Chapter 2: Interaction of Radiation with Matter
2.1 Interaction of Beta Particles
Key Aspects of Beta Interactions

- Collisional interactions of beta particles with matter.
- Specific energy loss of beta particles.
- Dependence of the specific energy loss on the effective Z of absorbing material and the energy of the beta particles.
- Mass stopping, what and why?
- Radiative energy loss of beta particles.
- Relative importance of collisional and radiative energy loss.
- Fraction of energy loss due to Bremsstrahlung process and its implementation to shielding design for beta particles.
- Range of beta particles.
- Backscattering of beta particles.
Sources of Beta Particles

• **Beta particle** is *an ordinary electron*. Many atomic and nuclear processes result in the emission of beta particles.

• One of the most common source of beta particles is the **beta decay** of nuclides, in which
  
  **Beta decay**
  
  \[ \frac{A}{Z}X \rightarrow \frac{A}{Z+1}Y + \ _{-1}^0 \beta + \bar{\nu} \]

  **Beta-plus decay**
  
  \[ \frac{A}{Z}X \rightarrow \frac{A}{Z-1}Y + \ _{1}^0 \beta + \nu \]

  **Electron capture**
  
  \[ \frac{A}{Z}X + e^- \rightarrow \frac{A}{Z-1}Y + \nu \]
Most of the radiation effects are initiated by the interactions of electrons and especially LOW energy electrons with water molecules.
Monte Carlo Simulation of Electron Paths. This simulation is of 15 KeV electrons in fayalite (Fe$_2$SiO$_4$). Distances are given in nanometers (1000 nm = 1 µm). Paths of backscattered electrons are in red; those of absorbed electrons in blue. One should remember that this slice through a three-dimensional volume. This model was run using the Casino software described at


http://www4.nau.edu/microanalysis/Microprobe-SEM/Signals.html
Beta radiation detected in an isopropanol cloud chamber (after insertion of an artificial source strontium-90)

https://en.wikipedia.org/wiki/Beta_particle
FIGURE 13.1. Chemical development of a 4-keV electron track in liquid water, calculated by Monte Carlo simulation. Each dot in these stereo views gives the location of one of the active radiolytic species, OH, H$_3$O$^+$, e$_{aq}^-$, or H, at the times shown. Note structure of track with spurs, or clusters of species, at early times. After $10^{-7}$ s, remaining species continue to diffuse further apart, with relatively few additional chemical reactions. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)
Interactions of Beta Particles
Mechanisms of Energy Loss by Electrons

Ionization and excitation:
Beta particles may interact with orbital electrons through the electric fields surrounding these charged particles, which leads to excitation and ionization.

Ionization process can be modeled as an inelastic collision, the energy loss by the electron and the kinetic energy carried by the ejected electron is related by

\[ E_k = E_{\text{loss}} - \phi \]

where \( \phi \) is the ionization potential of the absorbing medium.
Specific Energy Loss of Beta Particles

Specific energy loss: the linear rate of energy loss by an electron through excitation and ionization, which is given by

\[
\frac{dE}{dx} = \frac{2\pi q^4 NZ \times (3 \times 10^9)^4}{E_m \beta^2 (1.6 \times 10^{-6})^2} \left\{ \ln \left[ \frac{E_m E_k \beta^2}{I^2 (1 - \beta^2)} \right] - \beta^2 \right\} \text{MeV/cm}
\]

where
- \( q \) = charge on the electron, \( 1.6 \times 10^{-19} \text{ C} \),
- \( N \) = number of absorber atoms per cm\(^3\),
- \( Z \) = atomic number of the absorber,
- \( NZ \) = number of absorber electrons per cm\(^3\) = 3.88 \times 10^{20} \text{ for air at } 0^\circ \text{ and } 76 \text{ cm Hg},
- \( E_m \) = energy equivalent of electron mass, \( 0.51 \text{ MeV} \),
- \( E_k \) = kinetic energy of the beta particle, \( \text{MeV} \),
- \( \beta \) = \( v/c \),
- \( I \) = mean ionization and excitation potential of absorbing atoms, \( \text{MeV} \),
- \( I = 8.6 \times 10^{-5} \text{ for air; for other substances, } I = 1.35 \times 10^{-5}Z \).
Mechanisms of Energy Loss

Energy expenditure for creating ion pairs in media:

The average energy needed for creating an ion pair is normally \textbf{2 to 3 times greater} than the corresponding electron binding energy in the absorbing medium.

The deviation between the ionization energy and the average energy required to create an ion pair is due to the \textit{excitation of the atoms}, which does not lead to ionization.

### Table 5.1. Average Energy Lost by a Beta Particle in the Production of an Ion Pair

<table>
<thead>
<tr>
<th>Gas</th>
<th>Ionization potential</th>
<th>Mean energy expenditure per ion pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>H\textsubscript{2}</td>
<td>13.6 eV</td>
<td>36.6 eV</td>
</tr>
<tr>
<td>He</td>
<td>24.5</td>
<td>41.5</td>
</tr>
<tr>
<td>N\textsubscript{2}</td>
<td>14.5</td>
<td>34.6</td>
</tr>
<tr>
<td>O\textsubscript{2}</td>
<td>13.6</td>
<td>30.8</td>
</tr>
<tr>
<td>Ne</td>
<td>21.5</td>
<td>36.2</td>
</tr>
<tr>
<td>A</td>
<td>15.7</td>
<td>26.2</td>
</tr>
<tr>
<td>Kr</td>
<td>14.0</td>
<td>24.3</td>
</tr>
<tr>
<td>Xe</td>
<td>12.1</td>
<td>21.9</td>
</tr>
<tr>
<td>Air</td>
<td></td>
<td>33.7</td>
</tr>
<tr>
<td>CO\textsubscript{2}</td>
<td>14.4</td>
<td>32.9</td>
</tr>
<tr>
<td>CH\textsubscript{4}</td>
<td>14.5</td>
<td>27.3</td>
</tr>
<tr>
<td>C\textsubscript{2}H\textsubscript{2}</td>
<td>11.6</td>
<td>25.7</td>
</tr>
<tr>
<td>C\textsubscript{2}H\textsubscript{4}</td>
<td>12.2</td>
<td>26.3</td>
</tr>
<tr>
<td>C\textsubscript{2}H\textsubscript{6}</td>
<td>12.8</td>
<td>24.6</td>
</tr>
</tbody>
</table>

Cember, Introduction to Health Physics, Fourth Edition
Specific Ionization

In the context of radiation protection and health physics, it is normally important to specify the effect of the energy deposition by a beta particle in terms of the number of ion pairs created by the particle after traveling through a unit path length – the specific ionization.

\[ S.I. = \frac{dE/dx \text{ eV/cm}}{w \text{ eV/ip}} \]

where \( w \) is the average energy expenditure required to create a ion pair.

**Figure 5.7.** Relationship between beta particle energy and specific ionization of air.

Cember, Introduction to Health Physics, Fourth Edition
Most of the radiation effects are initiated by the interactions of electrons and especially LOW energy electrons with water molecules.

Physical and biological responses to ionizing radiation. Ionizing radiation causes damage either directly by damaging the molecular target or indirectly by ionizing water, which in turn generates free radicals that attack molecular targets. The physical steps that lead to energy deposition and free radical formation occur within $10^{-5}$ to $10^{-6}$ seconds, while the biological expression of the physical damage may occur seconds or decades later.
Specific Energy Loss of Beta Particles

An example:

What is the specific ionization resulting from the passage of a 0.1-MeV beta particle through standard air?

Solution:

The specific energy loss is given by

$$\frac{dE}{dx} = \frac{2\pi q^4 NZ \times (3 \times 10^9)^4}{E_m\beta^2(1.6 \times 10^{-6})^2} \ln \left[ \frac{E_mE_k\beta^2}{I^2(1-\beta^2)} \right] - \beta^2 \text{ MeV/cm}$$

To use the equation, one needs to find $\beta$ as the following

$$E_k = m_0c^2 \left[ \frac{1}{\sqrt{1-\beta^2}} - 1 \right]$$

so $\beta^2 = 0.3010$. 
Specific Energy Loss of Beta Particles

An example (continued)

What is the specific ionization resulting from the passage of a 0.1-MeV beta particle through standard air?

The specific energy loss is then

\[ \frac{dE}{dx} = \frac{2\pi (1.6 \times 10^{-19})^4 \times 3.88 \times 10^{20} \times (3 \times 10^9)^4}{0.51 \times 0.3010 \times (1.6 \times 10^{-6})^2} \]

\[ \times \left\{ \ln \left[ \frac{0.51 \times 0.1 \times 0.3010}{(8.6 \times 10^{-5})^2 (1 - 0.3010)} \right] - 0.3010 \right\} \text{ MeV/cm} = 4.75 \times 10^{-3} \text{ MeV/cm}. \]

For air, \( w = 34 \text{ eV/ip} \). Therefore, the specific ionization is

\[ \text{S.I.} = \frac{4750 \text{ eV/cm}}{34 \text{ eV/cm}} = 140 \text{ ip/cm}. \]
Key Aspects of Beta Interactions

- Collisional interactions of beta particles with matter.
- Specific energy loss of beta particles.
- Dependence of the specific energy loss on the effective Z of absorbing material and the energy of the beta particles.
- Mass stopping, what and why?
- Radiative energy loss of beta particles.
- Relative importance of collisional and radiative energy loss.
- Fraction of energy loss due to Bremsstrahlung process and its implementation to shielding design for beta particles.
- Range of beta particles.
- Backscattering of beta particles.
Mass Stopping Power

It is also common to specify the energy loss of beta particles in a medium in terms of mass stopping power, which given by

\[
S = \frac{\text{specific energy loss (MeV/cm)}}{\text{density (g/cm}^3\text{)}} = \frac{\text{dE/dx}}{\rho} (\text{MeV} \cdot \text{cm}^2/\text{g})
\]

where \( \rho \) is the density of the absorbing medium.

Why mass stopping power?
Remarks on the Mass Stopping Power

• Mass stopping power does not differ greatly for materials with similar atomic compositions.

• Mass stopping power for water can be scaled by density and used for tissue, plastics, hydrocarbons, and other materials that consist primarily of light elements.

In health physics, it is sometimes important to show the mass stopping power of different absorbers relative to that of air – the relative mass stopping power

\[ \rho_m = \frac{S_{\text{medium}}}{S_{\text{air}}} \approx \frac{S_{\text{medium}} \left( \frac{\text{MeV}}{\text{g/cm}^2} \right)}{3.67 \left( \frac{\text{MeV}}{\text{g/cm}^2} \right)} \]
Mass Stopping Power

An example:

What is the relative (to air) mass stopping power of graphite, density = 2.25 g/cm\(^3\), for a 0.1-MeV beta particle?

The **mass stopping power** is given by

\[
S = \frac{\text{specific energy loss (MeV/cm)}}{\text{density (g/cm}^3\text{)}} = \frac{dE}{dx} \frac{\text{MeV}}{\rho \text{cm}^2/g}
\]

where \(\rho\) is the density of the absorbing medium.

where

\[
\frac{dE}{dx} = \frac{2\pi q^4 NZ \times (3 \times 10^9)^4}{E_m \beta^2 (1.6 \times 10^{-6})^2} \left\{ \ln \left[ \frac{E_m E_k \beta^2}{I^2 (1 - \beta^2)} \right] - \beta^2 \right\} \text{MeV/cm}
\]

The electron density \(NZ\) is given by

\[
NZ = \frac{6.02 \times 10^{23} \text{ atoms/mol}}{12 \text{ g/mol}} \times \frac{2.25 \text{ g/cm}^3}{6 \text{ electrons/atom}}
\]

\[
= 6.77 \times 10^{23} \text{ electrons/cm}^3,
\]
Mass Stopping Power

An example (continued):

and \( I = 1.35 \times 10^{-5} \times 6 = 8.1 \times 10^{-5} \text{ MeV.} \)

Therefore

\[
\frac{dE}{dx} = \frac{2\pi (1.6 \times 10^{-19})^4 \times 6.77 \times 10^{23} \times (3 \times 10^9)^4}{0.51 \times 0.3010 \times (1.6 \times 10^{-6})^2} \times \left\{ \ln \left[ \frac{0.51 \times 0.1 \times 0.3010}{(8.1 \times 10^{-5})^2 (1 - 0.3010)} \right] - 0.3010 \right\} \frac{\text{MeV}}{\text{cm}} = 8.33 \frac{\text{MeV}}{\text{cm}}.
\]

The mass stopping power is given as

\[
S(\text{graphite}) = \frac{\frac{dE}{dx}}{\rho} = \frac{8.33 \text{ MeV/cm}}{2.25 \text{ g/cm}^3} = 3.70 \frac{\text{MeV}}{\text{g/cm}^2}.
\]

and therefore

\[
\rho_m = \frac{S_{\text{medium}}}{S_{\text{air}}} = \frac{3.70 \frac{\text{MeV}}{\text{g/cm}^2}}{3.67 \frac{\text{MeV}}{\text{g/cm}^2}} = 1.01.
\]
Remarks on the Mass Stopping Power

- In a gas, \(-dE/dx\) depends on pressure, but \(-dE/(\rho dx)\) does not, because dividing by the density exactly compensates for the pressure.

- Generally, **heavy atoms** are **less efficient** in terms of mass stopping power for slowing down charged particles, because many of their electrons are too tightly bound in the inner shells to participate effectively in the absorption of beta energy. For example, for Pb (Z=82) \(-dE/\rho dx = 17.5 \ \text{MeV cm}^2\text{g}^{-1}\) for 10-MeV protons. (~ 47 MeV cm\(^2\) g\(^{-1}\) for water for 10 MeV protons).
Interactions of Charged Particles
Mass Stopping Power for Compounds

H. Stopping Power in Compounds
The mass collision stopping power, the mass radiative stopping power, and their sum
the mass stopping power can all be well approximated for intimate mixtures of ele-
ments, or for chemical compounds, through the assumption of Bragg’s Rule (ICRU,
1984a). It states that atoms contribute nearly independently to the stopping power,
and hence their effects are additive. This neglects the influence of chemical binding
on I, as noted in Section III.A. In terms of the weight fractions $f_{Z_1}, f_{Z_2}$, of elements
of atomic numbers $Z_1, Z_2$, etc. present in a compound or mixture, the mass stopping
power $(dT/\rho dx)_{\text{mix}}$ can be written as

$$
\left( \frac{dT}{\rho dx} \right)_{\text{mix}} = f_{Z_1} \left( \frac{dT}{\rho dx} \right)_{Z_1} + f_{Z_2} \left( \frac{dT}{\rho dx} \right)_{Z_2} + \cdots
$$

(8.20)

where all stopping powers refer to a common kinetic energy and type of charged
particle.
Restricted Mass Stopping Power

I. Restricted Stopping Power
The mass collision stopping power \( \frac{d\tau}{\rho dx} \), expresses the average rate of energy loss by a charged particle in all hard, as well as soft, collisions. The \( \delta \)-rays resulting from hard collisions may be energetic enough to carry kinetic energy a significant distance away from the track of the primary particle. More importantly, if one is calculating the dose in a small object or thin foil transversed by charged particles (as will be discussed in Section V.A), the use of the mass collision stopping power will overestimate the dose, unless the escaping \( \delta \)-rays are replaced (i.e., unless \( \delta \)-ray CPE exists).

The restricted stopping power is that fraction of the collision stopping power that includes all the soft collisions plus those hard collisions resulting in \( \delta \) rays with energies less than a cutoff value \( \Delta \). The mass restricted stopping power in MeV cm\(^2\)/g, will be symbolized here as \( \frac{d\tau}{\rho dx}_\Delta \).
Mass Stopping Power

\[ -\frac{dE}{dx} \quad (\text{MeV cm}^2 \text{g}^{-1}) \]

\[ \text{ENERGY (eV)} \]

Fig. 6.1 Mass stopping power of water for low-energy electrons.

Turner, Atoms, Radiation and Radiation Protection, 3’rd edition
Single Collision Energy-Loss Spectrum

Fig. 5.3 Single-collision energy-loss spectra for 50-eV and 150-eV electrons and 1-MeV protons in liquid water. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)

Turner, Atoms, Radiation and Radiation Protection, 3’rd edition
Remarks on Mass Stopping Power

• It takes ~22 eV to produce an e-i pair in water. At low energy, the specific energy loss of electron is increasing with energy. This does NOT agree with Beth’s formula,

\[
\frac{dE}{dx} = \frac{2\pi q^4 NZ \times (3 \times 10^9)^4}{E_m\beta^2(1.6 \times 10^{-6})^2} \left\{ \ln \left[ \frac{E_mE_k\beta^2}{I^2(1 - \beta^2)} \right] - \beta^2 \right\} \text{MeV/cm}
\]

• An 10 keV electron produces ~450 secondary electrons through cascade of ionization events.

• In water, most of ionization events are induced by electrons with E<100 eV.
**FIGURE 13.1.** Chemical development of a 4-keV electron track in liquid water, calculated by Monte Carlo simulation. Each dot in these stereo views gives the location of one of the active radiolytic species, OH, H$_3$O$^+$, e$_{aq}$, or H, at the times shown. Note structure of track with spurs, or clusters of species, at early times. After 10$^{-7}$ s, remaining species continue to diffuse further apart, with relatively few additional chemical reactions. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)
Key Aspects of Beta Interactions

• Collisional interactions of beta particles with matter.

• Specific energy loss of beta particles.

• Dependence of the specific energy loss on the effective Z of absorbing material and the energy of the beta particles.

• Mass stopping, what and why?

• Radiative energy loss of beta particles.

• Relative importance of collisional and radiative energy loss.

• Fraction of energy loss due to Bremsstrahlung process and its implementation to shielding design for beta particles.

• Range of beta particles.

• Backscattering of beta particles.
Radiative Energy Loss of Beta Particles – Bremsstrahlung

- **Bremsstrahlung** occurs when a beta particle is deflected or accelerated in the forced field of nucleus.
Radiative Energy Loss of Beta Particles – Bremsstrahlung

Part of the energy possessed by the beta particle is emitted in the form of photons. The rate of energy loss is proportional to the square of the instantaneous acceleration experienced by the beta particle.

\[-\left(\frac{dE}{dx}\right)_r = \frac{NEZ(Z + 1)e^4}{137m_0^2c^4} \left(4 \ln \frac{2E}{m_0c^2} - \frac{4}{3}\right)\]

E: kinetic energy of the beta particle,
N: number of absorber atoms per cm\(^3\),
Z: atomic number of the absorber,
m\(_0\): mass of an electron
Radiative Energy Loss of Beta Particles – Bremsstrahlung

Key Aspects of Beta Interactions

• Collisional interactions of beta particles with matter.
• Specific energy loss of beta particles.
• Dependence of the specific energy loss on the effective $Z$ of absorbing material and the energy of the beta particles.
• Mass stopping, what and why?
• Radiative energy loss of beta particles.
• Relative importance of collisional and radiative energy loss.
• Fraction of energy loss due to Bremsstrahlung process and its implementation to shielding design for beta particles.
• Range of beta particles.
• Backscattering of beta particles.
Characteristics of Bremsstrahlung Process

- **Bremsstrahlung** process becomes increasingly important at higher energy, say in the MeV range.

- The efficiency of bremsstrahlung in elements **varies nearly as** $Z^2$ (In comparison, the energy loss due to ionization and excitation is proportional to $Z$).

- In MeV energy range, the rate of energy loss through bremsstrahlung **increases nearly linearly with beta energy**, whereas $(-dE/dx)$ by ionization and excitation increases only with the logarithm of beta energy.

- The ratio between the energy loss due to ionization-excitation and **bremsstrahlung** is approximately given by

\[
\frac{(-dE/dx)_{\text{bremsstrahlung}}}{(-dE/dx)_{\text{ionization–excitation}}} \approx \frac{ZE_\beta \text{ (MeV)}}{800}
\]
## Radiative Energy Loss of Beta Particles – Bremsstrahlung

<table>
<thead>
<tr>
<th>Kinetic Energy</th>
<th>$\beta^2$</th>
<th>$\frac{1}{\rho} \left( \frac{dE}{dx} \right)_{col}$ (MeV cm² g⁻¹)</th>
<th>$\frac{-1}{\rho} \left( \frac{dE}{dx} \right)_{rad}$ (MeV cm² g⁻¹)</th>
<th>$\frac{-1}{\rho} \left( \frac{dE}{dx} \right)_{tot}$ (MeV cm² g⁻¹)</th>
<th>Rad</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 eV</td>
<td>0.00004</td>
<td>4.0</td>
<td>—</td>
<td>4.0</td>
<td>4</td>
</tr>
<tr>
<td>30</td>
<td>0.00012</td>
<td>44.</td>
<td>—</td>
<td>44.</td>
<td>44</td>
</tr>
<tr>
<td>50</td>
<td>0.00020</td>
<td>170.</td>
<td>—</td>
<td>170.</td>
<td>170</td>
</tr>
<tr>
<td>75</td>
<td>0.00029</td>
<td>272.</td>
<td>—</td>
<td>272.</td>
<td>272</td>
</tr>
<tr>
<td>100</td>
<td>0.00039</td>
<td>314.</td>
<td>—</td>
<td>314.</td>
<td>314</td>
</tr>
<tr>
<td>200</td>
<td>0.00078</td>
<td>298.</td>
<td>—</td>
<td>298.</td>
<td>298</td>
</tr>
<tr>
<td>500 eV</td>
<td>0.00195</td>
<td>194.</td>
<td>—</td>
<td>194.</td>
<td>194</td>
</tr>
<tr>
<td>1 keV</td>
<td>0.00390</td>
<td>126.</td>
<td>—</td>
<td>126.</td>
<td>126</td>
</tr>
<tr>
<td>2</td>
<td>0.00778</td>
<td>77.5</td>
<td>—</td>
<td>77.5</td>
<td>77.5</td>
</tr>
<tr>
<td>5</td>
<td>0.0193</td>
<td>42.6</td>
<td>—</td>
<td>42.6</td>
<td>42.6</td>
</tr>
<tr>
<td>10</td>
<td>0.0380</td>
<td>23.2</td>
<td>—</td>
<td>23.2</td>
<td>23.2</td>
</tr>
<tr>
<td>25</td>
<td>0.0911</td>
<td>11.4</td>
<td>—</td>
<td>11.4</td>
<td>11.4</td>
</tr>
<tr>
<td>50</td>
<td>0.170</td>
<td>6.75</td>
<td>—</td>
<td>6.75</td>
<td>6.75</td>
</tr>
<tr>
<td>75</td>
<td>0.239</td>
<td>5.08</td>
<td>—</td>
<td>5.08</td>
<td>5.08</td>
</tr>
<tr>
<td>100</td>
<td>0.301</td>
<td>4.20</td>
<td>—</td>
<td>4.20</td>
<td>4.20</td>
</tr>
<tr>
<td>200</td>
<td>0.483</td>
<td>2.84</td>
<td>0.006</td>
<td>2.85</td>
<td>2.85</td>
</tr>
<tr>
<td>500</td>
<td>0.745</td>
<td>2.06</td>
<td>0.010</td>
<td>2.07</td>
<td>2.07</td>
</tr>
<tr>
<td>700 keV</td>
<td>0.822</td>
<td>1.94</td>
<td>0.013</td>
<td>1.95</td>
<td>1.95</td>
</tr>
<tr>
<td>1 MeV</td>
<td>0.886</td>
<td>1.87</td>
<td>0.017</td>
<td>1.89</td>
<td>1.89</td>
</tr>
<tr>
<td>4</td>
<td>0.987</td>
<td>1.91</td>
<td>0.065</td>
<td>1.98</td>
<td>1.98</td>
</tr>
<tr>
<td>7</td>
<td>0.991</td>
<td>1.93</td>
<td>0.084</td>
<td>2.02</td>
<td>2.02</td>
</tr>
<tr>
<td>10</td>
<td>0.998</td>
<td>2.00</td>
<td>0.183</td>
<td>2.18</td>
<td>2.18</td>
</tr>
<tr>
<td>100</td>
<td>0.999+</td>
<td>2.20</td>
<td>2.40</td>
<td>4.60</td>
<td>4.60</td>
</tr>
<tr>
<td>1000 MeV</td>
<td>0.999+</td>
<td>2.40</td>
<td>26.3</td>
<td>28.7</td>
<td>28.7</td>
</tr>
</tbody>
</table>

From Atoms, Radiation, and Radiation Protection, James E Turner, p140
Characteristics of Bremsstrahlung

The total linear energy loss of beta particles is given by

\[ (-dE/dx)_{\text{total}} = (-dE/dx)_{\text{bremsstrahlung}} + (-dE/dx)_{\text{ionization-excitation}} \]
Energy Loss by Bremsstrahlung

- For beta particles to stop completely, the fraction of energy loss by Bremsstrahlung process is approximately given by

\[ f_\beta = 3.5 \times 10^{-4} Z E_m, \]  

(5.1)

where
- \( f_\beta \) = the fraction of the incident beta energy converted into
- \( Z \) = atomic number of the absorber,
- \( E_m \) = maximum energy of the beta particle, MeV.
Energy Loss by Bremsstrahlung

An example

A very small source (physically) of \(3.7 \times 10^{10}\) Bq (1 Ci) of \(^{32}\)P is inside a lead shield just thick enough to prevent any beta particles from emerging. What is the bremsstrahlung energy flux at a distance of 10 cm from the source (neglect attenuation of the bremsstrahlung by the beta shield)?

Solution:

The fraction of energy emitted in the form of bremsstrahlung is

\[
f_\beta = 3.5 \times 10^{-4} \quad ZE_m = 3.5 \times 10^{-4} \times 82 \times 1.71 = 0.049.
\]

The total amount of kinetic energy carried by the electrons emitted by the source is

\[
E_\beta \text{ (MeV/s)} = \frac{1}{3} \frac{E_{\text{max}} \text{MeV}}{\beta} \times 3.7 \times 10^{10} \frac{\beta}{\text{s}}
\]
Energy Loss by Bremsstrahlung

An example (continued)

For health physics purposes, it is assumed that all the bremsstrahlung photons are of the beta particle’s maximum energy, $E_{\text{max}}$. The photon flux $\phi$ of bremsstrahlung photons at a distance $r$ cm from a point source of beta particles whose activity is $3.7 \times 10^{10}$ Bq (1 Ci) is therefore given as

\[
\phi = \frac{f E_\beta}{4\pi r^2 E_{\text{max}}}
\]

\[
= \frac{0.049 \times \frac{1}{3} \times 1.71 \text{ MeV/\beta}}{4\pi \times (10 \text{ cm})^2 \times 1.71 \text{ MeV/photon}} \times \frac{3.7 \times 10^{10} \text{ \beta}}{\text{s}} = 4.8 \times 10^5 \text{ photons/s/cm}^2.
\]
Energy Loss by Bremsstrahlung – X-ray Production

The fraction of energy emitted in the form of bremsstrahlung is
Energy Loss by Bremsstrahlung – X-ray Production

Typical energy spectrum for photons generated with an X-ray tube.

Cember, Introduction to Health Physics, Fourth Edition
Backscattering

The fact that electrons often undergo large-angle deflections along their tracks leads to the phenomenon of *backscattering*. An electron entering one surface of an absorber may undergo sufficient deflection so that it re-emerges from the surface through which it entered. These backscattered electrons do not deposit all their energy in the absorbing medium and therefore can have a significant effect on the response of detectors designed to measure the energy of externally incident electrons. Electrons that backscatter in the detector “entrance window” or dead layer will escape detection entirely.

Knoll, Radiation Detection and measurements, p47.
Monte Carlo Simulation of Electron Paths. This simulation is of 15 KeV electrons in fayalite (Fe$_2$SiO$_4$). Distances are given in nanometers (1000 nm = 1 µm). Paths of backscattered electrons are in red; those of absorbed electrons in blue. One should remember that this slice through a three-dimensional volume. This model was run using the Casino software described at http://www.gel.usherbrooke.ca/casino/What.html.

http://www4.nau.edu/microanalysis/Microprobe-SEM/Signals.html
Backscattering

Backscattering is most pronounced for electrons with low incident energy and absorbers with high atomic number. Figure 2.17 shows the fraction $\eta$ of monoenergetic electrons that are backscattered from thick slabs of various materials, as a function of incident energy $E$.

**(Figure 2.17)** Fraction $\eta$ of normally incident electrons that are backscattered from thick slabs of various materials, as a function of incident energy $E$. (From Tabata et al.\textsuperscript{27})

Knoll, Radiation Detection and measurements, p49.
Positron Interactions

The coulomb forces that constitute the major mechanism of energy loss for both electrons and heavy charged particles are present for either positive or negative charge on the particle. Whether the interaction involves a repulsive or attractive force between the incident particle and orbital electron, the impulse and energy transfer for particles of equal mass are about the same. Therefore, the tracks of positrons in an absorber are similar to those of normal negative

Positrons differ significantly, however, in that the annihilation radiation described in Chapter 1 is generated at the end of the positron track. Because these 0.511 MeV photons are very penetrating compared with the range of the positron, they can lead to the deposition of energy far from the original positron track.
Key Aspects of Beta Interactions

• Collisional interactions of beta particles with matter.

• Specific energy loss of beta particles.

• Dependence of the specific energy loss on the effective Z of absorbing material and the energy of the beta particles.

• Mass stopping, what and why?

• Radiative energy loss of beta particles.

• Relative importance of collisional and radiative energy loss.

• Fraction of energy loss due to Bremsstrahlung process and its implementation to shielding design for beta particles.

• Backscattering of beta particles.

• Range of beta particles.
Tracks of Beta Particles in Absorbing Medium

- Since beta particles have the same mass as the orbital electrons, they are easily scattered during collision and therefore follow **tortuous paths** in absorbing medium.
- The electrons are “**wondering**” more significantly near the end of their tracks.
- **Energy-loss interactions** are more sparsely distributed at the beginning of the track.

**Figure 6.7.** Calculated tracks (projected into the plane of the figure) of 800-keV electrons in water. Each electron starts moving horizontally toward the right from the point 0 on the vertical axis.

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p150
Range of Beta Particles

The range of beta particles is defined as the absorber thickness required to ensure that almost no beta particle can penetrate the entire thickness.

Fig. 5.1. Experimental arrangement for absorption measurements on beta particles.

Fig. 5.2. Absorption curve (aluminum absorbers) of $^{210}$Bi beta particles, 1.17 MeV. The broken line represents the mean background counting rate.
Range of Beta Particles

- The absorber **half-thickness** is \( \sim 1/8 \) the range of the beta rays.

- The **range-energy relationship** is typically determined experimentally by using different beta sources.

For beta particles having a fixed maximum energy, what does the stopping power of an absorber depend on?

---

Fig. 5.3. Range-energy curves for beta rays in various substances. (Adapted from *Radiological Health Handbook*, Office of Technical Services, Washington, 1960.)
Density Thickness and the Range of Beta Particles

• The stopping power of an absorber is proportional to the number of electrons in the path of the beta particle – the areal density of electrons (the number of electrons per cm²) in the absorber.

• The areal electron density is approximately proportional to the density × linear thickness of the absorber.

stopping power \( \propto \) areal density of electrons \( \propto \) density \( \times \) linear thickness

• Therefore, for assessing the attenuation of beta particles in absorbing media, we can use the density thickness defined as

\[
density\ thickness = \text{density} \times \text{linear thickness} \\
or \quad t_d = \rho (\text{g/cm}^3) \times t_l (\text{cm})
\]
Density Thickness and the Range of Beta Particles

The use of the **density thickness** allows one to specify the stopping power of an absorber independently of its material, given that the materials of interest have similar atomic compositions — **absorbers with similar density thicknesses should have similar stopping power for beta particles.**

![Range-energy curve for beta particles](image)

**Fig. 5.4.** Range-energy curve for beta particles. The range is expressed in units of density thickness (From *Radiation Health Handbook*, Office of Technical Services, Washington, 1960).
Density Thickness and Electron Range

The range of beta particles as a function of their maximum energy can be approximately given by

\[ R = 412 \ E^{1.265 - 0.0954 \ \ln E} \]

for \( 0.01 \leq E \leq 2.5 \ \text{MeV} \),

\[ \ln E = 6.63 - 3.2376(10.2146 - \ln R)^{\frac{1}{2}} \]

for \( R \leq 1200 \),

\[ R = 530 \ E - 106 \]

for \( E > 2.5 \ \text{MeV} \), \( R > 1200 \),

where \( R = \text{range, mg/cm}^2 \)
\( E = \text{maximum beta-ray energy, MeV} \).

The range-energy relationship is often used by health physicists as an aid in identifying an unknown beta-emitting contaminant.
Energy Release of Beta Decay

An example

\[ \frac{32}{15}P \rightarrow \frac{32}{16}S + \frac{0}{-1}e \]

The corresponding energy release is given by

\[ Q = M_P - M_d - M_e = 0.001837 \text{ AMU} \]

or equivalently

\[ Q = 1.71 \text{ MeV} \]

Similar to the case of alpha decay, the energy shared by the recoil nucleus is

\[ M_e/(M_p+M_e) \times Q \]

... So the electron generated will be mono-energetic...
The energy release is shared by all three daughter products. Due to the relatively large mass of the daughter nucleus, it attains only a small fraction of the energy. Therefore, the kinetic energy of the beta particle is

$$E_{\beta^-} \approx Q - E_{\nu}$$
Density Thickness and Electron Range

The experimental setup for measuring the range of electrons from an unknown beta source.

![Experimental Setup Diagram]

**Fig. 5.5.** Measuring the range of an unknown beta particle to identify the isotope.
Density Thickness and Electron Range

For example, if the range of the beta rays (the total density thickness of the absorbing material required to fully stop the beta rays) is determined as

\[
\text{Range} = 1.7 \frac{\text{mg}}{\text{cm}^2} + 1.29 \frac{\text{mg}}{\text{cm}^2} + 30 \frac{\text{mg}}{\text{cm}^2} = 32.99 \frac{\text{mg}}{\text{cm}^2}.
\]

The maximum energy of the beta rays can be determined, by using the universal range-energy curve, to be \(\sim 0.17\text{MeV}\). Therefore, the beta emitter is likely to be \(^{14}\text{C}\) that emits beta particles with a maximum energy of 0.155\text{MeV}. 
Density Thickness and the Range of Beta Particles

The use of the **density thickness** allows one to specify the stopping power of an absorber independently of its material, given that the materials of interest have similar atomic compositions – **absorbers with similar density thicknesses should have similar stopping power for beta particles.**

![Graph showing range-energy curve for beta particles.](image)

**Fig. 5.4.** Range-energy curve for beta particles. The range is expressed in units of density thickness (From *Radiation Health Handbook*, Office of Technical Services, Washington, 1960).