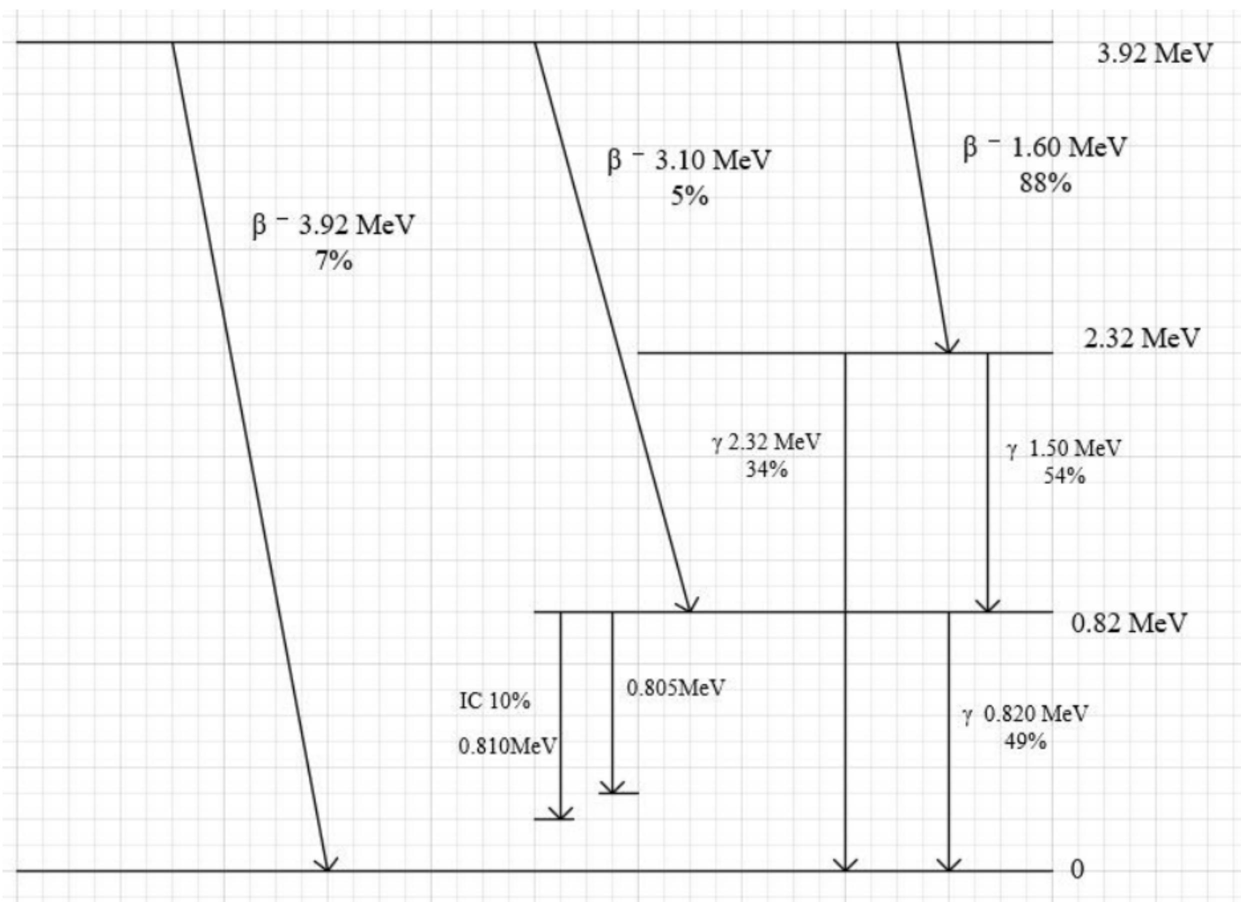
1. 3.27 (25 points total)
   A parent nuclide decays by beta-particle emission into a stable daughter. The major radiations, energies (MeV), and frequencies are:
   β^-: 3.92 max (7%), 3.10 max (5%), 1.60 max (88%)
   γ: 2.32 (34%), 1.50 (54%), 0.820 (88%)
   e^-: 0.818, 0.805

   (a) Draw the decay scheme.



   (b) What is the maximum energy that the antineutrino can recieve in this decay?

$$E_{\bar{\nu},max} = E_{\beta^-,max} = 3.92 \text{ MeV}$$

(c) What is the value of the internal conversion coefficient?

Textbook Eq. 3.31

$$\begin{aligned}\alpha &= \frac{N_e}{N_\gamma}\\ 5\%\ \beta^- &\rightarrow .82 \text{ MeV}\\ 54\%\ \gamma &\rightarrow .82 \text{ MeV}\end{aligned}$$

Total 59% in to .82 MeV energy state

$$\begin{aligned}N_\gamma &= 49\%\\ \Longrightarrow\ N_e &= 10\%\\ \alpha &= \frac{49}{10}\\ &= 4.9\end{aligned}$$

(d) Estimate the L-shell electron binding energy of the daughter nuclide.

$$\begin{aligned}E_e &= E^* - E_B\\ E^* &= .82 \text{ MeV}\\ E_B &= .818 \text{ MeV}\\ E_e &= .82 - .818\\ &= .002 \text{ MeV}\end{aligned}$$

(e) Would daughter X rays be expected also? Why or why not?
Yes. The daughter atom would be missing a K-shell or L-shell electron after internal conversion. When an outer shell electron moves in to fill this hole, the energy difference could be released as X rays (or Auger electrons).

2. 3.36 (25 points total)
   The isotope ${}^{126}_{53}\text{I}$ can decay by EC, β^-, and β^+ transitions.

   (a) Calculate the Q values for the three modes of decay to the ground states of the daughter nuclei.

   From Table 3.1 in the textbook (Eq. 3.35, 3.25, and 3.41)

$$\begin{aligned} Q_{EC} &= \Delta P - \Delta D - E_B \\ Q_{\beta^-} &= \Delta P - \Delta D \\ Q_{\beta^+} &= \Delta P - \Delta D - 2mc^2 \end{aligned}$$

   From App. D in the textbook

$$\begin{aligned} \Delta\,{}^{126}_{53}\text{I} &= -87.9 \\ \Delta\,{}^{126}_{52}\text{Te} &= -90.05 \\ \Delta\,{}^{126}_{54}\text{Xe} &= -89.15 \end{aligned}$$

   i. EC

$$\begin{aligned} {}^{126}_{53}\text{I} + {}^{0}_{-1}\beta^- &\rightarrow {}^{126m}_{52}\text{Te} + {}^{0}_{0}\gamma \\ Q_{EC} &= \Delta P - \Delta D - E_B \end{aligned}$$

   Lawrence Berkeley National Lab Electron Binding Energies data

$$\begin{aligned} E_{B,min} &= 0.033169 \text{ MeV} \\ Q_{EC} &= -87.9 - (-90.05) - 0.033169 \\ &= \end{aligned}$$

   ii. β^-

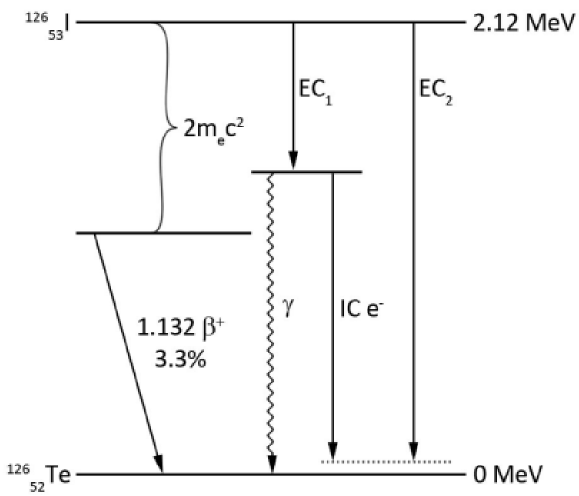
$$\begin{aligned} {}^{126}_{53}\text{I} &\rightarrow {}^{126}_{54}\text{Xe} + {}^{0}_{-1}\beta^- + {}^{0}_{0}\bar{\nu} \\ Q_{\beta^-} &= \Delta P - \Delta D \\ Q_{\beta^-} &= -87.9 - (-89.15) \\ &= 1.25 \text{ MeV} \end{aligned}$$

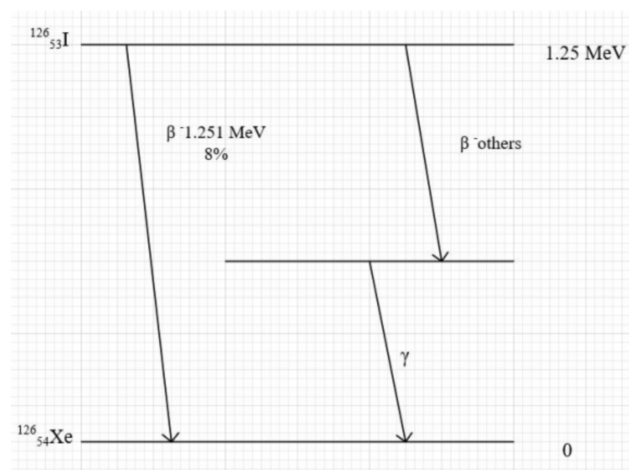
   iii. β^+

$${}^{126}_{53}\text{I} \rightarrow {}^{126}_{52}\text{Te} + {}^{0}_{+1}\beta^+ + {}^{0}_{0}\nu$$

$$\begin{aligned} Q_{\beta+} &= \Delta P - \Delta D - 2mc^2 \\ 2mc^2 &= 2 \times .511 \text{ MeV} \\ &= 1.022 \\ Q_{\beta+} &= -87.9 - (-90.05) - 1.022 \\ &= 1.128 \text{ MeV} \end{aligned}$$

(b) Draw the decay scheme.





(c) What kinds of radiation can one expect from a ^{126}I source?
X-ray, Gamma-rays, β^-, β^+, as well as 511 keV annihilation photons can be expected.

3. 4.17 (20 points total)
$^{210}_{82}$Pb decays by β^- with a half-life of 22 years to $^{210}_{83}$Bi.
$^{210}_{83}$Bi decays by β^- with a half-life of 5 days to $^{210}_{84}$Po.
A sample contains 30 MBq of ^{210}Pb and 15 MBq of ^{210}Bi at time $t = 0$.

$$\begin{aligned}
22 \text{ years} &= 8035.5 \text{ days} \\
\lambda_{Pb} &= \frac{\ln 2}{8035.5 \text{ days}} \\
&= 8.63 \times 10^{-5} \frac{1}{\text{days}} \\
\lambda_{Bi} &= \frac{\ln 2}{5 \text{ days}} \\
&= 0.139 \frac{1}{\text{days}}
\end{aligned}$$

(a) Calculate the activity of ^{210}Bi at time $t = 10$ days.

Textbook Eq. 4.37:

$$A_{Bi} = A_{Pb}(1 - e^{-\lambda_{Bi} t}) + A^0_{Bi} e^{-\lambda_{Bi} t}$$

$$\begin{aligned}
A_{Pb} &= A^0_{Pb} e^{-\lambda_{Pb} t} \\
A_{Bi} &= A^0_{Pb} e^{-\lambda_{Pb} t}(1 - e^{-\lambda_{Bi} t}) + A^0_{Bi} e^{-\lambda_{Bi} t} \\
A_{Bi}(t = 10 \text{ days}) &= 26.2 \text{ MBq}
\end{aligned}$$

(b) If the sample was originally pure ^{210}Pb, then how old is it at time $t = 0$?

Textbook Eq. 4.37:

$$\begin{aligned}
A_{Bi} &= A^0_{Pb} e^{-\lambda_{Pb} t}(1 - e^{-\lambda_{Bi} t}) + A^0_{Bi} e^{-\lambda_{Bi} t} \\
0 &= 30 e^{-\lambda_{Pb} t}(1 - e^{-\lambda_{Bi} t}) + 15 e^{-\lambda_{Bi} t} \\
t &\simeq -5 \text{ days}
\end{aligned}$$

The sample is approximately 5 days old.

4. 4.24 (20 points total)
   Starting with a 40 mg sample of Ra-226:

$$\begin{aligned}
\lambda_{Ra} &= \frac{\ln 2}{t_{\frac{1}{2}Ra}} \\
&\simeq 1.377 \times 10^{-11} \text{ s} \\
\lambda_{Ra} &\simeq 2.100 \times 10^{-6} \text{ s}
\end{aligned}$$

$$\begin{aligned}
N_{Ra}(0) &= \text{m}_{Ra} \cdot \frac{N_A}{\text{MM}_{Ra}} \\
N_A &\simeq 6.02 \times 10^{23} \ \frac{\text{atoms}}{\text{mol}} \\
\text{MM}_{Ra} &\simeq 226 \ \frac{\text{grams}}{\text{mol}} \\
N_{Ra}(0) &= 1.066 \times 10^{20}
\end{aligned}$$

$$\begin{aligned}
N_{Ra}(t) &= N_{Ra}(0) \cdot e^{-\lambda_{Ra} t} \\
N_{Rn} &= \frac{N_{Ra}(0)\lambda_{Ra}}{\lambda_{Rn} - \lambda_{Ra}} \cdot (e^{-\lambda_{Ra} t} - e^{-\lambda_{Rn} t}) \\
Q_{Ra} &= \lambda_{Ra} \cdot N_{Ra} \\
Q_{Rn} &= \lambda_{Rn} \cdot N_{Rn}
\end{aligned}$$

$$\begin{aligned}
Q_{Rn} &= \lambda_{Rn} \cdot N_{Rn} \\
&= \lambda_{Rn} \cdot \frac{N_{Ra}(0)\lambda_{Ra}}{\lambda_{Rn} - \lambda_{Ra}} \cdot (e^{-\lambda_{Ra} t} - e^{-\lambda_{Rn} t})
\end{aligned}$$

   (a) (5 points) How long will it take for 10mCi of Rn-222 to build up?

$$\begin{aligned}
10 \text{ mCi} &= 3.7 \times 10^8 \text{ Bq} \\
3.7 \times 10^8 &= Q_{Rn} \\
&= \lambda_{Rn} \cdot \frac{N_{Ra}(0)\lambda_{Ra}}{\lambda_{Rn} - \lambda_{Ra}} \cdot (e^{-\lambda_{Ra} t} - e^{-\lambda_{Rn} t}) \\
e^{-\lambda_{Ra} t} - e^{-\lambda_{Rn} t} &= .2523 \\
\lambda_{Ra} t &<<< \lambda_{Rn} t \\
e^{-\lambda_{Ra} t} &\simeq 1 \\
t &= -\frac{\ln(1 - .2523)}{\lambda_{Rn}} \\
&\simeq 1.384 \times 10^5 \text{ s}
\end{aligned}$$

$$\simeq 38.45 \text{ hours}$$
$$\simeq 1.602 \text{ days}$$

(b) What will be the activity of Rn-222 after 2 years?

$$\begin{aligned} Q_{Rn} &= \lambda_{Rn} \cdot \frac{N_{Ra}(0)\lambda_{Ra}}{\lambda_{Rn} - \lambda_{Ra}} \cdot (e^{-\lambda_{Ra}t} - e^{-\lambda_{Rn}t}) \\ &= 1.465^9 \text{ Bq} \\ &= .0396 \text{ Ci} \end{aligned}$$

(c) What will be the activity of Rn-222 after 1000 years?

$$\begin{aligned} Q_{Rn} &= \lambda_{Rn} \cdot \frac{N_{Ra}(0)\lambda_{Ra}}{\lambda_{Rn} - \lambda_{Ra}} \cdot (e^{-\lambda_{Ra}t} - e^{-\lambda_{Rn}t}) \\ &= 9.510^8 \text{ Bq} \\ &= .0257 \text{ Ci} \end{aligned}$$

(d) (5 points) What is the ratio of the specific activity of Rn-222 to that of Ra-226?

$$\begin{aligned} \text{Specific Activity} &= \frac{\text{activity}}{\text{mass}} \\ \text{Ratio} &= \frac{\lambda_{Rn}}{\text{MM}_{Rn}} \cdot \frac{\text{MM}_{Ra}}{\lambda_{Ra}} \\ &= 155314 \end{aligned}$$

5. 4.26 (20 points total)

(a) Verify that the maximum activity, $A_1 + A_2$, occurs at time t.

$$t = \frac{1}{\lambda_2 - \lambda_1} \ln \frac{\lambda_2^2}{2\lambda_1\lambda_2 - \lambda_1^2}$$

$$\begin{aligned}
A_1 &= A_0 e^{-\lambda_1 t} \\
A_2 &= \frac{A_0\lambda_2}{\lambda_2 - \lambda_1} \cdot (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \\
A_1 + A_2 &= A_0 e^{-\lambda_1 t} + \frac{A_0\lambda_2}{\lambda_2 - \lambda_1} \cdot (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \\
&= A_0 \left(\left(1 + \frac{\lambda_2}{\lambda_2 - \lambda_1}\right) e^{-\lambda_1 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} \right) \\
\frac{\partial}{\partial t}(A_1 + A_2) &= A_0 \left(\frac{\lambda_2^2}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} - \left(\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1} + \lambda_1 \right) e^{-\lambda_1 t} \right) \\
&= 0 \\
\left(\frac{\lambda_1\lambda_2}{\lambda_2 + \lambda_1} + \lambda_1 \right) e^{-\lambda_1 t} &= \frac{\lambda_2^2}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} \\
\ln\left(\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1} - \lambda_1 \right) - \lambda_1 t &= \ln\left(\frac{\lambda_2^2}{\lambda_2 + \lambda_1} \right) - \lambda_2 t \\
\lambda_2 t - \lambda_1 t &= \ln\left(\frac{\lambda_2^2}{\lambda_2 - \lambda_1} \right) - \ln\left(\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1} + \lambda_1 \right) \\
&= \ln\left(\frac{\lambda_2^2}{\lambda_2 - \lambda_1} \right) - \ln\left(\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1} + \lambda_1 \frac{\lambda_2 - \lambda_1}{\lambda_2 - \lambda_1} \right) \\
&= \ln\left(\frac{\lambda_2^2}{\lambda_2 - \lambda_1} \right) - \ln\left(\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1} + \frac{\lambda_1\lambda_2 - \lambda_1^2}{\lambda_2 - \lambda_1} \right) \\
&= \ln\left(\frac{\lambda_2^2}{\lambda_2 - \lambda_1} \right) - \ln\left(\frac{2\lambda_1\lambda_2 - \lambda_1^2}{\lambda_2 - \lambda_1} \right) \\
t &= \frac{1}{\lambda_2 - \lambda_1} \ln\left(\frac{\lambda_2^2}{\lambda_2 - \lambda_1} \frac{\lambda_2 - \lambda_1}{2\lambda_1\lambda_2 - \lambda_1^2} \right) \\
&= \frac{1}{\lambda_2 - \lambda_1} \ln \frac{\lambda_2^2}{2\lambda_1\lambda_2 - \lambda_1^2}
\end{aligned}$$

□

(b) Show that the time of maximum total activity occurs earlier than the time of maximum daughter activity, A_2, for $t_1 > t_2$.

$$A_2 = \frac{A_0\lambda_2}{\lambda_2 - \lambda_1} \cdot (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

$$
\begin{aligned}
\frac{\partial}{\partial t} A_2 &= \frac{A_0 \lambda_2}{\lambda_2 - \lambda_1} \cdot (\lambda_2 e^{-\lambda_2 t} - \lambda_1 e^{-\lambda_1 t}) \\
&= 0 \\
\lambda_1 e^{-\lambda_1 t} &= \lambda_2 e^{-\lambda_2 t} \\
\ln(\lambda_1) - \lambda_1 t &= \ln(\lambda_2) - \lambda_2 t \\
(\lambda_2 - \lambda_1) t &= \ln(\lambda_2) - \ln(\lambda_1) \\
t &= \frac{1}{\lambda_2 - \lambda_1} \ln \frac{\lambda_2}{\lambda_1}
\end{aligned}
$$

$$
t_1 > t_2 \implies \lambda_1 < \lambda_2
$$

$$
\begin{aligned}
\frac{1}{\lambda_2 - \lambda_1} \ln \frac{\lambda_2^2}{2\lambda_1\lambda_2 - \lambda_1^2} &\stackrel{?}{<} \frac{1}{\lambda_2 - \lambda_1} \ln \frac{\lambda_2}{\lambda_1} \\
\frac{\lambda_2^2}{2\lambda_1\lambda_2 - \lambda_1^2} &\stackrel{?}{<} \frac{\lambda_2}{\lambda_1} \\
\lambda_1 \lambda_2^2 &\stackrel{?}{<} \lambda_2 (2\lambda_1\lambda_2 - \lambda_1^2) \\
&\stackrel{?}{<} 2\lambda_1\lambda_2^2 - \lambda_1^2\lambda_2 \\
\lambda_1^2 \lambda_2 &\stackrel{?}{<} \lambda_1 \lambda_2^2 \\
\lambda_1 &< \lambda_2
\end{aligned}
$$

□

(c) Does the maximum daughter activity, A_2 occur at the time t shown below when there is no equilibrium?

$$
t = \frac{1}{\lambda_2 - \lambda_1} \ln \frac{\lambda_2}{\lambda_1}
$$

This fact was shown above. No assumptions need be made relating λ_1 or λ_2.

6. 4.28 (20 points total)
   The average mass of potassium in the human body is about 140 g. From the abundance and half-life given for ^{40}K, estimate the average activity of ^{40}K in the body.
   Abundance: .0118%
   Half-Life: 1.28×10^9 years

$$140 \text{ grams} \times .0118\% = .01652 \text{ grams } ^{40}\text{K}$$

$$\begin{aligned} N &= .01652 \text{ grams} \times \frac{6.02 \times 10^{23} \text{ atoms}}{40 \text{ grams}} \\ &= 2.486 \times 10^{20} \text{ atoms} \end{aligned}$$

$$\begin{aligned} \lambda &= \frac{\ln 2}{t_{\frac{1}{2}}} \\ &= \frac{\ln 2}{1.28 \times 10^9 \text{ years}} \\ &= \frac{\ln 2}{4.04 \times 10^{16} \text{ seconds}} \\ &= 1.72 \times 10^{-17} \frac{1}{\text{second}} \end{aligned}$$

$$\begin{aligned} A &= N \times \lambda \\ A &= 2.486 \times 10^{20} \cdot 1.72 \times 10^{-17} \text{ Bq} \\ &= 4276 \text{ Bq} \end{aligned}$$