Chapter 4: Interaction of Radiation with Matter

4.1 Interaction of Beta Particles

Key Aspects of Beta Interactions

- Collisional interactions of beta particles with matter.
- Specific energy loss of beta particles.
- Dependence of the specific energy loss on the effective Z of absorbing material and the energy of the beta particles.
- Mass stopping, what and why?
- Radiative energy loss of beta particles.
- Relative importance of collisional and radiative energy loss.
- Fraction of energy loss due to Bremsstrahlung process and its implementation to shielding design for beta particles.
- Range of beta particles.
- Backscattering of beta particles.

Mechanisms of Energy Loss by Electrons

Ionization and excitation:

Beta particles may **interact with orbital electrons through the electric fields** surrounding these charged particles, which leads to excitation and ionization.

Ionization process can be modeled as a **inelastic collision**, the energy loss by the electron and the kinetic energy carried by the ejected electron is related by

$$E_k = E_{loss} - \phi$$

where ϕ is the **ionization potential** of the absorbing medium.

Specific Energy Loss of Beta Particles

Specific energy loss: the linear rate of energy loss by an electron through excitation and ionization, which is given by

$$\frac{dE}{dx} = \frac{2\pi q^4 NZ \times (3 \times 10^9)^4}{E_m \beta^2 (1.6 \times 10^{-6})^2} \left\{ \ln \left[\frac{E_m E_k \beta^2}{I^2(1-\beta^2)} \right] - \beta^2 \right\} \frac{\text{MeV}}{\text{cm}}$$

where q = charge on the electron, 1.6×10^{-19} C,
N = number of absorber atoms per cm^3,
Z = atomic number of the absorber,
NZ = number of absorber electrons per cm^3 = 3.88×10^{20} for air at 0° and 76 cm Hg,
E_m = energy equivalent of electron mass, 0.51 MeV,
E_k = kinetic energy of the beta particle, MeV,
β = v/c,
I = mean ionization and excitation potential of absorbing atoms, MeV,
I = 8.6×10^{-5} for air; for other substances, $I = 1.35 \times 10^{-5} Z$.

Mechanisms of Energy Loss

Energy expenditure for creating ion pairs in media:

The average energy needed for creating an ion pair is normally **2 to 3 times greater** that the corresponding electron binding energy in the absorbing medium.

TABLE 5.1. AVERAGE ENERGY LOST BY A BETA PARTICLE IN THE PRODUCTION OF AN ION PAIR

Gas	Ionization potential	Mean energy expenditure per ion pair
H_2	13.6 eV	36.6 eV
He	24.5	41.5
N_2	14.5	34.6
O_2	13.6	30.8
Ne	21.5	36.2
A	15.7	26.2
Kr	14.0	24.3
Xe	12.1	21.9
Air		33.7
CO_2	14.4	32.9
CH_4	14.5	27.3
C_2H_2	11.6	25.7
C_2H_4	12.2	26.3
C_2H_6	12.8	24.6

The deviation between the ionization energy and the average energy required to create an ion pair is due to the **excitation of the atoms**, which does not lead to ionization.

Cember, Introduction to Health Physics, Fourth Edition

Specific Ionization

In the context of radiation protection and health physics, it is normally important to specify the effect of the energy deposition by a bate particle in terms of the number of ion pairs created by the particle after traveling through a unit path length – the **specific ionization**.

$$\text{S.I.} = \frac{\mathrm{d}E/\mathrm{d}x \ \text{eV/cm}}{w \ \text{eV/ip}}$$

where ***w*** is the average energy expenditure required to create a ion pair.

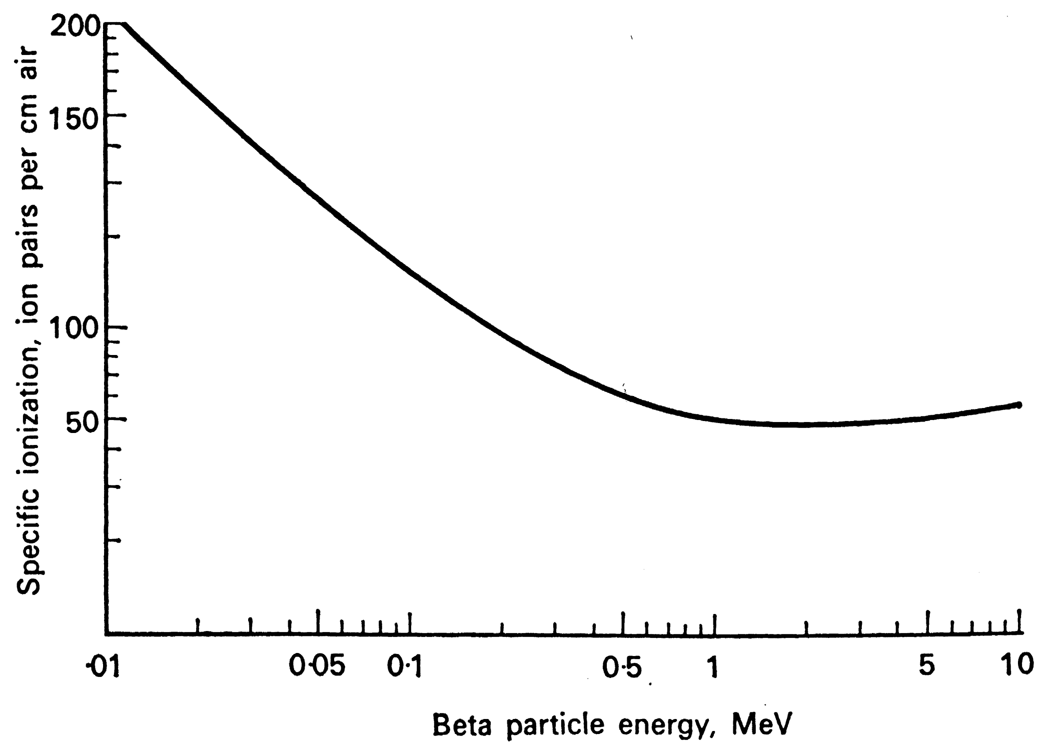

FIG 5.7. Relationship between beta particle energy and specific ionization of air.

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Key Aspects of Beta Interactions

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Mass Stopping Power

It is also common to specify the energy loss of beta particles in a medium in terms of **mass stopping power**, which given by

$$S = \frac{\text{specific energy loss}(MeV/\text{cm})}{\text{density}(\text{g}/\text{cm}^3)} = \frac{\text{dE}/\text{dx}}{\rho}(MeV \cdot \text{cm}^2/\text{g})$$

where ρ is the density of the absorbing medium.

In health physics, it is sometimes important to show the mass stopping power of different absorbers relative to that of air – the **relative mass stopping power**

$$\rho_m = \frac{S_{medium}}{S_{air}} \approx \frac{S_{medium}\left(\frac{MeV}{g/cm^2}\right)}{3.67\left(\frac{MeV}{g/cm^2}\right)}$$

Why **mass stopping power**?

Mass Stopping Power

An example:

What is the relative (to air) mass stopping power of graphite, density = 2.25 g/cm^3, for a 0.1-MeV beta particle?

The **mass stopping power** is given by

$$S = \frac{\text{specific energy loss}(MeV/\text{cm})}{\text{density}(\text{g}/\text{cm}^3)} = \frac{\text{dE}/\text{dx}}{\rho}(MeV \cdot \text{cm}^2/\text{g})$$

where ρ is the density of the absorbing medium.

where

$$\frac{dE}{dx} = \frac{2\pi q^4 \, NZ \times (3 \times 10^9)^4}{E_m \beta^2 (1.6 \times 10^{-6})^2} \left\{ \ln \left[\frac{E_m E_k \beta^2}{I^2 (1 - \beta^2)} \right] - \beta^2 \right\} \frac{\text{MeV}}{\text{cm}}$$

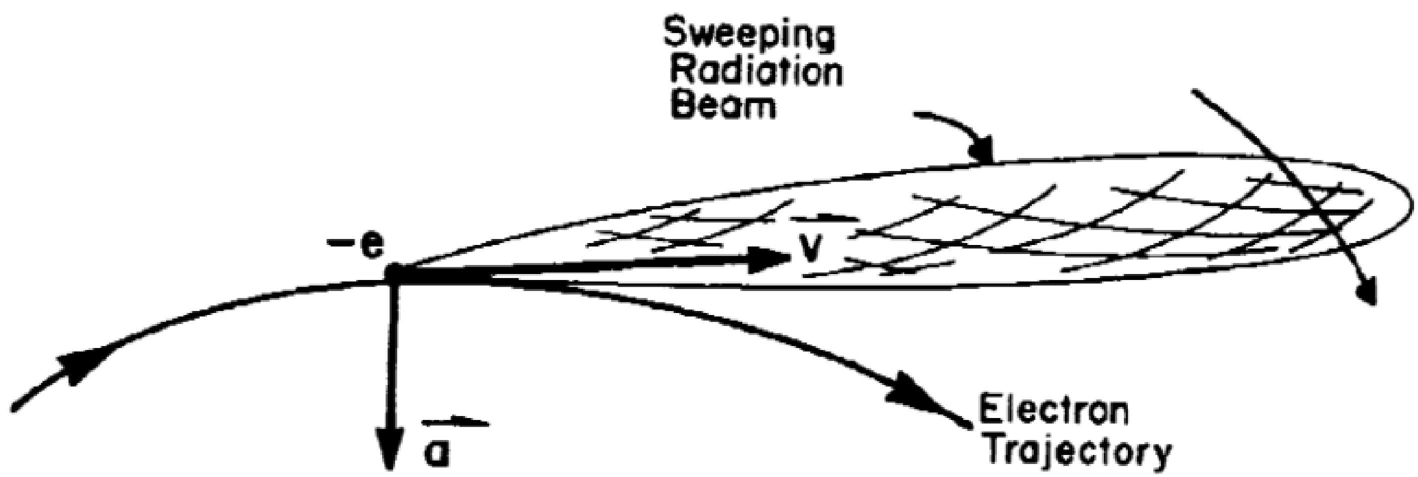
The electron density *NZ* is given by

$$NZ = \frac{6.02 \times 10^{23} \dfrac{\text{atoms}}{\text{mol}} \times 2.25 \dfrac{\text{g}}{\text{cm}^3} \times 6 \dfrac{\text{electrons}}{\text{atom}}}{12 \dfrac{\text{g}}{\text{mol}}}$$

$$= 6.77 \times 10^{23} \text{ electrons/cm}^3,$$

Radiative Energy Loss of Beta Particles – Bremsstrahlung

- **Bremsstrahlung** occurs when a beta particle is deflected or accelerated in the forced field of nucleus.



Radiative Energy Loss of Beta Particles – Bremsstrahlung

Part of the energy possessed by the beta particle is emitted in the form of **photons**. The **rate of energy loss** is proportional to the **square of the instantaneous acceleration** experienced by the beta particle.

$$-\left(\frac{dE}{dx}\right)_r = \frac{NEZ(Z+1)e^4}{137 m_0^2 c^4}\left(4\ln\frac{2E}{m_0c^2} - \frac{4}{3}\right)$$

Radiative Energy Loss of Beta Particles – Bremsstrahlung

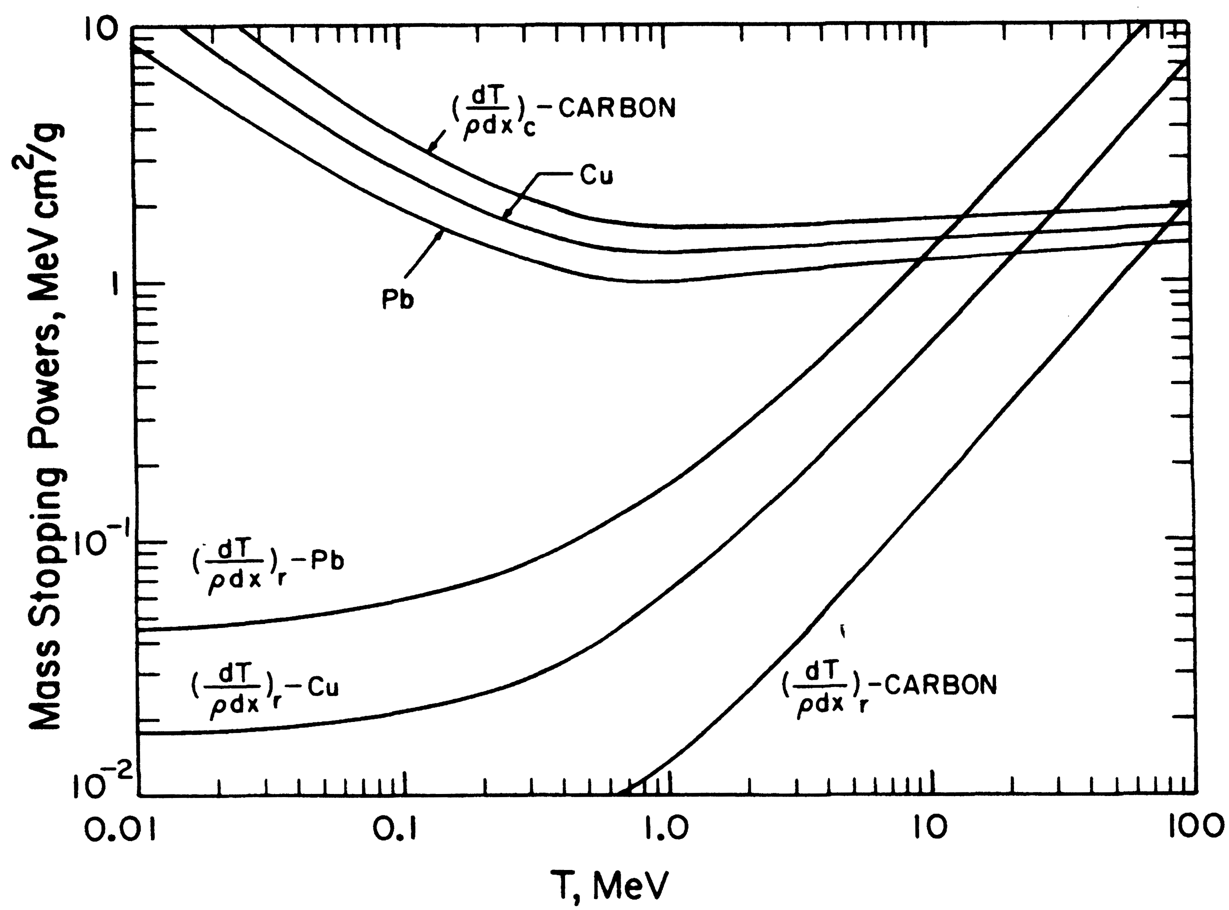


FIGURE 8.6. Mass radiative and collision stopping powers for electrons (and approximately for positrons) in C, Cu, and Pb. (From data of Bichsel, 1968).

Characteristics of Bremsstrahlung Process

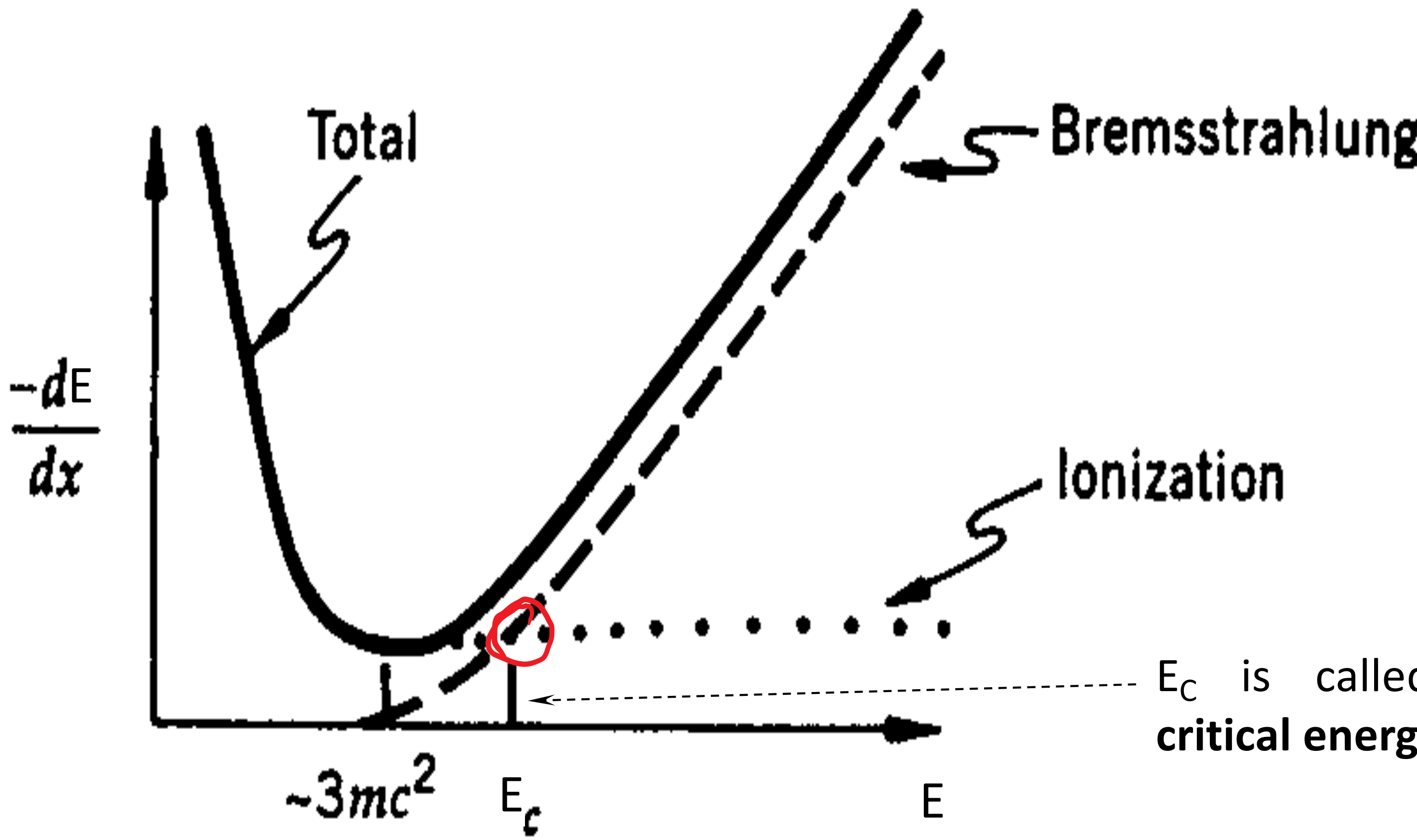
- **Bremsstrahlung** process becomes increasingly important at higher energy, say in the MeV range.
- The efficiency of bremsstrahlung in elements **varies nearly as Z^2** (In comparison, the energy loss due to ionization and excitation is proportional to Z).
- In MeV energy range, the rate of energy loss through bremsstrahlung **increases nearly linearly with beta energy,** whereas (-dE/dx) by ionization and excitation increases only with the logarithm of beta energy.
- **The ratio between the energy loss due to ionization-excitation and bremsstrahlung** is approximately given by

$$\frac{(-dE/dx)_{bremsstrahlung}}{(-dE/dx)_{ionization-excitation}} \approx \frac{ZE_{\beta}(MeV)}{800}$$

Characteristics of Bremsstrahlung

The total linear energy loss of beta particles is given by

$$\left(-dE/dx\right)_{total} = \left(-dE/dx\right)_{bremsstrahlung} + \left(-dE/dx\right)_{ionization-excitation}$$



E_C is called the **critical energy**

Radiative Energy Loss of Beta Particles – Bremsstrahlung

TABLE 6.1. Electron Collisonal, Radiative, and Total Mass Stopping Powers; and Range in Water

Kinetic Energy	β^2	$\frac{1}{\rho}\left(\frac{dE}{dx}\right)_{col}^{-}$ (MeV cm^2 g^{-1})	$-\frac{1}{\rho}\left(\frac{dE}{dx}\right)_{rad}^{-}$ (MeV cm^2 g^{-1})	$-\frac{1}{\rho}\left(\frac{dE}{dx}\right)_{tot}^{-}$ (MeV cm^2 g^{-1})	Rad Y
10 eV	0.00004	4.0	—	4.0	
30	0.00012	44.	—	44.	
50	0.00020	170.	—	170.	
75	0.00029	272.	—	272.	
100	0.00039	314.	—	314.	
200	0.00078	298.	—	298.	
500 eV	0.00195	194.	—	194.	
1 keV	0.00390	126.	—	126.	
2	0.00778	77.5	—	77.5	
5	0.0193	42.6	—	42.6	
10	0.0380	23.2	—	23.2	0.(
25	0.0911	11.4	—	11.4	0.(
50	0.170	6.75	—	6.75	0.(
75	0.239	5.08	—	5.08	0.(
100	0.301	4.20	—	4.20	0.(
200	0.483	2.84	0.006	2.85	0.(
500	0.745	2.06	0.010	2.07	0.(
700 keV	0.822	1.94	0.013	1.95	0.(
1 MeV	0.886	1.87	0.017	1.89	0.(
4	0.987	1.91	0.065	1.98	0.(
7	0.991	1.93	0.084	2.02	0.(
10	0.998	2.00	0.183	2.18	0.(
100	0.999+	2.20	2.40	4.60	0.:
1000 MeV	0.999+	2.40	26.3	28.7	0.7

From Atoms, Radiation, and Radiation Protection, James E Turner, p140

Energy Loss by Bremsstrahlung

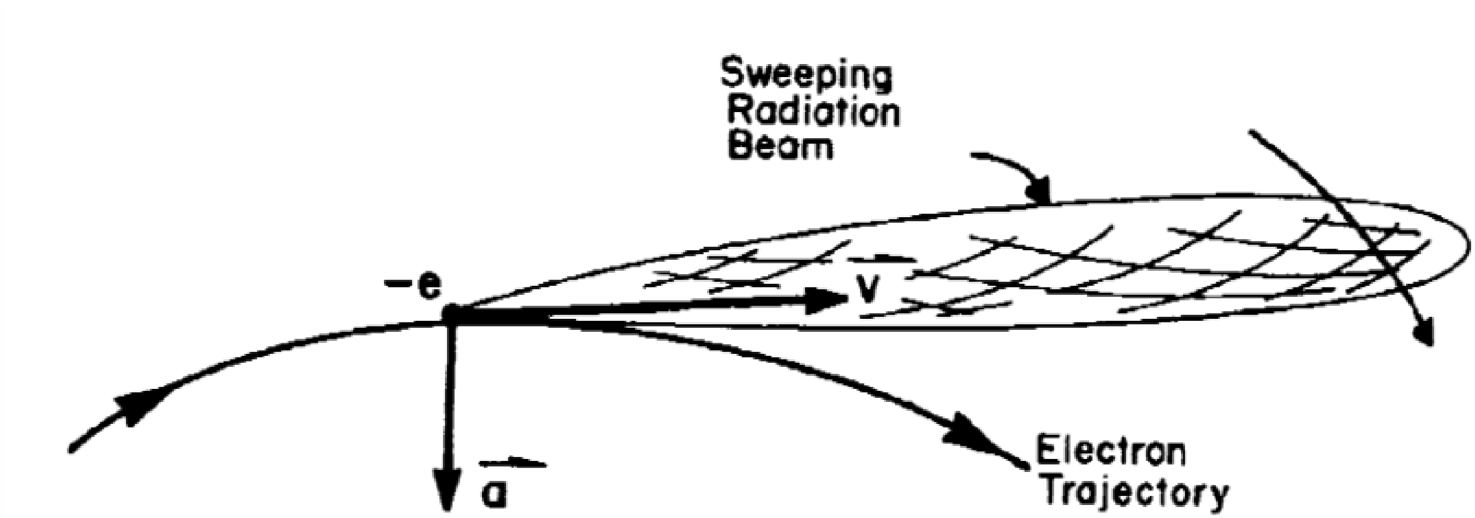
- For beta particles to stop completely , the **fraction of energy loss** by **Bremsstrahlung** process is approximately given by

$$f_\beta = 3.5 \times 10^{-4} ZE_m, \qquad (5.1$$

where f_β = the fraction of the incident beta energy converted into p

Z = atomic number of the absorber,

E_m = maximum energy of the beta particle, MeV.



Energy Loss by Bremsstrahlung

An example

A very small source (physically) of 3.7×10^{10}Bq (1 Ci) of ^{32}P is inside a lead shield just thick enough to prevent any beta particles from emerging. What is the bremsstrahlung energy flux at a distance of 10 cm from the source (neglect attenuation of the bremsstrahlung by the beta shield)?

Solution:

The fraction of energy emitted in the form of bremsstrahlung is

$$f_\beta = 3.5 \times 10^{-4} ZE_m = 3.5 \times 10^{-4} \times 82 \times 1.71 = 0.049.$$

The total amount of kinetic energy carried by the electrons emitted by the source is

$$E_\beta \text{ (MeV/s)} = \frac{1}{3} \frac{E_{\max}\text{MeV}}{\beta} \times 3.7 \times 10^{10} \frac{\beta}{\text{s}}$$

Energy Loss by Bremsstrahlung

An example (continued)

For health physics purposes, it is assumed that all the bremsstrahlung photons are of the beta particle's maximum energy, E_{max}. The photon flux ϕ of bremsstrahlung photons at a distance r cm from a point source of beta particles whose activity is 3.7×10^{10} Bq (1 Ci) is therefore given as

$$\phi = \frac{f E_\beta}{4\pi r^2 E_{max}}$$

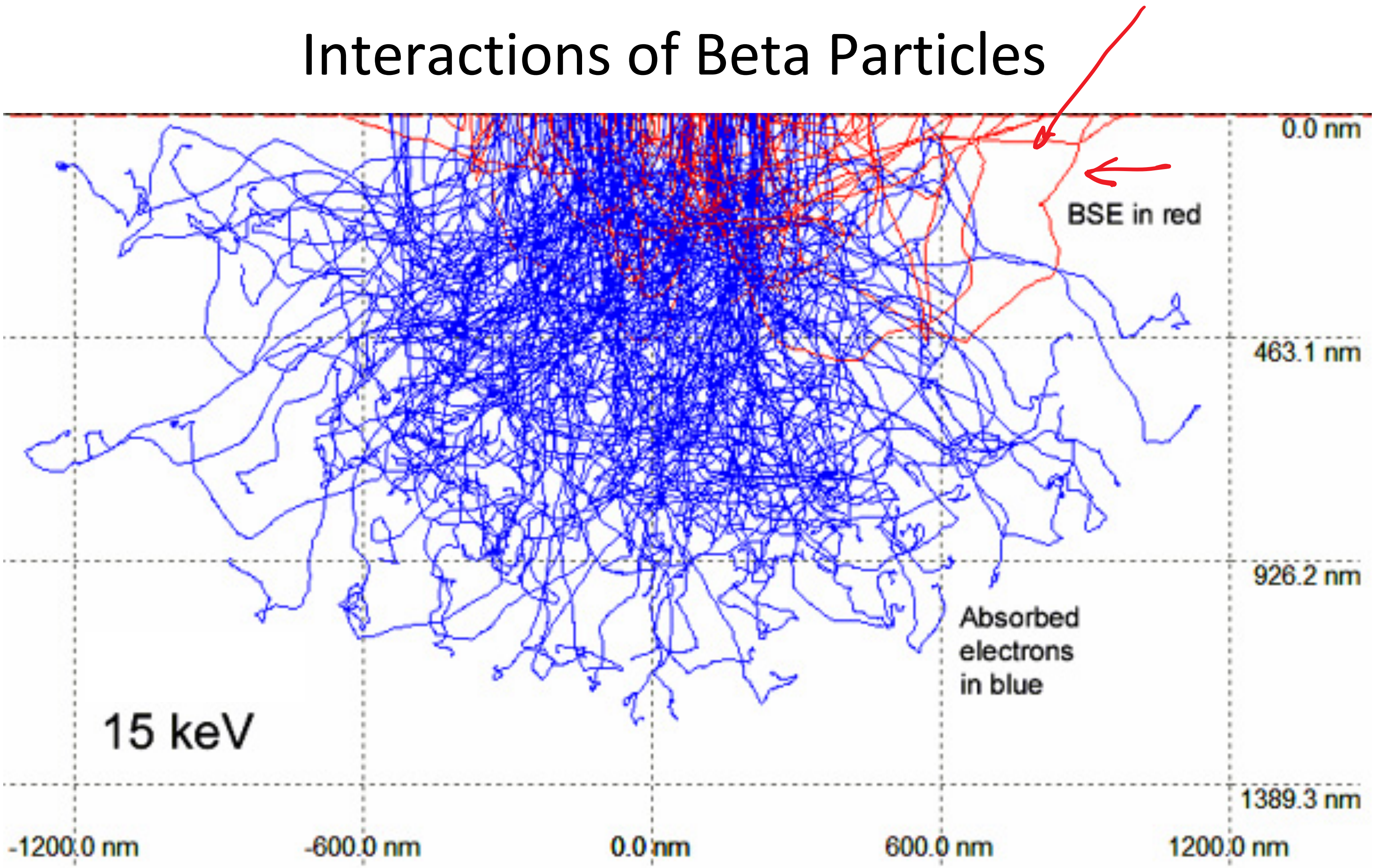
$$= \frac{0.049 \times \frac{1}{3} \times 1.71 \frac{\text{MeV}}{\beta} \times 3.7 \times 10^{10} \frac{\beta}{\text{s}}}{4\pi \times (10 \text{ cm})^2 \times 1.71 \text{ MeV/photon}} = 4.8 \times 10^5 \frac{\text{photons/s}}{\text{cm}^2}.$$

Backscattering

The fact that electrons often undergo large-angle deflections along their tracks leads to the phenomenon of *backscattering*. An electron entering one surface of an absorber may undergo sufficient deflection so that it re-emerges from the surface through which it entered. These backscattered electrons do not deposit all their energy in the absorbing medium and therefore can have a significant effect on the response of detectors designed to measure the energy of externally incident electrons. Electrons that backscatter in the detector "entrance window" or dead layer will escape detection entirely.

Knoll, Radiation Detection and measurements, p47.

Interactions of Beta Particles



Monte Carlo Simulation of Electron Paths. This simulation is of 15 KeV electrons in fayalite (Fe_2SiO_4). Distances are given in nanometers (1000 nm = 1 µm). Paths of backscattered electrons are in red; those of absorbed electrons in blue. One should remember that this slice through a three-dimensional volume. This model was run using the Casino software described at http://www.gel.usherbrooke.ca/casino/What.html.

http://www4.nau.edu/microanalysis/Microprobe-SEM/Signals.html

Backscattering

Backscattering is most pronounced for electrons with low incident energy and absorbers with high atomic number. Figure 2.17 shows the frac[illegible] of monoenergetic elec-

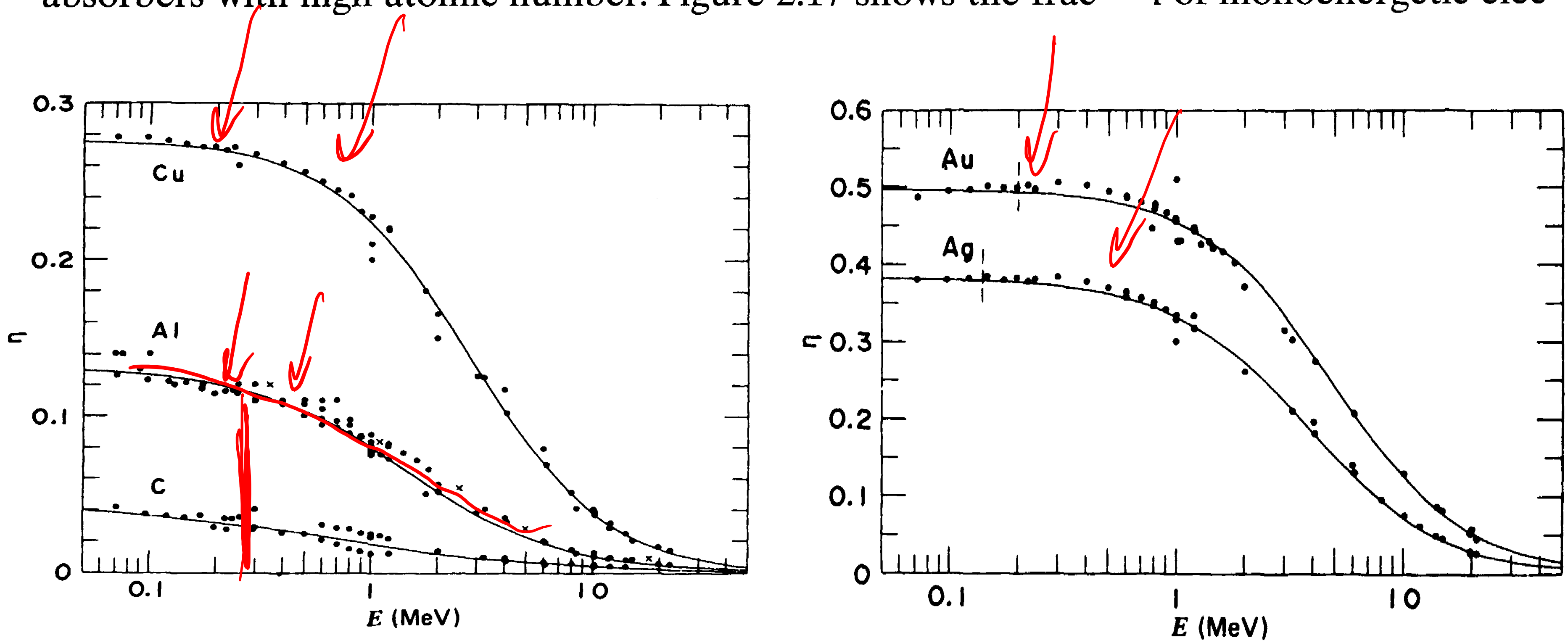


Figure 2.17 Fraction η of normally incident electrons that are backscattered from thick slabs of various materials, as a function of incident energy E. (From Tabata et al.[27])

Knoll, Radiation Detection and measurements, p49.

4.2 Interaction of Heavy Charged Particles

Energy Loss Mechanisms

- Heavy charged particles loss energy **primarily though the ionization and excitation** of atoms.

- Heavy charged particles can transfer only a small fraction of its energy in a single collision. Its deflection in collision is almost negligible. Therefore heavy charged particles **travel in a almost straight paths** in matter, **losing energy continuously** through a large number of collisions with atomic electrons.

- At low velocity, a heavy charged particle may losses a negligible amount of energy in **nuclear collisions**. It may also pick up free electrons along its path, which reduces it net charge.

Energy Loss Mechanisms

For heavy charged particles, the **maximum energy that can be transferred** in a single collision is given by the **conservation of energy and momentum**:

$$\tfrac{1}{2}MV^2 = \tfrac{1}{2}MV_1^2 + \tfrac{1}{2}mv_1^2$$

$$MV = MV_1 + mv_1.$$

where M and m are the mass of the heavy charged particle and the electron. V is the initial velocity of the charged particle. V_1 and v_1 are the velocities of both particles after the collision.

The **maximum energy transfer** is therefore given by

$$Q_{\max} = \frac{1}{2}MV^2 - \frac{1}{2}MV_1^2 = \frac{4mME}{(M+m)^2}$$

Maximum Energy Loss by a Single Collision

For a more general case, which includes the relativistic effect, the **maximum energy transferred by a single collision** is

$$Q_{\max} = \frac{2\gamma^2 m V^2}{1 + 2\gamma m/M + m^2/M^2}$$

where $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = V/c$, and c is the speed of light

Maximum Energy Loss by a Single Collision

TABLE 5.1. Maximum Possible Energy Transfer, Q_{max}, in Proton Collision with Electron

Proton Kinetic Energy E (MeV)	Q_{max} (MeV)	Maximum Percentage Energy Transfer $100Q_{max}/E$
0.1	0.00022	0.22
1	0.0022	0.22
10	0.0219	0.22
100	0.229	0.23
10^3	3.33	0.33
10^4	136.	1.4
10^5	1.06×10^4	10.6
10^6	5.38×10^5	53.8
10^7	9.21×10^6	92.1

Single Collision Energy-Loss Spectrum

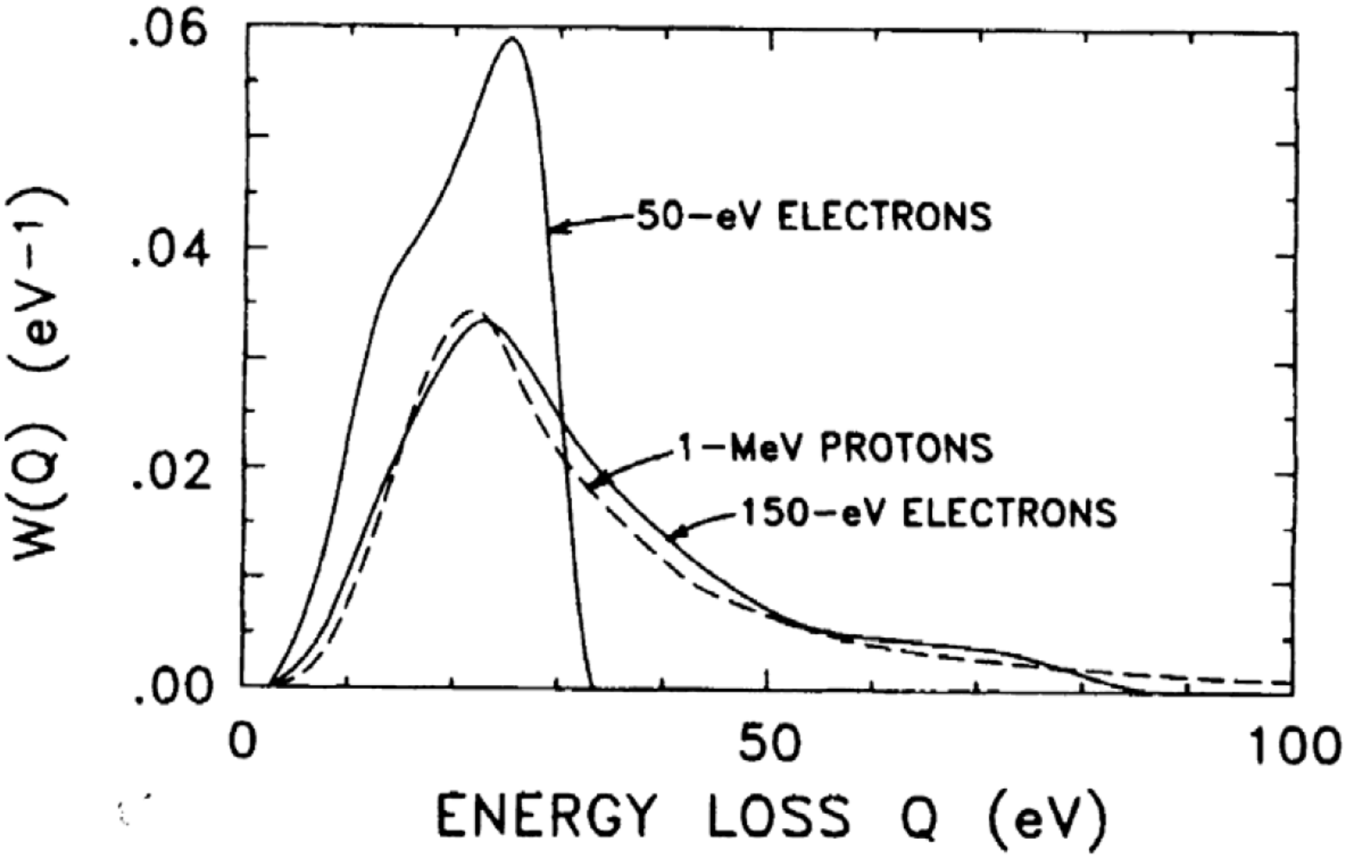


Fig. 5.3 Single-collision energy-loss spectra for 50-eV and 150-eV electrons and 1-MeV protons in liquid water. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)

Linear Stopping Power of a Medium for Heavy Charged Particles (revisited)

The **linear stopping power** of a medium is given by the Bethe formula,

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2\beta^2}\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2\right].$$

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2}\ln\frac{mV^2}{hf}.$$

k_0 = 8.99×10^9 N m^2 C^{-2} (Appendix C),
z = atomic number of the heavy particle,
e = magnitude of the electron charge,
n = number of electrons per unit volume in the medium,
m = electron rest mass,
c = speed of light in vacuum,
β = V/c = speed of the particle relative to c,
I = mean excitation energy of the medium.

Mean Excitation Energies

The main excitation energy (I) for an element having atomic number Z, can be approximately given by

$$I \cong \begin{cases} 19.0 \text{ eV}, \ Z = 1 \text{ (hydrogen)} \\ 11.2 + 11.7\, Z \text{ eV}, \ 2 \le Z \le 13 \\ 52.8 + 8.71\, Z \text{ eV}, \ Z > 13. \end{cases}$$

For compound or mixture,

If there are N_i atoms cm^{-3} of an element with atomic number Z_i and mean excitation energy I_i, then in formula (5.23) one makes the replacement

mean excitation energy for compound

$$n \ln I = \sum_i N_i Z_i \ln I_i,$$

Limitation of the Bethe Formula

Since almost all analytical descriptions of the behavior of heavy charged particles are based on the Bathe formula, it is important to realize the limitation of this formula.

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2\beta^2}\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2\right]$$

- ☞ Bethe formula is **valid for high energies** as long as the inequality $\gamma m/M \ll 1$ holds.
- ☞ At **low energy**, it fails because the term **$\ln[2mc^2\beta^2/I(1-\beta^2)]-\beta^2$ eventually becomes negative** giving a negative value for the stopping power.
- ☞ It does not account for the fact that at low energies, **a charged particle may capture electrons as it moves**, this will reduce its net charge and reduce the stopping power of the medium.
- ☞ The dependence of the Bethe formula on z^2 implies that a pair of particles, with the same amount of mass but opposite charge, have the same stopping power and range. **Departures from this predication** has been measured and theoretically predicted.

Interactions of Photons with Matter

Reading Material:

- ☞ Chapter 5 in <<Introduction to Health Physics>>, Third edition, by Cember.
- ☞ Chapter 8 in <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.
- ☞ Chapter 2 in <<Radiation Detection and Measurements>>, Third Edition, by G. F. Knoll.

Classification of Photon Interactions

Table 1. Classification of elementary photon interactions.

Type of interaction / Interaction with:	Absorption	Scattering: Elastic (Coherent)	Scattering: Inelastic (Incoherent)
Atomic electrons	**Photoelectric effect** $\sigma_{pe} \begin{cases} \sim Z^4 (L.E.) \\ \sim Z^5 (H.E.) \end{cases}$	Rayleigh scattering $\sigma_R \sim Z^2$ $(L.E.)$	**Compton scattering** $\sigma_C \sim Z$
Nucleus	Photonuclear reactions $(\gamma,n),(\gamma,p)$, photofission, etc. $\sigma_{ph.n.} \sim Z$ $(h\nu \geq 10\text{MeV})$	Elastic nuclear scattering $(\gamma,\gamma) \sim Z^2$	Inelastic nuclear scattering (γ,γ')
Electric field surrounding charged particles	**Electron-positron pair production in field of nucleus,** $\sigma_{pair} \sim Z^2$ $(h\nu \geq 1.02\text{MeV})$		

Photoelectric Effect – Absorption Edges

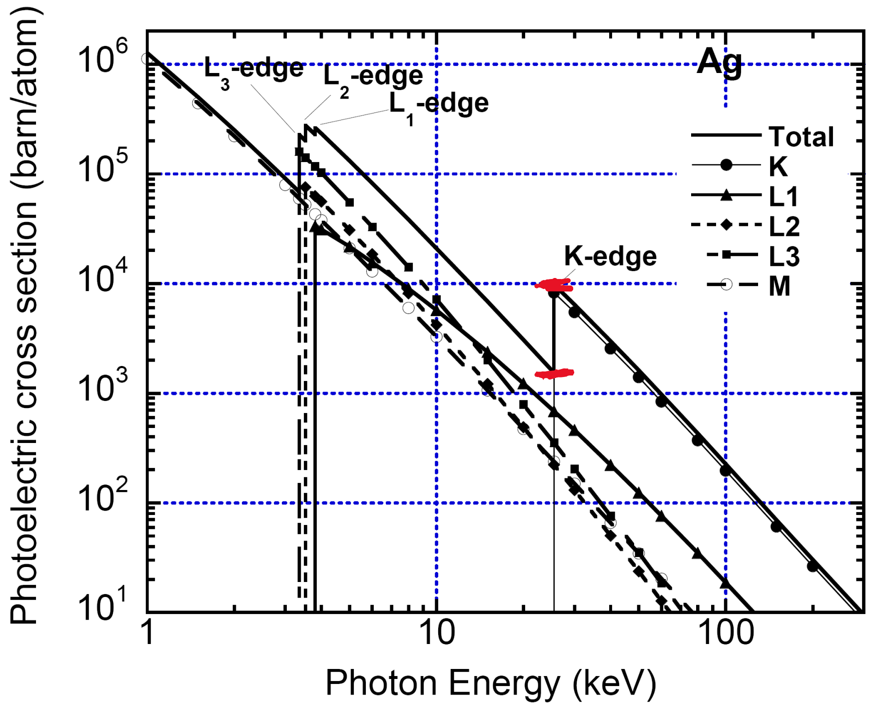


Figure 2: Total and partial atomic photoeffect of Ag.

- Requires **sufficient photon energy** for P.E. interaction.
- Interaction probability decreases dramatically with increasing energy.
- P.E. interaction is significant only for low energy photons, when the photon energy is close to the binding energies of the target atoms.

Photoelectric Effect Cross Section

Probability of photoelectric absorption per atom is

$$\sigma \propto \begin{cases} \dfrac{Z^4}{(hv)^{3.5}} & low\ energy \\ \dfrac{Z^5}{(hv)^{3.5}} & high\ energy \end{cases}$$

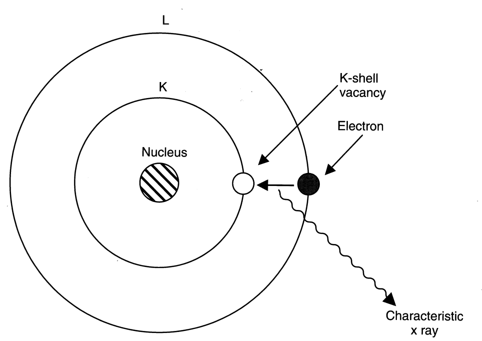
☞ The interaction cross section for photoelectric effect **depends strongly on Z**.

☞ Photoelectric effect is **favored at lower photon energies**. It is the major interaction process for photons at low hundred keV energy range.

Page 49, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

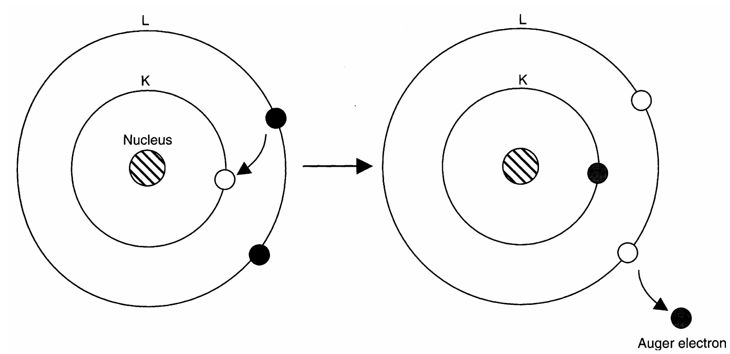
Relaxation Process after Photoelectric Effect

☞ The excited atoms will **de-excite** through one of the following processes:



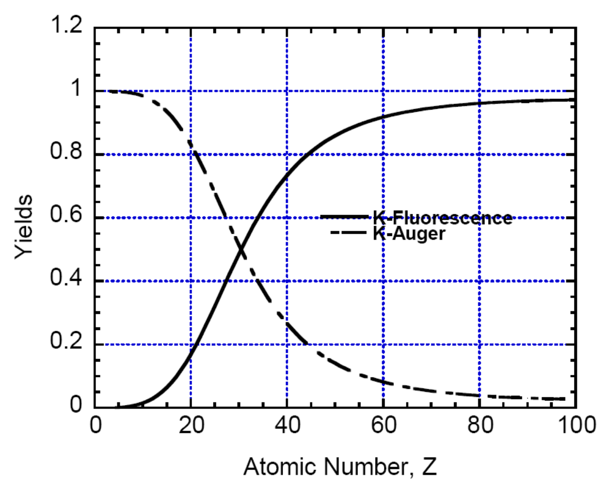
Generation of characteristic X-rays

Competing Processes



Generation of Auger electrons

☞ **Auger electron** emission dominates in **low-Z** elements. **Characteristic X-ray** emission dominates in **higher-Z** elements.



Auger Electrons

The relative probability of the emission of characteristic radiation to the emission of an Auger electron is called the fluorescent yield, ω:

$$\omega_K = \frac{\text{Number K x ray photons emitted}}{\text{Number K shell vacancies}} \quad (3\text{-}12)$$

Values for ω_K are given in Table 3-1. We see that for large Z values fluorescent radiation is favored, while for low values of Z Auger electrons tend to be produced.

From this table we see that if a nucleus with Z = 40 had a K shell hole, then on the average 0.74 fluorescent photons and 0.26 Auger electrons would be emitted.

TABLE 3-1
Fluorescent Yield

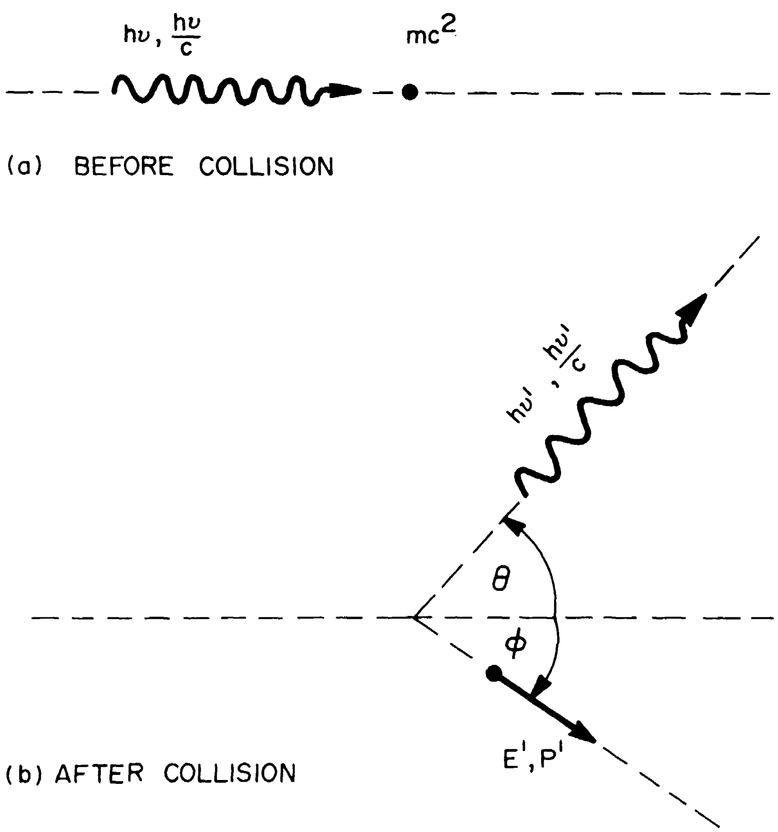
Z	ω_K	Z	ω_K	Z	ω_K
10	0	40	.74	70	.92
15	.05	45	.80	75	.93
20	.19	50	.84	80	.95
25	.30	55	.88	85	.95
30	.50	60	.89	90	.97
35	.63	65	.90		

From Evans (E1)

Basic Kinematics in Compton Scattering

The **energy transfer** in Compton scattering may be derived as the following:

- Assuming that the **electron binding energy is small** compared with the energy of the incident photon – **elastic scattering**.
- Write out the **conservation of energy and momentum**:



Conservation of energy

$$h\nu + mc^2 = h\nu' + E'$$

Conservation of momentum

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + P'\cos\varphi$$

$$\frac{h\nu'}{c}\sin\theta = P'\sin\varphi$$

Energy Transfer in Compton Scattering

If we assume that the electron is free and at rest, the scattered gamma ray has an energy

Initial photon energy, *v*: photon frequency

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1-\cos\theta)},$$

mass of electron

Scattering angle

and the photon transfers part of its energy to the electron (assumed to be at rest before the collision), which is known as the ***recoil electron***. Its energy is simply

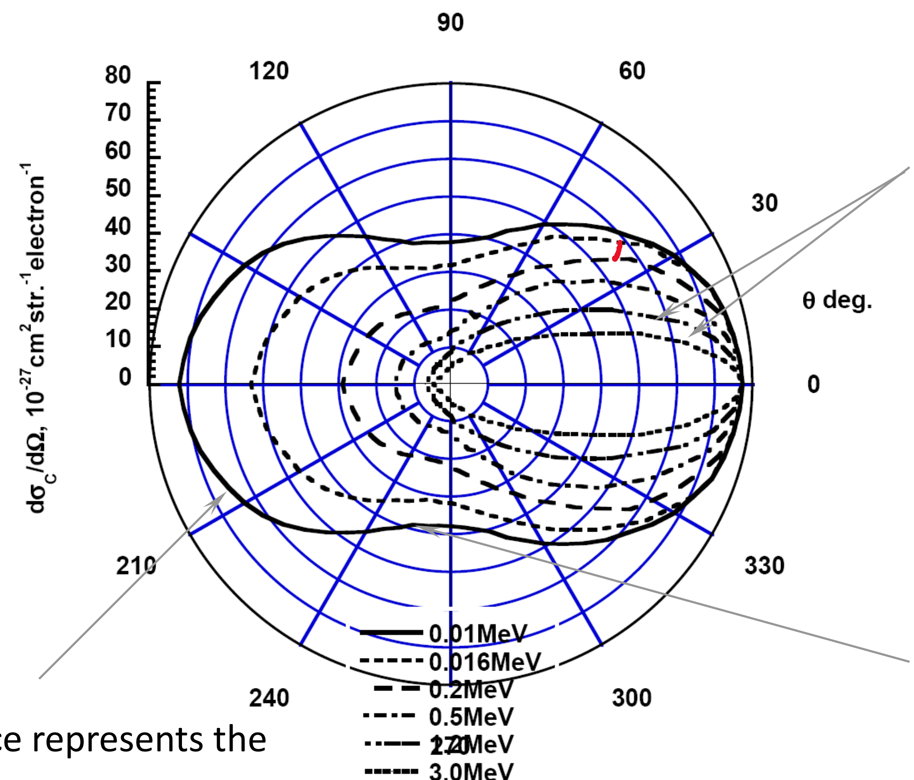
$$E_{recoil} = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1-\cos(\theta))}$$

assuming the binding energy of the electron is negligible.

In the simplified elastic scattering case, there is an one-to-one relationship between scattering angle and energy loss!!

Reading: Page 51, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Angular Distribution of the Scattered Gamma Rays



Radial distance represents the differential cross section.

Incident photons with ***higher energies*** tend to scatter with smaller angles (forward scattering).

Incident photons with ***lower energy*** (a few hundred keV) have greater chance of undergoing large angle scattering (back scattering).

The higher the energy carried by an incident gamma ray, the more likely that the gamma ray undergoes forward scattering …

Energy Distribution of Compton Recoil Electrons

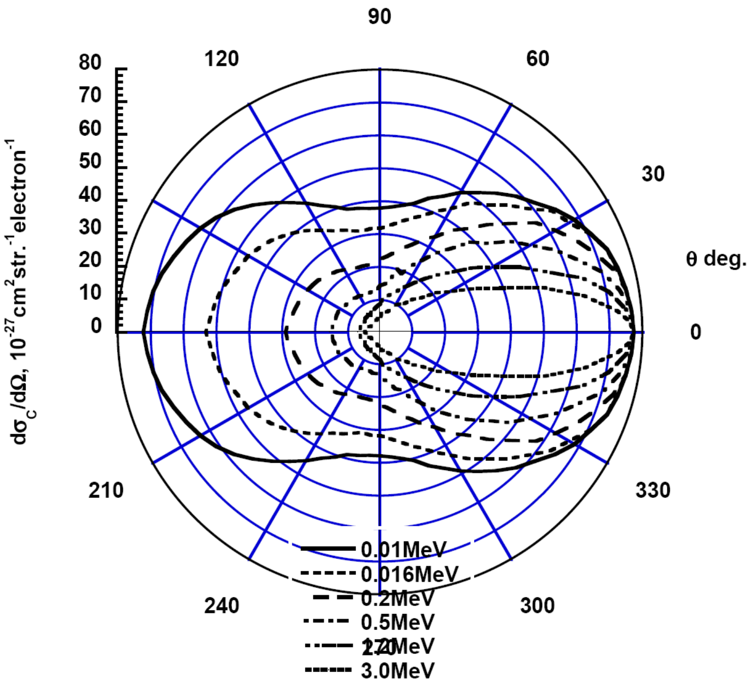
Klein-Nishina formula can be used to calculate the expected **energy spectrum of recoil electrons** as the following:

$$\frac{d\sigma}{dE_{recoil}} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{d\theta}\frac{d\theta}{dE_{recoil}}$$

and the probability that a recoil electron possesses an energy between E_{recoil}-ΔE/2 and E_{recoil}+ΔE/2 is given by

$$\propto \Delta E \cdot \frac{d\sigma}{dE_{recoil}}$$

Application of the Klein-Nishina Formula (1)



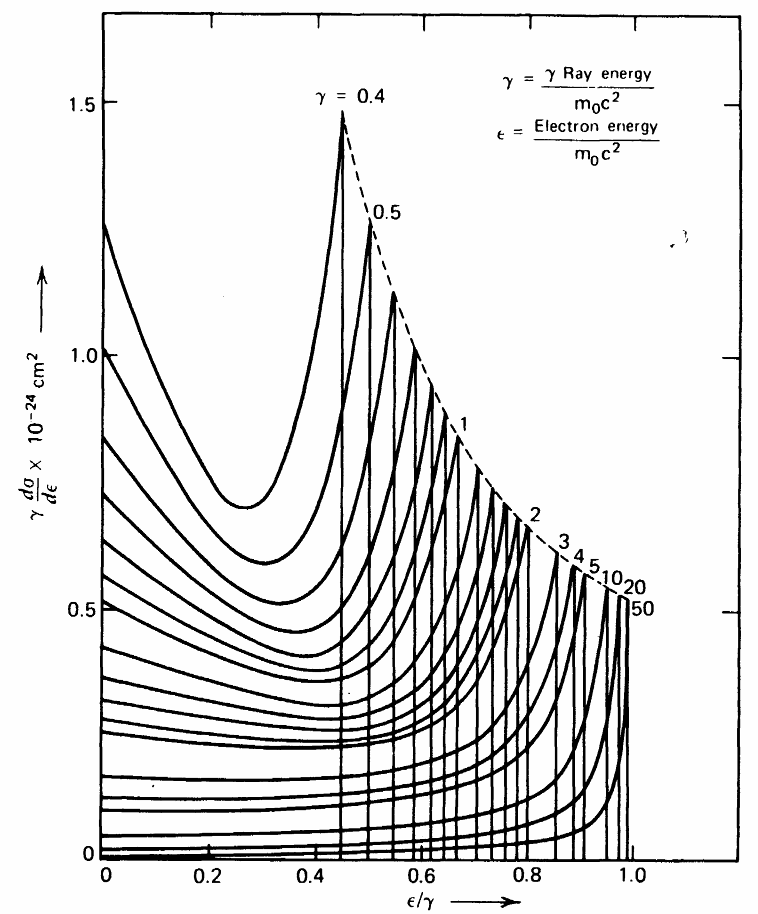


Figure 10.1 Shape of the Compton continuum for various gamma-ray energies. (From S. M. Shafroth (ed.), *Scintillation Spectroscopy of Gamma Radiation.* Copyright 1964 by Gordon & Breach, Inc. By permission of the publisher.)

☞ The energy distribution of the recoil electrons derived using the Klein-Nishina formula is closely related to **the energy spectrum measured with "small" detectors** (in particular, the so-called **Compton continuum**).

From Page 311, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Linear Attenuation Coefficient through Compton Scattering

The **differential Compton cross section** given by the Klein-Nishina Formula can also be related to another important parameter for gamma ray dosimetry – **the linear attenuation coefficient**.

$$\sigma_{linear} = NZ\sigma \quad (m^{-1}),$$

which is the **probability of a photon interacting with the absorber through Compton scattering while traversing a unit distance**. NZ is the electron density of the absorber materials (number of electrons per m^3)

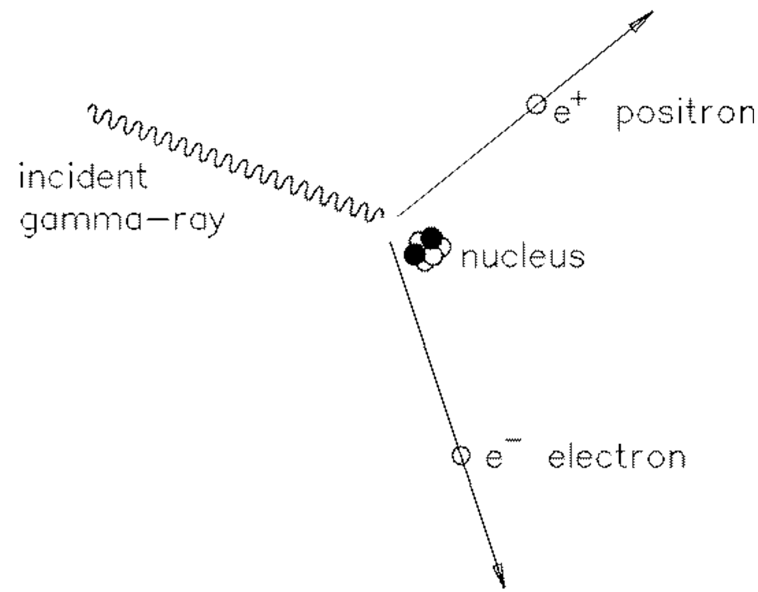
Note that σ is the Compton scattering cross section per electron and is given by

$$\sigma = 2\pi \int_{\Omega} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \cdot \sin\theta \cdot d\theta \;\; (m^2) \,.$$

Pair Production

Definition:

Pair production refers to the creation of an electron-positron pair by an incident gamma ray in the vicinity of a nucleus.



Characteristics

☞ The minimum energy required is

$$E_\gamma \geq 2m_e c^2 + \frac{2m_e^2 c^2}{m_{nucleus}} \approx 2m_e c^2 = 1.022MeV$$

☞ The process is more probable with a ***heavy nucleus*** and incident ***gamma rays with higher energies***.

☞ The positrons emitted will soon **annihilate** with ordinary electrons near by and produces two 511keV gamma rays.

Photonuclear Reaction

- A photon can be absorbed by an atomic nucleus and knock out a nucleon. This process is called **photonuclear reaction**. For example,

$$^{9}_{4}Be + hv \Rightarrow ^{8}_{4}Be + ^{1}_{0}n, \quad Q-value: -1.666 MeV$$

$$^{2}_{1}H + hv \Rightarrow ^{1}_{1}H + ^{1}_{0}n, \quad Q-value: -2.226 MeV$$

- The photon **must possess enough energy to overcome the nuclear binding energy**, which is generally several MeV.
- The threshold, or the minimum photon energy required, for (γ,p) reaction is generally higher than that for (γ,n) reactions. Since the repulsive Coulomb barrier that a proton must overcome to escape from the nucleus.
- Other nuclear reaction s are also possible, such as (γ, 2n), (γ, np), (γ, α) and photon induced fission reaction.

Interaction of Photons in Matter

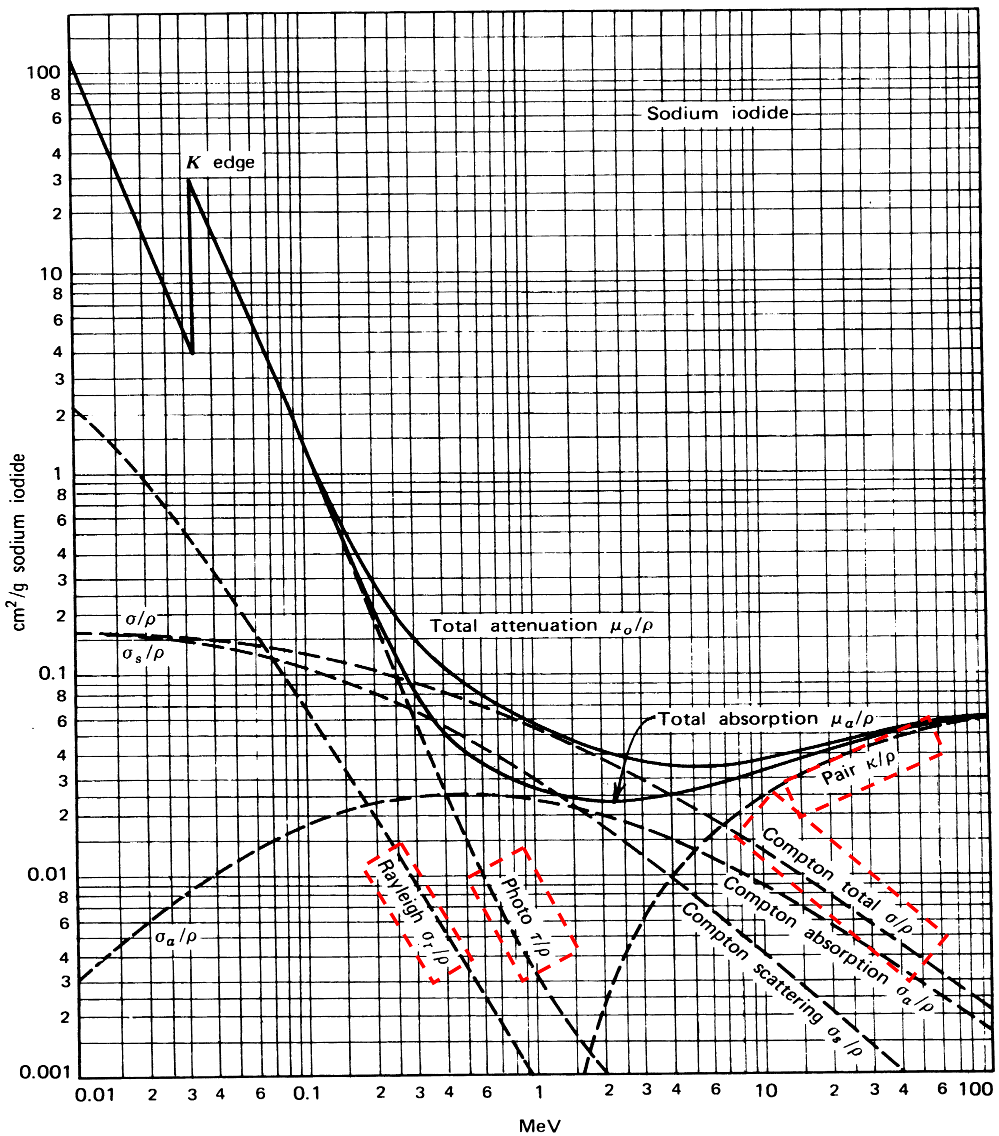


Figure 2.18 Energy dependence of the various gamma–ray interaction processes in sodium iodide. (From *The Atomic Nucleus* by R. D. Evans. Copyright 1955 by the McGraw-Hill Book Company. Used with permission.)

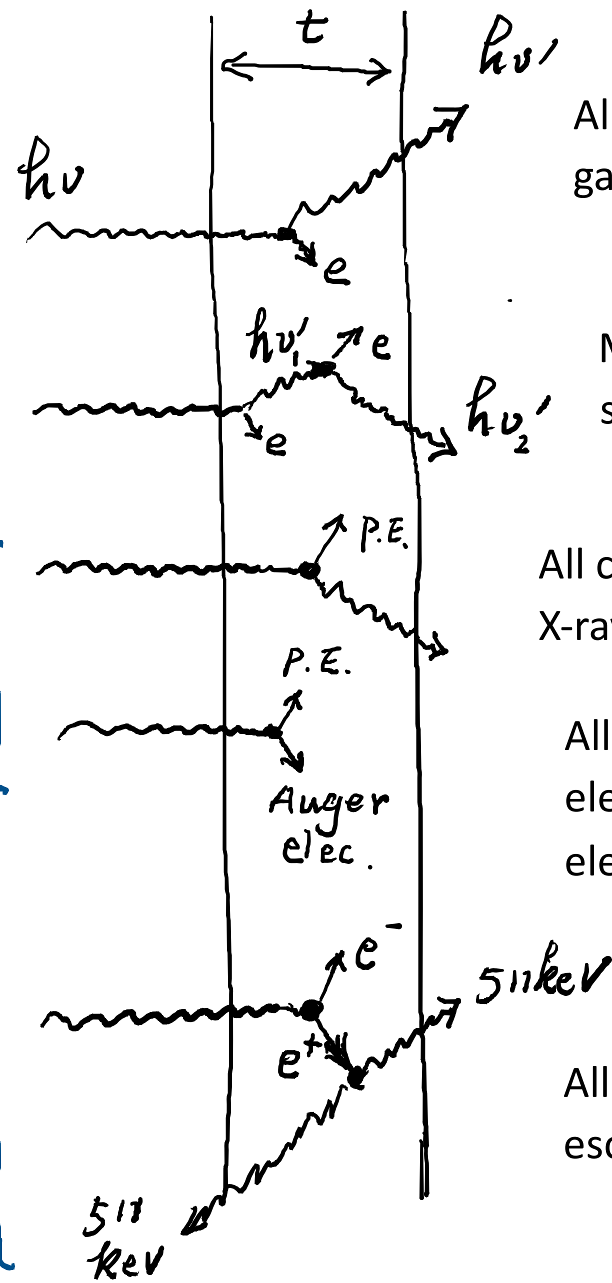
From Page 50, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Linear attenuation coefficient

and

Atomic attenuation coefficient

Energy Transfer by a Gamma Ray Beam



Compton scattering

- All Compton scattered gamma rays escaped
- Multiple Compton scattering ignored

Photoelectric effect

- All characteristic X-rays escaped
- All photoelectrons, auger electrons and Compton recoil electrons are absorbed

Pair production

- All annihilation gamma rays escaped

Energy-Transfer Coefficient

The **total mass energy transfer coefficient** is given by

The fraction of energy that is carried away by characteristic x-rays following photoelectric effect.

The fraction of energy that is carried away by the two 511keV gamma rays generated by the annihilation of the positron.

$$\frac{\mu_{tr}}{\rho} = \frac{\tau}{\rho}\left(1 - \frac{\delta}{h\upsilon}\right) + \frac{\sigma}{\rho}\left(\frac{E_{avg}}{h\upsilon}\right) + \frac{\kappa}{\rho}\left(1 - \frac{2mc^2}{h\upsilon}\right)$$

The fraction of energy that is transferred to recoil electron through Compton scattering.

For a parallel beam of monochromatic gamma rays transmitting through a unit distance in an absorbing material, **the energy-transfer coefficient is the fraction of energy that was originally carried by the incident gamma ray beam and transferred into the kinetic energy of secondary electron** inside the absorber.

Comparison Between Linear Attenuation Coefficient and Energy Absorption Coefficient

The fraction of photons removed from the beam after traveling through a unit distance.

The fraction of energy carried by the photons being absorbed in material.

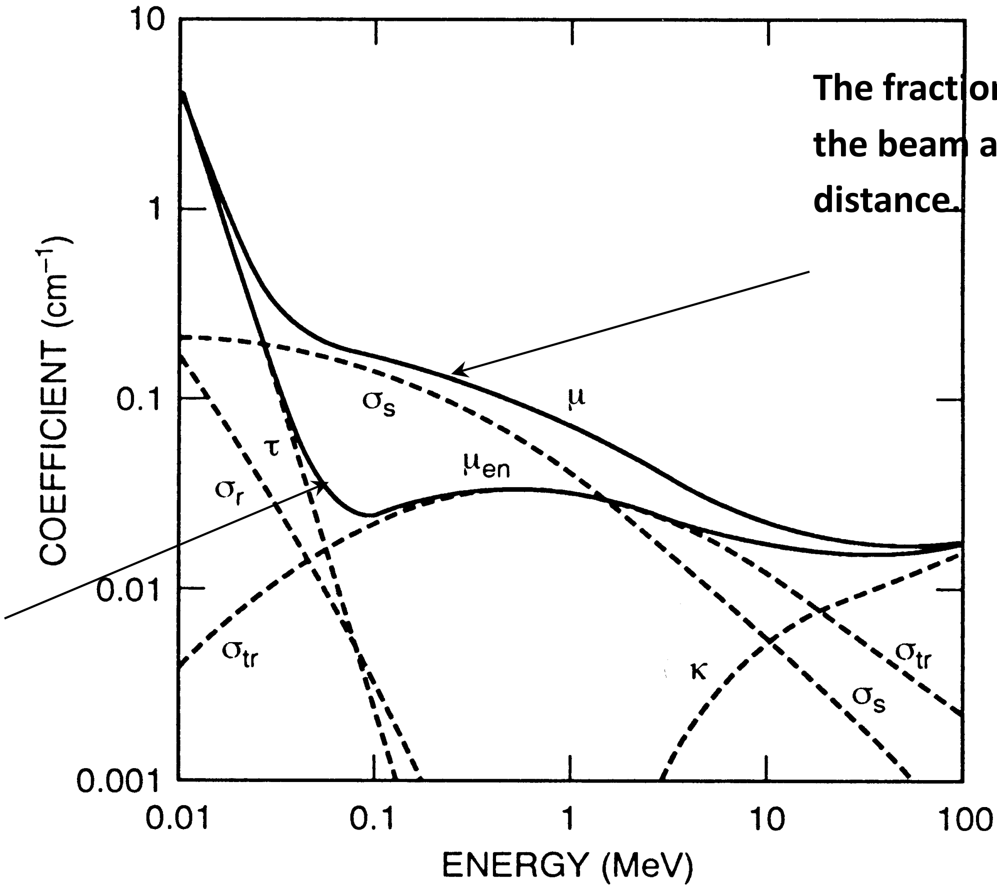


FIGURE 8.13. Linear attenuation and energy-absorption coefficients as functions of energy for photons in water.

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p195

Energy-Transfer and Energy-Absorption Coefficients

The photon fluence Φ: the number of photons cross a unit area perpendicular to the beam.

The photon fluence rate: the number of photons per unit area per unit time.

$$\dot{\Phi} = \frac{d\Phi}{dt} (m^{-2}s^{-1})$$

The energy fluence Ψ (Jm^{-2}): the amount of energy passes per unit area perpendicular to the beam.

The energy fluence rate (Jm^{-2}s^{-1}): the amount of energy transfer per unit area per unit time.

$$\dot{\Psi} = \frac{d\Psi}{dt} (Jm^{-2}s^{-1})$$

Calculation of Energy Transfer and Energy Absorption

For simplicity, we consider an idealized case, in which

- ☞ Photons are assumed to be monoenergetic and in broad parallel beam.
- ☞ Multiple Compton scattering of photons is negligible.
- ☞ Virtually all fluorescence and bremsstrahlung photons escape from the absorber.
- ☞ All secondary electrons (Auger electrons, photoelectrons and Compton electrons) generated are stopped in the slab.

Under these conditions, the transmitted energy intensity (the amount of energy transmitted through a unit area within each second) can be given by

$$\dot{\Psi} = \dot{\Psi}_0 e^{-\mu_{en} x}$$

Calculation of Energy Transfer and Energy Absorption

Assuming $\mu_{en}x<<1$, which is consistent with the thin slab approximation and the energy fluence rate carried by the incident gamma ray beam is $\dot{\Psi}_0(J\cdot cm^{-2}\cdot s^{-1})$. Then the energy absorbed in the thin slab per second over a unit cross section area is given by

$$\dot{\Psi}_0\mu_{en}x \ \ (J\cdot cm^{-2}\cdot s^{-1})$$

The rate of energy absorbed in the slab of area A (cm^2) and thickness *x* is

$$A\dot{\Psi}_0\mu_{en}x \ (J\cdot s^{-1})$$

Given the density of the material is ρ, the rate of energy absorption per unit mass (**Dose Rate**) in the slab is

$$\dot{D}=\frac{A(cm^2)\cdot\dot{\Psi}_0(J\cdot cm^{-2}\cdot s^{-1})\cdot\mu_{en}(cm^{-1})\cdot x\ (cm)}{\rho(g\cdot cm^{-3})\cdot A(cm^2)\cdot x(cm)},$$

$$Dose\ rate\ in\ the\ absorber:\ \ \dot{D}=\dot{\Psi}_0\frac{\mu_{en}}{\rho}(J\cdot g^{-3}\cdot s^{-1})$$

Interactions of Neutrons with Matter

Reading Material:

- ☞ Chapter 5 in <<Introduction to Health Physics>>, Third edition, by Cember.
- ☞ Chapter 9 in <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.
- ☞ Chapter 2 in <<Radiation Detection and Measurements>>, Third Edition, by G. F. Knoll.

Elastic Scattering of Neutrons

Kinematics of neutron scattering:

☞ Energy transfer as a function of scattering angle.

☞ Angular distribution of scattered neutrons.

☞ Energy spectrum of scattered neutrons.

☞ Average logarithm energy decrement of a neutron in multiple scattering.

Elastic Scattering of Neutrons

The **elastic scattering** plays an important role in neutron energy measurements. For example, a proton-neutron telescope illustrated below can be used to accurately measure the spectrum of neutrons in a collimated beam.

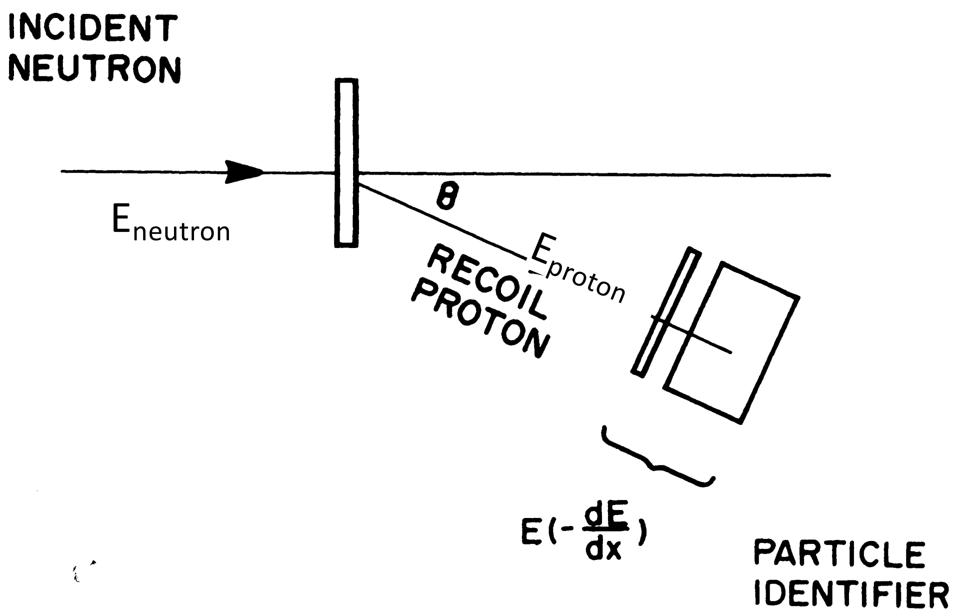


FIGURE 10.36. Arrangement of proton-recoil telescope for measuring spectrum neutron beam.

$$E_{proton} = E_{neutron} \cos^2 \theta$$

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p281

Elastic Scattering of Neutrons

The **maximum energy** that a neutron of mass M and kinetic energy E_n can transfer to a nucleus of mass m in a single elastic collision given by

$$E_{\max} = E_n \frac{4Mm}{(M+m)^2}$$

TABLE 9.4. Maximum Fraction of Energy Lost, Q_{max}/E_n from Eq. (9.3), by Neutron in Single Elastic Collision with Various Nuclei

Nucleus	Q_{max}/E_n
$^{1}_{1}H$	1.000
$^{2}_{1}H$	0.889
$^{4}_{2}He$	0.640
$^{9}_{4}Be$	0.360
$^{12}_{6}C$	0.284
$^{16}_{8}O$	0.221
$^{56}_{26}Fe$	0.069
$^{118}_{50}Sn$	0.033
$^{238}_{92}U$	0.017

Energy Transfer in Compton Scattering (Revisited)

If we assume that the electron is free and at rest, the scattered gamma ray has an energy

Initial photon energy, v: photon frequency

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos\theta)},$$

mass of electron

Scattering angle

and the photon transfers part of its energy to the electron (assumed to be at rest before the collision), which is known as the ***recoil electron***. Its energy is simply

$$E_{recoil} = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos(\theta))}$$

assuming the binding energy of the electron is negligible.

In the simplified elastic scattering case, there is an one-to-one relationship between scattering angle and energy loss!!

Reading: Page 51, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Angular Distribution of the Scattered Gamma Rays (Revisited)

The differential scattering cross section – **the probability of a photon scattered into a unit solid angle around the scattering angle θ, when passing normally through a layer of material containing one electron per unit area.**

$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2\left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2\left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right)\left(m^2sr^{-1}\right)$$

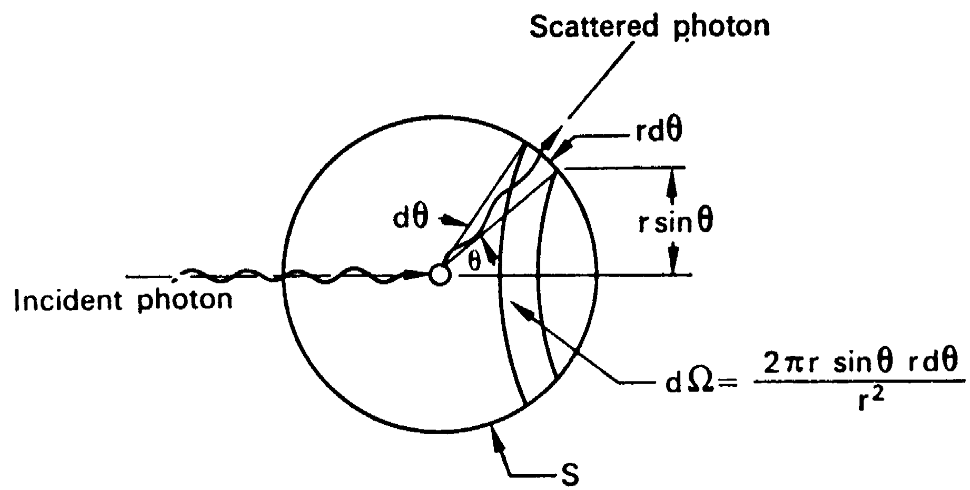


FIG. 5.15. Compton scattering diagram to illustrate differential scattering cross section. S is a sphere of unit radius whose center is the scattering electron.

Elastic Scattering of Neutrons

- The **elastic scattering** is the dominating mechanism whereby fast neutrons deliver dose to tissue.

- The recoil nuclei are essentially ionizing particles traveling in media and losing their energy through ionization and excitation.

- As we will see later, over 85% of the “first-collision” dose in soft tissue (composed of H, C, O and N) arises from n-p scattering for neutron energy below 10MeV.

Elastic Scattering of Neutrons

Kinematics of neutron scattering:

☞ Energy transfer as a function of scattering angle.

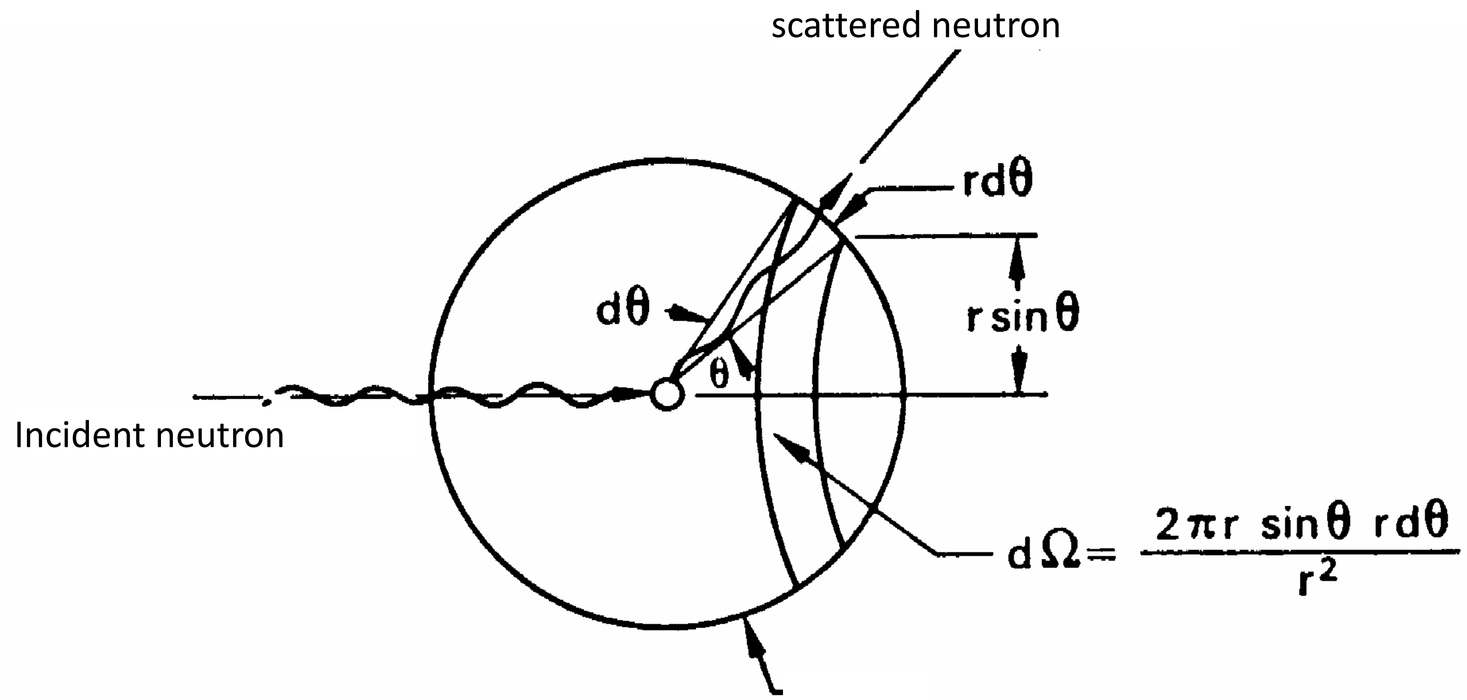
☞ Angular distribution of scattered neutrons.

☞ Energy spectrum of scattered neutrons.

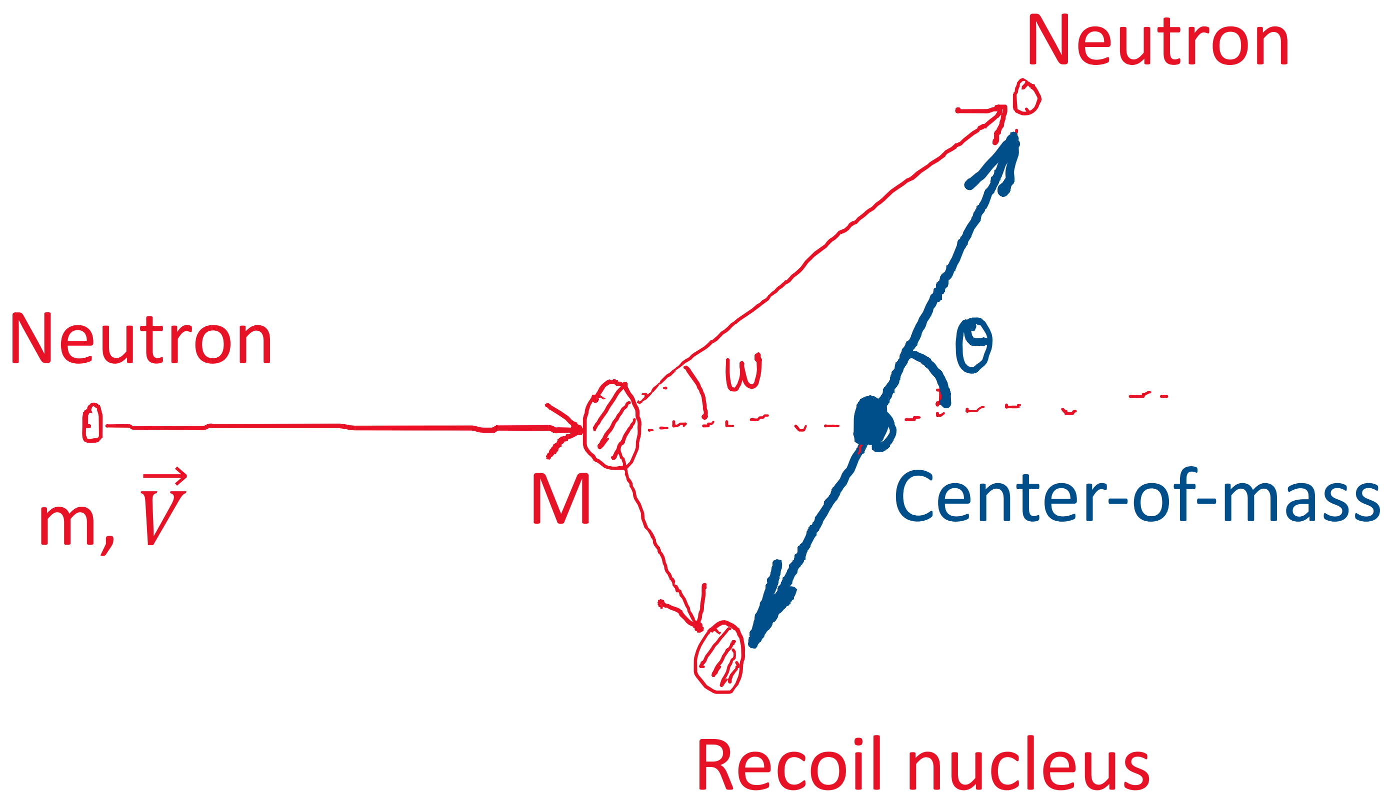
☞ Average logarithm energy decrement of a neutron in multiple scattering.

Angular Distributions of the Scattered Neutrons

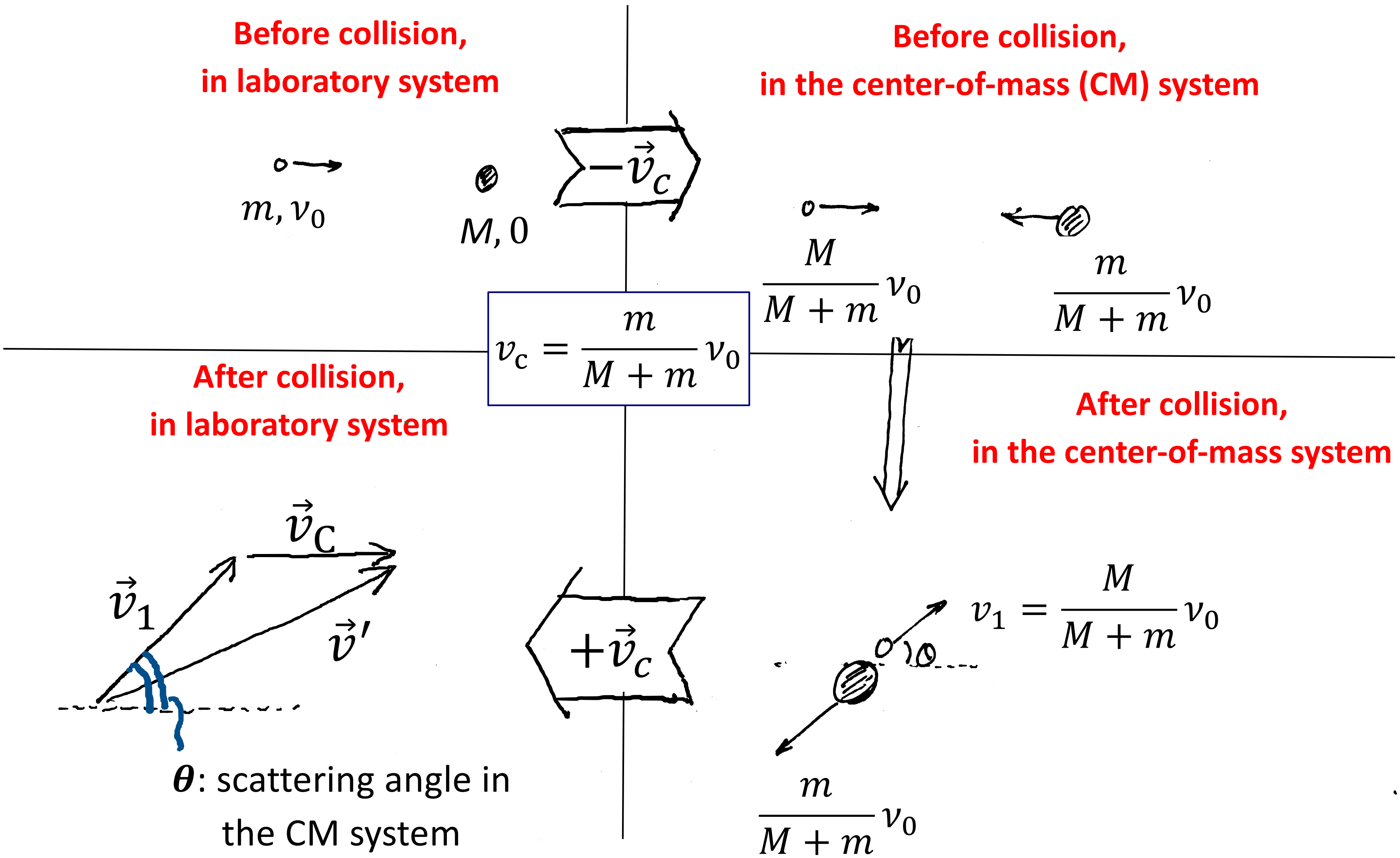
☞ For neutron energies up to 10MeV, it is experimentally observed that the scattering of neutrons is **isotropic** in the **center-of-mass coordinate system**. The neutron and the recoil nuclei are scattered with equal probability in any direction in this 3-D coordinate system.



Angular Distributions of the Scattered Neutrons



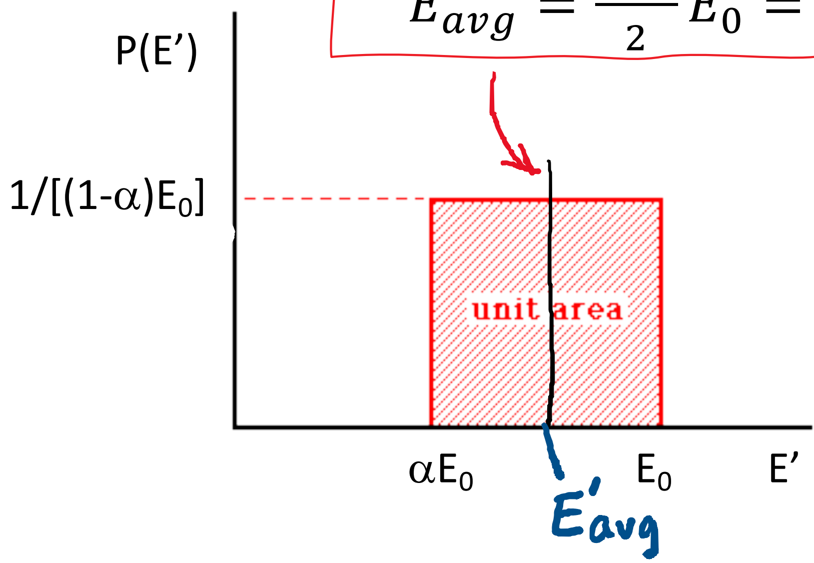
Angular Distributions of the Scattered Neutrons



Energy Spectrum of Scattered Neutrons

Average energy carried by the scattered neutron:

$$E'_{avg} = \frac{1+\alpha}{2} E_0 = \frac{1+\frac{(M-m)^2}{(M+m)^2}}{2} E_0 = \frac{M^2+m^2}{(M+m)^2} \cdot E_0$$



$$\alpha = \frac{(M-m)^2}{(M+m)^2}$$

Average energy transferred to the recoil nucleus:

$$E_{avg\_energy\_loss} = E_0 - E'_{avg} = \frac{2Mm}{(M+m)^2} \cdot E_0$$

Fast- and Thermal-Diffusion Lengths

The **fast-diffusion length**: the average straight line distance covered by fast neutrons traveling in a given medium.

The **thermal-diffusion length**: the average distance covered by thermalized neutrons before it is absorbed. It is measured by the thickness of a slowing down medium that attenuates the beam of thermal neutrons by a factor of e. Thus the attenuation of a beam of thermal neutrons by a substance of thickness t (cm), whose thermal diffusion length is L (cm) is given by

$$n = n_0 e^{-t/L}$$

TABLE 5.6. Fast and Thermal Diffusion Lenghts of Selected Materials

Substance	Fast Diffusion Length, cm	Thermal Diffusion Length, cm
H_2O	5.75	2.88
D_2O	11	171
Be	9.9	24
C (graphite)	17.3	50

Interaction of Slow Neutrons (E<0.5eV)

- Significant interactions include ***elastic scattering*** and ***neutron induced nuclear reactions***.
- Due to the low neutron energy, very little energy can be transferred by elastic scattering.
- The more significant effect of elastic scattering is to ***slow down*** slow neutrons and turn them into ***thermal neutrons*** (average E<0.025eV at room temperature).
- Neutron absorption followed by the immediate emission of a gamma ray photon and other particles.

Interaction of Slow Neutrons (E<0.5eV)

- The most important interactions between slow neutrons and absorbing materials are ***neutron-induced reactions***, such as (n,γ), (n,α), (n,p) and (n, fission) etc. These interactions lead to more prominent signatures for neutron detection.

$$\text{neutron} + \text{target nucleus} \Rightarrow \begin{cases} \text{recoil nucleus} \\ \text{protons} \\ \text{alpha particles} \\ \text{fission fragments} \end{cases}$$

Neutron Induced Reactions

$$ {}_0^1n + {}_1^1H \rightarrow {}_1^2H + {}_0^0\gamma $$

- Neutron absorption followed by the immediate emission of a gamma ray photon.
- Since the thermal neutron has negligible energy by comparison, the gamma photon has the energy Q=2.22MeV released by the reaction, which represents the binding energy of the deuteron.
- The capture cross section per atom is 0.33barn.
- When tissue is exposed to thermal neutrons, this reaction provides a source of gamma rays that delivers dose to the tissue.

Neutron Induced Reactions

$$ {}_0^1n + {}_7^{14}N \rightarrow {}_6^{14}C + {}_1^1p $$

- Cross section for thermal neutron is 1.70 barns.
- Q=0.626MeV.
- Since the range of the proton and the ^{14}C nucleus are relatively small, their energy is deposited locally at the site where the neutron was captured.
- Capture by hydrogen and nitrogen are the only two processes through which neutron deliver a significant does to soft tissue.

Neutron Induced Reactions

$$ {}_{0}^{1}n + {}_{11}^{23}Na \rightarrow {}_{11}^{24}Na + {}_{0}^{0}\gamma $$

- Cross section for thermal neutron is 0.534 barns.
- Q=0.626MeV.
- ^{24}Na undergo radioactive decay with the emission of two gamma rays, having energies of 2.75MeV and 1.37MeV per disintegration.
- Since ^{23}Na is a normal constituent of blood, activation of blood sodium can be sued as a dosimetric tool when persons are exposed to relatively high does of neutrons, for example, in a criticality accident.

Energetics of Threshold Reactions

- Consider the following reaction

$$ {}^{1}_{0}n + {}^{32}_{16}S \rightarrow {}^{32}_{15}P + {}^{1}_{1}p $$

- The neutrons must have an energy of above a certain threshold to enable this reaction.
- These reactions are called **endothermic reactions**, in which energy is converted into mass and therefore Q<0.

Energetics of Threshold Reactions

Energy release : $Q = M_1 + M_2 - (M_3 + M_4)$ (1)

Conservation of energy: $E_1 = E_3 + E_4 + Q \Rightarrow E_4 = E_1 - E_3$ (2)

Conservation of momentum: $p_1 = p_3 + p_4 \Rightarrow (2M_1E_1)^{1/2} = (2M_3E_3)^{1/2} + (2M_4E_4)^{1/2}$ (3)

Substitute (2) into (3), we have

$$(2M_1E_1)^{1/2} = (2M_3E_3)^{1/2} + (2M_4(E_1 - E_3))^{1/2}. \quad (4)$$

After some algebraic manipulations,

$$E_3 - \frac{2(M_1M_3E_1)^{1/2}}{M_3+M_4}\sqrt{E_3} - \frac{(M_4-M_1)E_1+M_4Q}{M_3+M_4} = 0\,. \quad (5)$$

For E_3 to take a real value, we need to have

$$\left[-\frac{2(M_1M_3E_1)^{1/2}}{M_3+M_4}\right]^2 - 4\left[-\frac{(M_4-M_1)E_1+M_4Q}{M_3+M_4}\right] \geq 0,$$

or

$$\frac{M_1M_3E_1}{(M_3+M_4)^2} + \frac{(M_4-M_1)E_1+M_4Q}{M_3+M_4} \geq 0, \quad (6)$$

which finally leads to

$$E_i \geq -Q\left(1 + \frac{M_1}{M_3+M_4-M_1}\right). \quad (7)$$

Neutron Activation

☞ Considering the decay of the radioactive daughters and the constant bombardment by incident neutrons, the net rate of increase of radioactive atoms is given by

$$\frac{dN}{dt} = \phi\sigma n - \lambda N,$$

where ϕ = flux, neutrons per cm^2 per s,
σ = activation cross section, cm^2,
λ = transformation constant of the induced activity,
N = number of radioactive atoms,
n = number of target atoms.

☞ The radioactivity induced by neutron activation (the number of disintegration of the activated daughter atoms per second) is given by

$$\lambda N = \phi\sigma n(1 - e^{-\lambda t})$$

Neutron Activation

- The radioactivity induced by neutron activation (the number of disintegration of the activated daughter atoms per second) is given by

$$\lambda N = \phi \sigma n(1 - e^{-\lambda t})$$

- The **saturation activity** is given by $\phi\sigma n$. For an infinitely long irradiation time, it represents the maximum obtainable activity with any given neutron flux.

- The analysis leading to these results is identical to that used for analyzing the secular equilibrium for radioactive decay chains, in which the daughter has a much shorter decay time than that of the parent.

Neutron Activation

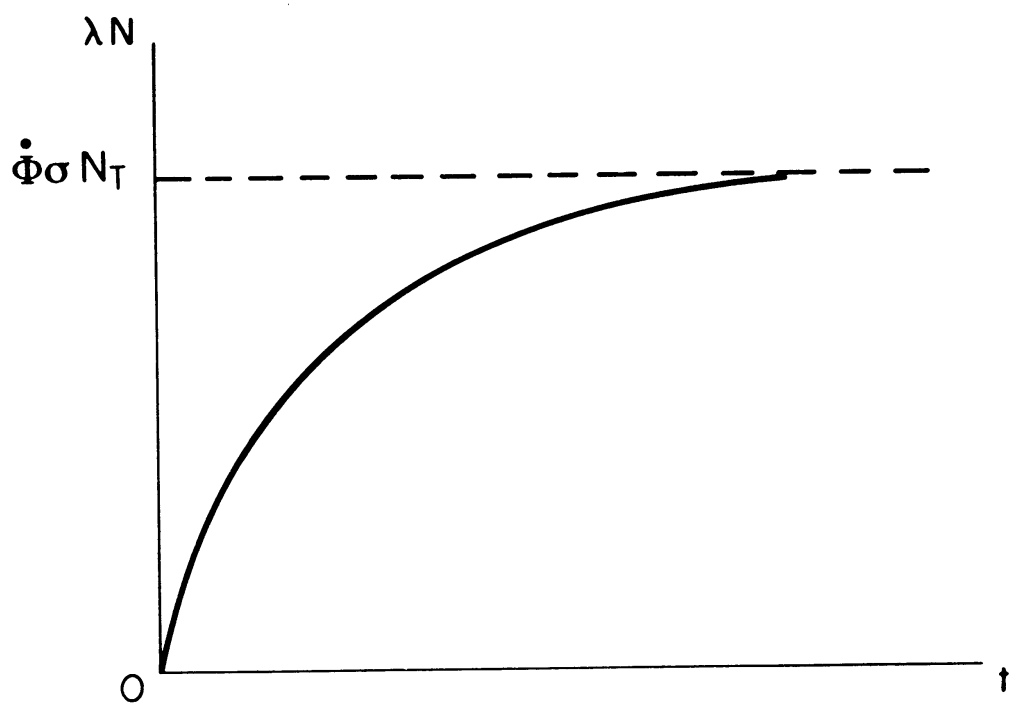


FIGURE 9.8. Buildup of induced activity λN, as given by Eq. (9.36), during neutron irradiation at constant fluence rate.