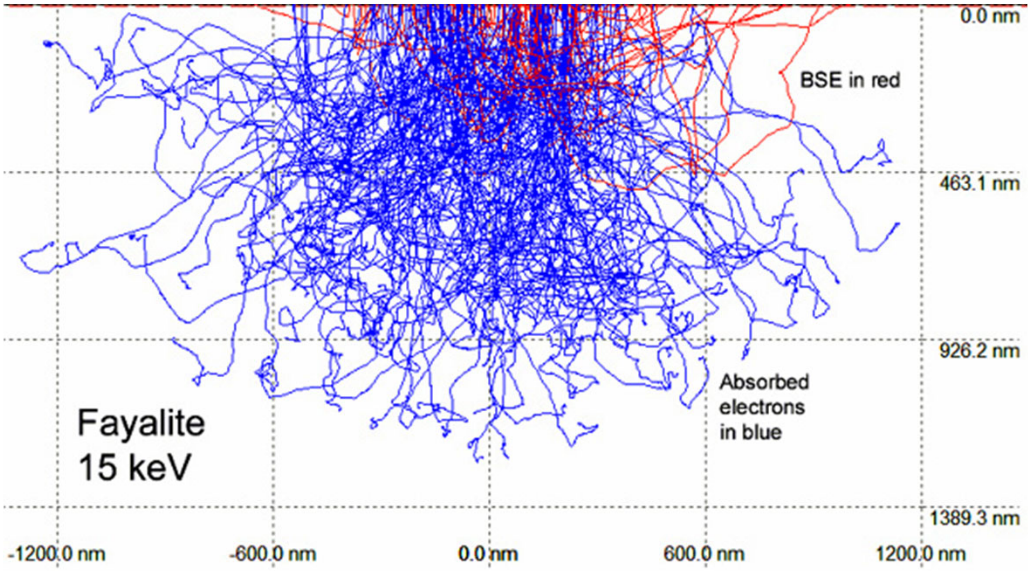
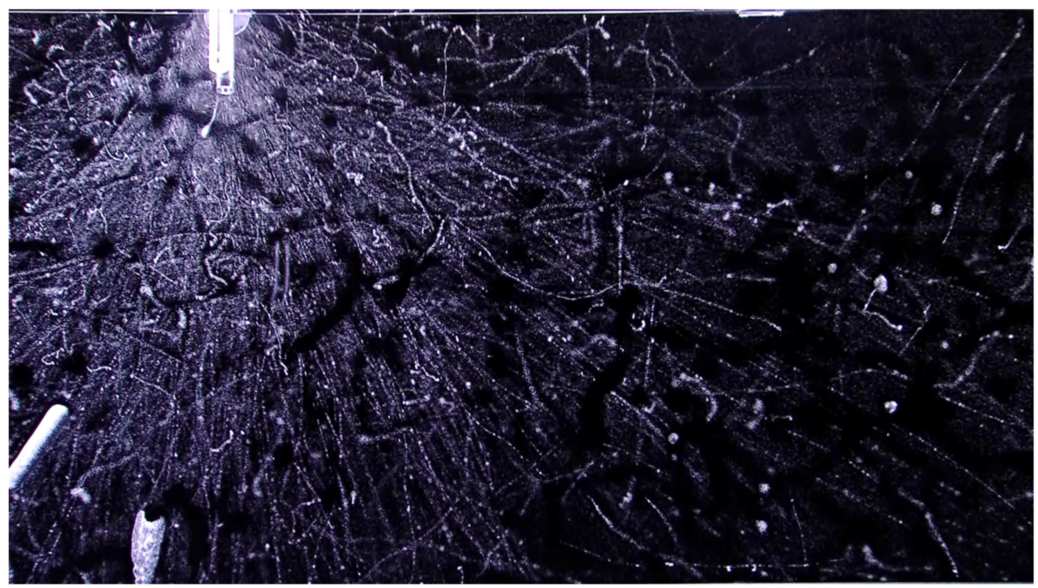
Chapter 2: Interaction of Radiation with Matter

Physical behavior of beta particles traveling in media

Interactions of Beta Particles





Beta radiation detected in an isopropanol cloud chamber (after insertion of an artificial source strontium-90)

https://en.wikipedia.org/wiki/Beta_particle

What are the significant interactions of beta particles with matter?

Mechanisms of Energy Loss by Electrons

Ionization and excitation:

Beta particles may **interact with orbital electrons through the electric fields** surrounding these charged particles, which leads to excitation and ionization.

Ionization process can be modeled as an **inelastic collision**, the energy loss by the electron and the kinetic energy carried by the ejected electron is related by

$$E_k = E_{loss} - \phi$$

where ϕ is the **ionization potential** of the absorbing medium.

Specific Energy Loss of Beta Particles

Specific energy loss: the linear rate of energy loss by an electron through excitation and ionization, which is given by

$$\frac{dE}{dx} = \frac{2\pi q^4 \, NZ \times (3 \times 10^9)^4}{E_m \beta^2 (1.6 \times 10^{-6})^2} \left\{ \ln \left[\frac{E_m E_k \beta^2}{I^2 (1 - \beta^2)} \right] - \beta^2 \right\} \frac{\text{MeV}}{\text{cm}}$$

where q = charge on the electron, 1.6×10^{-19} C,
N = number of absorber atoms per cm^3,
Z = atomic number of the absorber,
NZ = number of absorber electrons per cm^3 = 3.88×10^{20} for air at 0° and 76 cm Hg,
E_m = energy equivalent of electron mass, 0.51 MeV,
E_k = kinetic energy of the beta particle, MeV,
β = v/c,
I = mean ionization and excitation potential of absorbing atoms, MeV,
I = 8.6×10^{-5} for air; for other substances, $I = 1.35 \times 10^{-5} Z$.

Mechanisms of Energy Loss

Energy expenditure for creating ion pairs in media:

The average energy needed for creating an ion pair is normally **2 to 3 times greater** that the corresponding electron binding energy in the absorbing medium.

TABLE 5.1. AVERAGE ENERGY LOST BY A BETA PARTICLE IN THE PRODUCTION OF AN ION PAIR

Gas	Ionization potential	Mean energy expenditure per ion pair
H_2	13.6 eV	36.6 eV
He	24.5	41.5
N_2	14.5	34.6
O_2	13.6	30.8
Ne	21.5	36.2
A	15.7	26.2
Kr	14.0	24.3
Xe	12.1	21.9
Air		33.7
CO_2	14.4	32.9
CH_4	14.5	27.3
C_2H_2	11.6	25.7
C_2H_4	12.2	26.3
C_2H_6	12.8	24.6

The deviation between the ionization energy and the average energy required to create an ion pair is due to the **excitation of the atoms**, which does not lead to ionization.

Cember, Introduction to Health Physics, Fourth Edition

Specific Ionization

In the context of radiation protection and health physics, it is normally important to specify the effect of the energy deposition by a bate particle in terms of the number of ion pairs created by the particle after traveling through a unit path length – the **specific ionization**.

$$\text{S.I.} = \frac{\mathrm{d}E/\mathrm{d}x \ \text{eV/cm}}{w \ \text{eV/ip}}$$

where ***w*** is the average energy expenditure required to create a ion pair.

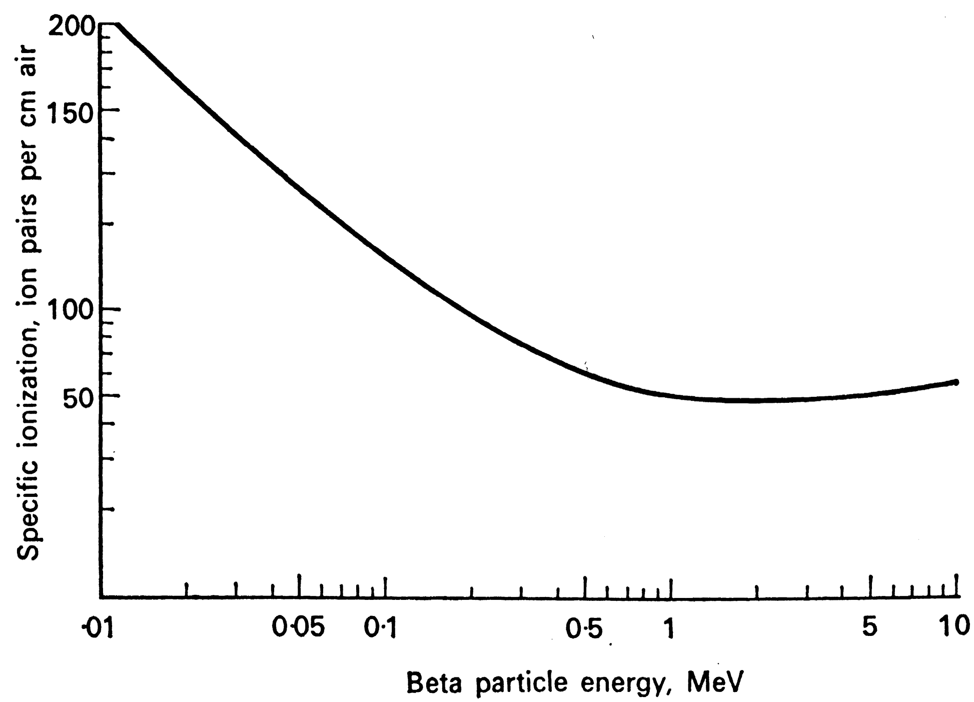


FIG 5.7. Relationship between beta particle energy and specific ionization of air.

Cember, Introduction to Health Physics, Fourth Edition

Specific Energy Loss of Beta Particles

An example:

What is the specific ionization resulting from the passage of a 0.1-MeV beta particle through standard air?

Solution:

The specific energy loss is given by

$$\frac{dE}{dx} = \frac{2\pi q^4 NZ \times (3 \times 10^9)^4}{E_m \beta^2 (1.6 \times 10^{-6})^2} \left\{ \ln \left[\frac{E_m E_k \beta^2}{I^2 (1-\beta^2)} \right] - \beta^2 \right\} \frac{\text{MeV}}{\text{cm}}$$

To use the equation, one needs to find β as the following

$$E_k = m_0 c^2 \left[\frac{1}{\sqrt{(1-\beta^2)}} - 1 \right] \qquad \text{so} \qquad \beta^2 = 0.3010.$$

Mass Stopping Power

It is also common to specify the energy loss of beta particles in a medium in terms of **mass stopping power**, which given by

$$S = \frac{\text{specific energy loss}(MeV/\text{cm})}{\text{density}(\text{g}/\text{cm}^3)} = \frac{\text{dE}/\text{dx}}{\rho}(MeV \cdot \text{cm}^2/\text{g})$$

where ρ is the density of the absorbing medium.

Why **mass stopping power**?

Mass Stopping Power

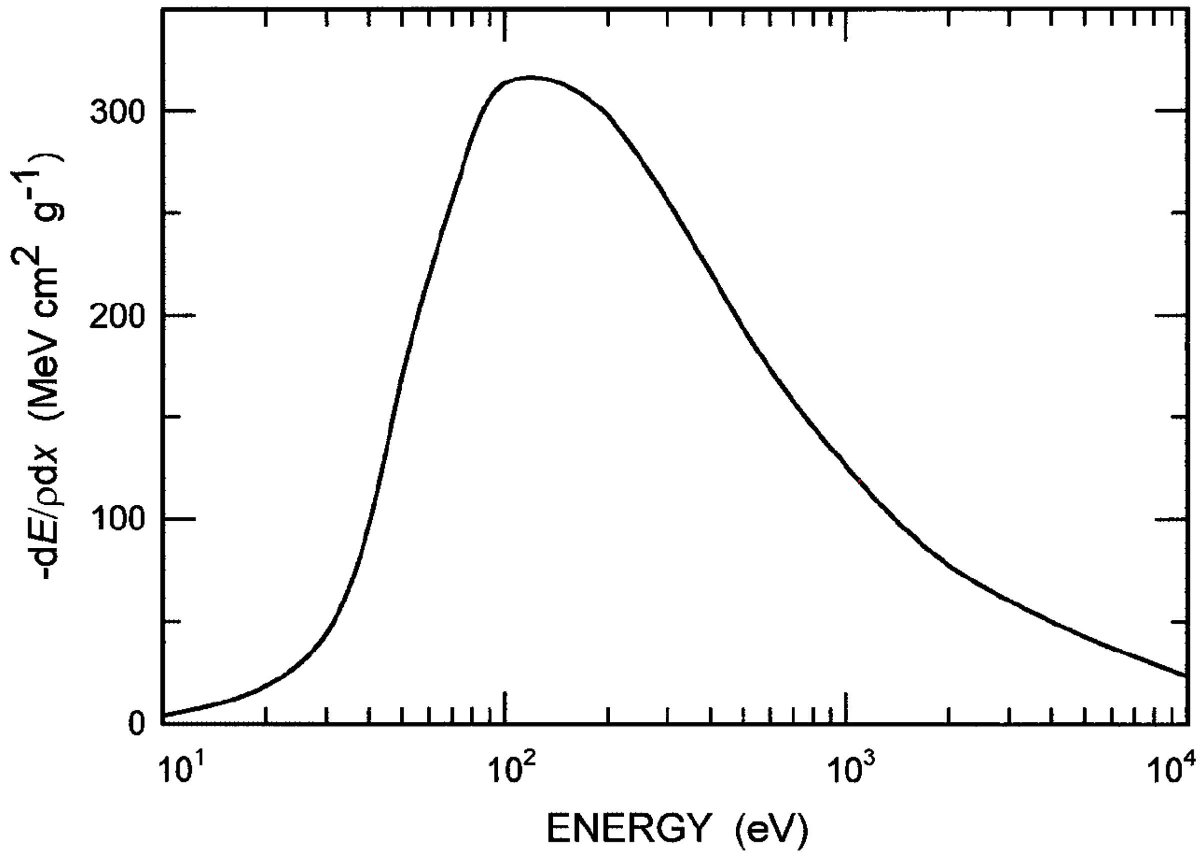


Fig. 6.1 Mass stopping power of water for low-energy electrons.

Turner, Atoms, Radiation and Radiation Protection, 3'rd edition

Remarks on Mass Stopping Power

- It takes ~22 eV to produce an e-i pair in water. At low energy, the specific energy loss of electron is increasing with energy. This does NOT agree with Beth's formula,

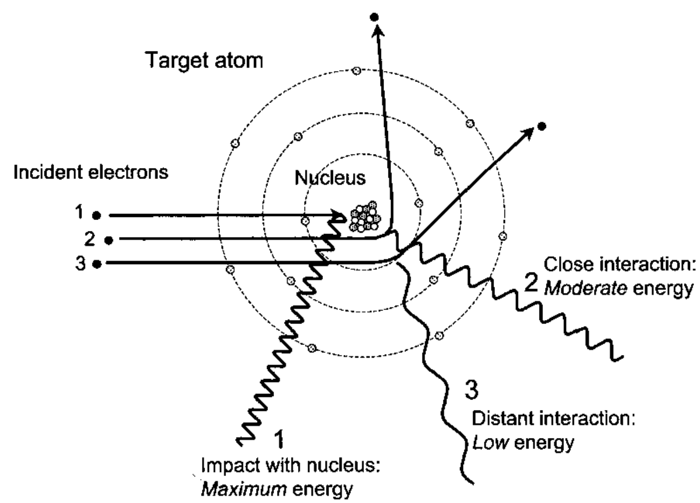
$$\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{2\pi q^4\,\mathrm{NZ} \times (3 \times 10^9)^4}{E_m \beta^2 (1.6 \times 10^{-6})^2} \left\{ \ln\left[\frac{E_m E_k \beta^2}{I^2(1-\beta^2)}\right] - \beta^2 \right\} \frac{\mathrm{MeV}}{\mathrm{cm}}$$

- A 10 keV electron produces ~450 secondary electrons through cascade of ionization events.

- In water, most of ionization events are induced by electrons with E<100 eV.

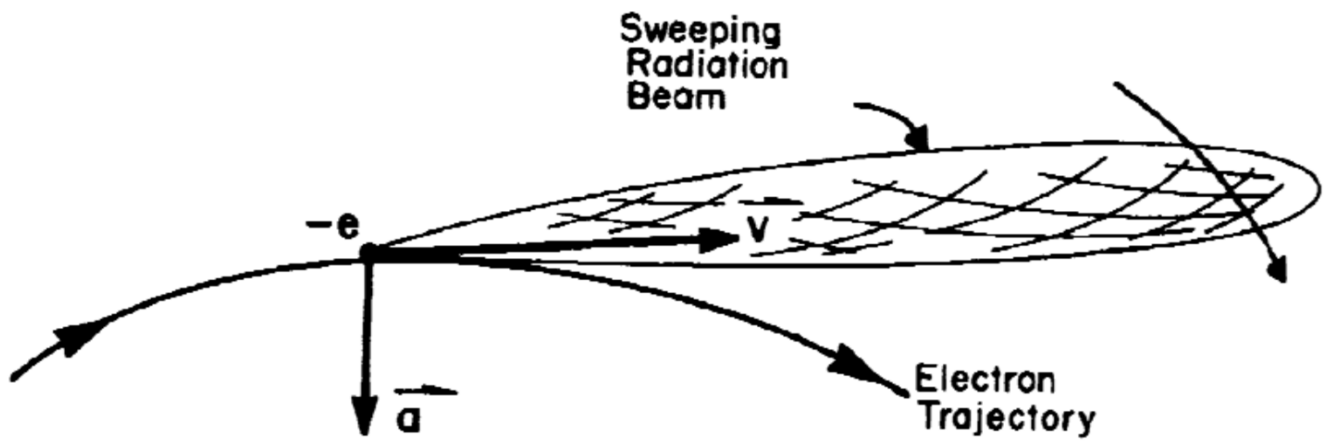
X-ray Generation – Bremsstrahlung

- Target nucleus positive charge ($Z \cdot p^+$) attracts incident e^-
- Deceleration of an incident e^- occurs in the proximity of the target atom nucleus
- Energy lost by e^- is gained by the EM photon (x-ray) generated
  - The impact parameter distance, the closest approach to the nucleus by the e^- determines the amount of E loss
  - The Coulomb force of attraction varies strongly with distance ($\propto 1/r^2$); ↓ distance → ↑ deceleration and E loss → ↑ photon E
  - Direct impact on the nucleus determines the maximum x-ray E (E_{max})



Radiative Energy Loss of Beta Particles – Bremsstrahlung

- **Bremsstrahlung** occurs when a beta particle is deflected or accelerated in the forced field of nucleus.



Radiative Energy Loss of Beta Particles – Bremsstrahlung

Part of the energy possessed by the beta particle is emitted in the form of **photons**. The **rate of energy loss** is proportional to the **square of the instantaneous acceleration** experienced by the beta particle.

$$-\left(\frac{dE}{dx}\right)_r = \frac{NEZ(Z+1)e^4}{137 m_0^2 c^4}\left(4 \ln \frac{2E}{m_0 c^2} - \frac{4}{3}\right)$$

E: kinetic energy of the beta particle,

N: number of absorber atoms per cm^3,

Z: atomic number of the absorber,

m_0: mass of an electron

e: charge of an electron

c: speed of light

Glenn Knoll, Radiation Detection and Measurement, Fourth Edition

Radiative Energy Loss of Beta Particles – Bremsstrahlung

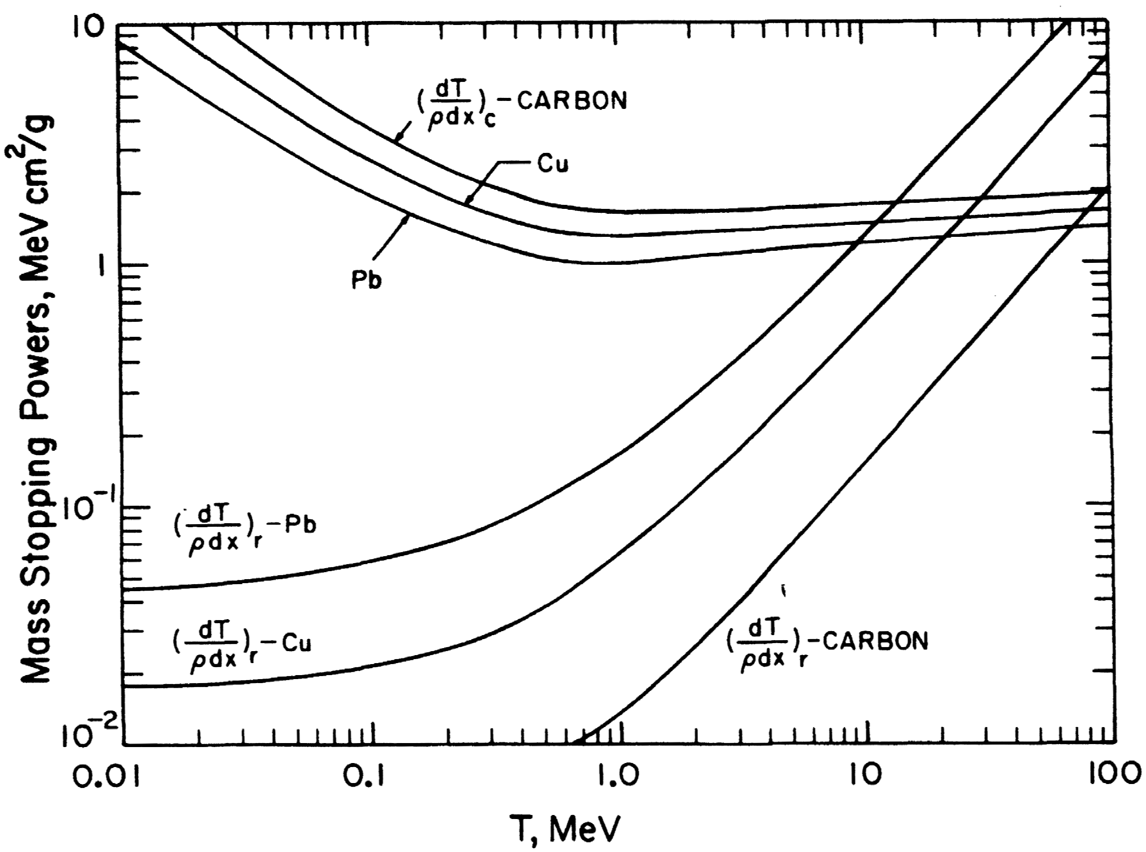


FIGURE 8.6. **Mass radiative and collision stopping powers for electrons (and approximately for positrons) in C, Cu, and Pb. (From data of Bichsel, 1968).**

2.2 Interaction of Heavy Charged Particles

Overview

Energy Loss Mechanisms

- Heavy charged particles loss energy **primarily though the ionization and excitation** of atoms.

- Heavy charged particles can transfer only a small fraction of their energy in a single collision. Its deflection in a collision is almost negligible. Therefore, heavy charged particles **travel in almost straight paths** in the absorber, **losing energy continuously** through a large number of collisions with atomic electrons.

- At low velocity, a heavy charged particle may losses a negligible amount of energy in **nuclear collisions**. It may also pick up free electrons along its path, which reduces it net charge.

Energy Loss Mechanisms

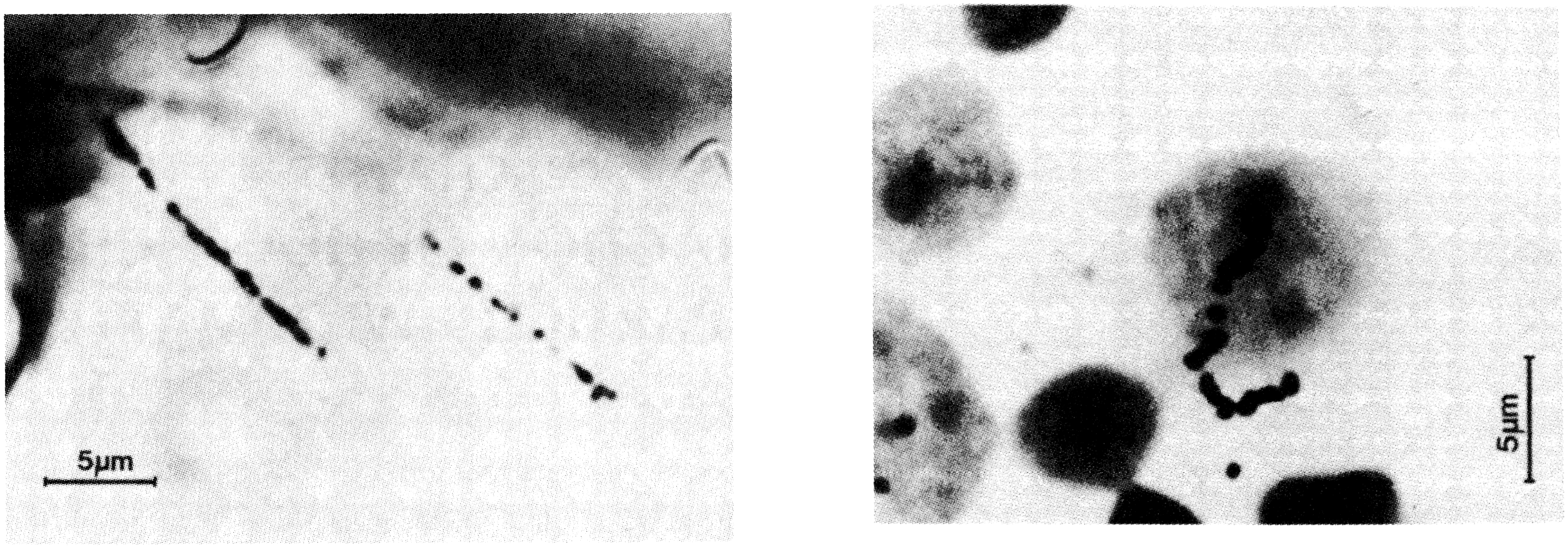


FIGURE 5.1. (Top) Alpha-particle autoradiograph of rat bone after inhalation of ^{241}Am. Biological preparation by R. Masse and N. Parmentier. (Bottom) Beta-particle autoradiograph of isolated rat-brain nucleus. The ^{14}C-thymidine incorporated in the nucleolus is located at the track origin of the electron emitted by the tracer element. Biological preparation by M. Wintzerith and P. Mandel. (Courtesy R. Rechenmann and E. Wittendorp-Rechenmann, Laboratoire de Biophysique des Rayonnements et de Methodologie INSERM U.220, Strasbourg, France.)

How do heavy charged particles lose energy in absorbing media?

Energy Loss Mechanisms

- Heavy charged particles loss energy **primarily though the ionization and excitation** of atoms.

- Heavy charged particles can transfer only a small fraction of their energy in a single collision. Its deflection in a collision is almost negligible. Therefore, heavy charged particles **travel in almost straight paths** in the absorber, **losing energy continuously** through a large number of collisions with atomic electrons.

- At low velocity, a heavy charged particle may losses a negligible amount of energy in **nuclear collisions**. It may also pick up free electrons along its path, which reduces it net charge.

Energy Loss Mechanisms

For heavy charged particles, the **maximum energy that can be transferred** in a single collision is given by the **conservation of energy and momentum**:

$$\tfrac{1}{2}MV^2 = \tfrac{1}{2}MV_1^2 + \tfrac{1}{2}mv_1^2$$

$$MV = MV_1 + mv_1.$$

where M and m are the mass of the heavy charged particle and the electron. V is the initial velocity of the charged particle. V_1 and v_1 are the velocities of both particles after the collision.

The **maximum energy transfer** is therefore given by

$$Q_{\max} = \frac{1}{2}MV^2 - \frac{1}{2}MV_1^2 = \frac{4mME}{(M+m)^2}$$

Maximum Energy Loss by a Single Collision

For a more general case, which includes the relativistic effect, the **maximum energy transferred by a single collision** is

$$Q_{\max} = \frac{2\gamma^2 mV^2}{1 + 2\gamma m/M + m^2/M^2}$$

where $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = V/c$, and c is the speed of light

Maximum Energy Loss by a Single Collision

TABLE 5.1. Maximum Possible Energy Transfer, Q_{max}, in Proton Collision with Electron

Proton Kinetic Energy E (MeV)	Q_{max} (MeV)	Maximum Percentage Energy Transfer $100Q_{max}/E$
0.1	0.00022	0.22
1	0.0022	0.22
10	0.0219	0.22
100	0.229	0.23
10^3	3.33	0.33
10^4	136.	1.4
10^5	1.06×10^4	10.6
10^6	5.38×10^5	53.8
10^7	9.21×10^6	92.1

Paying attention to the single-collision energy-loss spectrum

Single Collision Energy-Loss Spectrum

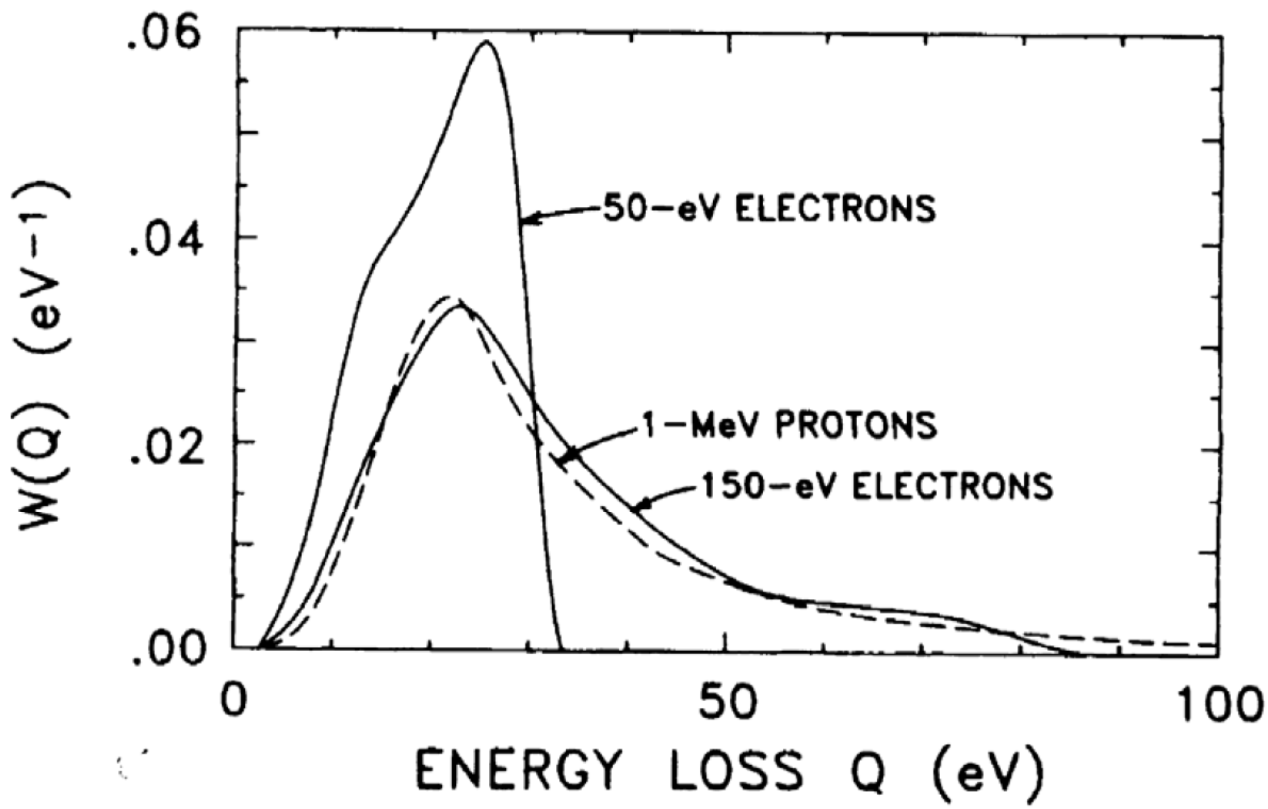


Fig. 5.3 Single-collision energy-loss spectra for 50-eV and 150-eV electrons and 1-MeV protons in liquid water. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)

Single Collision Energy-Loss Spectrum

Example

Estimate the probability that a 50-MeV proton will lose between 30 eV and 40 eV in a collision with an atomic electron in penetrating the soft tissue of the body.

Solution:

Soft tissue is similar in atomic composition to liquid water (Table 12.3), and so we use Fig. 5.3 to make the estimate.

As implied in the text, the energy-loss spectrum for 50-MeV protons is close to that for 1-MeV protons, except that it extends out to a different value of $Q_{\max}$.

Therefore,

$$W(Q)\Delta Q = (0.019\ \text{eV}^{-1})(40 - 30)\ \text{eV} = 0.19.$$

Thus, a 50-MeV proton has about a 20% chance of losing between 30 eV and 40 eV in a single electronic collision in soft tissue.

Linear Stopping Power for Heavy Charged Particles

The **linear stopping power** for heavy charged particles may be estimated using the single collision energy-loss spectra discussed previously.

For a given type of charged particle at a given energy, the stopping power is given by the product of (1) the probability μ per unit distance of travel that an electronic collision occurs and (2) the average energy loss per collision, Q_{avg}. The former is called the macroscopic cross section, or attenuation coefficient, and has the dimensions of inverse length. The latter is given by

$$Q_{\text{avg}} = \int_{Q_{\min}}^{Q_{\max}} Q W(Q)\, dQ,$$

where $Q_{\min}$ was introduced at the end of the last section.

Therefore, the **linear stopping power** is given by

$$-\frac{dE}{dx} = \mu Q_{\text{avg}} = \mu \int_{Q_{\min}}^{Q_{\max}} Q W(Q)\, dQ.$$

Can you derive the linear stopping power for heavy charged particles?

Linear Stopping Power of a Medium for Heavy Charged Particles (revisited)

The **linear stopping power** of a medium is given by the Bethe formula,

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2\beta^2}\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2\right].$$

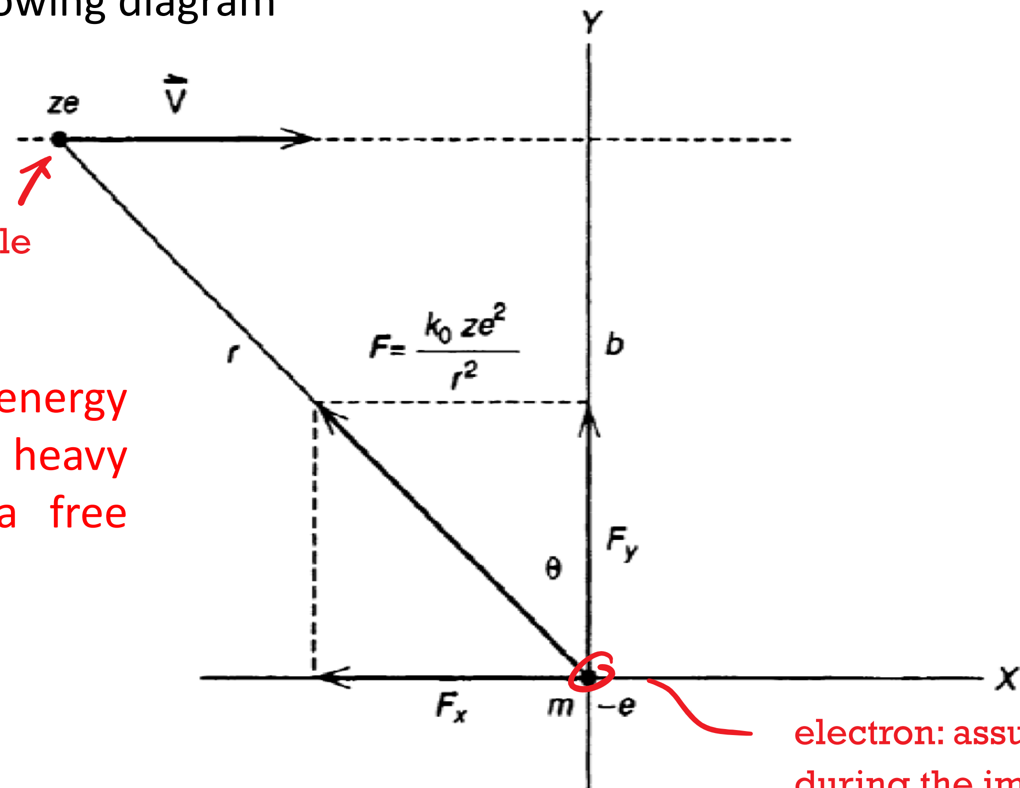
$k_0 = 8.99 \times 10^9$ N m^2 C^{-2} (Appendix C),
z = atomic number of the heavy particle,
e = magnitude of the electron charge,
n = number of electrons per unit volume in the medium,
m = electron rest mass,
c = speed of light in vacuum,
$\beta = V/c$ = speed of the particle relative to c,
I = mean excitation energy of the medium.

Linear Stopping Power – A Semiclassical Treatment

Consider the following diagram

Charged particle

Step 1: Deriving the energy transfer from the heavy charged particle to a free electron nearby.



electron: assumed stationery during the impact.

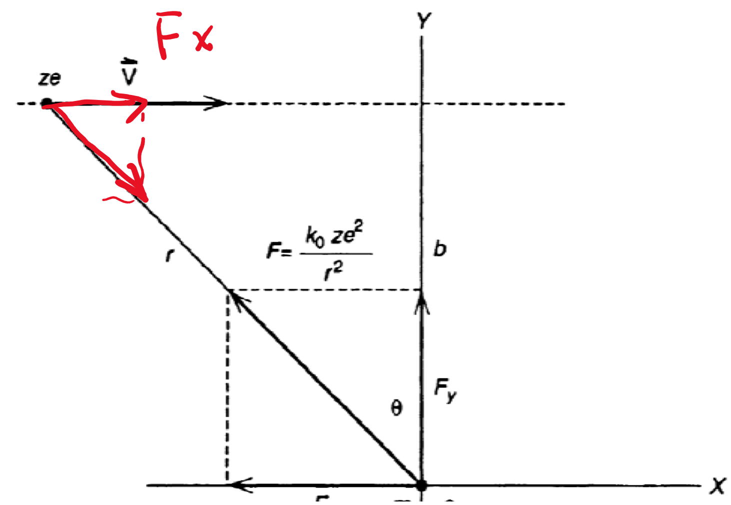
Fig. 5.4 Representation of the sudden collision of a heavy charged particle with an electron, located at the origin of *XY* coordinate axes shown. See text.

and assuming the electron is stationary during the collision…

P 117, <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.

Deriving the Linear Stopping Power for Heavy Charged Particles – A Semiclassical Treatment

The total momentum imparted to the electron is given by



$$p = \int_{-\infty}^{\infty} F_y \, dt = \int_{-\infty}^{\infty} F \cos\theta \, dt = k_0 z e^2 \int_{-\infty}^{\infty} \frac{\cos\theta}{r^2} \, dt.$$

Coulomb force $F = \dfrac{k_0 z e}{r^2}$ (5.11)

To carry out the integration, we let $t = 0$ represent the time at which the heavy charged particle crosses the Y-axis in Fig. 5.4. Since $\cos\theta = b/r$ and the integral is symmetric in time, we write

$$r = \sqrt{b^2 + V^2 t^2}$$

$$\int_{-\infty}^{\infty} \frac{\cos\theta}{r^2} \, dt = 2 \int_0^{\infty} \frac{b}{r^3} \, dt = 2b \int_0^{\infty} \frac{dt}{(b^2 + V^2 t^2)^{3/2}}$$

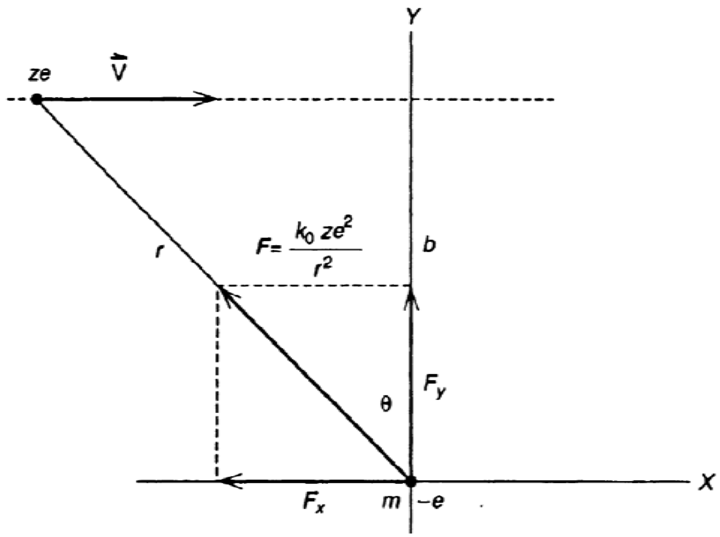
$$= 2b \left[\frac{t}{b^2 (b^2 + V^2 t^2)^{1/2}} \right]_0^{\infty} = \frac{2}{Vb}. \quad (5.12)$$

P 118, <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.

Linear Stopping Power – A Semiclassical Treatment

Combining this result with (5.11) gives, for the momentum transferred to the electron in the collision,[2)]

$$p = \frac{2k_0 z e^2}{Vb}. \qquad (5.13)$$

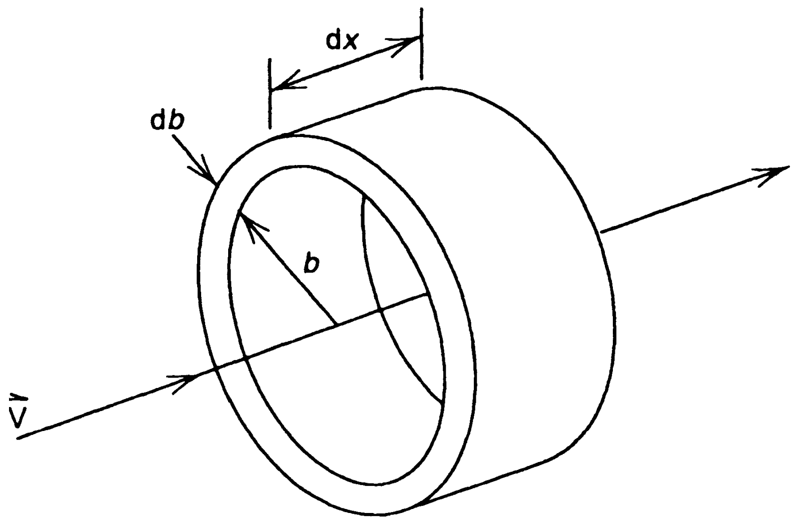


The energy transferred is

$$Q = \frac{p^2}{2m} = \frac{2k_0^2 z^2 e^4}{mV^2 b^2}.$$

P 118, <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.

Linear Stopping Power – A Semiclassical Treatment



Step 2: Integrate the energy transfer to all the electrons surrounding the path of the heave charged particle.

Fig. 5.5 Annular cylinder of length dx centered about path of heavy charged particle. See text.

In traversing a distance dx in a medium having a uniform density of n electrons per unit volume, the heavy particle encounters $2\pi nb\,db\,dx$ electrons at impact parameters between b and $b + db$, as indicated in Fig. 5.5. The energy lost to these electrons per unit distance traveled is therefore $2\pi nQb\,db$. The total linear rate of

Therefore, the total linear rate of energy-loss is given by

$$-\frac{dE}{dx} = 2\pi n \int_{Q_{min}}^{Q_{max}} Qb\,db = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2} \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2} \ln \frac{b_{max}}{b_{min}}. \quad (5.15)$$

P 118, <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.

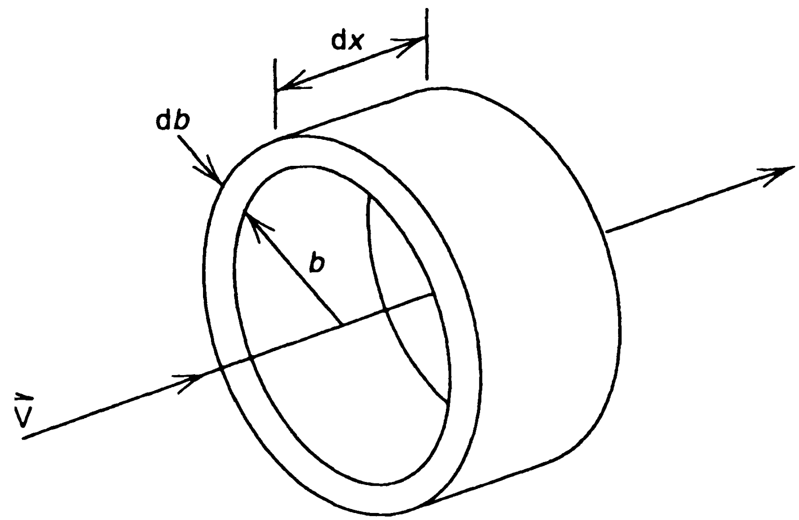
Linear Stopping Power – A Semiclassical Treatment

The maximum value of the impact parameter can be estimated from the physical principle that a quantum transition is likely only when the passage of the charged particle is rapid compared with the period of motion of the atomic electron. We denote the latter time by $1/f$, where f is the orbital frequency. The duration of the collision is of the order of b/V. Thus, the important impact parameters are restricted to values approximately given by

~ duration of the collision

$$\boxed{\frac{b}{V}} < \boxed{\frac{1}{f}} \quad \text{or} \quad b_{\max} \sim \frac{V}{f}.$$

period of the orbital electron

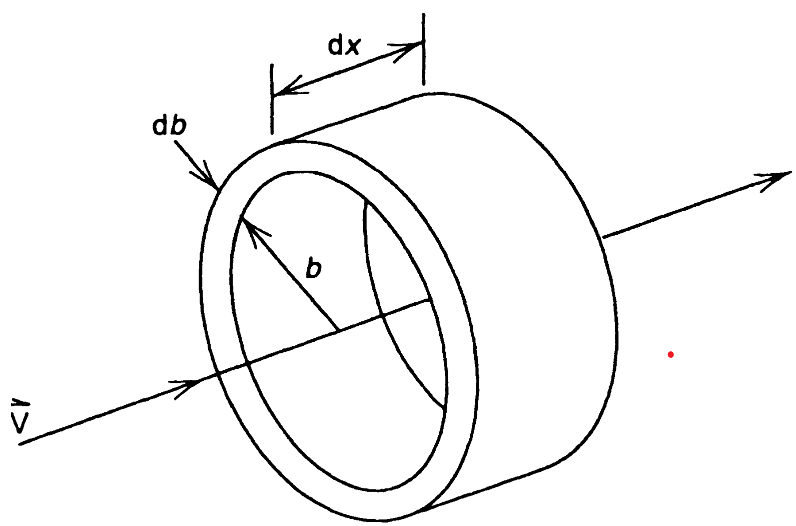


P 118, <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.

Linear Stopping Power – A Semiclassic Treatment

For the minimum impact parameter, the analysis implies that the particles' positions remain separated by a distance b_{min} at least as large as their de Broglie wavelengths during the collision. This condition is more restrictive for the less massive electron than for the heavy particle. In the rest frame of the latter, the electron has a de Broglie wavelength $\lambda = h/mV$, since it moves approximately with speed V relative to the heavy particle. Accordingly, we choose

$$b_{\mathrm{min}} \sim \frac{h}{mV}. \tag{5.17}$$



Linear Stopping Power – A Semiclassic Treatment

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 2\pi n \int_{Q_{\min}}^{Q_{\max}} Qb\,\mathrm{d}b = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2} \int_{b_{\min}}^{b_{\max}} \frac{\mathrm{d}b}{b} = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2} \ln \frac{b_{\max}}{b_{\min}}. \quad (5.15)$$

Combining the relations (5.15), (5.16), and (5.17) gives the semiclassical formula for stopping power,

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2} \ln \frac{mV^2}{hf}. \quad (5.18)$$

Linear Stopping Power – A Semiclassical Treatment

Example

Calculate the maximum and minimum impact parameters for electronic collisions for an 8-MeV proton. To estimate the orbital frequency f, assume that it is about the same as that of the electron in the ground state of the He^+ ion.

$$b_{\min} \sim \frac{h}{mV}.$$

$$\frac{b}{V} < \frac{1}{f} \quad \text{or} \quad b_{\max} \sim \frac{V}{f}.$$

P 119, <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.

Linear Stopping Power – A Semiclassical Treatment

Solution

$$b_{\max} \sim \frac{V}{f}.$$

The proton velocity is given by $V = (2T/M)^{1/2}$, where T is the kinetic energy and M is the mass:

$$V = \left[\frac{2 \times 8\ \text{MeV} \times 1.60 \times 10^{-13}\,\text{J MeV}^{-1}}{1.67 \times 10^{-27}\ \text{kg}}\right]^{1/2} = 3.92 \times 10^{7}\ \text{m s}^{-1}. \tag{5.19}$$

The orbital frequency of the electron in the ground state of He^{+} can be found by using Eqs. (2.8) and (2.9) with $Z = 2$ and $n = 1$:

$$f = \frac{v_1}{2\pi r_1} = \frac{4.38 \times 10^{6}\ \text{m s}^{-1}}{2\pi \times 2.65 \times 10^{-11}\ \text{m}} = 2.63 \times 10^{16}\ \text{s}^{-1}. \tag{5.20}$$

Equation (5.16) gives for the maximum impact parameter

$$b_{\max} \sim \frac{V}{f} = \frac{3.92 \times 10^{7}\ \text{m s}^{-1}}{2.63 \times 10^{16}\ \text{s}^{-1}} = 1.49 \times 10^{-9}\ \text{m} = 15\ \text{Å}. \tag{5.21}$$

Linear Stopping Power – A Semiclassical Treatment

$$b_{\min} \sim \frac{h}{mV}.$$

The minimum impact parameter is, from Eq. (5.17),

$$b_{\min} \sim \frac{h}{mV} = \frac{6.63 \times 10^{-34}\ \mathrm{J\,s}}{9.11 \times 10^{-31}\ \mathrm{kg} \times 3.92 \times 10^{7}\ \mathrm{m\,s^{-1}}}$$

$$= 1.86 \times 10^{-11}\ \mathrm{m} = 0.19\ \text{Å}. \tag{5.22}$$

Linear Stopping Power for Heavy Charged Particles

The **linear stopping power** of a medium is given by the Bethe formula,

Combining the relations (5.15), (5.16), and (5.17) gives the semiclassical formula for stopping power,

speed of the charged particle

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2} \ln \frac{mV^2}{hf}. \tag{5.18}$$

frequency of the orbital electron

$k_0 = 8.99 \times 10^9$ N m^2 C^{-2}
z = atomic number of the heavy particle,
e = magnitude of the electron charge,
n = number of electrons per unit volume in the medium
m = electron rest mass,

MLJ1

Linear Stopping Power of a Medium for Heavy Charged Particles (revisited)

The **linear stopping power** of a medium is given by the Bethe formula,

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2\beta^2}\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2\right].$$

$$\boxed{-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2}\ln\frac{mV^2}{hf}.}$$

$k_0 = 8.99 \times 10^9$ N m^2 C^{-2} (Appendix C),
z = atomic number of the heavy particle,
e = magnitude of the electron charge,
n = number of electrons per unit volume in the medium,
m = electron rest mass,
c = speed of light in vacuum,
$\beta = V/c$ = speed of the particle relative to c,
I = mean excitation energy of the medium.

Slide 43

MLJ1 Meng, Ling Jian, 2/15/2021

Mean Excitation Energies

The main excitation energy (*I*) for an element having atomic number Z, can be approximately given by

$$I \cong \begin{cases} 19.0 \text{ eV}, \ Z = 1 \text{ (hydrogen)} \\ 11.2 + 11.7\, Z \text{ eV}, \ 2 \leq Z \leq 13 \\ 52.8 + 8.71\, Z \text{ eV}, \ Z > 13. \end{cases}$$

For compound or mixture,

If there are N_i atoms cm^{-3} of an element with atomic number Z_i and mean excitation energy I_i, then in formula (5.23) one makes the replacement

mean excitation energy for compound

$$n \ln I = \sum_i N_i Z_i \ln I_i,$$

mean excitation energy for a given element

What is the restricted stopping power?

Restricted Stopping Power

- Energy *lost* versus energy *absorbed*…
- **Restricted Stopping Power** is introduced to better associate the energy loss in a target with the energy actually absorbed there.

$$\left(-\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\Delta} = \mu \int_{Q_{\min}}^{\Delta} QW(Q)\,\mathrm{d}Q.$$

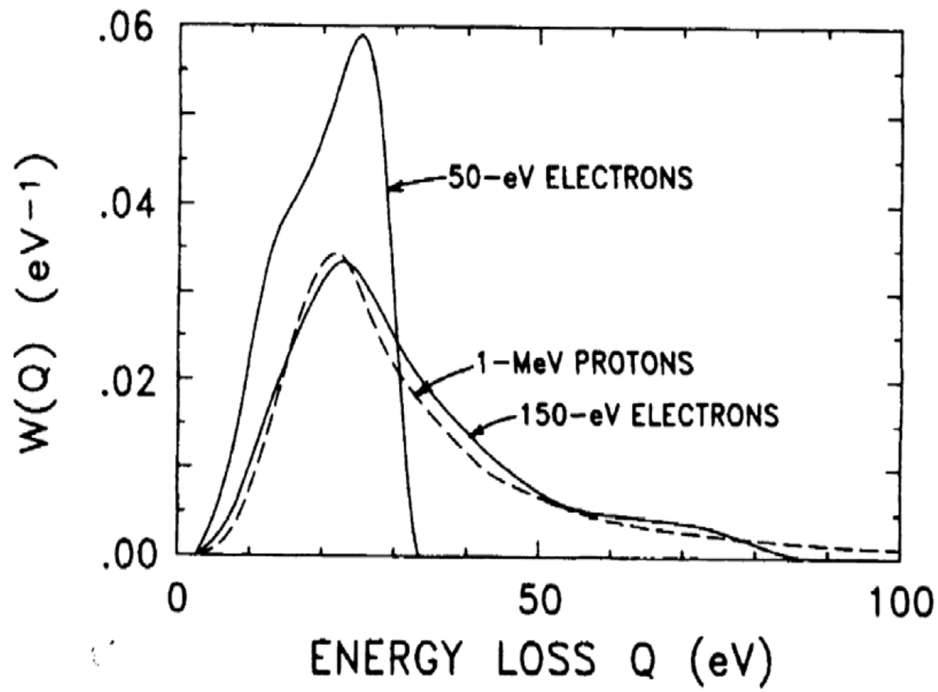


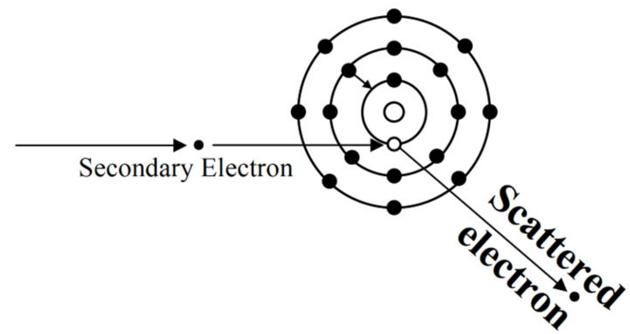
Fig. 5.3 Single-collision energy-loss spectra for 50-eV and 150-eV electrons and 1-MeV protons in liquid water. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)

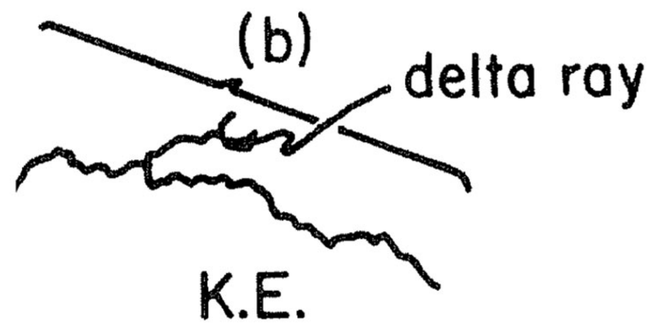
Restricted Stopping Power

In hard collisions, the scattered electron – or delta ray - can receive significant amounts of energy.

The delta ray can carry this energy a significant distance from the initial interaction site.

Need to separate this from the energy loss that is deposited locally.





The Rationale behind Restricted Stopping Power

- If we are interested in microscopic events, in which incident particles deposit energy in **local regions with finite sizes** …
- Since the predominate way for heavy charged particles to loss energy is to transferring its energies to energetic **delta-rays** …
- **If the range of the delta-ray is large compare to the dimension of the region-of-interest (ROI)**, it is likely that the energy carried by these delta-rays will not be fully deposited in the ROI.
- To account for this effect, **we will consider those delta-rays that carry energy less than a threshold**, this give rise to the Restrict Stopping Power.
- The value for the **threshold is typically determined by the dimension of the ROI** associated with the given application.

Restricted Stopping Power

Table 7.1 Restricted Mass Stopping Power of Water, $(-dE/\rho dx)_\Delta$ in MeV cm^2 g^{-1}, for Protons

Energy (MeV)	$\left(-\frac{dE}{\rho dx}\right)_{100\text{ eV}}$	$\left(-\frac{dE}{\rho dx}\right)_{1\text{ keV}}$	$\left(-\frac{dE}{\rho dx}\right)_{10\text{ keV}}$	$\left(-\frac{dE}{\rho dx}\right)_{\infty}$
0.05	910.	910.	910.	910.
0.10	711.	910.	910.	910.
0.50	249.	424.	428.	428.
1.00	146.	238.	270.	270.
10.0	24.8	33.5	42.2	45.9
100.	3.92	4.94	5.97	7.28

2.3 Interaction of Photons

Classification of Photon Interactions

Table 1. Classification of elementary photon interactions.

Type of interaction / Interaction with:	Absorption	Scattering: Elastic (Coherent)	Scattering: Inelastic (Incoherent)
Atomic electrons	**Photoelectric effect** $\sigma_{pe} \begin{cases} \sim Z^4 (L.E.) \\ \sim Z^5 (H.E.) \end{cases}$	Rayleigh scattering $\sigma_R \sim Z^2$ $(L.E.)$	**Compton scattering** $\sigma_C \sim Z$
Nucleus	Photonuclear reactions $(\gamma$,n),$(\gamma$,p), photofission, etc. $\sigma_{ph.n.} \sim Z$ $(h\nu \geq 10\text{MeV})$	Elastic nuclear scattering $(\gamma, \gamma) \sim Z^2$	Inelastic nuclear scattering (γ, γ')
Electric field surrounding charged particles	**Electron-positron pair production in field of nucleus,** $\sigma_{pair} \sim Z^2$ $(h\nu \geq 1.02\text{MeV})$		

Interaction of Photons in Matter

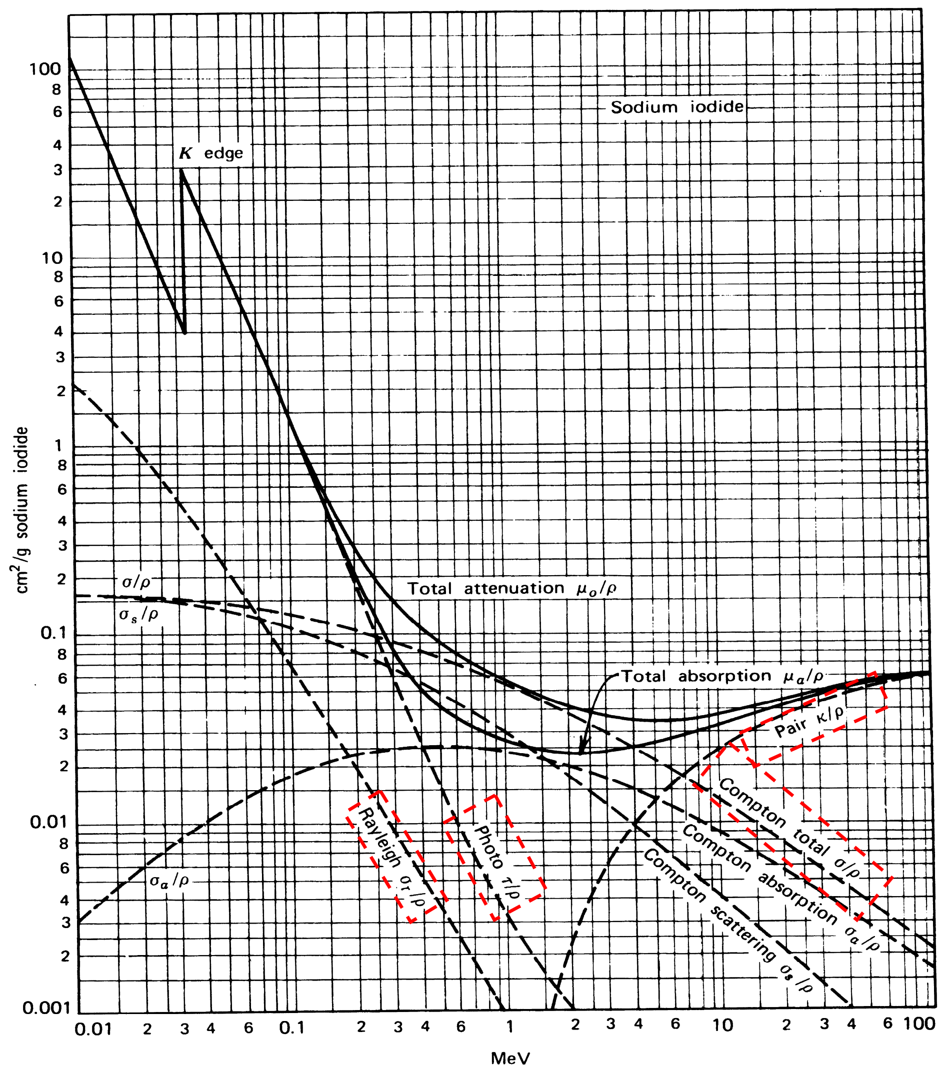


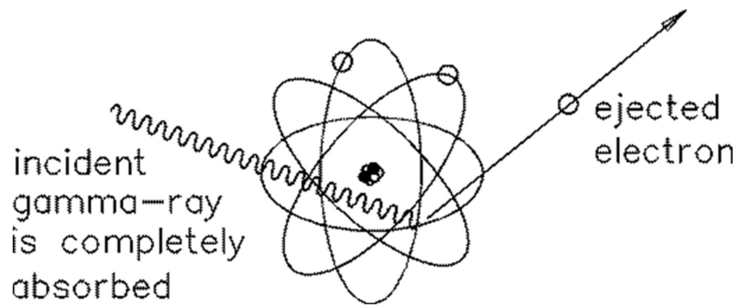
Figure 2.18 Energy dependence of the various gamma–ray interaction processes in sodium iodide. (From *The Atomic Nucleus* by R. D. Evans. Copyright 1955 by the McGraw-Hill Book Company. Used with permission.)

From Page 50, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Photoelectric Effect and its implication to shielding design and dosimetry calculation

Photoelectric Effect

In **photoelectric** process, an incident photon transfer its energy to an orbital electron, causing it to be ejected from the atom.



$$E_{e^-} = h\nu - E_b$$

h is the Planck's constant

ν is the frenquency of the photon

- Photoelectric interaction is **with the atom in a whole** and can not take place with free electrons.
- Photoelectric effect **creates a vacancy in one of the electron shells**, which leaves the atom at an excited state.

Photoelectric Effect Cross Section

Probability of photoelectric absorption per atom is

$$\sigma \propto \begin{cases} \dfrac{Z^4}{(h\nu)^{3.5}} & \textit{low energy} \\ \dfrac{Z^5}{(h\nu)^{3.5}} & \textit{high energy} \end{cases}$$

☞ The interaction cross section for photoelectric effect **depends strongly on Z**.

☞ Photoelectric effect is **favored at lower photon energies**. It is the major interaction process for photons at low hundred keV energy range.

Page 49, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Photoelectric Effect – Absorption Edges

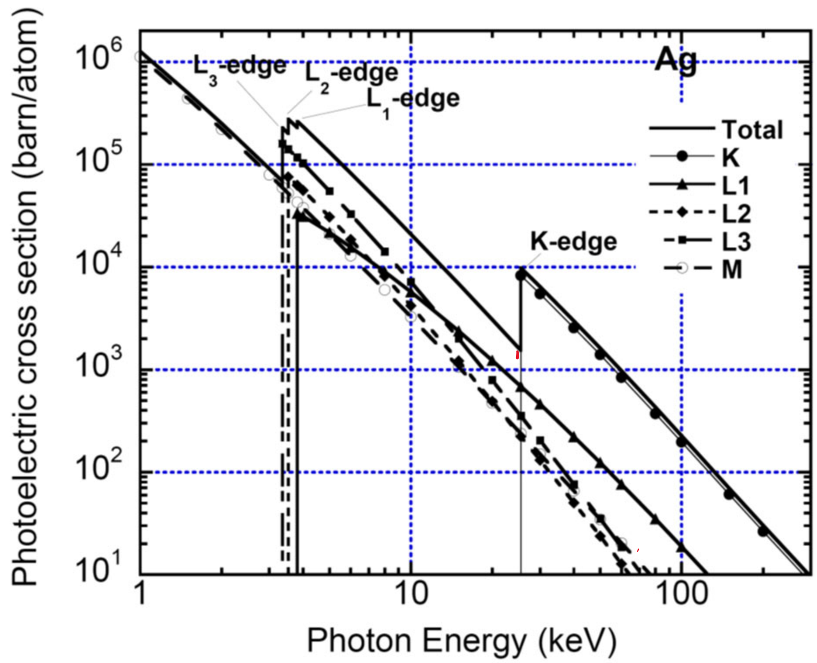
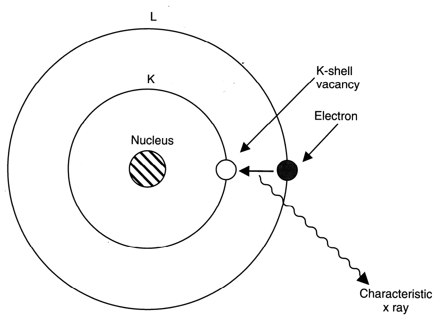


Figure 2: Total and partial atomic photoeffect of Ag.

- Requires **sufficient photon energy** for P.E. interaction.
- Interaction probability decreases dramatically with increasing energy.
- P.E. interaction is significant only for low energy photons, when the photon energy is close to the binding energies of the target atoms.

Relaxation Processes after Photoelectric Interaction

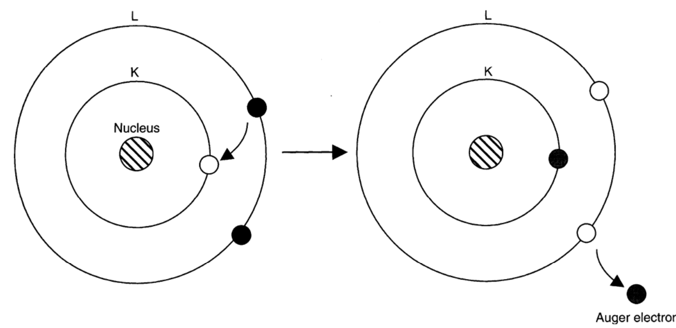
☞ The excited atoms will **de-excite** through one of the following processes:



Generation of characteristic X-rays

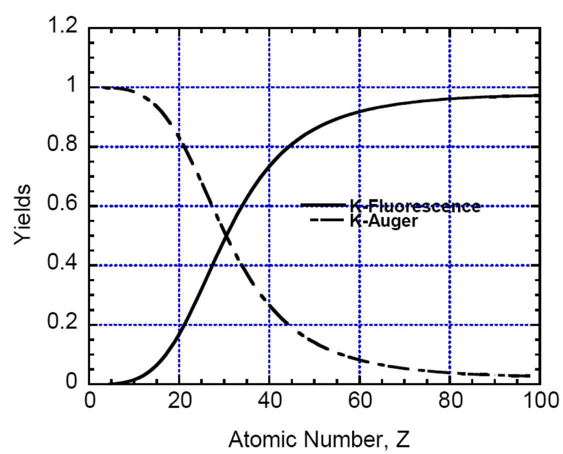
Competing

Processes



Generation of Auger electrons

☞ **Auger electron** emission dominates in **low-Z** elements. **Characteristic X-ray** emission dominates in **higher-Z** elements.



Auger Electrons

The relative probability of the emission of characteristic radiation to the emission of an Auger electron is called the fluorescent yield, ω:

$$\omega_K = \frac{\text{Number K x ray photons emitted}}{\text{Number K shell vacancies}} \tag{3-12}$$

Values for ω_K are given in Table 3-1. We see that for large Z values fluorescent radiation is favored, while for low values of Z Auger electrons tend to be produced.

From this table we see that if a nucleus with $Z = 40$ had a K shell hole, then on the average 0.74 fluorescent photons and 0.26 Auger electrons would be emitted.

TABLE 3-1
Fluorescent Yield

Z	ω_K	Z	ω_K	Z	ω_K
10	0	40	.74	70	.92
15	.05	45	.80	75	.93
20	.19	50	.84	80	.95
25	.30	55	.88	85	.95
30	.50	60	.89	90	.97
35	.63	65	.90		

From Evans (E1)

A few things about Compton scattering

- **Angular distribution of the scattered photon and**
- **Energy transfer to recoil electrons.**

Compton Scattering

- In **Compton scattering**, the incident gamma ray photon is **deflected by an orbital electron** in the absorbing material.
- Part of the **energy** carried by the incident photon is **transferred to the target electron** in the atom, causing it to be ejected from the atom.

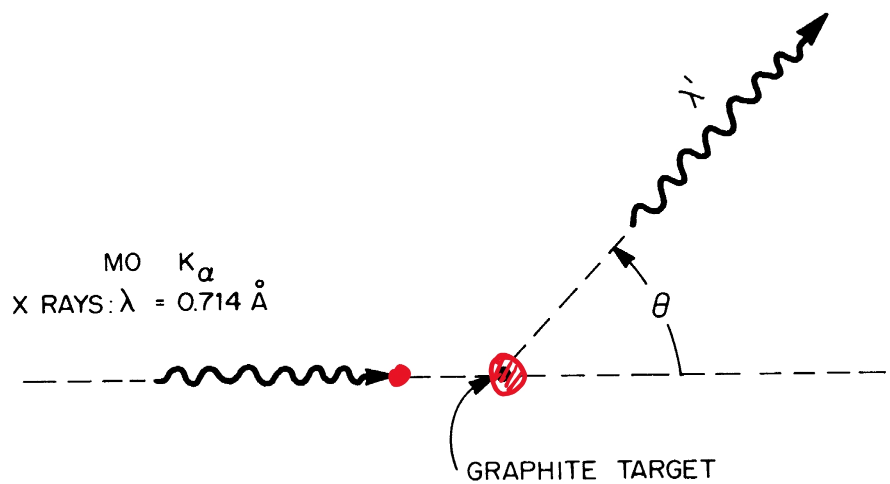
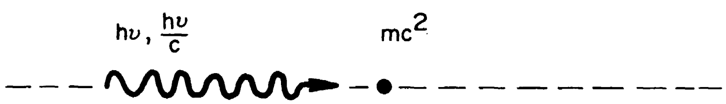


FIGURE 8.2. Compton measured the intensity of scattered photons as a function their wavelength λ′ at various scattering angles θ. Incident radiation was molybdenu K_α X rays, having a wavelength $\lambda = 0.714$ Å.

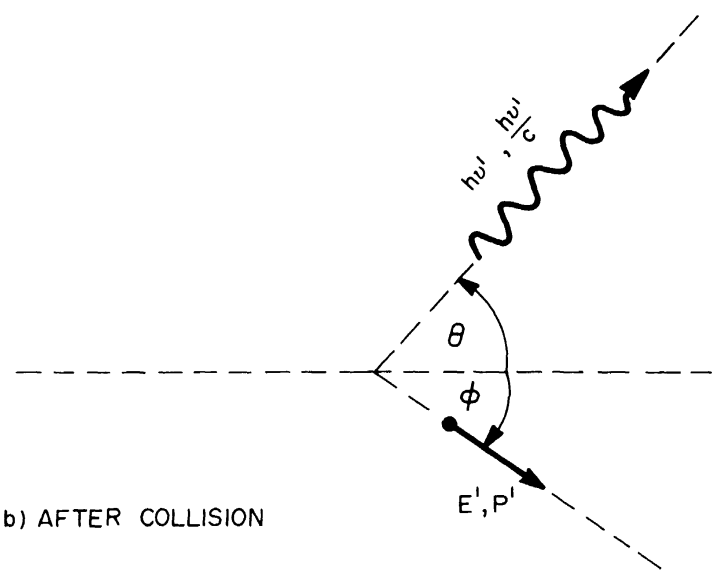
Kinematics in Compton Scattering

The **energy transfer** in Compton scattering may be derived as the following:

- Assuming that the **electron binding energy is small** compared with the energy of the incident photon – **elastic scattering**.
- Write out the **conservation of energy and momentum**:



(a) BEFORE COLLISION



(b) AFTER COLLISION

Conservation of energy

$$h\nu + mc^2 = h\nu' + E'$$

Conservation of momentum

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + P'\cos\varphi$$

$$\frac{h\nu'}{c}\sin\theta = P'\sin\varphi$$

Energy Transfer in Compton Scattering

If we assume that the electron is free and at rest, the scattered gamma ray has an energy

Initial photon energy, *v*: photon frequency

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos\theta)},$$

mass of electron

Scattering angle

and the photon transfers part of its energy to the electron (assumed to be at rest before the collision), which is known as the ***recoil electron***. Its energy is simply

$$E_{recoil} = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos(\theta))}$$

assuming the binding energy of the electron is negligible.

In the simplified elastic scattering case, there is a one-to-one relationship between scattering angle and energy loss!!

Reading: Page 51, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Energy Transfer in Compton Scattering

- The **maximum energy carried by the recoil electron** is obtained by setting θ to 180°,

$$E_{\max} = \frac{2h\nu}{2 + mc^2/h\nu}$$

- The maximum energy transfer is exemplified by the **Compton edge** in measured gamma ray energy spectra.

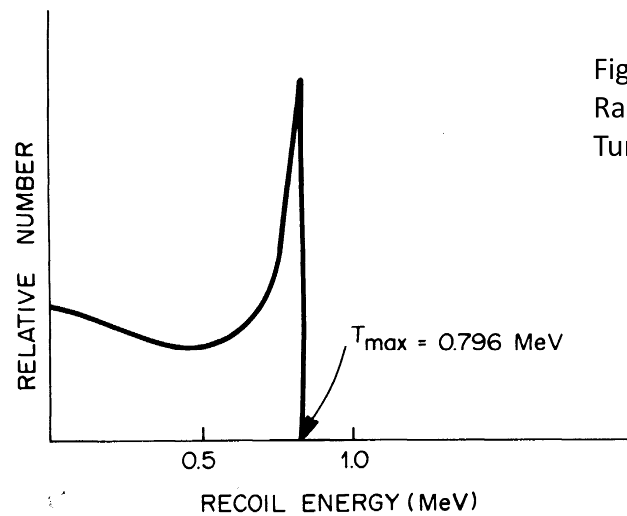
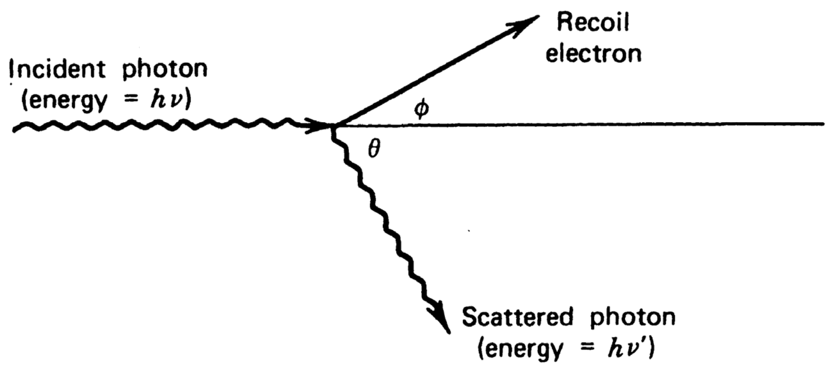


FIGURE 8.5. Relative number of Compton recoil electrons as a function of their en
for 1-MeV photons.

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p180.

Derivation of the Relationship Between Scattering Angle and Energy Loss

The relation between energy the scattering angle and energy transfer are derived based on the ***conservation of energy and momentum***:



$$\vec{p}_{hv} + \vec{p}_{e} = \vec{p}_{hv'} + \vec{p}_{e'}$$

$$E_{hv} + E_{e} = E_{hv'} + E_{e'}$$

Are those terms truly zero?

Compton Scattering with Non-stationary Electrons – Doppler Broadening

- ☞ It is so far assumed that (a) the ***electron is free and stationary*** and (b) the ***incident photon is unpolarized***.

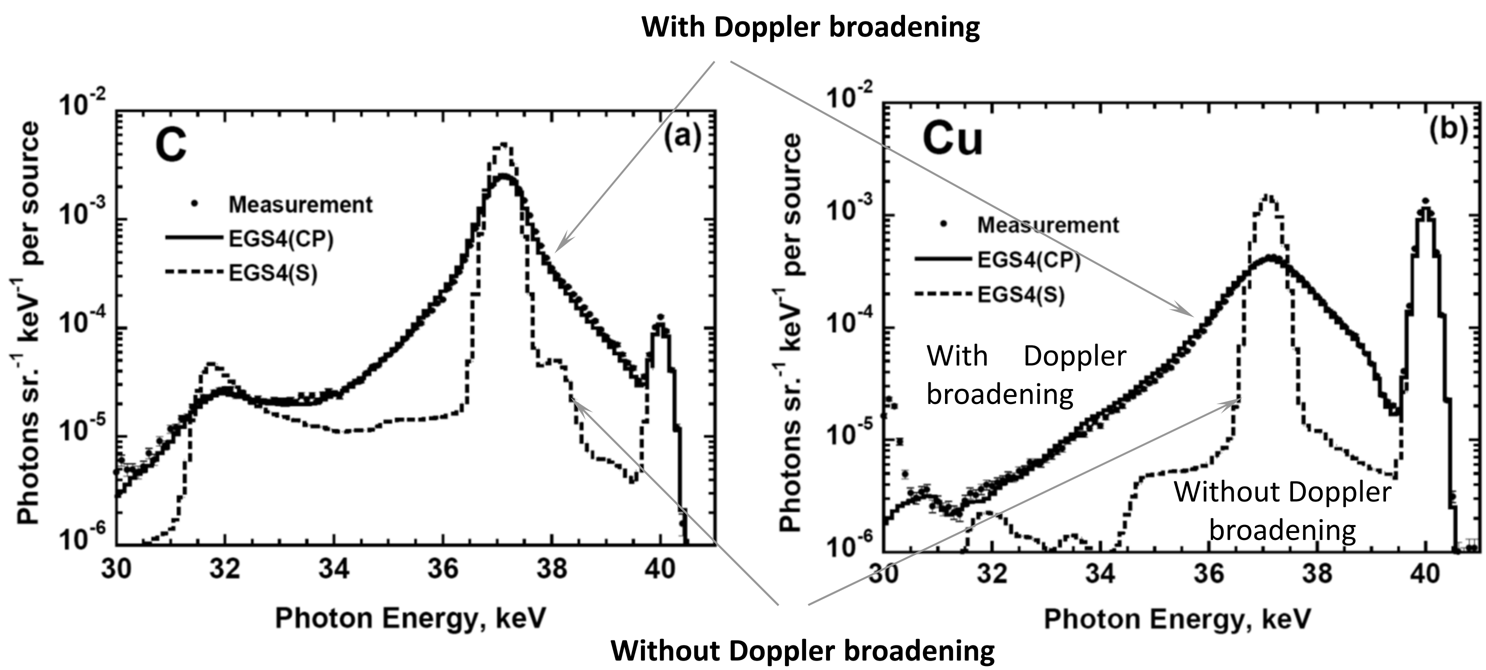
- ☞ When an incident photon is reflected by a ***non-stationary electron***, for example an bond electron, an extra uncertainty is added to the energy of the scattered photon. This extra uncertainty is called **Doppler broadening**.

$$hv' = \frac{hv}{1 + \frac{hv}{m_0 c^2}(1 - \cos(\theta))} \pm \boxed{\sigma(hv')}$$

The one-to-one relationship between scattering angle and energy loss holds only when incident photon energy is far greater than the bonding energy of the electron…

Compton Scattering with Non-stationary Electrons – Doppler Broadening

Comparison of the energy spectra for the photons scattered by C and Cu samples. $E_{h\nu}$=40keV, θ=90 degrees



The **Doppler broadening** is stronger in Cu than in C because of the Cu electrons have greater bonding energy.

Angular Distribution of the Scattered Gamma Rays

The **differential scattering cross section** of per electron is given by the **Klein-Nishina** formula:

$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2\left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2\left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right), \quad (m^2 sr^{-1})$$

$where$

$\alpha = \frac{hv}{m_0c^2}$ and $r_e = \frac{k_0e^2}{m_0c^2}$ is the classic electron radius (2.818×10^{-15} m)

Angular Distribution of the Scattered Gamma Rays

The differential scattering cross section per electron – **the probability of a photon scattered into a unit solid angle around the a given scattering angle θ, when the incident photon is passing normally through a thin layer of scattering material that contains one electron per unit area.**

$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2\left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2\left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right)(m^2sr^{-1})$$

where $\alpha = \frac{hv}{m_0c^2}$ and $r_e = \frac{k_0e^2}{m_0c^2}$ is the classic electron radius (2.818×10^{-15} m).

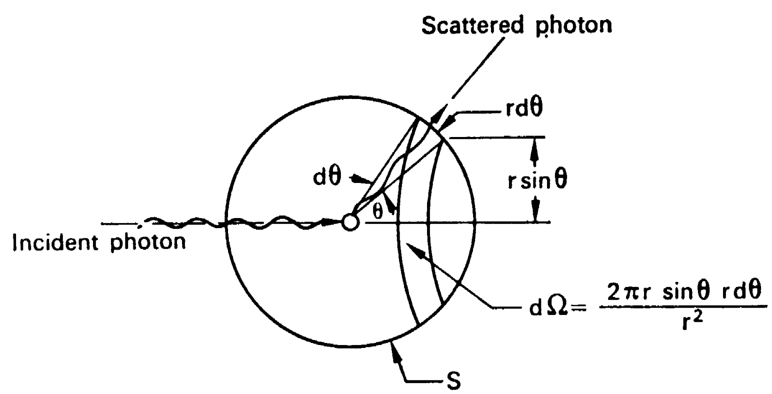
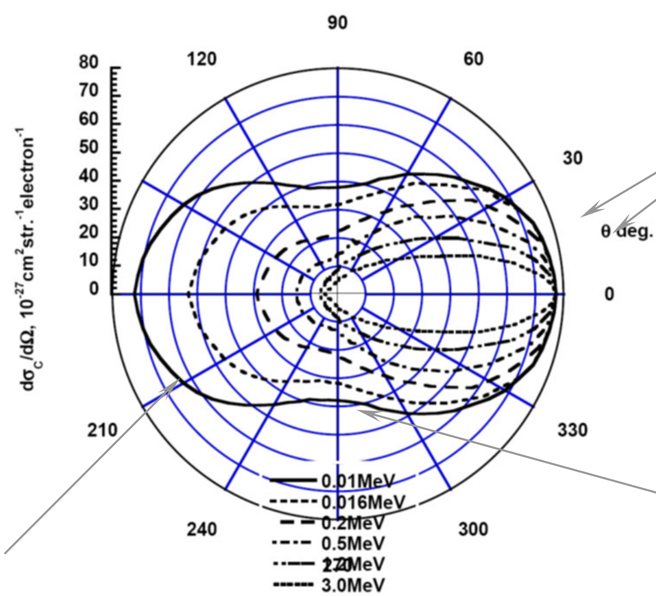


FIG. 5.15. Compton scattering diagram to illustrate differential scattering cross section. S is a sphere of unit radius whose center is the scattering electron.

Angular Distribution of the Scattered Gamma Rays



Radial distance represents the differential cross section.

Incident photons with ***higher energies*** tend to scatter with smaller angles (forward scattering).

Incident photons with ***lower energies*** (a few hundred keV) have a greater chance of undergoing large-angle scattering (backscattering).

The higher the energy carried by an incident gamma-ray, the more likely that the gamma-ray undergoes forward scattering …

Total Compton Collision Cross Section for an Electron

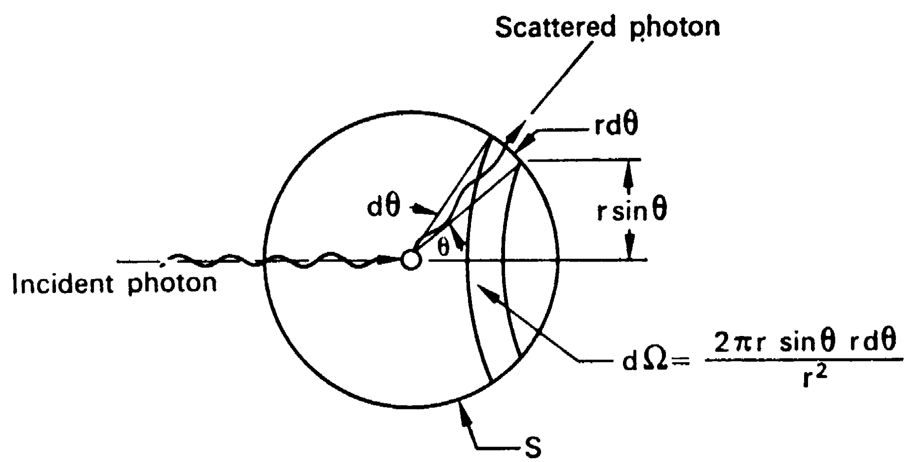
Compton Collision Cross Section is defined as the total cross section per electron for Compton scattering. It can be derived by integrating the differential cross section, $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$, over 4 π solid angle.

Since

$$d\Omega = 2\pi \sin\theta \, d\theta,$$

then the Compton scattering cross section per electron is given by

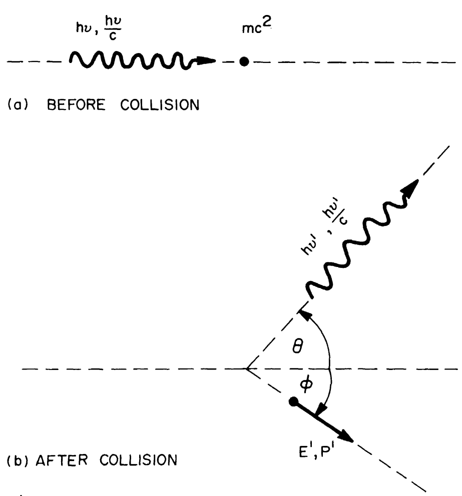
$$\sigma = 2\pi \int_{\Omega} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \cdot \sin\theta \cdot d\theta \ \ (cm^2)$$



Note that the Compton scattering cross section per electron is given in unit of cm^2.

Energy Distribution of Compton Recoil Electrons

Given the Klein-Nishina formula, how do we derive the **energy spectrum of recoil electrons**? In other words, how do we derive the probability of a gamma-ray undergoing a Compton scatting and transferring an energy falling into an energy window, $E_{recoil} \in \left[E' - \frac{1}{2}\Delta E, E' + \frac{1}{2}\Delta E\right]$?



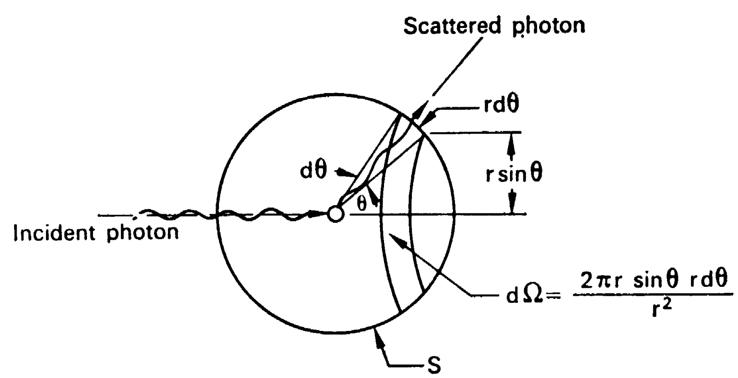


FIG. 5.15. Compton scattering diagram to illustrate differential scattering cross section. S is a sphere of unit radius whose center is the scattering electron.

$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2 \left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2 \left(\frac{1+\cos^2\theta}{2}\right)\left(1 + \frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right)(cm^2 sr^{-1})$$

Energy Distribution of Compton Recoil Electrons

Klein-Nishina formula can be used to derive the **energy spectrum of recoil electrons** as the following:

$$\frac{d\sigma}{dE_{recoil}} = \frac{d\sigma}{d\Omega} \cdot \frac{d\Omega}{d\theta} \cdot \frac{d\theta}{dE_{recoil}} (\mathrm{m}^2 \cdot \mathrm{keV}^{-1})$$

If a gamma-ray underwent a Compton Scattering, then probability of the gamma-ray transferring a given amount of energy falling into a small energy window, $E_{recoil} \in \left[E' - \frac{1}{2}\Delta E, E' + \frac{1}{2}\Delta E\right]$ would be proportional to

$$p \propto \Delta E \cdot \left(\frac{d\sigma}{dE_{recoil}}\right)\Bigg|_{E'}$$

Energy Distribution of Compton Recoil Electrons

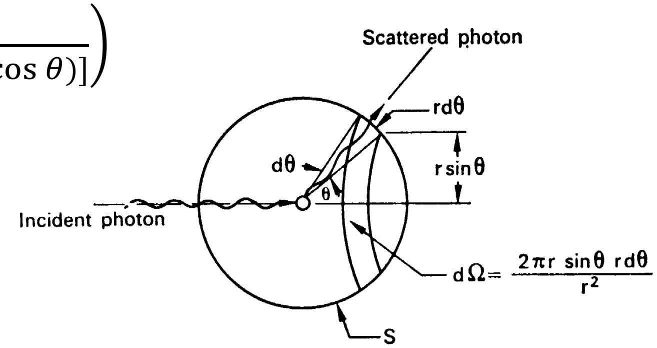
The energy distribution for the recoil electrons could be derived with the following differential cross section

$$\frac{d\sigma}{dE_{recoil}} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{d\theta}\frac{d\theta}{dE_{recoil}}$$

The three partial derivative terms on the right-hand side of the equation can be derived from the following relationships:

- From Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2\left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2\left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right)$$



- From Compton equation:

$$E_{recoil} = hv - hv' = hv - \frac{hv}{1+\frac{hv}{m_0c^2}(1-\cos\theta)} \quad \Rightarrow$$

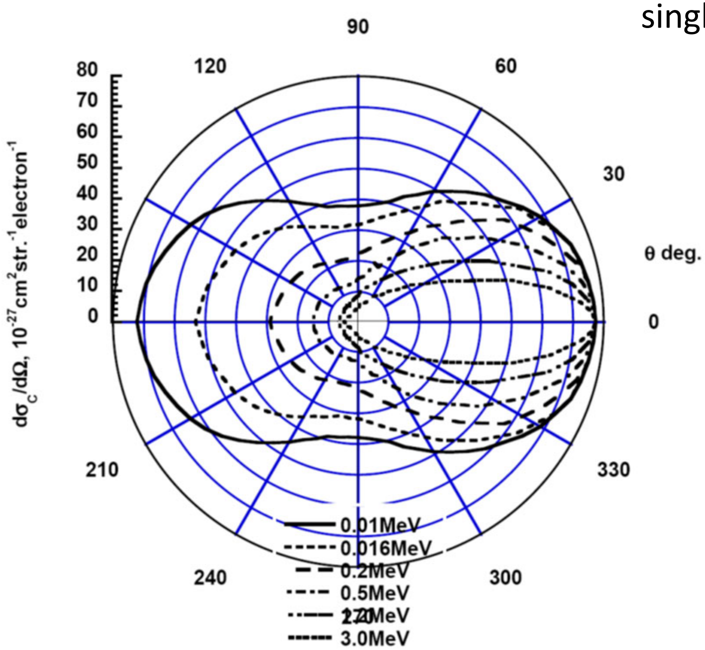
$$\frac{d\theta}{dE_{recoil}} = -\frac{m_0c^2}{(hv-E_{recoil}^2)\cdot\sin\theta} = \frac{m_0c^2}{(hv)^2\cdot\sin\theta}\left[1+\frac{hv}{m_0c^2}(1-\cos\theta)\right]^2$$

- From the known scattering geometry: $d\Omega = 2\pi\sin\theta\, d\theta \;\Rightarrow\; \frac{d\Omega}{d\theta} = 2\pi\sin\theta$

Energy Distribution of Compton Recoil Electrons

Remember that the maximum amount of energy that a photon can transfer to an electron in a single Compton scattering is given by:

$$E_{\max} = \frac{2h\nu}{2 + mc^2/h\nu}$$



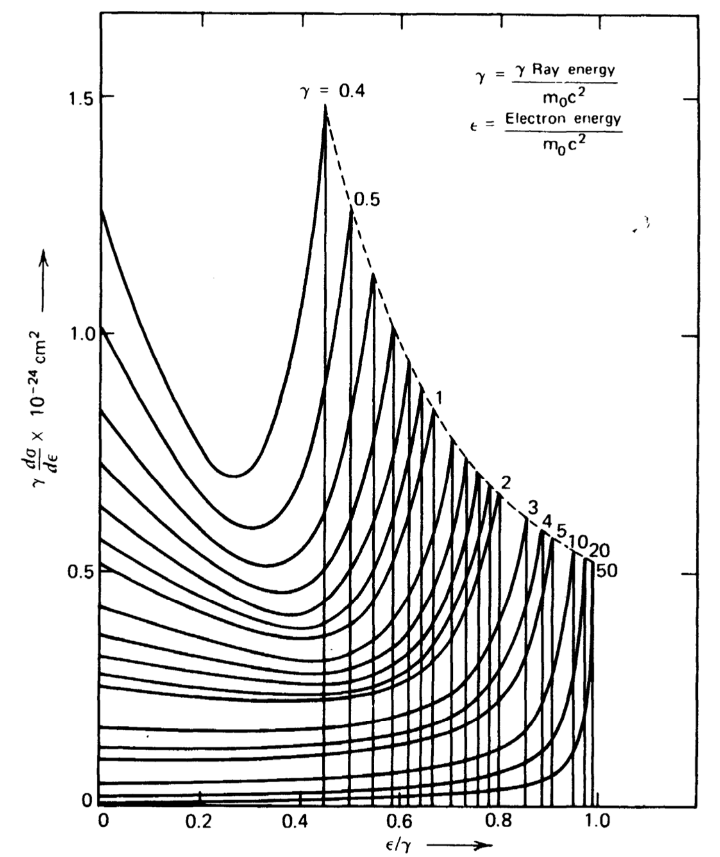


Figure 10.1 Shape of the Compton continuum for various gamma-ray energies. (From S. M. Shafroth (ed.), *Scintillation Spectroscopy of Gamma Radiation.* Copyright 1964 by Gordon & Breach, Inc. By permission of the publisher.)

- The energy distribution of the recoil electrons derived using the Klein-Nishina formula is closely related to **the energy spectrum measured with "small" detectors** (in particular, the so-called **Compton continuum**).

From Page 311, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Average fraction of energy transfer to the recoil electron through a single Compton Collision

Average recoil electron energy E_{avg_recoil} is of special interest for dosimetry is the, since it is an approximation of the **radiation dose delivered by each photon through a single Compton scattering interaction**.

The **average fraction of energy transfer to the recoil electron** through a single Compton scattering is given by

$$\frac{E_{avg\_recoil}}{h\nu} = \int_{E_{recoil}} \frac{E_{recoil}}{h\nu} \cdot \left[\left(\frac{d\sigma}{dE_{recoil}}\right)/\sigma\right] \cdot dE_{recoil} \ ,$$

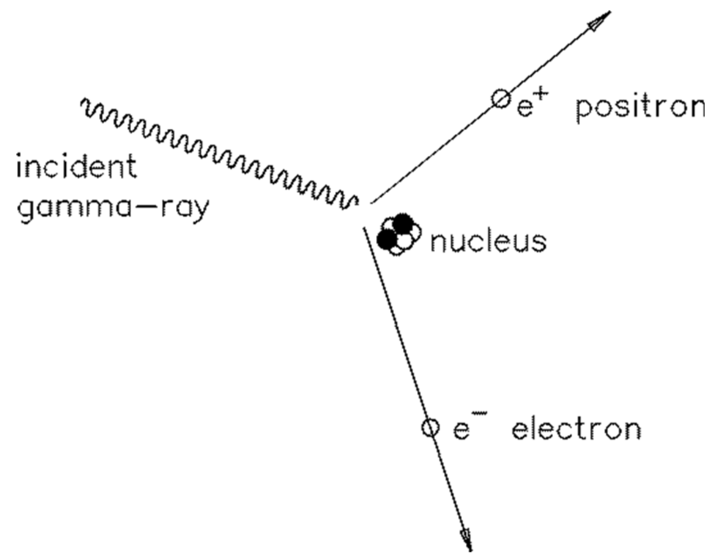
where σ is the Compton scattering cross section per electron and is given by

$$\sigma = 2\pi \int_{\Omega} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \cdot \sin\theta \cdot d\theta \ \ (m^2) \ .$$

Pair Production

Definition:

Pair production refers to the creation of an electron-positron pair by an incident gamma ray in the vicinity of a nucleus.



Characteristics

☞ The minimum energy required is

$$E_\gamma \geq 2m_e c^2 + \frac{2m_e^2 c^2}{m_{nucleus}} \approx 2m_e c^2 = 1.022 MeV$$

☞ The process is more probable with a ***heavy nucleus*** and incident ***gamma rays with higher energies***.

☞ The positrons emitted will soon **annihilate** with ordinary electrons near by and produces two 511keV gamma rays.

Photonuclear Reaction

☞ A photon can be absorbed by an atomic nucleus and knock out a nucleon. This process is called **photonuclear reaction**. For example,

$$^{9}_{4}Be + hv \Rightarrow {}^{8}_{4}Be + {}^{1}_{0}n, \quad Q-value: -1.666MeV$$
$$^{2}_{1}H + hv \Rightarrow {}^{1}_{1}H + {}^{1}_{0}n, \quad Q-value: -2.226MeV$$

☞ The photon **must possess enough energy to overcome the nuclear binding energy**, which is generally several MeV.

☞ The threshold, or the minimum photon energy required, for (γ,p) reaction is generally higher than that for (γ,n) reactions. Since the repulsive Coulomb barrier that a proton must overcome to escape from the nucleus.

☞ Other nuclear reactions are also possible, such as (γ, 2n), (γ, np), (γ, α) and photon-induced fission reaction.

Interaction of Photons in Matter

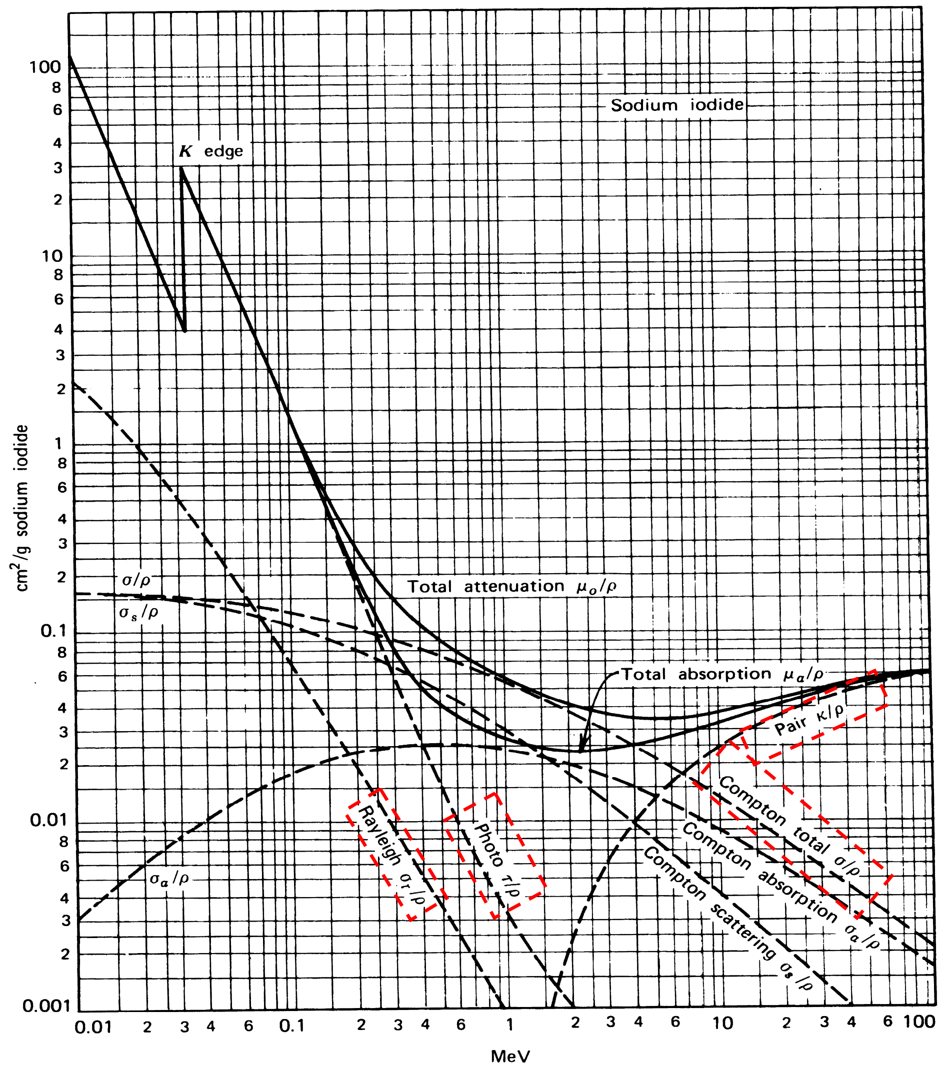
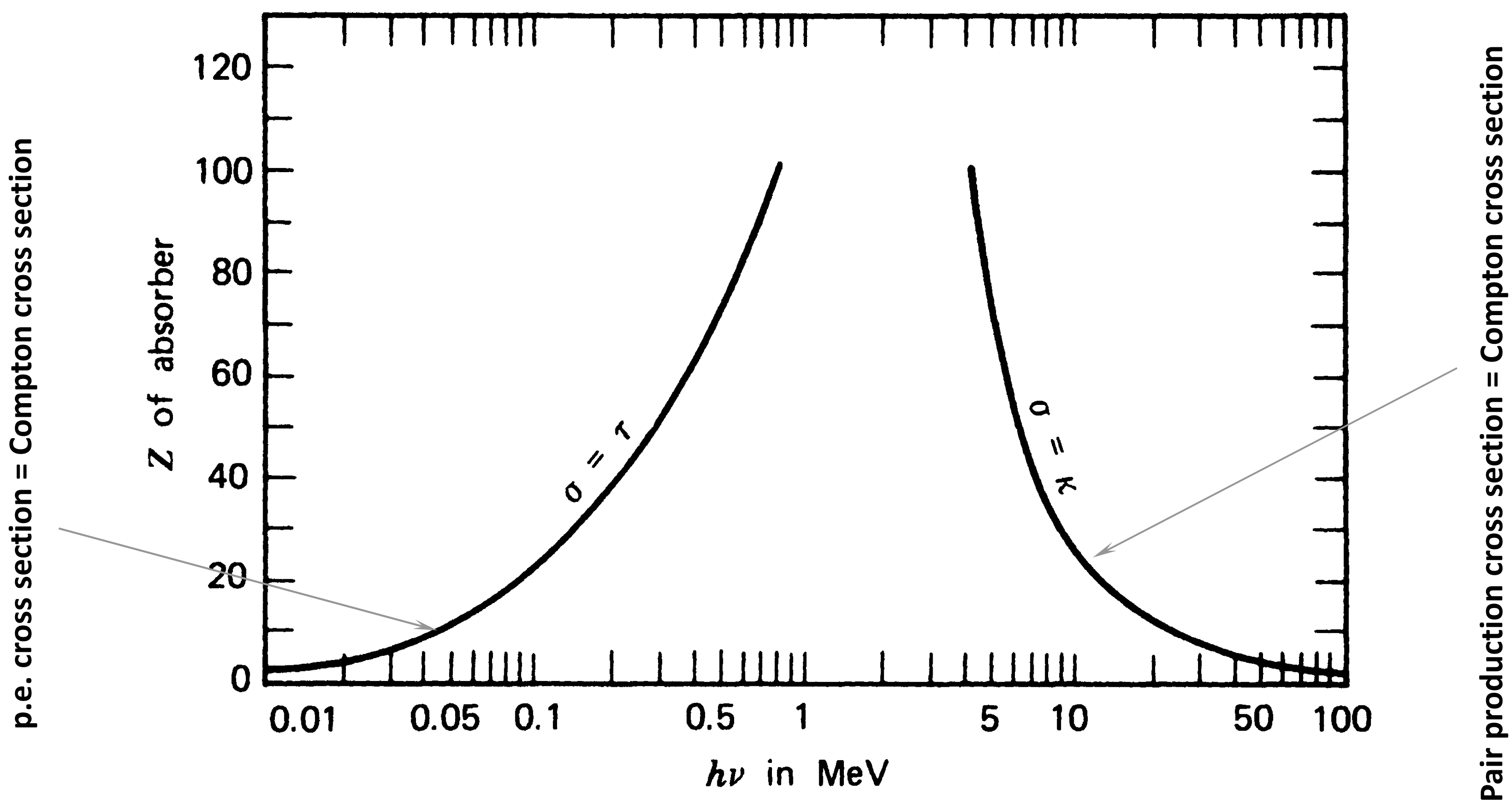


Figure 2.18 Energy dependence of the various gamma–ray interaction processes in sodium iodide. (From *The Atomic Nucleus* by R. D. Evans. Copyright 1955 by the McGraw-Hill Book Company. Used with permission.)

From Page 50, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

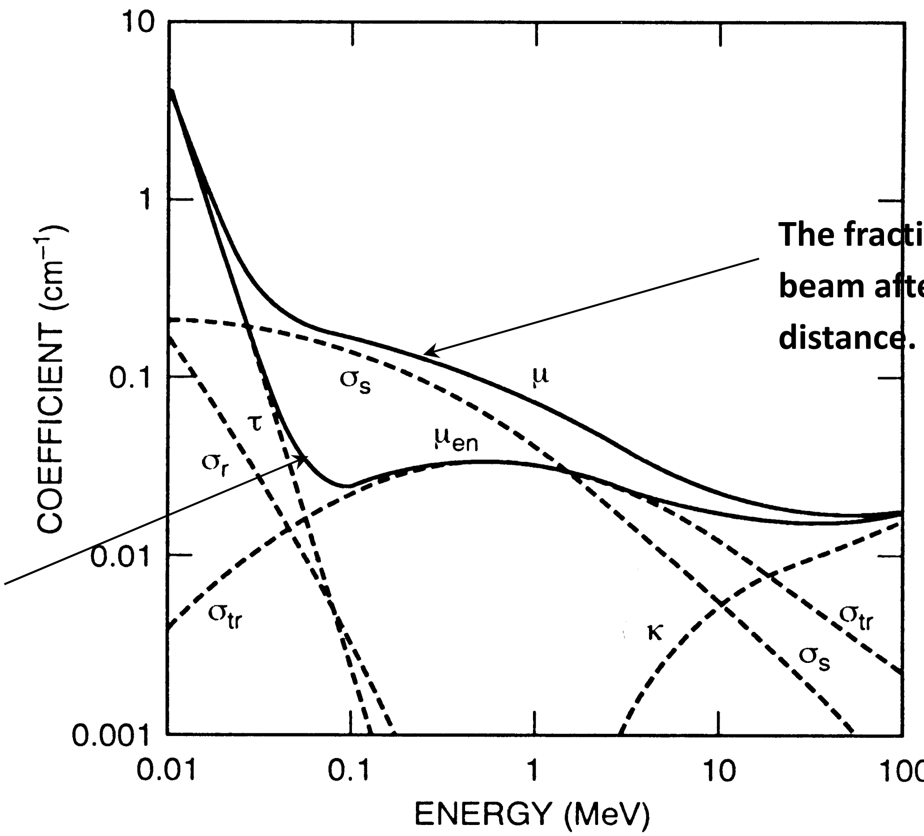
The Relative Importance of the Three Major Types of X and Gamma Ray Interactions



Page 52, Chapter 2 in Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

What are the energy transfer coefficient and energy absorption coefficient? and their implication to dose calculation?

Comparison Between Linear Attenuation Coefficient and Energy Absorption Coefficient



The fraction of photons removed from the beam after traveling through a unit distance.

The fraction of energy carried by the photons being absorbed in material.

FIGURE 8.13. Linear attenuation and energy-absorption coefficients as functions of energy for photons in water.

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p195

Linear Attenuation Coefficients for Gamma-Rays

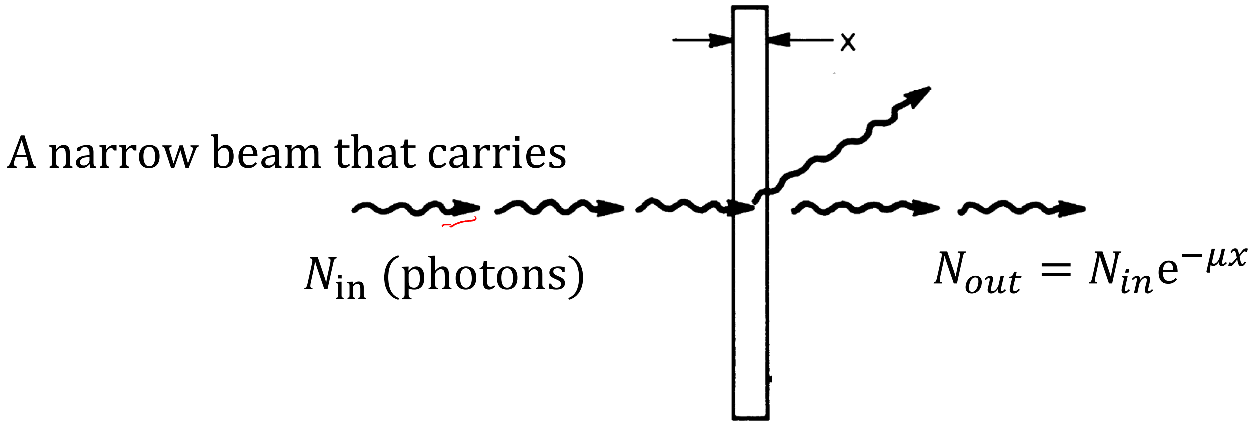


FIGURE 8.7. Illustration of "good" scattering geometry for measuring linear attenuatior coefficient μ. Photons from a *narrow* beam that are absorbed or scattered by the ab sorber do not reach a small detector placed in beam line some distance away.

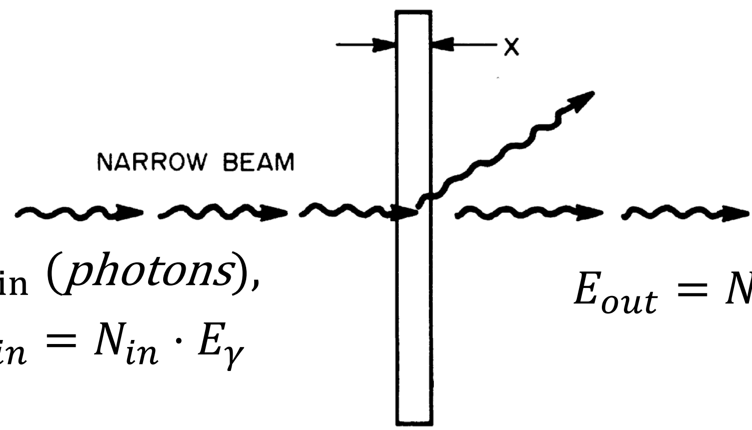
☞ Considering a thin slab of absorbing media, the number of photons being removed/attenuated from the beam through the distance x is given by

$$dN = N_{in} - N_{out} = N_{in}(1 - e^{-\mu x}) = \mu N_{in} dx$$

where $\boldsymbol{\mu}$ is the linear attenuation coefficient (the probability that a photon is removed from the beam by the absorbing media per unit distance it travels)

$$\mu = \tau_{photoelectric} + \sigma_{Compton} + \kappa_{pair}$$

Linear Attenuation Coefficients for Gamma-Rays



N_{in} (*photons*),

$E_{in} = N_{in} \cdot E_{\gamma}$

$E_{out} = N_{out} \cdot E_{\gamma} = N_{in} \cdot \mathrm{e}^{-\mu x} \cdot E_{\gamma}$

Instead, if we consider the gamma-ray beam as a flow of energy, where $E_{in} = N_{in} \cdot E_{\gamma}$, for each unit distance that the beam transmitted through, what is the fraction of the incident energy that is transferred to the secondary electrons within the absorbing media?

Linear Attenuation Coefficients

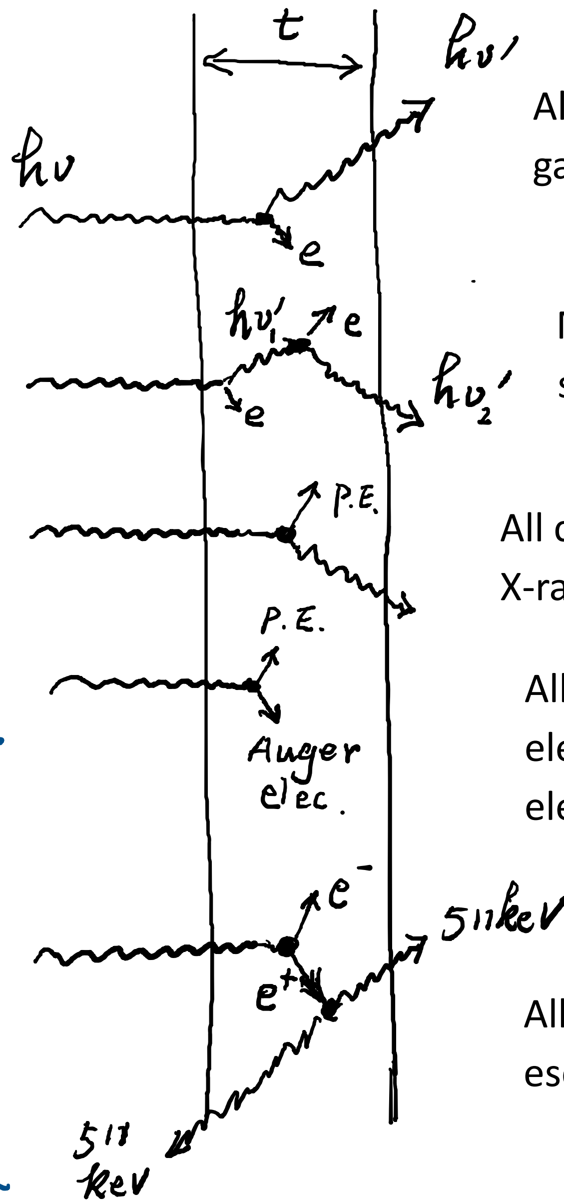
Linear attenuation coefficient μ is the probability that a photon is attenuated (removed from the beam) in the absorber per unit distance it travels,

$$\mu = \tau_{photoelectric} + \sigma_{Compton} + \kappa_{pair}$$

Can we define a **linear energy transfer coefficient**, μ_{tr}, which is the fraction of the energy carried by the incident beam that is subsequently transferred to the absorber per unit distance the beam is transmitted through?

$$\mu_{tr} = \tau_{photoelectric} \times ? + \sigma_{Compton} \times ?? + \kappa_{pair} \times ???$$

Energy Transfer by a Gamma-Ray Beam



Compton scattering

All Compton scattered gamma rays escaped

Multiple Compton scattering ignored

Photoelectric effect

All characteristic X-rays escaped

All photoelectrons, auger electrons and Compton recoil electrons are absorbed

Pair production

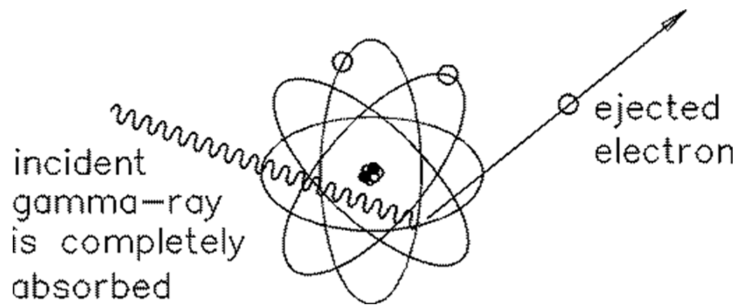
All annihilation gamma rays escaped

What is Energy Transfer Coefficients?

For a parallel beam of monochromatic gamma rays transmitting through a unit distance in an absorbing material, **the energy-transfer coefficient is the fraction of energy that was originally carried by the incident gamma-ray beam and transferred into the kinetic energy of secondary electrons per unit distance of travel**.

Photoelectric Effect

In **photoelectric** process, an incident photon transfer its energy to an orbital electron, causing it to be ejected from the atom.



$$E_{e^-} = h\nu - E_b$$

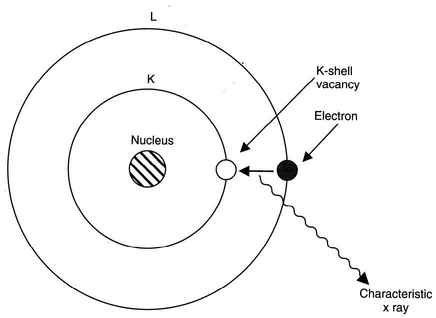
h is the Planck's constant

ν is the frenquency of the photon

- Photoelectric interaction is **with the atom in a whole** and can not take place with free electrons.
- Photoelectric effect **creates a vacancy in one of the electron shells**, which leaves the atom at an excited state.

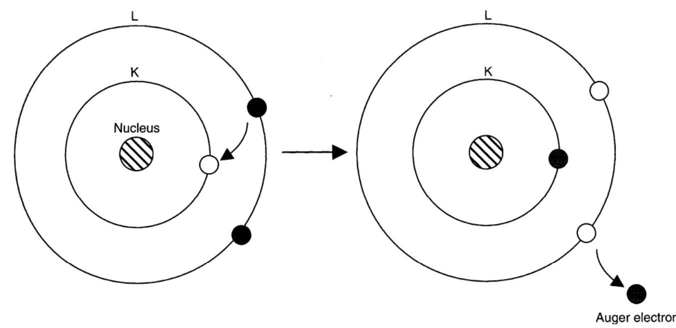
Relaxation Process after Photoelectric Effect

☞ The excited atoms will **de-excite** through one of the following processes:



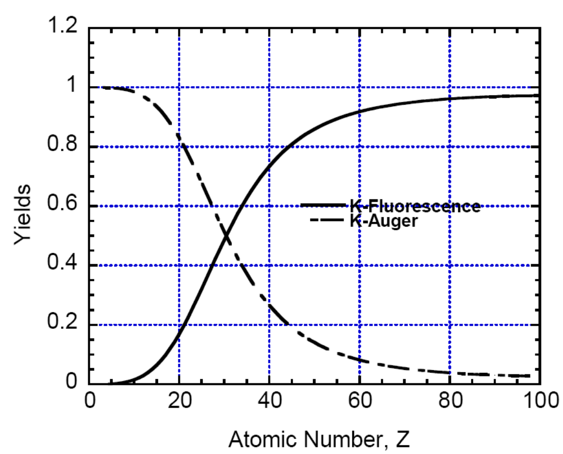
Generation of characteristic X-rays

Competing Processes



Generation of Auger electrons

☞ **Auger electron** emission dominates in **low-Z** elements. **Characteristic X-ray** emission dominates in **higher-Z** elements.



Energy-Transfer Coefficients through Photoelectric Effect

Based on these considerations, we can define the **mass energy transfer coefficient**, which is the fraction of photons that interact by photoelectric absorption per g·cm^{-2},

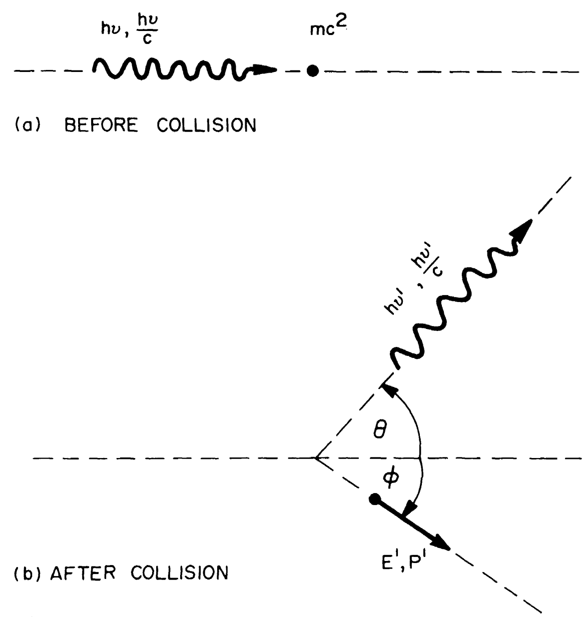
$$\tau_{tr} = \tau\left(1 - \frac{\delta}{h\upsilon}\right)$$

δ is the fraction of the gamma ray energy got converted into characteristic x-rays following the photoelectric interaction.

Note that the photoelectrons and Auger electrons may also lead to secondary photon emission through Bremsstrahlung. So the energy transfer coefficient defined above does not fully describe **energy absorption** in the slab. We will return to this point later.

Energy-Transfer Coefficients Through Compton Scattering

For Compton scattering of monoenergetic photons, the mass energy transfer coefficient



$$\sigma_{tr} = \sigma\left(\frac{E_{avg}}{h\upsilon}\right)$$

The factor $E_{avg}/h\nu$ is the fraction of the incident photon energy that is converted into the initial kinetic energy of Compton electrons.

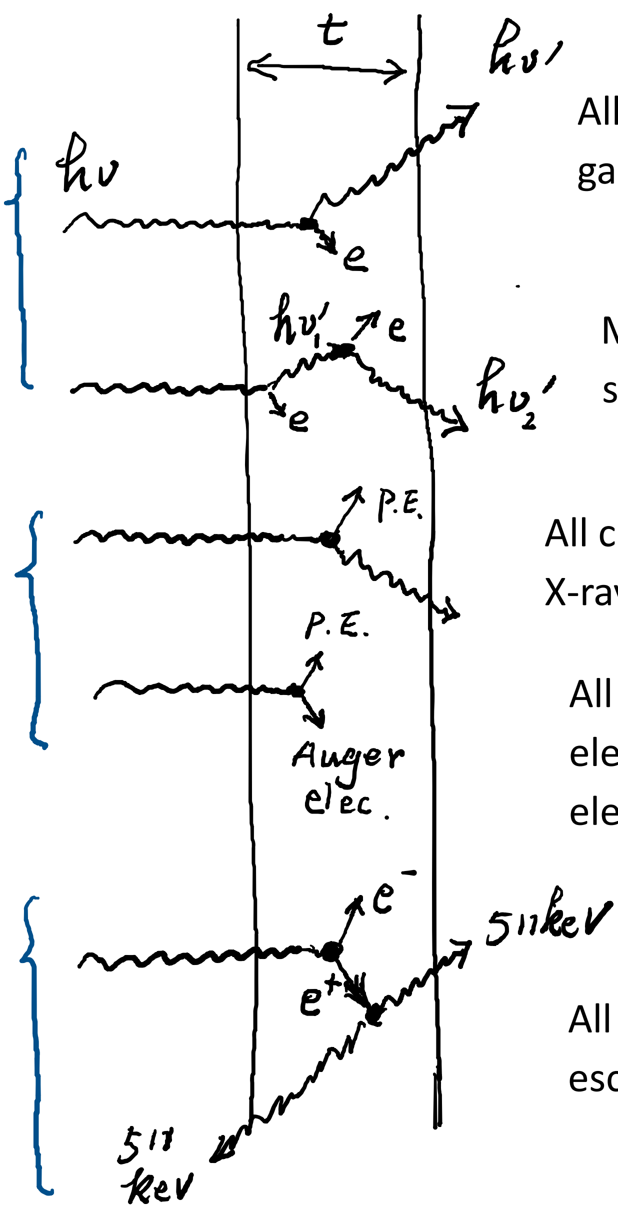
As with the photoelectric effect, the above mass energy transfer coefficient does not take into account the subsequent photon emission due to bremsstrahlung.

Energy Transfer by a Gamma Ray Beam

Compton scattering

Photoelectric effect

Pair production



All Compton scattered gamma rays escaped

Multiple Compton scattering ignored

All characteristic X-rays escaped

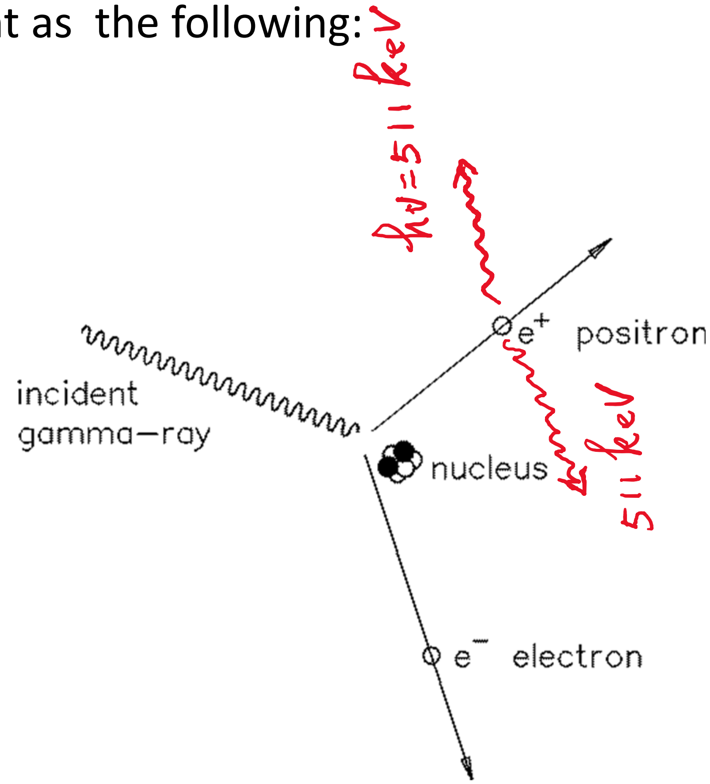
All photoelectrons, auger electrons and Compton recoil electrons are absorbed

All annihilation gamma rays escaped

Energy-Transfer Coefficient through Pair Production

For the **pair production process**, the initial energy carried by the electron-positron pair is $h\nu$-$2mc^2$. Therefore, the energy transfer coefficient for pair production is related to the mass attenuation coefficient as the following:

$$\kappa_{tr} = \kappa\left(1 - \frac{2m_e c^2}{h\nu}\right)$$



Energy-Transfer Coefficient

The **total energy transfer coefficient** is given by

The fraction of energy that is carried away by characteristic x-rays following the photoelectric effect.

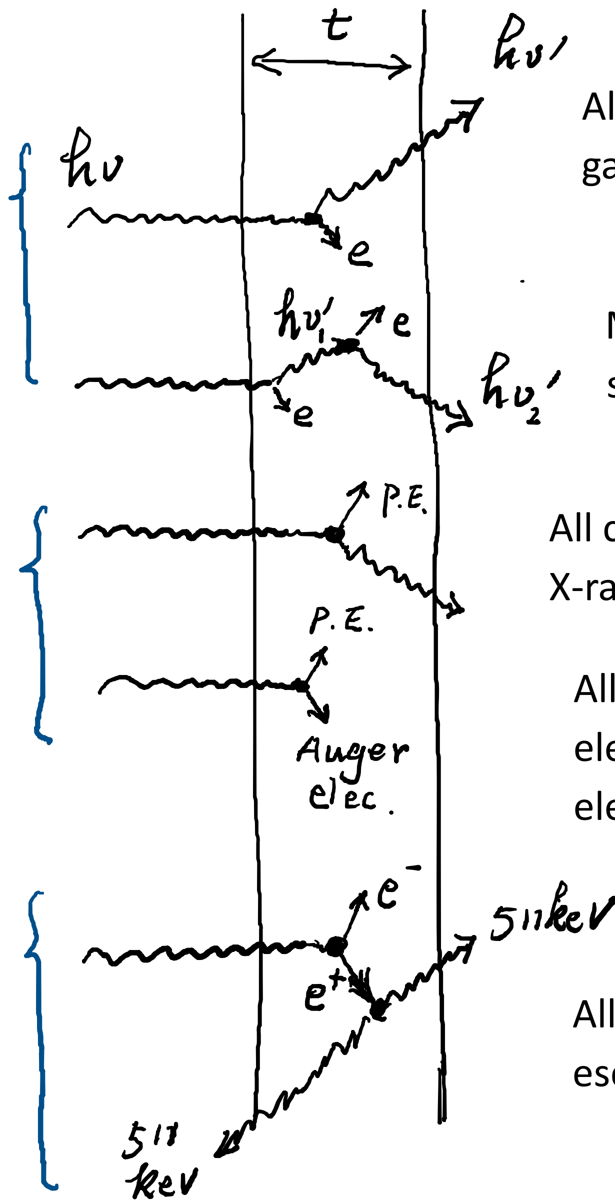
The fraction of energy that is carried away by the two 511keV gamma-rays generated during the annihilation of the positron.

$$\mu_{tr} = \tau\left(1 - \frac{\delta}{h\upsilon}\right) + \sigma\left(\frac{E_{avg}}{h\upsilon}\right) + \kappa\left(1 - \frac{2mc^2}{h\upsilon}\right)$$

The average fraction of energy that is transferred to recoil electron through Compton scattering.

For a parallel beam of monochromatic gamma rays transmitting through a unit distance in an absorbing material, **the energy-transfer coefficient is the fraction of energy that was originally carried by the incident gamma ray beam and subsequently transferred into the kinetic energy of secondary electron** inside the absorber.

Energy Transfer by a Gamma Ray Beam



Compton scattering

All Compton scattered gamma rays escaped

Multiple Compton scattering ignored

Photoelectric effect

All characteristic X-rays escaped

All photoelectrons, auger electrons and Compton recoil electrons are absorbed

Pair production

All annihilation gamma rays escaped

What is Energy Absorption Coefficients?

The **total mass energy transfer coefficient** is given by

$$\mu_{tr} = \tau\left(1 - \frac{\delta}{h\upsilon}\right) + \sigma\left(\frac{E_{avg}}{h\upsilon}\right) + \kappa\left(1 - \frac{2mc^2}{h\upsilon}\right)$$

Consider the fraction of energy that may be carried away by the subsequent bremsstrahlung photons, one can define the **mass energy-absorption coefficient** as

$$\boxed{\mu_{en} = \mu_{tr}(1 - g)}$$

where g is the average fraction of energy of the initial kinetic energy transferred to electrons that is subsequently emitted as bremsstrahlung photons.

For a parallel beam of monochromatic gamma rays transmitting through a unit distance in an absorbing material, **the energy-absorption coefficient is the fraction of energy that was originally carried by the incident gamma ray beam and eventually absorbed** inside the absorber.

Energy Loss by Bremsstrahlung

- For beta particles to stop in a given medium, the **fraction of energy loss** by **Bremsstrahlung** process is approximately given by

$$f_{\beta} = 3.5 \times 10^{-4}\, Z E_{m},$$

where
f_{β} = the fraction of the incident beta energy converted into photons,
Z = atomic number of the absorber,
E_{m} = maximum energy of the beta particle, MeV.

Mass Energy Absorption Coefficient

Mass attenuation coefficient

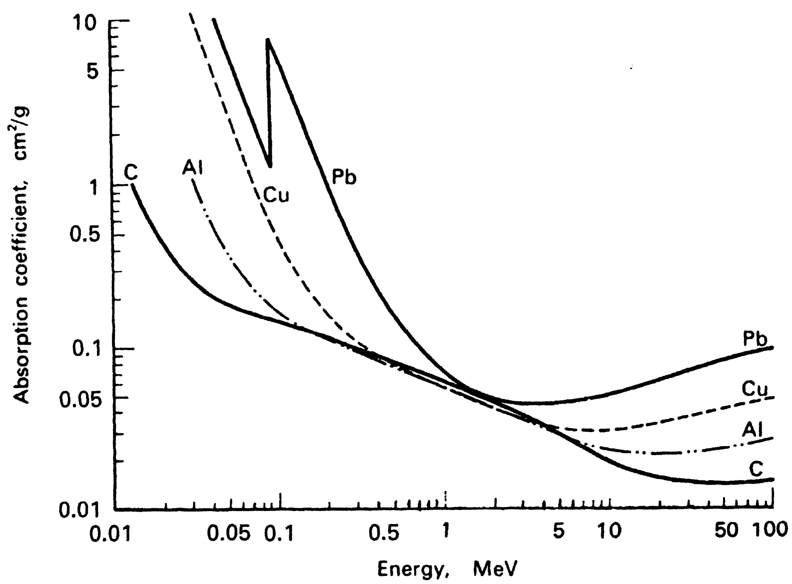


Figure 5.12. Curves illustrating the systematic variation of attenuation coefficient with atomic number of absorber and with quantum energy.

Mass energy absorption coefficient

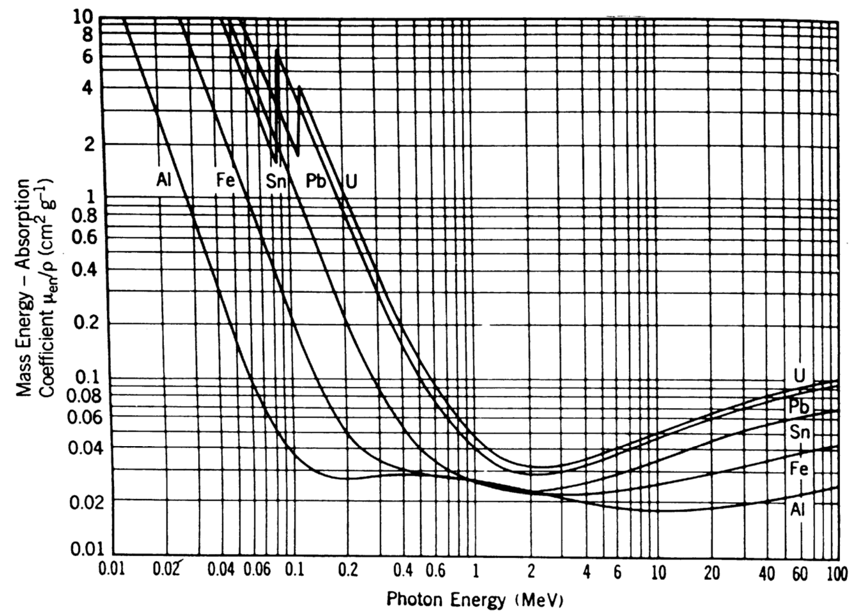


FIGURE 8.11. Mass energy-absorption coefficients for various elements. [Reprinted with permission from K. Z. Morgan and J. E. Turner, eds., *Principles of Radiation Protection*, Wiley, New York (1967). Copyright 1967 by John Wiley & Sons.]

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p195

Mass Energy Absorption Coefficient

Mass attenuation coefficient

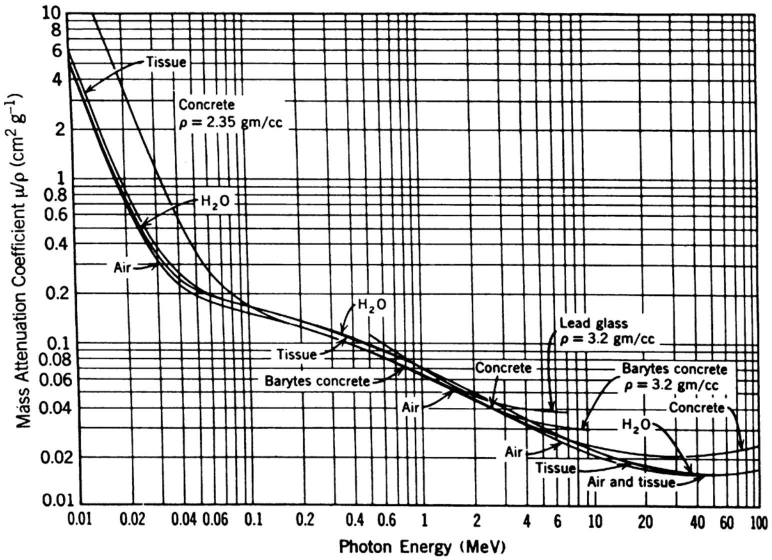


FIGURE 8.9. Mass attenuation coefficients for various materials. [Reprinted with permission from K. Z. Morgan and J. E. Turner, eds., *Principles of Radiation Protection*, Wiley, New York (1967). Copyright 1967 by John Wiley & Sons.]

Mass energy absorption coefficient

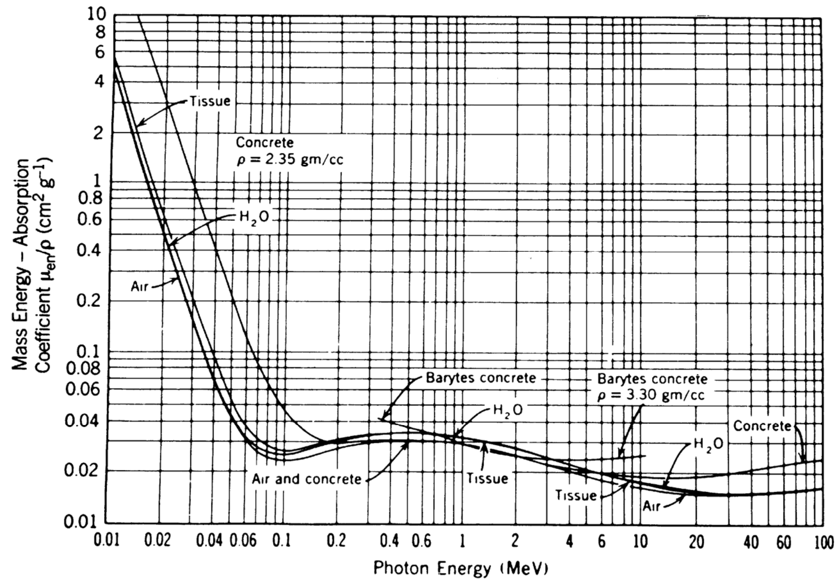


FIGURE 8.12. Mass energy-absorption coefficients for various materials. [Reprinted with permission from K. Z. Morgan and J. E. Turner, eds., *Principles of Radiation Protection*, Wiley, New York (1967). Copyright 1967 by John Wiley & Sons.]

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p195

Energy-Transfer and Energy Absorption Coefficients

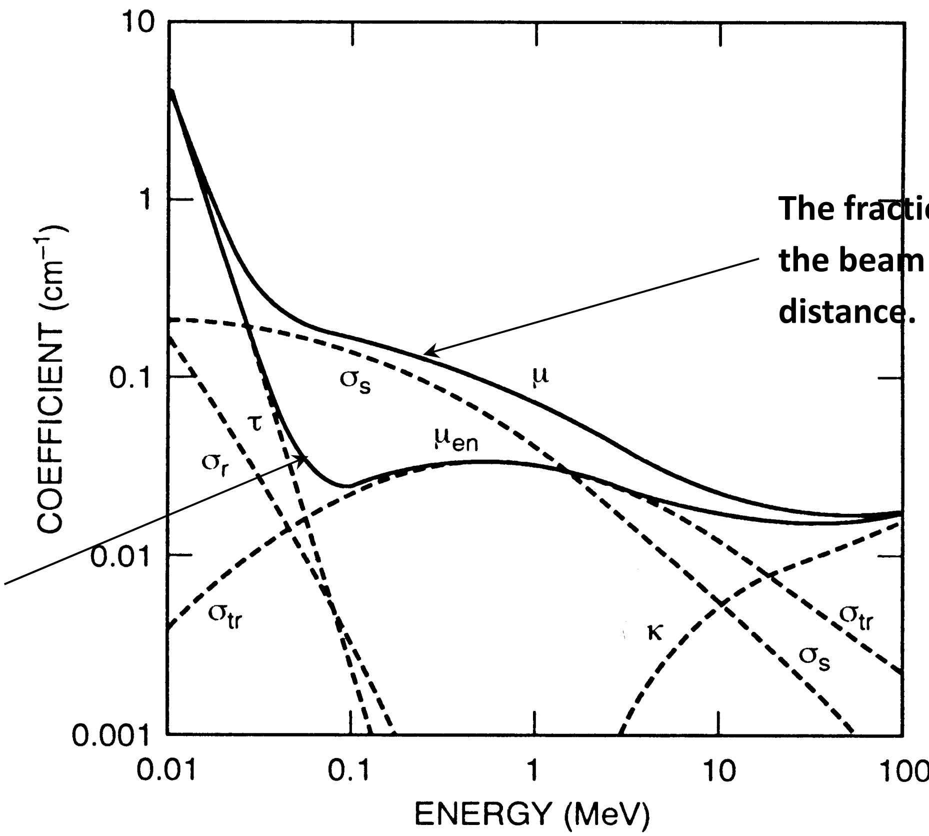
TABLE 8.3. Mass Attenuation, Mass Energy-Transfer, and Mass Energy-Absorption Coefficients ($cm^2\ g^{-1}$) for Photons in Water and Lead

Photon Energy (MeV)	Water			Lead		
	μ/ρ	μ_{tr}/ρ	μ_{en}/ρ	μ/ρ	μ_{tr}/ρ	μ_{en}/ρ
0.01	5.33	4.95	4.95	131.	126.	126.
0.10	0.171	0.0255	0.0255	5.55	2.16	2.16
1.0	0.0708	0.0311	0.0310	0.0710	0.0389	0.0379
10.0	0.0222	0.0163	0.0157	0.0497	0.0418	0.0325
100.0	0.0173	0.0167	0.0122	0.0931	0.0918	0.0323

Source: Based on P. D. Higgins, F. H. Attix, J. H. Hubbell, S. M. Seltzer, M. J. Berger, and C. H. Sibata, *Mass Energy-Transfer and Mass Energy-Absorption Coefficients, Including In-Flight Positron Annihilation for Photon Energies 1 keV to 100 MeV,* NISTIR 4680, National Institute of Standards and Technology, Gaithersburg, MD (1991).

- As expected, bremsstrahlung is relatively unimportant for photon energy of less that ~10MeV, whilst it accounts for a significant difference between the mass energy-transfer coefficient and the mass energy absorption coefficient.

Comparison Between Linear Attenuation Coefficient and Energy Absorption Coefficient



The fraction of photons removed from the beam after traveling through a unit distance.

The fraction of energy carried by the photons being absorbed in material.

FIGURE 8.13. Linear attenuation and energy-absorption coefficients as functions of energy for photons in water.

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p195

Calculation of Energy Transfer and Energy Absorption

For simplicity, we consider an idealized case, in which

- ☞ Photons are assumed to be monoenergetic and in broad parallel beam.
- ☞ Multiple Compton scattering of photons is negligible.
- ☞ Virtually all fluorescence and bremsstrahlung photons escape from the absorber.
- ☞ All secondary electrons (Auger electrons, photoelectrons and Compton electrons) generated are stopped in the slab.

Under these conditions, the transmitted energy intensity (the amount of energy transmitted through a unit area within each second) can be given by

$$\dot{\Psi} = \dot{\Psi}_0 e^{-\mu_{en} x}$$

Calculation of Energy Transfer and Energy Absorption

Assuming $\mu_{en}x \ll 1$, which is consistent with the thin slab approximation and the energy fluence rate carried by the incident gamma ray beam is $\dot{\Psi}_0 (J \cdot cm^{-2} \cdot s^{-1})$. Then the energy absorbed in the thin slab per second over a unit cross section area is given by

$$\Delta\Psi \approx \dot{\Psi}_0 \cdot \mu_{en} \cdot x \ \ (J \cdot cm^{-2} \cdot s^{-1})$$

The rate of energy absorbed in the slab of area A (cm^2) and thickness *x* is

$$A\dot{\Psi}_0 \mu_{en} x \ (J \cdot s^{-1})$$

Given the density of the material is ρ, the rate of energy absorption per unit mass (**Dose Rate**) in the slab is

$$\dot{D} = \frac{A(cm^2)\cdot\dot{\Psi}_0(J\cdot cm^{-2}\cdot s^{-1})\cdot\mu_{en}(cm^{-1})\cdot x\,(cm)}{\rho(g\cdot cm^{-3})\cdot A(cm^2)\cdot x(cm)},$$

$$\boxed{Dose\ rate\ in\ the\ absorber:\ \ \dot{D} = \dot{\Psi}_0 \frac{\mu_{en}}{\rho}\,(J \cdot g^{-1} \cdot s^{-1})}$$

Interactions of Neutrons with Matter

Reading Material:

- Chapter 5 in <<Introduction to Health Physics>>, Third edition, by Cember.
- Chapter 9 in <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.
- Chapter 2 in <<Radiation Detection and Measurements>>, Third Edition, by G. F. Knoll.

Interactions of Neutrons with Matters

Neutrons can interact with an atomic nuclei through

- **Elastic scattering**: the total kinetic energy is conserved – the energy loss by the neutron is equal to the kinetic energy of the recoil nucleus.

- **Inelastic scattering**: the nucleus absorbs some energy internally and is left to an excited state.

- **(Thermal) neutron capture**: the neutron is captured or absorbed by a nucleus, leading to a reaction such as (n,p), (n,2n), (n,α) or (n,γ). The reaction changes the atomic number and/or atomic mass number of the struck nucleus.

Elastic Scattering of Neutrons

Elastic scattering is the most important process for slowing down fast neutrons. Due to the rapid increase in the probability of neutron capture, the neutrons, once slowed down, will eventually be captured by target nuclei.

Here we will discuss several aspects of neutron scattering in matter:

☞ Maximum energy transfer.

☞ Angular distribution of scattered neutrons.

☞ Energy distribution of scattered neutrons.

☞ Average logarithm energy decrement of a neutron in multiple scattering.

Elastic Scattering of Neutrons

The **elastic scattering** plays an important role in neutron energy measurements. For example, a proton-neutron telescope illustrated below can be used to accurately measure the spectrum of neutrons in a collimated beam.

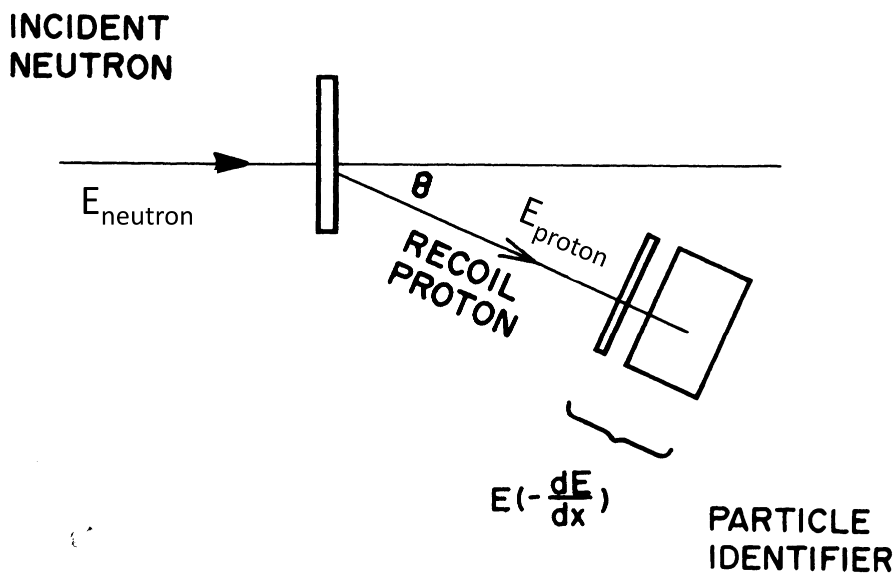


FIGURE 10.36. Arrangement of proton-recoil telescope for measuring spectrum neutron beam.

$$E_{proton} = E_{neutron} \cos^2 \theta$$

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p281

Elastic Scattering of Neutrons

The **maximum energy** that a neutron of mass M and kinetic energy E_n can transfer to a nucleus of mass m in a single elastic collision given by

$$E_{\max} = E_n \frac{4Mm}{(M+m)^2}$$

TABLE 9.4. Maximum Fraction of Energy Lost, Q_{max}/E_n from Eq. (9.3), by Neutron in Single Elastic Collision with Various Nuclei

Nucleus	Q_{max}/E_n
$^{1}_{1}H$	1.000
$^{2}_{1}H$	0.889
$^{4}_{2}He$	0.640
$^{9}_{4}Be$	0.360
$^{12}_{6}C$	0.284
$^{16}_{8}O$	0.221
$^{56}_{26}Fe$	0.069
$^{118}_{50}Sn$	0.033
$^{238}_{92}U$	0.017

Elastic Scattering of Neutrons

- The **elastic scattering** is the dominating mechanism whereby fast neutrons deliver dose to tissue.

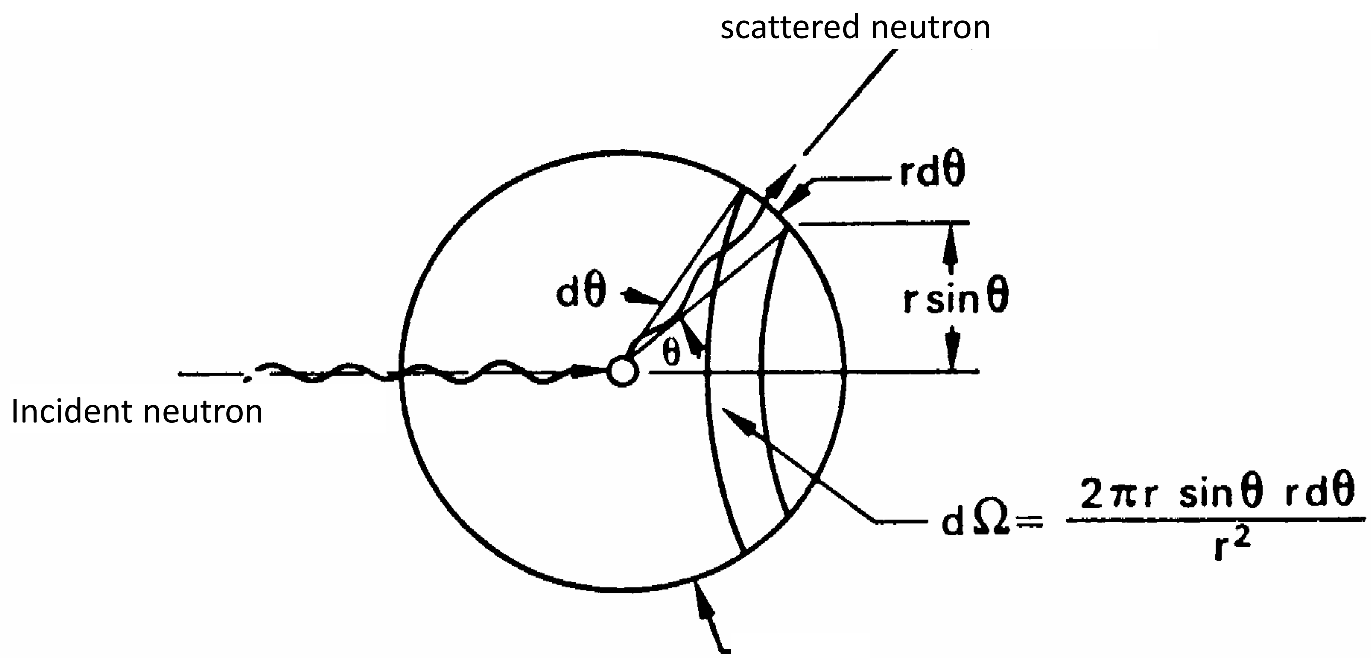
- The recoil nuclei are essentially ionizing particles traveling in media and losing their energy through ionization and excitation.

- As we will see later, over 85% of the “first-collision” dose in soft tissue (composed of H, C, O and N) arises from n-p scattering for neutron energy below 10MeV.

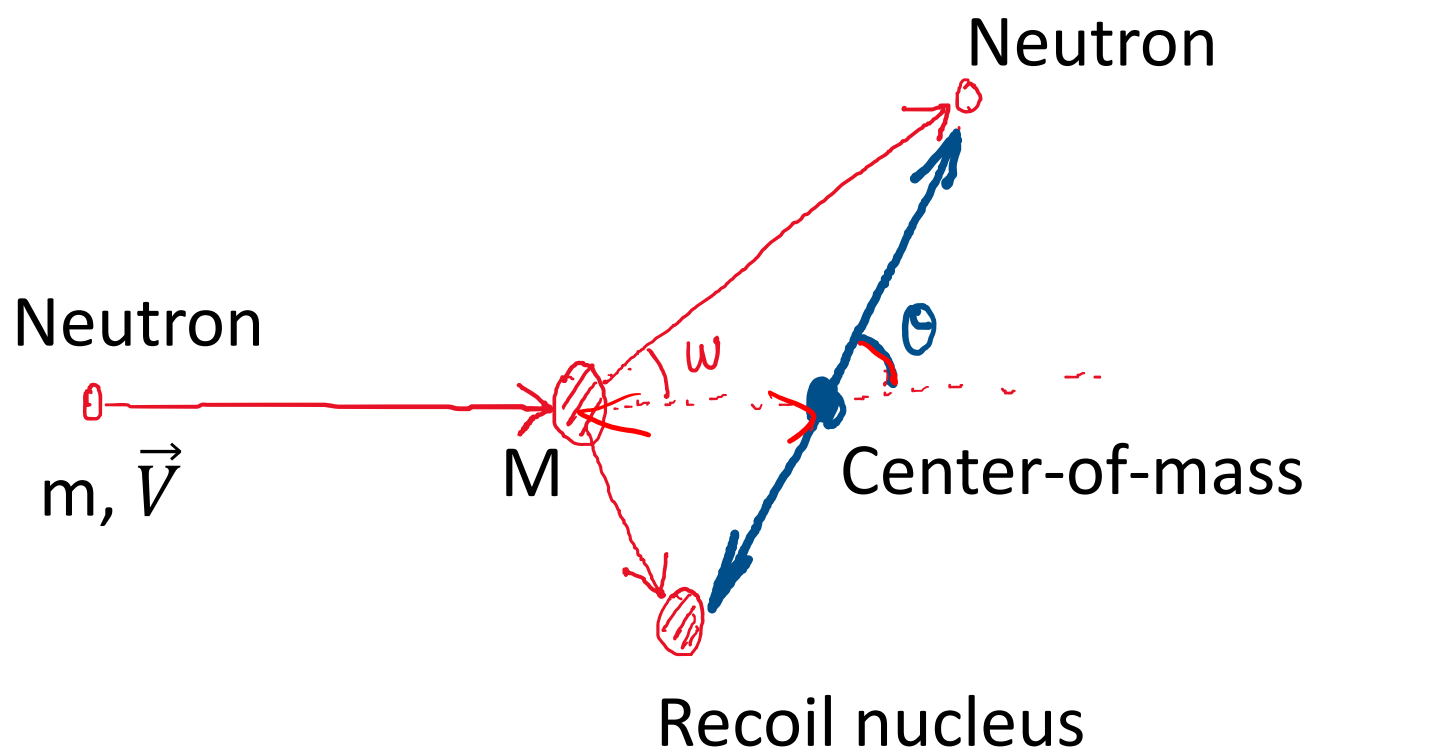
How do we derive the average energy transfer per single elastic collision between an energetic neutron and a target nucleus?

Angular Distributions of the Scattered Neutrons

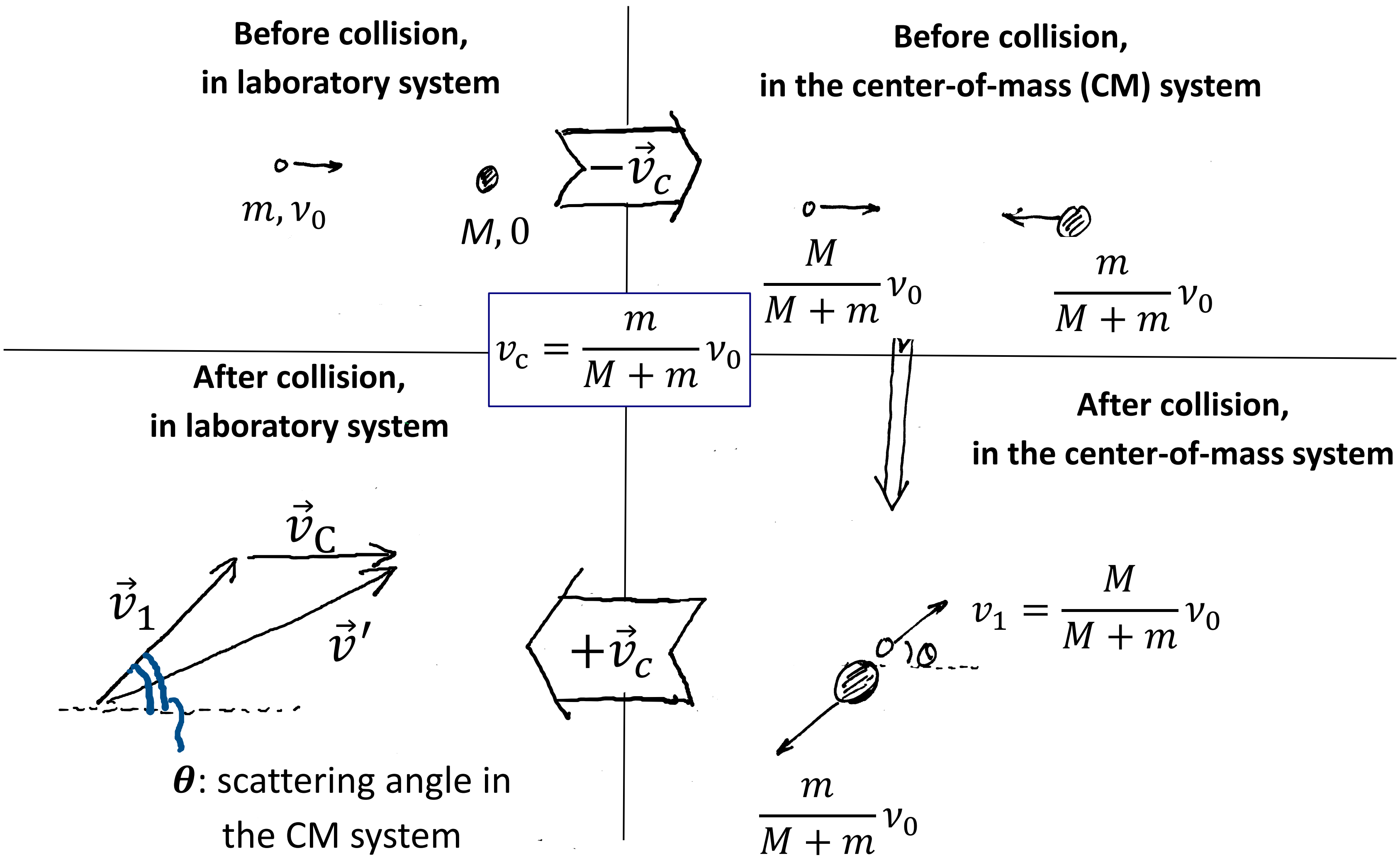
☞ For neutron energies up to 10MeV, it is experimentally observed that the scattering of neutrons is **isotropic** in the **center-of-mass coordinate system**. The neutron and the recoil nuclei are scattered with equal probability in any direction in this 3-D coordinate system.



Angular Distributions of the Scattered Neutrons



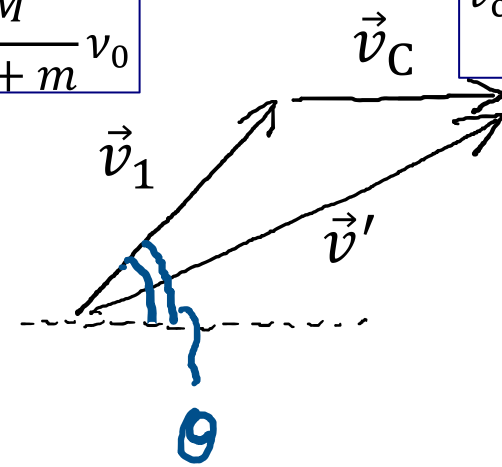
Angular Distributions of the Scattered Neutrons



Angular Distributions of the Scattered Neutrons

$$\boxed{v_1 = \frac{M}{M+m} v_0}$$

$$\boxed{v_c = \frac{m}{M+m} v_0}$$



The speed of the scattered neutron within the laboratory frame is

$$\vec{v}' = \vec{v}_1 + \vec{v}_c.$$

Therefore

$$v'^2 = v_1^2 + v_c^2 - 2 v_1 v_c \cos(\pi - \theta)$$
$$= v_1^2 + v_C^2 + 2 v_1 v_C \cos\theta.$$

The kinetic energy of the scattered neutron, E', is

$$E' = \frac{1}{2} m v'^2,$$

and

$$\frac{E'}{E_0} = \frac{1}{v_0^2}\left[\left(\frac{M}{M+m} v_0\right)^2 + \left(\frac{m}{M+m} v_0\right)^2 + 2 \frac{M}{M+m} \frac{m}{M+m} v_0^2 \cos\theta\right].$$

Therefore,

$$\boxed{\frac{E'}{E_0} = \frac{M^2 + m^2 + 2Mm\cos\theta}{(M+m)^2}}.$$

Energy Spectrum of the Scattered Neutrons

Since the scattering in the CM system is isotropic, the probability of the scattered neutron falling into an angular interval $[\theta, \theta + \mathrm{d}\theta]$ is

$$p(\theta) \cdot \mathrm{d}\theta = [2\pi\, r^2 \sin\theta\, \mathrm{d}\theta]/(4\pi r^2) = \frac{1}{2}\sin\theta \cdot \mathrm{d}\theta.$$

The probability of the outgoing neutron carrying a kinetic energy falling into a given window $[E', E' + \mathrm{d}E']$ is given by

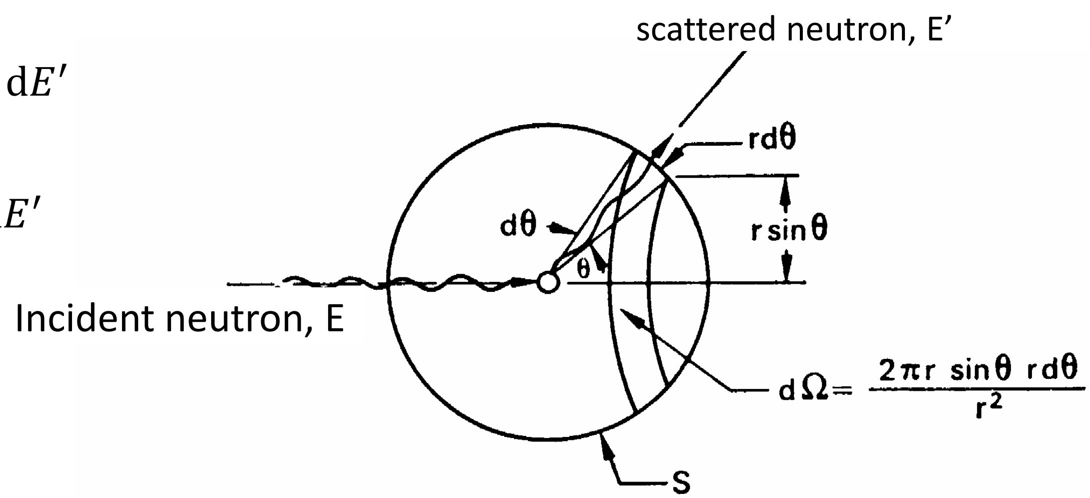
$$p(E')\,\mathrm{d}E' = -p(\theta) \cdot \mathrm{d}\theta$$

$$= -\frac{1}{2}\sin\theta \cdot \mathrm{d}\theta = -\frac{1}{2}\sin\theta \cdot \left(\frac{dE'}{d\theta}\right)^{-1} \cdot \mathrm{d}E'$$

$$= -\frac{1}{2}\sin\theta \left[E_0 \frac{2Mm}{(\mathrm{M}+m)^2}(-\sin\theta)\right]^{-1} \cdot \mathrm{d}E'$$

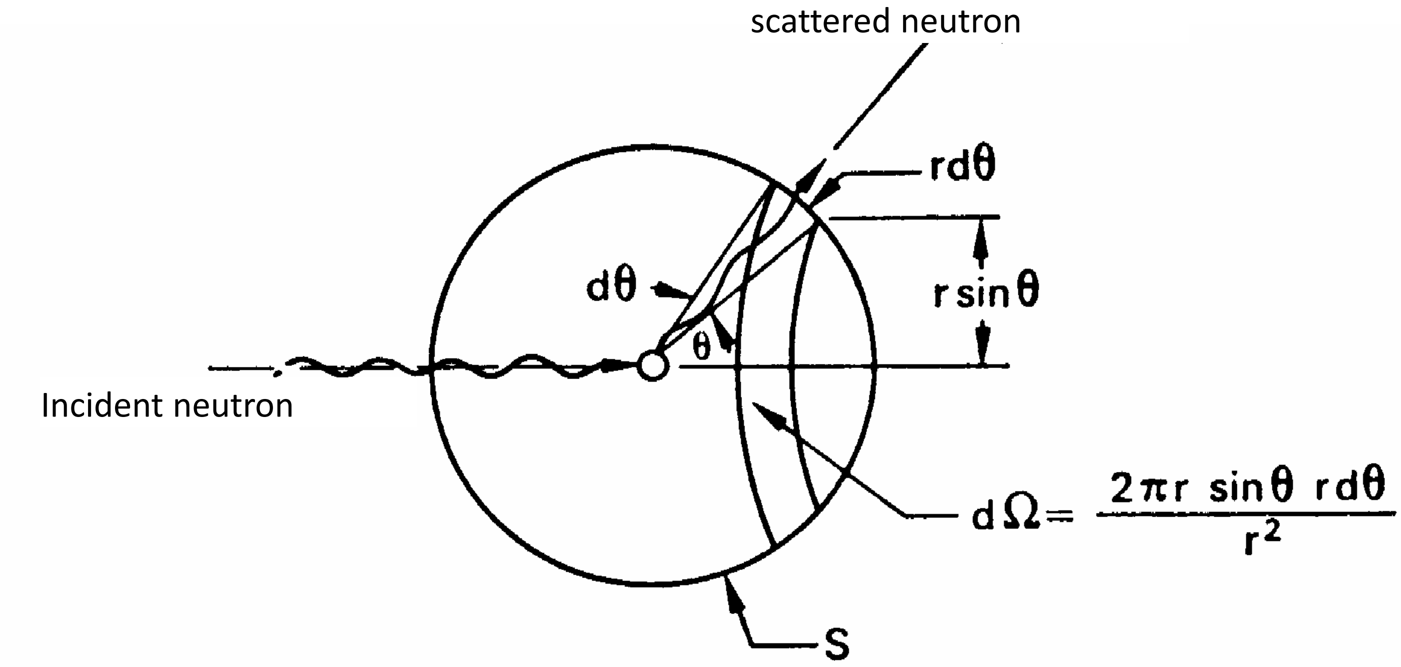
$$= \frac{1}{2}\sin\theta \cdot \frac{(M+m)^2}{2Mm\sin\theta} \cdot \frac{1}{E_0} \cdot \mathrm{d}E'$$

$$= \frac{(M+m)^2}{4Mm} \cdot \frac{1}{E_0} \cdot \mathrm{d}E'$$



$$\boxed{p(E')\,\mathrm{d}E' = \frac{(M+m)^2}{4Mm} \cdot \frac{1}{E_0} \cdot \mathrm{d}E'}$$

Angular Distributions of the Scattered Neutrons



Since the scattering of the neutron is isotropic in the CM system, the probability of the scattered neutron going into an angular interval dθ can be written as

$$p(\theta)\cdot d\theta = [2\pi \sin\theta \cdot d\theta]/4\pi = \frac{1}{2}\sin\theta \cdot d\theta$$

Energy Spectrum of the Scattered Neutrons

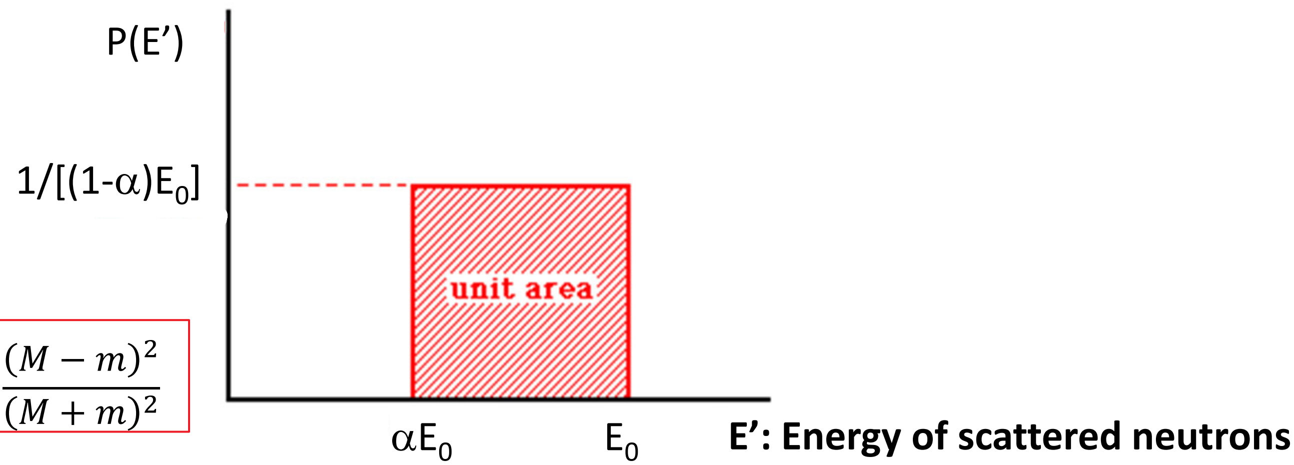
The probability of the outgoing neutron carrying kinetic energy falling into a given window $[E', E' + \mathrm{d}E']$ is given by

$$p(E')\,\mathrm{d}E' = \frac{(M+m)^2}{4Mm} \cdot \frac{1}{E_0} \cdot \mathrm{d}E' = \frac{1}{1-\alpha} \cdot \frac{1}{E_0} \cdot \mathrm{d}E' \,,$$

where

$$\alpha = \frac{(M-m)^2}{(M+m)^2} \,.$$

Energy Spectrum of Scattered Neutrons



$$\alpha = \frac{(M-m)^2}{(M+m)^2}$$

The fraction of energy carried by the scattered neutron is

$$\frac{E'}{E_0} = \frac{M^2 + m^2 + 2Mm\cos\theta}{(M+m)^2}$$

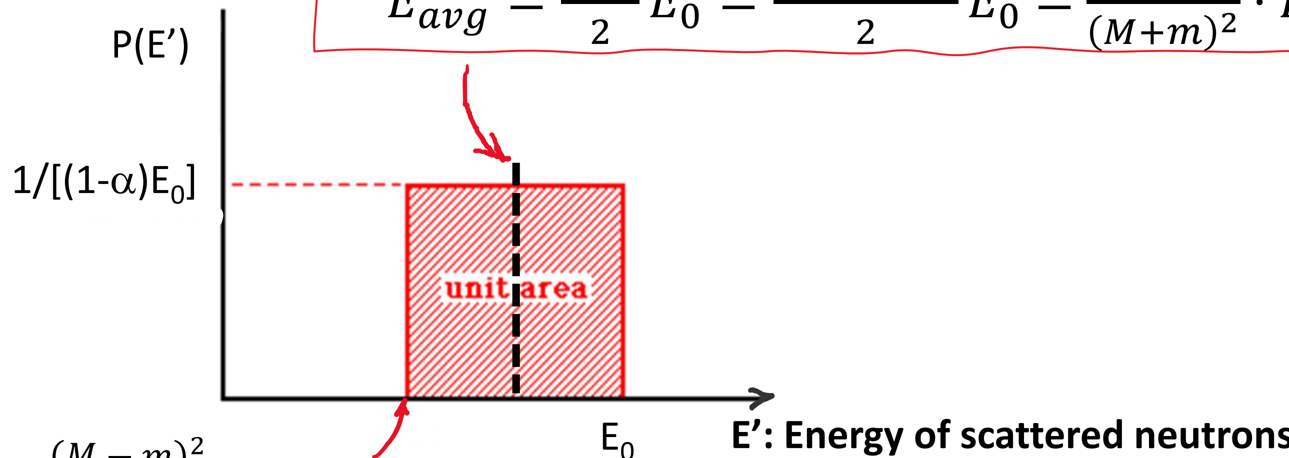
The distribution of the energy of the scattered neutrons is given by

$$p(E') = \frac{1}{1-\alpha}\frac{1}{E_0}, E' \subset [\alpha E_0, E_0].$$

Energy Spectrum of Scattered Neutrons

Average energy carried by the scattered neutron:

$$E'_{avg} = \frac{1+\alpha}{2} E_0 = \frac{1+\frac{(M-m)^2}{(M+m)^2}}{2} E_0 = \frac{M^2+m^2}{(M+m)^2} \cdot E_0$$



$$\frac{(M-m)^2}{(M+m)^2} E_0 \equiv \alpha E_0$$

αE_0

Average energy transferred to the recoil nucleus:

$$E_{avg\_energy\_loss} = E_0 - E'_{avg} = \frac{2Mm}{(M+m)^2} \cdot E_0$$

Important neutron capture reactions

Interaction of Slow Neutrons (E<0.5eV)

- The most important interactions between slow neutrons and absorbing materials are ***neutron-induced reactions***, such as (n,γ), (n,α), (n,p) and (n, fission) etc. These interactions lead to more prominent signatures for neutron detection.

$$\text{neutron} + \text{target nucleus} \Rightarrow \begin{cases} \text{recoil nucleus} \\ \text{protons} \\ \text{alpha particles} \\ \text{fission fragments} \end{cases}$$

Neutron Induced Reactions

$$ {}_{0}^{1}n + {}_{1}^{1}H \rightarrow {}_{1}^{2}H + {}_{0}^{0}\gamma $$

- Neutron absorption followed by the immediate emission of a gamma-ray photon.
- Since the thermal neutron has negligible energy by comparison, the gamma photon has the energy Q=2.22 MeV released by the reaction, which represents the binding energy of the deuteron.
- The capture cross section per atom is 0.33 barn.
- When tissue is exposed to thermal neutrons, this reaction provides a source of gamma rays that delivers dose to the tissue.

Neutron Induced Reactions

$$ {}_0^1n + {}_7^{14}N \rightarrow {}_6^{14}C + {}_1^1p $$

- Cross section for thermal neutron is 1.70 barns.
- Q=0.626MeV.
- Since the range of the proton and the ^{14}C nucleus are relatively small, their energy is deposited locally at the site where the neutron was captured.
- Capture by hydrogen and nitrogen are the only two processes through which neutron deliver a significant does to soft tissue.

Neutron Induced Reactions

$$ {}_{0}^{1}n + {}_{2}^{3}He \rightarrow {}_{1}^{3}H + {}_{1}^{1}p $$

- Cross section for thermal neutron is 5330 barns.
- Q=765keV.
- Commonly used in proportional counters for fast neutron detection.

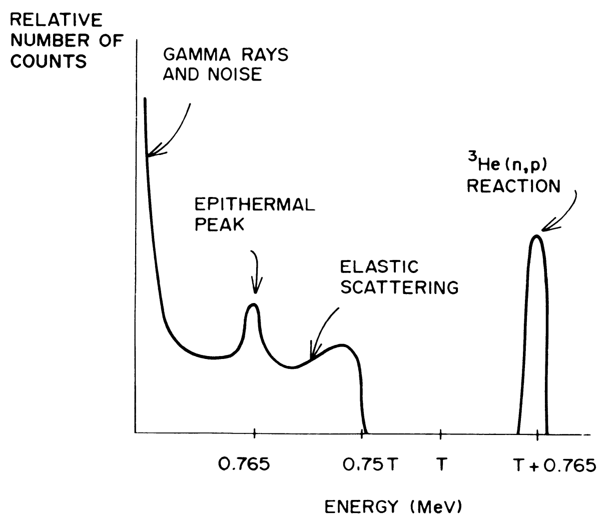


FIGURE 10.35. Pulse-height spectrum from ^{3}He proportional counter for monoenergetic neutrons of energy T.

Neutron Induced Reactions

$$^{1}_{0}n+^{6}_{3}Li\rightarrow^{4}_{2}He+^{3}_{1}H$$

- Cross section for thermal neutron is 940 barns.
- Q=4.78MeV.
- Widely used for thermal neutron detection.

Neutron sensitive LiI scintillator can be made or Li can be added to other scintillator to register neutrons.

^{6}Li is 7.42% abundant and Li enriched in the isotope ^{6}Li is available.

Neutron Induced Reactions

$$ {}^{1}_{0}n + {}^{23}_{11}Na \rightarrow {}^{24}_{11}Na + {}^{0}_{0}\gamma $$

- Cross section for thermal neutron is 0.534 barns.
- Q=0.626MeV.
- ^{24}Na undergo radioactive decay with the emission of two gamma rays, having energies of 2.75MeV and 1.37MeV per disintegration with a half-life of 15 hours.
- Since ^{23}Na is a normal constituent of blood, activation of blood sodium can be sued as a dosimetry tool when persons are exposed to relatively high does of neutrons, for example, in a criticality accident.

Threshold Reactions

Energetics of Threshold Reactions

☞ Consider the following reaction

$$ {}_{0}^{1}n + {}_{16}^{32}S \rightarrow {}_{15}^{32}P + {}_{1}^{1}p $$

☞ The neutrons must have an energy of above a certain threshold to enable this reaction.

☞ These reactions are called **endothermic reactions**, in which energy is converted into mass and therefore Q<0.

Energetics of Threshold Reactions

Example :

Calculate the threshold energy for the reaction $^{32}S(n,p)^{32}P$

Solution:

The reaction

$$^{1}_{0}n + ^{32}_{16}S \rightarrow ^{1}_{1}H + ^{32}_{15}P$$

The atomic mass difference is given by

$$\Delta = A_{^1H} + A_{^{32}S} - A_n - A_{^{32}P}$$

$$= -0.9276 \text{ MeV}$$

The threshold energy is

$$E_{th} = -Q\left(1 + \frac{M_1}{M_3 + M_4 - M_1}\right) = 0.9276\left(1 + \frac{1}{1+32-1}\right)$$

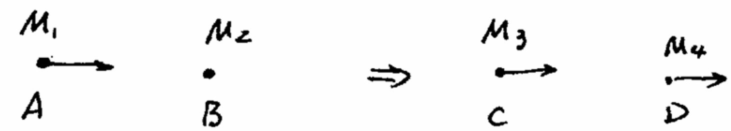
$$= 0.957 \text{ MeV}.$$

Energetics of Threshold Reactions

Nuclide	Natural Abundance (%)	Mass Difference $\Delta = M - A$ (MeV) (at. mass – at. mass No.)	Type of Decay	Half-Life	Major Radiations, Energies (MeV), and Frequency per Disintegration (%)
${}^{22}_{11}Na$	—	–5.182	β^+ 89.8% EC 10.2%	2.602 y	β^+ : 0.545 max (avg 0.215) γ : 0.511 (180%, $\gamma^\pm$), 1.275 (100%), Ne X rays
${}^{23}_{11}Na$	100.	–9.528	—	—	—
${}^{24}_{11}Na$	—	–8.418	β^-	15.00 h	β^- : 1.390 max (avg 0.554) γ : 1.369 (100%), 2.754 (100%)
${}^{24}_{12}Mg$	78.60	–13.933	—	—	—
${}^{26}_{12}Mg$	11.3	–16.214	—	—	—
${}^{26}_{13}Al$	—	–12.211	β^+ 81.8% EC 18.2%	7.16×10^5 y	β^+ : 1.174 max (avg 0.544) γ : 0.511 (164%, $\gamma^\pm$), 1.130 (2.5%), 1.809 (100%), Mg X rays
${}^{26m}_{13}Al$	—	–11.982	β^+	6.4 s	β^+ : 3.21 max γ : 0.511 (200%, $\gamma^\pm$)
${}^{32}_{15}P$	—	–24.303	β^-	14.29 d	β^- : 1.710 max (avg 0.695) No γ
${}^{32}_{16}S$	95.0	–26.013	—	—	—
${}^{35}_{16}S$	—	–28.847	β^-	87.44 d	β^- : 0.167 max (avg 0.0488) No γ
${}^{37}_{16}S$	—	–27.0	β^-	5.06 min	β^- : 1.6 max (90%) 4.7 max (10%) γ : 3.09 (90%)
${}^{38}_{16}S$	—	–26.8	β^-	2.87 h	β^- : 1.1 max (95%), 3.0 max (5%) γ : 1.88 (95%) Daughter radiations from ${}^{38}Cl$

Energetics of Threshold Reactions

Consider a head-on collision



- A particle of mass M_1 strikes another particle of mass M_2 initially at rest.

  The identities of the particles are changed by the reaction. So there will generally be different masses M_3 and M_4 after the encounter.

- The change in rest mass/energy is

$$Q = M_1 + M_2 - (M_3 + M_4), \qquad (1)$$

which is negative for endothermic reactions. (assuming (a). both A and B are in ground states. (b) C and D are also in their ground states. Otherwise we would need to replace Q by $Q - E_{excit}$)

Energetics of Threshold Reactions

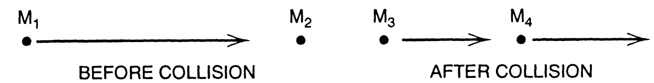


FIGURE 9.7. Schematic representation of a head-on collision producing a nuclear reaction in which the identity of the particles can change.

- The conservation of energy requires

$$E_1 = E_3 + E_4 - Q \qquad (2)$$

where E_3 and E_4 are the kinetic energy of the moving particles.

Conservation of momentum requires

$$P_1 = P_3 + P_4 \qquad (3)$$

We consider the non-relativistic case, where $E = p^2/2m$.

So one can eliminate E_4 by using $E_4 = P_4^2/2M_4$ and write

$$E_4 = \frac{P_4^2}{2M_4} = \frac{1}{2M_4}(P_1 - P_3)^2$$

Substitute

$$P_1 = (2M_1E_1)^{1/2} \quad \text{and} \quad P_3 = (2M_3E_3)^{1/2}$$

We have

$$E_4 = \frac{1}{M_4}\left[M_1E_1 - 2(M_1M_3)^{1/2}(E_1E_3)^{1/2} + M_3E_3\right] \qquad (4)$$

Energetics of Threshold Reactions

Substitute <4> into <2> and rearrange terms. we have

$$E_1 = E_3 + \frac{M_1}{M_4} E_1 - \frac{2(M_1 M_3)^{1/2}(E_1 E_3)^{1/2}}{M_4} + \frac{M_3 E_3}{M_4} - Q$$

So that

$$E_3 - \frac{2(M_1 M_3 E_1)^{1/2}}{M_3+M_4}\sqrt{E_3} - \frac{(M_4 - M_1)E_1 + M_4 Q}{M_3 + M_4} = 0 \qquad (5)$$

(5) is a quadratic equation of $\sqrt{E_3}$, having the form of

$$E_3 - 2A\sqrt{E_3} - B = 0$$

The two roots of this equation are

$$E_3 = \left[\frac{2A \pm \sqrt{4A^2 + 4B}}{2}\right]^2 = B + 2A^2\left(1 \pm \frac{1}{A}\sqrt{A^2+B}\right)$$

For E_3 to be real, $A^2 + B \geq 0$, therefore

$$\left[\frac{(M_1 M_3 E_1)^{1/2}}{M_3 + M_4}\right]^2 + \frac{(M_4 - M_1)E_1 + M_4 Q}{M_3 + M_4} \geq 0$$

Or

$$E_1 \geq -Q\left(1 + \frac{M_1}{M_3 + M_4 - M_1}\right)$$

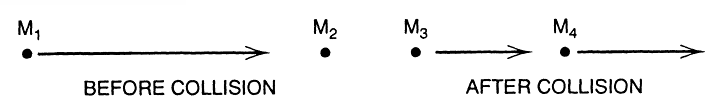


FIGURE 9.7. Schematic representation of a head-on collision producing a nuclear reaction in which the identity of the particles can change.

Neutron Activation

☞ For **endothermic reactions**, the minimum energy carried by the neutron (the **threshold energy**) can be derived based on the conservation of energy and momentum:

$$E_1 = E_3 + E_4 - Q$$

$$p_1 = p_3 + p_4$$

☞ The **threshold energy** is slightly greater that the Q value (the mass difference before and after the reaction).

$$E_{th} = -Q\left(1 + \frac{M_1}{M_3 + M_4 - M_1}\right)$$

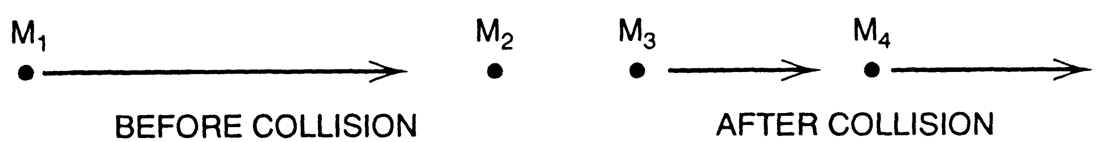


FIGURE 9.7. Schematic representation of a head-on collision producing a nuclear reaction in which the identity of the particles can change.

Energetics of Threshold Reactions

Energy release : $Q = M_1 + M_2 - (M_3 + M_4)$ (1)

Conservation of energy: $E_1 = E_3 + E_4 + Q \Rightarrow E_4 = E_1 - E_3 (2)$

Conservation of momentum: $p_1 = p_3 + p_4 \Rightarrow (2M_1E_1)^{1/2} = (2M_3E_3)^{1/2} + (2M_4E_4)^{1/2}$ (3)

Substitute (2) into (3), we have

$$(2M_1E_1)^{1/2} = (2M_3E_3)^{1/2} + (2M_4(E_1 - E_3))^{1/2}. \quad (4)$$

Rearranging the terms in (4),

$$E_3 - \frac{2(M_1M_3E_1)^{1/2}}{M_3+M_4}\sqrt{E_3} - \frac{(M_4-M_1)E_1+M_4Q}{M_3+M_4} = 0\,. \quad (5)$$

Solving (5) and considering E_3 should take a real value, we need to have

$$\left[-\frac{2(M_1M_3E_1)^{1/2}}{M_3+M_4}\right]^2 - 4\left[-\frac{(M_4-M_1)E_1+M_4Q}{M_3+M_4}\right] \geq 0,$$

or

$$\frac{M_1M_3E_1}{(M_3+M_4)^2} + \frac{(M_4-M_1)E_1+M_4Q}{M_3+M_4} \geq 0, \quad (6)$$

which finally leads to

$$E_i \geq -Q\left(1 + \frac{M_1}{M_3+M_4-M_1}\right). \quad (7)$$

Average Thermal Neutron Capture Cross Section

- ☞ The thermal neutron capture cross section for neutron reactions with a threshold usually increase steadily from zero at E_{th} to a maximum and then decline at higher energies.
- ☞ The neutron energy at which the cross section has approximately its average value is called the **effective threshold energy**, which is greater than E_{th}.

Energy Dependence of Thermal Neutron Absorption Cross section

- Capture cross sections for low energy neutrons generally decreases as the reciprocal of the velocity as the neutron energy increases (the **1/v law**).
- So if the capture cross section σ_0 is known for a given velocity v_0, then the cross section at velocity v can be estimated from the following relation,

$$\frac{\sigma}{\sigma_0} = \frac{v_0}{v} = \sqrt{\frac{E_0}{E}}$$

- This equation can be used for neutrons of energies up to 100eV or 1keV, depending on the absorbing nucleus.

Energy Dependence of Thermal Neutron Absorption Cross section

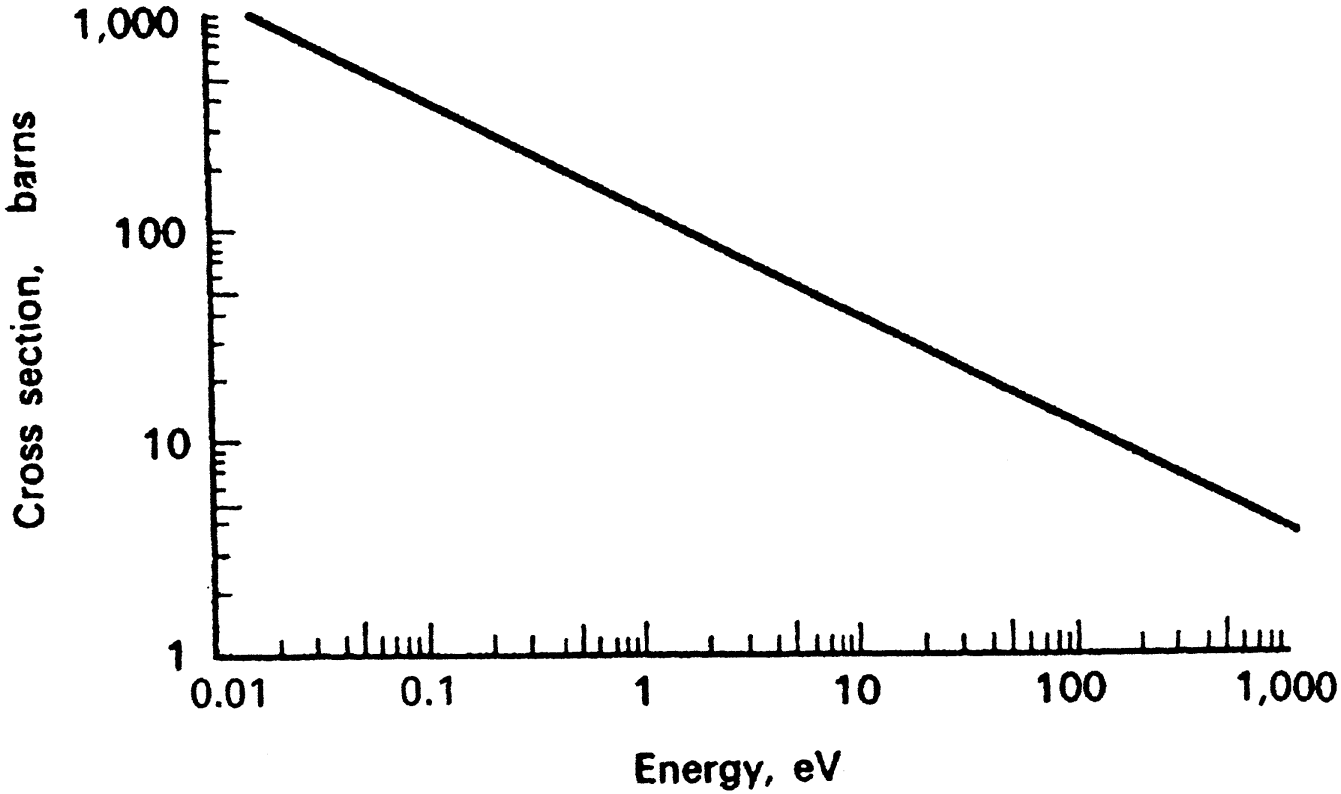


FIGURE 5.23. Neutron absorption cross section for boron, showing the validity of the $1/v$ law for neutrons from 0.02 to 1000 eV in energy. The equation of the curve is $\sigma = {}^{116}\!/\!\sqrt{E(eV)}$ barns.

Neutron Activation Analysis

Neutron Activation

☞ Neutron activation is the production of a radioactive isotope by the absorption of a neutron, such as in the (n,p) reaction.

☞ Neutron activation is important to health physicist for several reasons.

(a) Materials irradiated by neutrons may become radioactive. A radiation hazard may therefore persist after the irradiation by neutron is terminated.

(b) Neutron activation provides a convenient way to measure neutron flux.

(c) By spectroscopic examination of the induced radiation, quantitative analysis of the unknown samples is also possible.

Neutron Induced Reactions

$$ {}_{0}^{1}n + {}_{11}^{23}Na \rightarrow {}_{11}^{24}Na + {}_{0}^{0}\gamma $$

- Cross section for thermal neutron is 0.534 barns.
- Q=0.626MeV.
- ^{24}Na undergoes radioactive decay with the emission of two gamma rays, having energies of 2.75MeV and 1.37MeV per disintegration.
- Since ^{23}Na is a normal constituent of blood, activation of blood sodium can be used as a dosimetry tool when persons are exposed to relatively high doses of neutrons, for example, in a criticality accident.

Neutron Activation

- Considering the case that an object is irradiated by a constant flux, $\phi(n/cm^2/s)$, of neutrons, which activates a given types of atoms. Then the net rate of increase of radioactive (daughter) atoms is given by

$$\frac{dN}{dt} = \phi\sigma n - \lambda N,$$

where
ϕ = flux, neutrons per cm^2 per s,
σ = activation cross section, cm^2,
λ = transformation constant of the induced activity,
N = number of radioactive atoms,
n = number of target atoms.

- The radioactivity induced by neutron activation (the number of disintegration of the activated daughter atoms per second) is given by

$$\lambda N = \phi\sigma n(1 - e^{-\lambda t})$$

Neutron Activation

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where ϕ = flux, neutrons per cm^2 per s,
σ = activation cross section, cm^2,
λ = transformation constant of the induced activity,
N = number of radioactive atoms,
n = number of target atoms.

- The **saturation activity** is given by $\phi\sigma n$. For an infinitely long irradiation time, it represents the maximum obtainable activity with any given neutron flux.

- The analysis leading to these results is identical to that used for analyzing the secular equilibrium for radioactive decay chains, in which the daughter has a much shorter decay time than that of the parent.

Neutron Activation

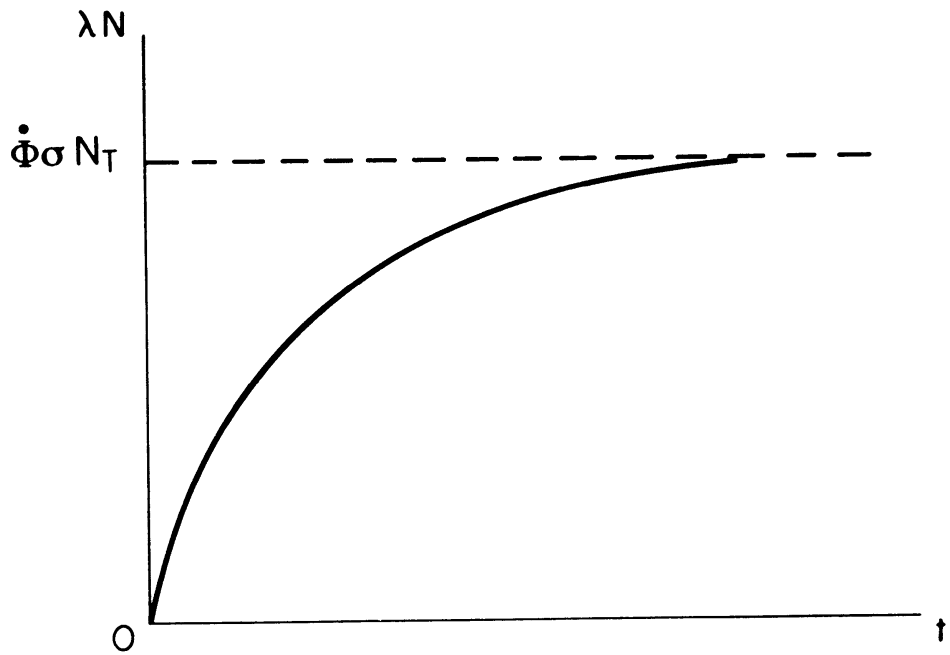

FIGURE 9.8. Buildup of induced activity λN, as given by Eq. (9.36), during neutron irradiation at constant fluence rate.

Example

A 3-g sample of ^{32}S is irradiated with fast neutrons having a constant fluence rate of 155 $cm^{-2}\,s^{-1}$. The cross section for the reaction $^{32}S(n,p)^{32}P$ is 0.200 barn, and the half-life of ^{32}P is $T = 14.3$ d. What is the maximum ^{32}P activity that can be induced? How many days are needed for the level of the activity to reach three quarters of the maximum?

Solution

The total number of target atoms is $N_T = \frac{3}{32} \times 6.02 \times 10^{23} = 5.64 \times 10^{22}$. The maximum (saturation) activity is $\dot{\Phi}\sigma N_T = (155\ cm^{-2}\,s^{-1})(0.2 \times 10^{-24}\ cm^2)(5.64 \times 10^{22}) = 1.75\ s^{-1} = 1.75$ Bq. [Expressed in curies, the saturation activity is $1.75/(3.7 \times 10^{10}) = 4.73 \times 10^{-11}$ Ci.] The time t needed to reach three-quarters of this value can be found from Eq. (9.36) by writing $\frac{3}{4} = 1 - e^{-\lambda t}$. Then $e^{-\lambda t} = \frac{1}{4}$ and $t = 2T = 28.6$ d. Note that the buildup toward saturation activity is analogous to the approach to secular equilibrium by the daughter of a long-lived parent (Sect. 4.4).

From Turner's textbook, Page 229

Example

Estimate the fraction of the ^{32}S atoms that would be consumed in the last example in 28.6 days.

Solution

The rate at which ^{32}S atoms are used up is $\dot{\Phi}\sigma N_T = 1.75\ s^{-1}$. Since $t = 28.6$ d $= 2.47 \times 10^6$ s, the number of ^{32}S atoms lost is $1.75 \times 2.47 \times 10^6 = 4.32 \times 10^6$. The fraction of ^{32}S atoms consumed, therefore, is $4.32 \times 10^6/(5.64 \times 10^{22}) = 7.66 \times 10^{-17}$, a negligible amount. Note that fractional burnup does not depend on the sam-

From Turner’s textbook, Page 230