37}_{16}$S	—	−27.0	β^-	5.06 min	β^- : 1.6 max (90%) 4.7 max (10%) γ : 3.09 (90%)
$^{38}_{16}$S	—	−26.8	β^-	2.87 h	β^- : 1.1 max (95%), 3.0 max (5%) γ : 1.88 (95%) Daughter radiations from ^{38}Cl

Energetics of Threshold Reactions

Consider a head-on collision



- A particle of mass M_1 strikes another particle of mass M_2 initially at rest.

  The identities of the particles are changed by the reaction. So there will generally be different masses M_3 and M_4 after the encounter.

- The change in rest mass/energy is

$$Q = M_1 + M_2 - (M_3 + M_4), \qquad (1)$$

which is negative for endothermic reactions. (assuming (a). both A and B are in ground states, (b) C and D are also in their ground states. Otherwise we would need to replace Q by $Q - E_{excit}$)

Energetics of Threshold Reactions

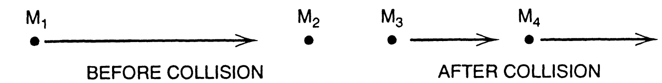


FIGURE 9.7. Schematic representation of a head-on collision producing a nuclear reaction in which the identity of the particles can change.

- The conservation of energy requires

$$E_1 = E_3 + E_4 - Q \qquad (2)$$

where E_3 and E_4 are the kinetic energy of the moving particles.

Conservation of momentum requires

$$p_1 = p_3 + p_4 \qquad (3)$$

We consider the non-relativistic case, where $E = p^2/2m$.

So one can eliminate E_4 by using $E_4 = p_4^2/2M_4$ and write

$$E_4 = \frac{p_4^2}{2M_4} = \frac{1}{2M_4}(p_1 - p_3)^2$$

Substitute

$$p_1 = (2M_1E_1)^{1/2} \quad \text{and} \quad p_3 = (2M_3E_3)^{1/2}$$

We have

$$E_4 = \frac{1}{M_4}\left[M_1E_1 - 2(M_1M_3)^{1/2}(E_1E_3)^{1/2} + M_3E_3\right] \qquad (4)$$

Energetics of Threshold Reactions

Substitute <4> into <2> and rearrange terms. we have

$$E_1 = E_3 + \frac{M_1}{M_4}E_1 - \frac{2(M_1 M_3)^{1/2}(E_1 E_3)^{1/2}}{M_4} + \frac{M_3 E_3}{M_4} - Q$$

So that

$$E_3 - \frac{2(M_1 M_3 E_1)^{1/2}}{M_3+M_4}\sqrt{E_3} - \frac{(M_4-M_1)E_1 + M_4 Q}{M_3+M_4} = 0 \qquad (5)$$

(5) is a quadratic equation of $\sqrt{E_3}$, having the form of

$$E_3 - 2A\sqrt{E_3} - B = 0$$

The two roots of this equation are

$$E_3 = \left[\frac{2A \pm \sqrt{4A^2+4B}}{2}\right]^2 = B + 2A^2\left(1 \pm \frac{1}{A}\sqrt{A^2+B}\right)$$

For E_3 to be real, $A^2 + B \geq 0$, therefore

$$\left[\frac{(M_1 M_3 E_1)^{1/2}}{M_3+M_4}\right]^2 + \frac{(M_4-M_1)E_1 + M_4 Q}{M_3+M_4} \geq 0$$

Or

$$E_1 \geq -Q\left(1 + \frac{M_1}{M_3+M_4-M_1}\right)$$

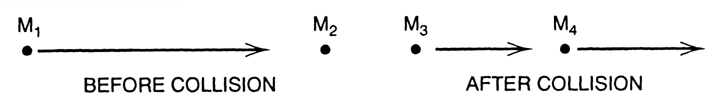


FIGURE 9.7. Schematic representation of a head-on collision producing a nuclear reaction in which the identity of the particles can change.

Energetics of Threshold Reactions

Energy release : $Q = M_1 + M_2 - (M_3 + M_4)$ (1)

Conservation of energy: $E_1 = E_3 + E_4 + Q \Rightarrow E_4 = E_1 - E_3 (2)$

Conservation of momentum: $p_1 = p_3 + p_4 \Rightarrow (2M_1E_1)^{1/2} = (2M_3E_3)^{1/2} + (2M_4E_4)^{1/2}$ (3)

Substitute (2) into (3), we have

$$(2M_1E_1)^{1/2} = (2M_3E_3)^{1/2} + (2M_4(E_1 - E_3))^{1/2}. \quad (4)$$

Rearranging the terms in (4),

$$E_3 - \frac{2(M_1M_3E_1)^{1/2}}{M_3+M_4}\sqrt{E_3} - \frac{(M_4-M_1)E_1+M_4Q}{M_3+M_4} = 0\,. \quad (5)$$

Solving (5) and considering E_3 should take a real value, we need to have

$$\left[-\frac{2(M_1M_3E_1)^{1/2}}{M_3+M_4}\right]^2 - 4\left[-\frac{(M_4-M_1)E_1+M_4Q}{M_3+M_4}\right] \geq 0,$$

or

$$\frac{M_1M_3E_1}{(M_3+M_4)^2} + \frac{(M_4-M_1)E_1+M_4Q}{M_3+M_4} \geq 0, \quad (6)$$

which finally leads to

$$E_i \geq -Q\left(1 + \frac{M_1}{M_3+M_4-M_1}\right). \quad (7)$$

Neutron Activation

- For **endothermic reactions**, the minimum energy carried by the neutron (the **threshold energy**) can be derived based on the conservation of energy and momentum:

$$E_1 = E_3 + E_4 - Q$$

$$p_1 = p_3 + p_4$$

- The **threshold energy** is slightly greater that the Q value (the mass difference before and after the reaction).

$$E_{th} = -Q\left(1 + \frac{M_1}{M_3 + M_4 - M_1}\right)$$

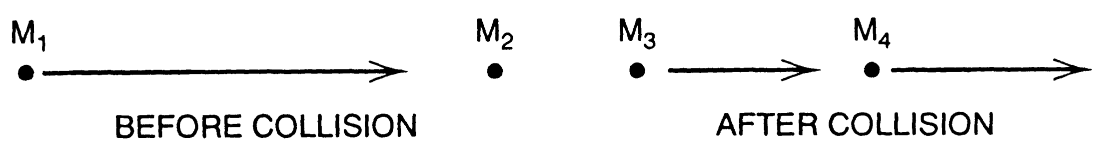


FIGURE 9.7. Schematic representation of a head-on collision producing a nuclear reaction in which the identity of the particles can change.

Average Thermal Neutron Capture Cross Section

- The thermal neutron capture cross section for neutron reactions with a threshold usually increase steadily from zero at E_{th} to a maximum and then decline at higher energies.
- The neutron energy at which the cross section has approximately its average value is called the **effective threshold energy**, which is greater than E_{th}.

Energy Dependence of Thermal Neutron Absorption Cross section

- Capture cross sections for low energy neutrons generally decreases as the reciprocal of the velocity as the neutron energy increases (the **1/v law**).
- So if the capture cross section σ_0 is known for a given velocity v_0, then the cross section at velocity v can be estimated from the following relation,

$$\frac{\sigma}{\sigma_0} = \frac{v_0}{v} = \sqrt{\frac{E_0}{E}}$$

- This equation can be used for neutrons of energies up to 100eV or 1keV, depending on the absorbing nucleus.

Energy Dependence of Thermal Neutron Absorption Cross section

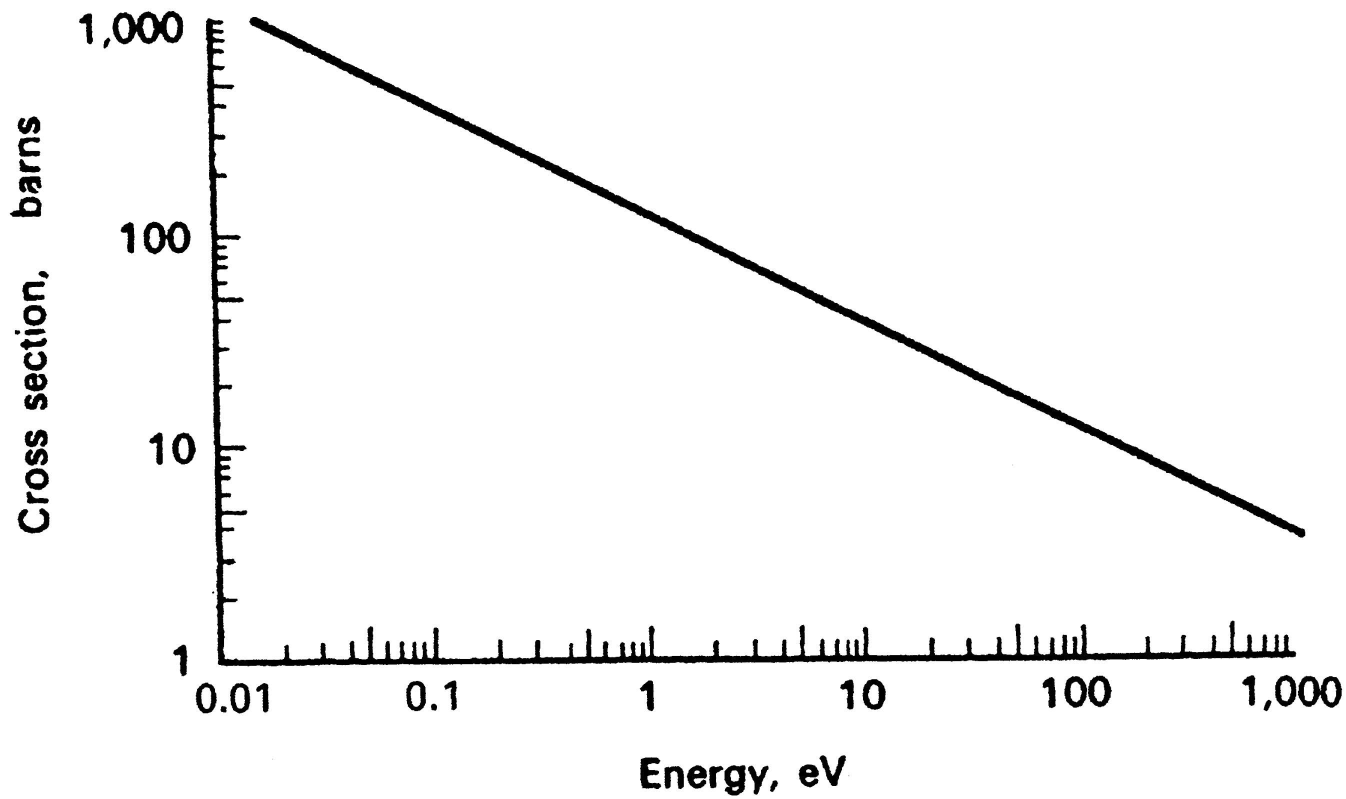


FIGURE 5.23. Neutron absorption cross section for boron, showing the validity of the $1/v$ law for neutrons from 0.02 to 1000 eV in energy. The equation of the curve is $\sigma = 116/\sqrt{E(eV)}$ barns.

Neutron Activation

☞ Neutron activation is the production of a radioactive isotope by the absorption of a neutron, such as in the (n,p) reaction.

☞ Neutron activation is important to health physicist for several reasons.

(a) Materials irradiated by neutrons may become radioactive. A radiation hazard may therefore persist after the irradiation by neutron is terminated.

(b) Neutron activation provides a convenient way to measure neutron flux.

(c) By spectroscopic examination of the induced radiation, quantitative analysis of the unknown samples is also possible.

Neutron Induced Reactions

$$ {}_0^1n + {}_1^1H \rightarrow {}_1^2H + {}_0^0\gamma $$

- Neutron absorption followed by the immediate emission of a gamma ray photon.
- Since the thermal neutron has negligible energy by comparison, the gamma photon has the energy Q=2.22MeV released by the reaction, which represents the binding energy of the deuteron.
- The capture cross section per atom is 0.33barn.
- When tissue is exposed to thermal neutrons, this reaction provides a source of gamma rays that delivers dose to the tissue.

Neutron Induced Reactions

$$^{1}_{0}n+^{14}_{7}N\rightarrow^{14}_{6}C+^{1}_{1}p$$

- Cross section for thermal neutron is 1.70 barns.
- Q=0.626MeV.
- Since the range of the proton and the ^{14}C nucleus are relatively small, their energy is deposited locally at the site where the neutron was captured.
- Capture by hydrogen and nitrogen are the only two processes through which neutron deliver a significant does to soft tissue.

Neutron Induced Reactions

$$ {}_{0}^{1}n + {}_{11}^{23}Na \rightarrow {}_{11}^{24}Na + {}_{0}^{0}\gamma $$

- Cross section for thermal neutron is 0.534 barns.
- Q=0.626MeV.
- ^{24}Na undergo radioactive decay with the emission of two gamma rays, having energies of 2.75MeV and 1.37MeV per disintegration.
- Since ^{23}Na is a normal constituent of blood, activation of blood sodium can be used as a dosimetric tool when persons are exposed to relatively high does of neutrons, for example, in a criticality accident.

Neutron Activation

☞ Considering the case that an object is irradiated by a constant flux, ϕ, of neutrons, which activates a given types of atoms. Then the net rate of increase of radioactive (daughter) atoms is given by

$$\frac{dN}{dt} = \phi\sigma n - \lambda N,$$

$$ {}_{0}^{1}n + {}_{11}^{23}Na \rightarrow {}_{11}^{24}Na + {}_{0}^{0}\gamma $$

where ϕ = flux, neutrons per cm^2 per s,
σ = activation cross section, cm^2,
λ = transformation constant of the induced activity,
N = number of radioactive atoms,
n = number of target atoms.

☞ The radioactivity induced by neutron activation (the number of disintegration of the activated daughter atoms per second) is given by

$$\lambda N = \phi\sigma n(1 - e^{-\lambda t})$$

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N = number of radioactive atoms,
n = number of target atoms.

☞ The **saturation activity** is given by $\phi \sigma n$. For an infinitely long irradiation time, it represents the maximum obtainable activity with any given neutron flux.

☞ The analysis leading to these results is identical to that used for analyzing the secular equilibrium for radioactive decay chains, in which the daughter has a much shorter decay time than that of the parent.

Neutron Activation

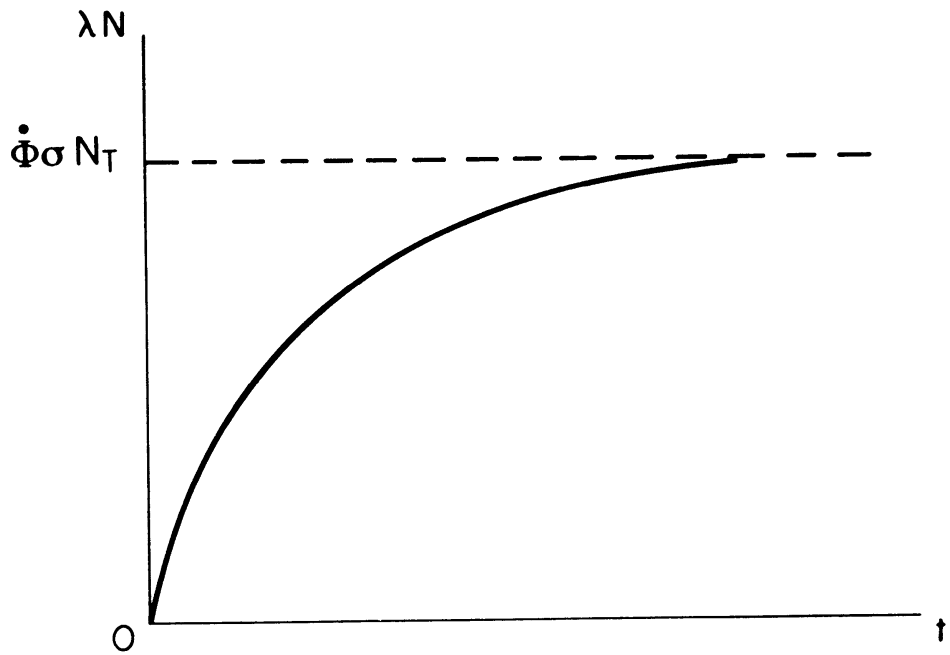

FIGURE 9.8. Buildup of induced activity λN, as given by Eq. (9.36), during neutron irradiation at constant fluence rate.

Neutron Activation

Example :

A sample contains an unknown quantity of chromium. It is irradiated for 1 week in a thermal neutron flux of 10^{11} n/cm²·s. The resulting ^{51}Cr gamma rays give a counting rate of 600 counts/min in a scintillation counter, whose efficiency is 10%.

How many grams of chromium were there in the original sample ?

Neutron Activation

Solution:

$${}^{1}_{0}n + {}^{50}_{24}Cr \longrightarrow {}^{51}_{24}Cr + \gamma$$

- The thermal neutron activation cross section of ${}^{51}Cr$ is 13.5 barns
- ${}^{50}Cr$ forms 4.31% by number of the naturally occuring Chromium atoms.
- ${}^{51}Cr$ decays by orbital electron capture with a half-life of 27.8 days and emits a 0.323 MeV gamma ray in 9.8% of the decay.
- The atomic weight of Cr is 52.01.

The activity of ^{51}Cr daughters is given by

$$\lambda \cdot N = \phi n \sigma (1-e^{-\lambda t})$$

where

N : the # of daughter atoms at time t.

λ : decay constant of the daughter

ϕ : neutron fluence rate

σ : thermal neutron activition cross section

So

$$10 \ C/s \ \frac{1}{10\%} = 10^{11} \ cm^{-2} \cdot s^{-1} \times 1.35 \times 10^{-23} \ cm^2/atom \times 9.8\% \times 4.31\% \times n \times (1-e^{-0.693/27.8 \times 7})$$

Therefore $n = 1.096 \times 10^{17}$ atoms.

Since there are 52.01 g/mole Cr, the mass of the sample was

$$\frac{1.096 \times 10^{17} \ atoms}{6.02 \times 10^{23} \ atoms/mole} \cdot \frac{52.01 \ g}{mole} = 9.46 \times 10^{-6} \ g.$$

Example

A 3-g sample of ^{32}S is irradiated with fast neutrons having a constant fluence rate of 155 cm^{-2} s^{-1}. The cross section for the reaction $^{32}\mathrm{S(n,p)}^{32}\mathrm{P}$ is 0.200 barn, and the half-life of ^{32}P is $T = 14.3$ d. What is the maximum ^{32}P activity that can be induced? How many days are needed for the level of the activity to reach three quarters of the maximum?

Solution

The total number of target atoms is $N_\mathrm{T} = \frac{3}{32} \times 6.02 \times 10^{23} = 5.64 \times 10^{22}$. The maximum (saturation) activity is $\dot{\Phi}\sigma N_\mathrm{T} = (155\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1})(0.2 \times 10^{-24}\ \mathrm{cm}^2)(5.64 \times 10^{22}) = 1.75\ \mathrm{s}^{-1} = 1.75$ Bq. [Expressed in curies, the saturation activity is $1.75/(3.7 \times 10^{10}) = 4.73 \times 10^{-11}$ Ci.] The time t needed to reach three-quarters of this value can be found from Eq. (9.36) by writing $\frac{3}{4} = 1 - e^{-\lambda t}$. Then $e^{-\lambda t} = \frac{1}{4}$ and $t = 2T = 28.6$ d. Note that the buildup toward saturation activity is analogous to the approach to secular equilibrium by the daughter of a long-lived parent (Sect. 4.4).

From Turner's textbook, Page 229

Example

Estimate the fraction of the ^{32}S atoms that would be consumed in the last example in 28.6 days.

Solution

The rate at which ^{32}S atoms are used up is $\dot{\Phi}\sigma N_T = 1.75\ s^{-1}$. Since $t = 28.6$ d $= 2.47 \times 10^6$ s, the number of ^{32}S atoms lost is $1.75 \times 2.47 \times 10^6 = 4.32 \times 10^6$. The fraction of ^{32}S atoms consumed, therefore, is $4.32 \times 10^6/(5.64 \times 10^{22}) = 7.66 \times 10^{-17}$, a negligible amount. Note that fractional burnup does not depend on the sam-

From Turner's textbook, Page 230