4.3 Interaction of Photons

Interactions of Photons with Matter

Reading Material:

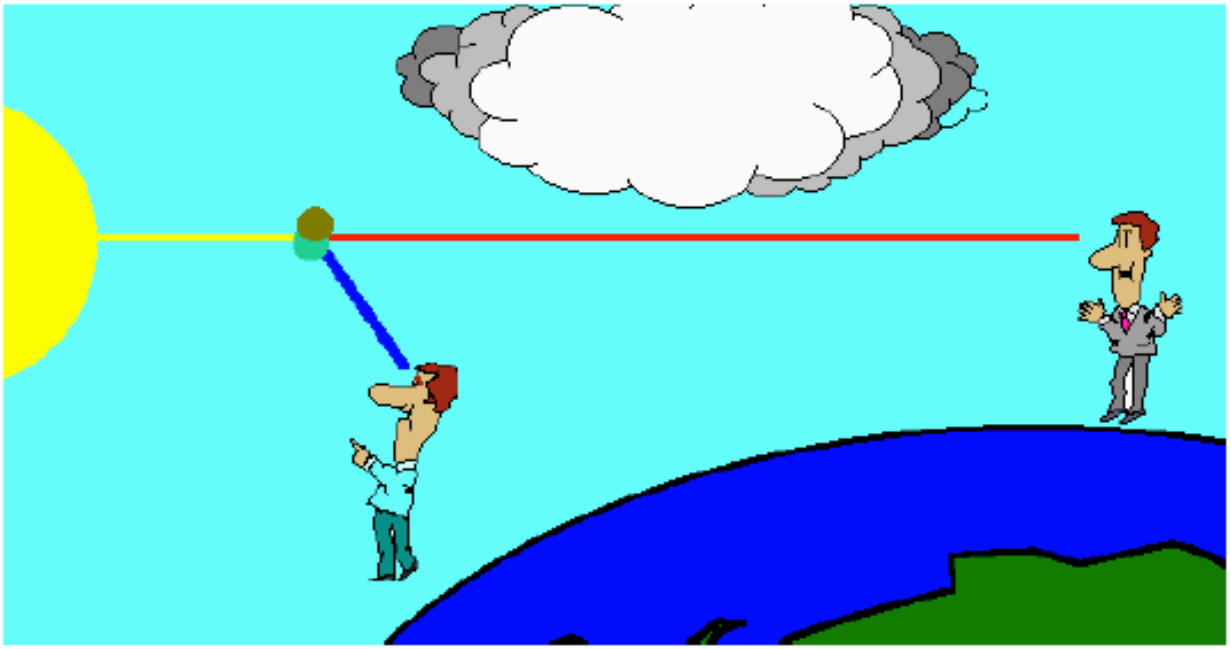
- Chapter 5 in <<Introduction to Health Physics>>, Third edition, by Cember.
- Chapter 8 in <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.
- Chapter 2 in <<Radiation Detection and Measurements>>, Third Edition, by G. F. Knoll.

Classification of Photon Interactions

Table 1. Classification of elementary photon interactions.

Type of interaction / Interaction with:	Absorption	Scattering	
		Elastic (Coherent)	Inelastic (Incoherent)
Atomic electrons	**Photoelectric effect** $\sigma_{pe} \begin{cases} \sim Z^4 (L.E.) \\ \sim Z^5 (H.E.) \end{cases}$	Rayleigh scattering $\sigma_R \sim Z^2$ $(L.E.)$	**Compton scattering** $\sigma_C \sim Z$
Nucleus	Photonuclear reactions $(\gamma,n),(\gamma,p)$, photofission, etc. $\sigma_{ph.n.} \sim Z$ $(h\nu \geq 10\text{MeV})$	Elastic nuclear scattering $(\gamma,\gamma) \sim Z^2$	Inelastic nuclear scattering (γ,γ')
Electric field surrounding charged particles	**Electron-positron pair production in field of nucleus,** $\sigma_{pair} \sim Z^2$ $(h\nu \geq 1.02\text{MeV})$		

Coherent or Raleigh Scattering



- Daytime sky looks blue on a clear day
- The sky looks red at sunrise and sunset

Why?

Blue light is scattered more efficiently than red

Elastic scattering ($\lambda_{\text{scattered light}} = \lambda_{\text{incoming light}}$)

Rayleigh scattering: $D_{\text{particle}} \leq \lambda$

•www.forbrf.lth.se/Rayleigh_scattering_2009__lecture_2_.pdf

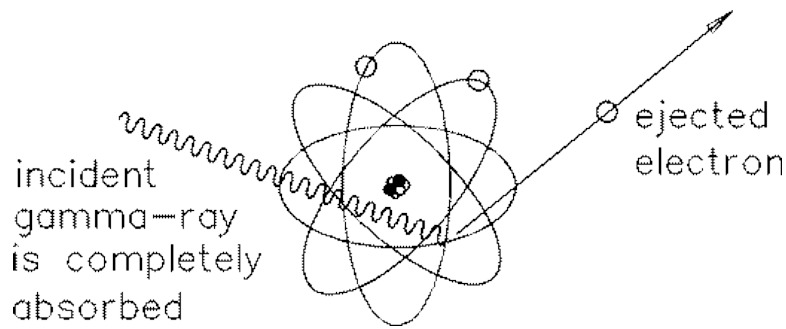
Coherent or Raleigh Scattering

Rayleigh scattering

- Rayleigh scattering results from the **interaction between the incident photons and the target atoms as a whole**.
- There is **no appreciable energy loss** by the photon to the atom.
- The **scattering angle** is very small.

Photoelectric Effect

In **photoelectric** process, an incident photon transfer its energy to an orbital electron, causing it to be ejected from the atom.



$$E_{e^-} = h\nu - E_b$$

h is the Planck's constant

ν is the frenquency of the photon

- ☞ Photoelectric interaction is **with the atom in a whole** and can not take place with free electrons.
- ☞ Photoelectric effect **creates a vacancy in one of the electron shells**, which leaves the atom at an excited state.

Photoelectric Effect – Absorption Edges

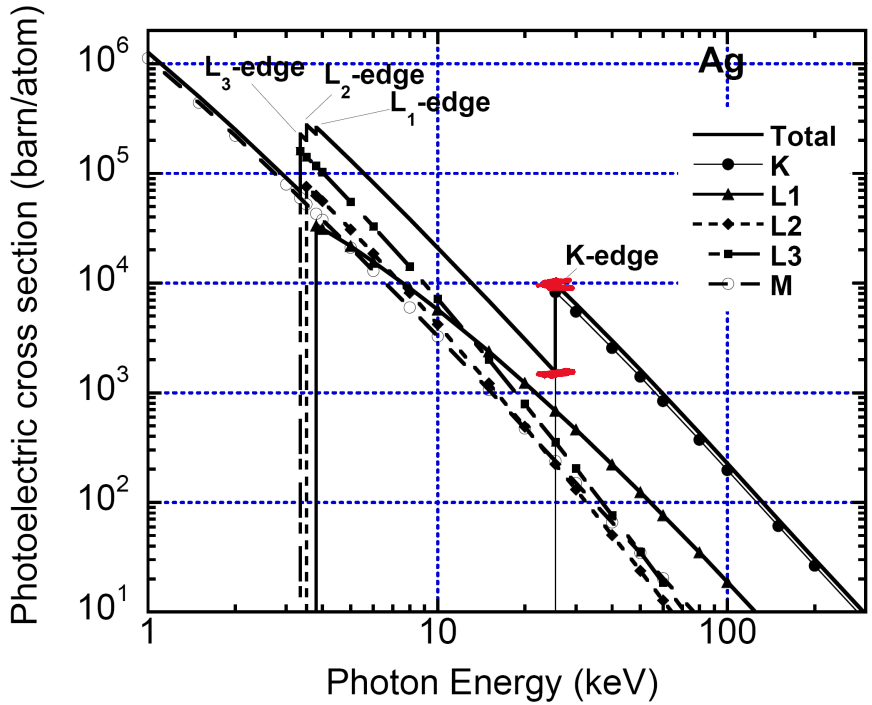


Figure 2: Total and partial atomic photoeffect of Ag.

- Requires **sufficient photon energy** for P.E. interaction.
- Interaction probability decreases dramatically with increasing energy.
- P.E. interaction is significant only for low energy photons, when the photon energy is close to the binding energies of the target atoms.

Photoelectric Effect Cross Section

Probability of photoelectric absorption per atom is

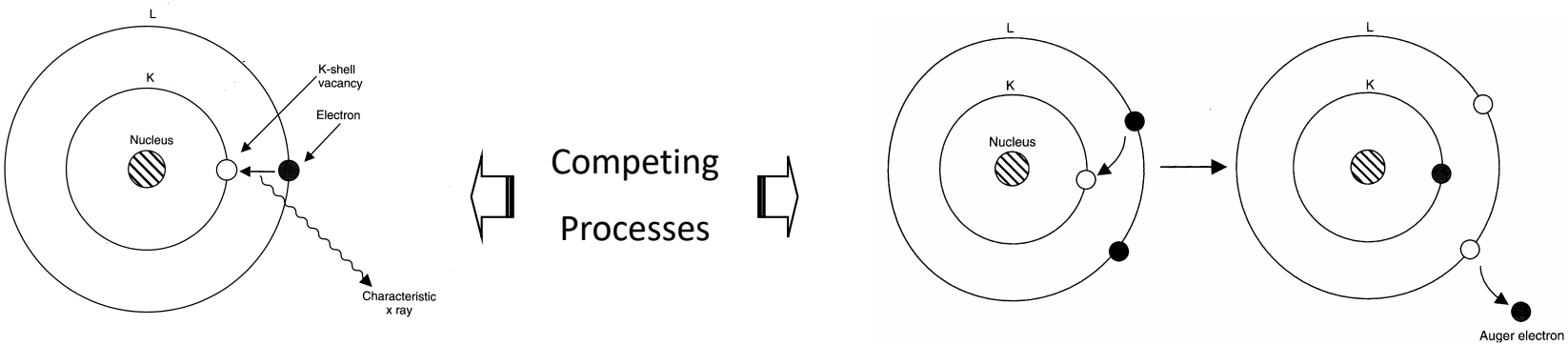
$$\sigma \propto \begin{cases} \dfrac{Z^4}{(h\nu)^{3.5}} & low\ energy \\ \dfrac{Z^5}{(h\nu)^{3.5}} & high\ energy \end{cases}$$

- The interaction cross section for photoelectric effect **depends strongly on Z**.
- Photoelectric effect is **favored at lower photon energies**. It is the major interaction process for photons at low hundred keV energy range.

Page 49, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Relaxation Process after Photoelectric Effect

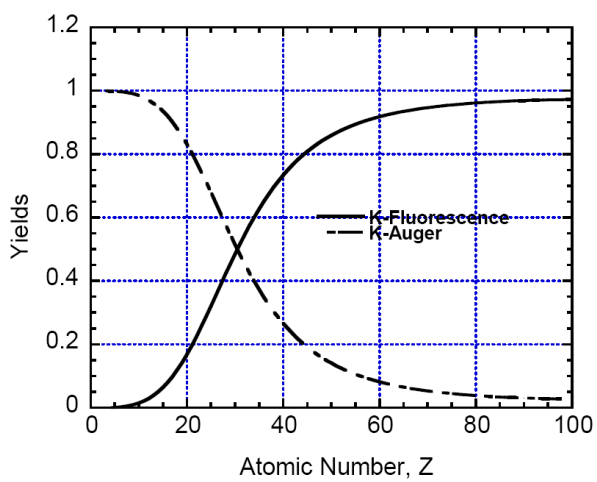
☞ The excited atoms will **de-excite** through one of the following processes:



Generation of characteristic X-rays

Generation of Auger electrons

☞ **Auger electron** emission dominates in **low-Z** elements. **Characteristic X-ray** emission dominates in **higher-Z** elements.



Auger Electrons

The relative probability of the emission of characteristic radiation to the emission of an Auger electron is called the fluorescent yield, ω:

$$\omega_K = \frac{\text{Number K x ray photons emitted}}{\text{Number K shell vacancies}} \qquad (3\text{-}12)$$

Values for ω_K are given in Table 3-1. We see that for large Z values fluorescent radiation is favored, while for low values of Z Auger electrons tend to be produced.

From this table we see that if a nucleus with Z = 40 had a K shell hole, then on the average 0.74 fluorescent photons and 0.26 Auger electrons would be emitted.

TABLE 3-1
Fluorescent Yield

Z	ω_K	Z	ω_K	Z	ω_K
10	0	40	.74	70	.92
15	.05	45	.80	75	.93
20	.19	50	.84	80	.95
25	.30	55	.88	85	.95
30	.50	60	.89	90	.97
35	.63	65	.90		

From Evans (E1)

Compton Scattering

- In **Compton scattering**, the incident gamma ray photon is **deflected by an orbital electron** in the absorbing material.
- Part of the **energy** carried by the incident photon is **transferred to the target electron** in the atom, causing it to be ejected from the atom.

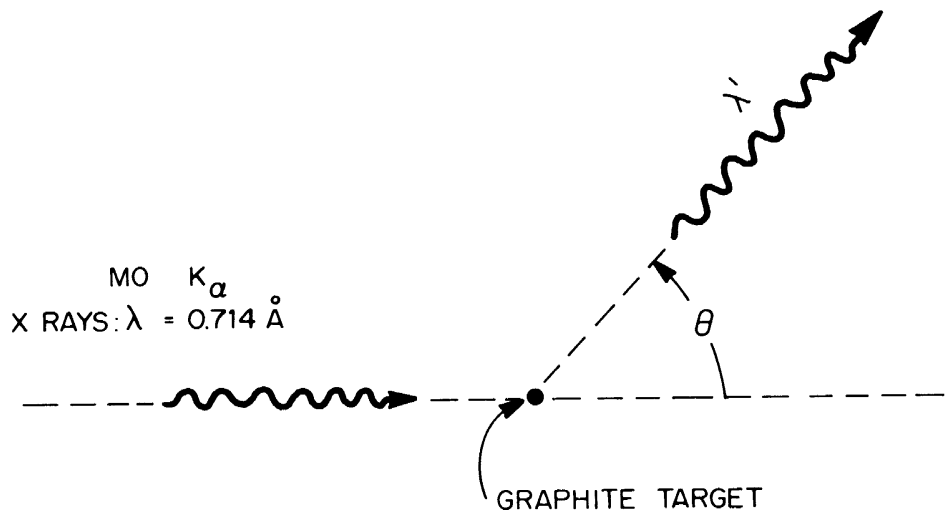
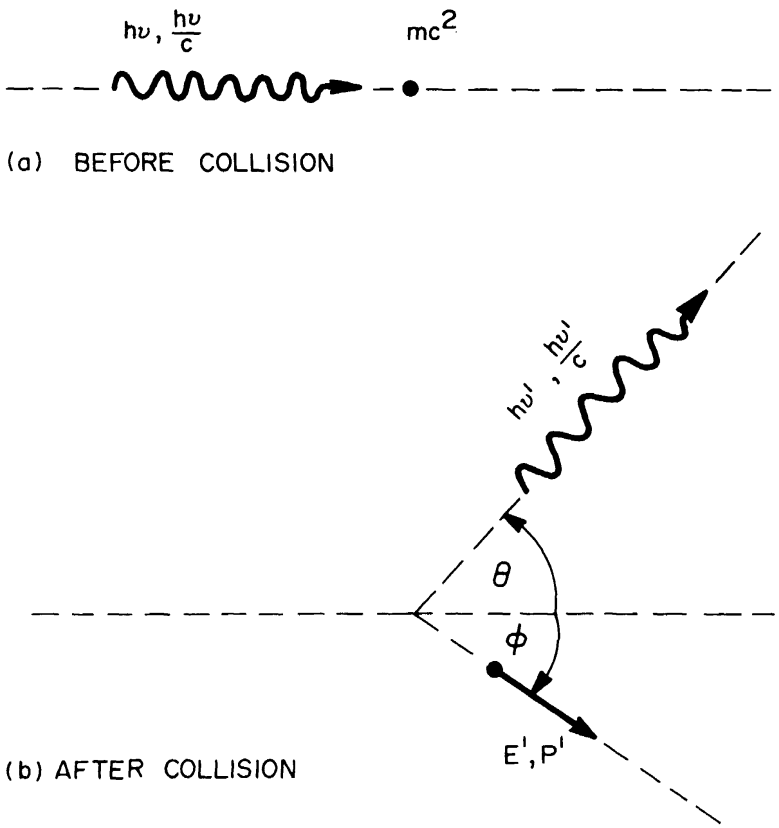


FIGURE 8.2. Compton measured the intensity of scattered photons as a function their wavelength λ′ at various scattering angles θ. Incident radiation was molybdenu K_α X rays, having a wavelength λ = 0.714 Å.

Basic Kinematics in Compton Scattering

The **energy transfer** in Compton scattering may be derived as the following:

- Assuming that the **electron binding energy is small** compared with the energy of the incident photon – **elastic scattering**.
- Write out the **conservation of energy and momentum**:



Conservation of energy

$$h\nu + mc^2 = h\nu' + E'$$

Conservation of momentum

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + P'\cos\varphi$$

$$\frac{h\nu'}{c}\sin\theta = P'\sin\varphi$$

Energy Transfer in Compton Scattering

If we assume that the electron is free and at rest, the scattered gamma ray has an energy

Initial photon energy, *v*: photon frequency

$$hv' = \frac{hv}{1 + \frac{hv}{m_0 c^2}(1 - \cos\theta)},$$

mass of electron

Scattering angle

and the photon transfers part of its energy to the electron (assumed to be at rest before the collision), which is known as the ***recoil electron***. Its energy is simply

$$E_{recoil} = hv - hv' = hv - \frac{hv}{1 + \frac{hv}{m_0 c^2}(1 - \cos(\theta))}$$

assuming the binding energy of the electron is negligible.

In the simplified elastic scattering case, there is an one-to-one relationship between scattering angle and energy loss!!

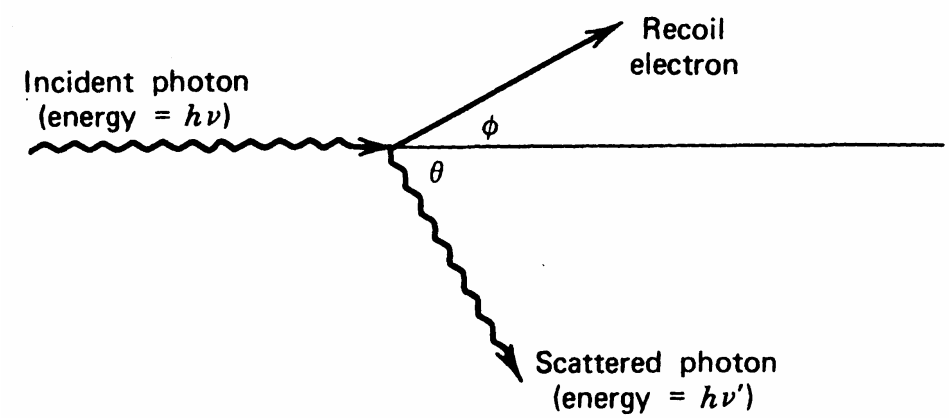
Reading: Page 51, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Energy Transfer in Compton Scattering

The scattering angles of the photon and the recoil electron is related by

$$\cot\frac{\theta}{2} = \left(1 + \frac{h\nu}{mc^2}\right)\tan\varphi$$

- The electron recoil angle is confined to the forward direction ($0 \le \varphi \le 90°$.
- The scattering angle of the photon can take any value between 0 and 180°.



Reading: Page 51, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Energy Transfer in Compton Scattering

- The **maximum energy carried by the recoil electron** is obtained by setting θ to 180°,

$$E_{\max} = \frac{2h\nu}{2 + mc^2/h\nu}$$

- The maximum energy transfer is exemplified by the **Compton edge** in measured gamma ray energy spectra.

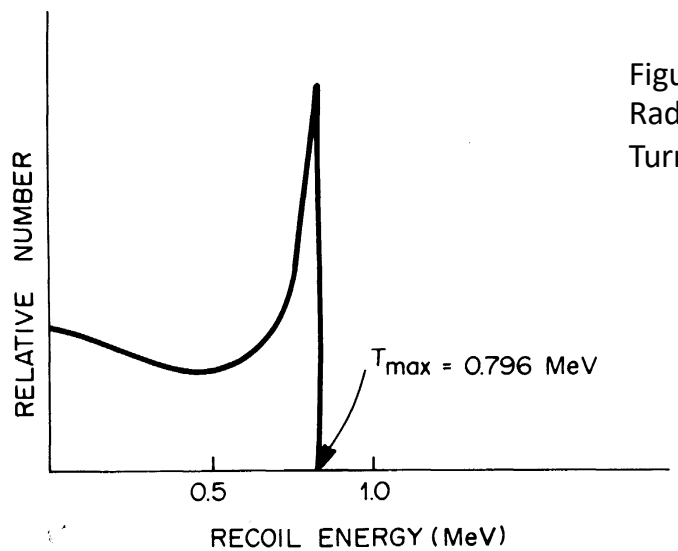


Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p180.

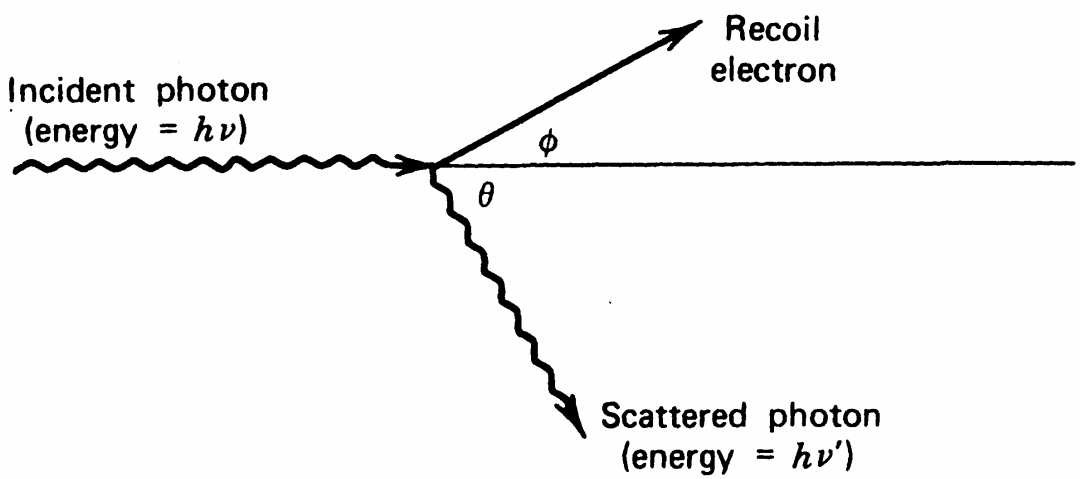
FIGURE 8.5. Relative number of Compton recoil electrons as a function of their en for 1-MeV photons.

Energy Transfer in Compton Scattering

Example

In the previous example a 1.332-MeV photon from ^{60}Co was scattered by an electron at an angle of 140°. Calculate the energy acquired by the recoil electron. What is the recoil angle of the electron? What is the maximum fraction of its energy that this photon could lose in a single Compton scattering?

$$\cot\frac{\theta}{2} = \left(1 + \frac{h\nu}{mc^2}\right)\tan\phi \qquad E_{recoil} = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1-\cos(\theta))}$$



Energy Transfer in Compton Scattering

Solution

Substitution into Eq. (8.19) gives the electron recoil energy,

$$T = 1.332 \frac{1-(-0.766)}{0.511/1.332 + 1 - (-0.766)} = 1.094 \text{ MeV}. \tag{8.25}$$

Note from Eq. (8.15) that $T + h\nu' = 1.332 \text{ MeV} = h\nu$, as it should. The angle of recoil of the electron can be found from Eq. (8.24). We have

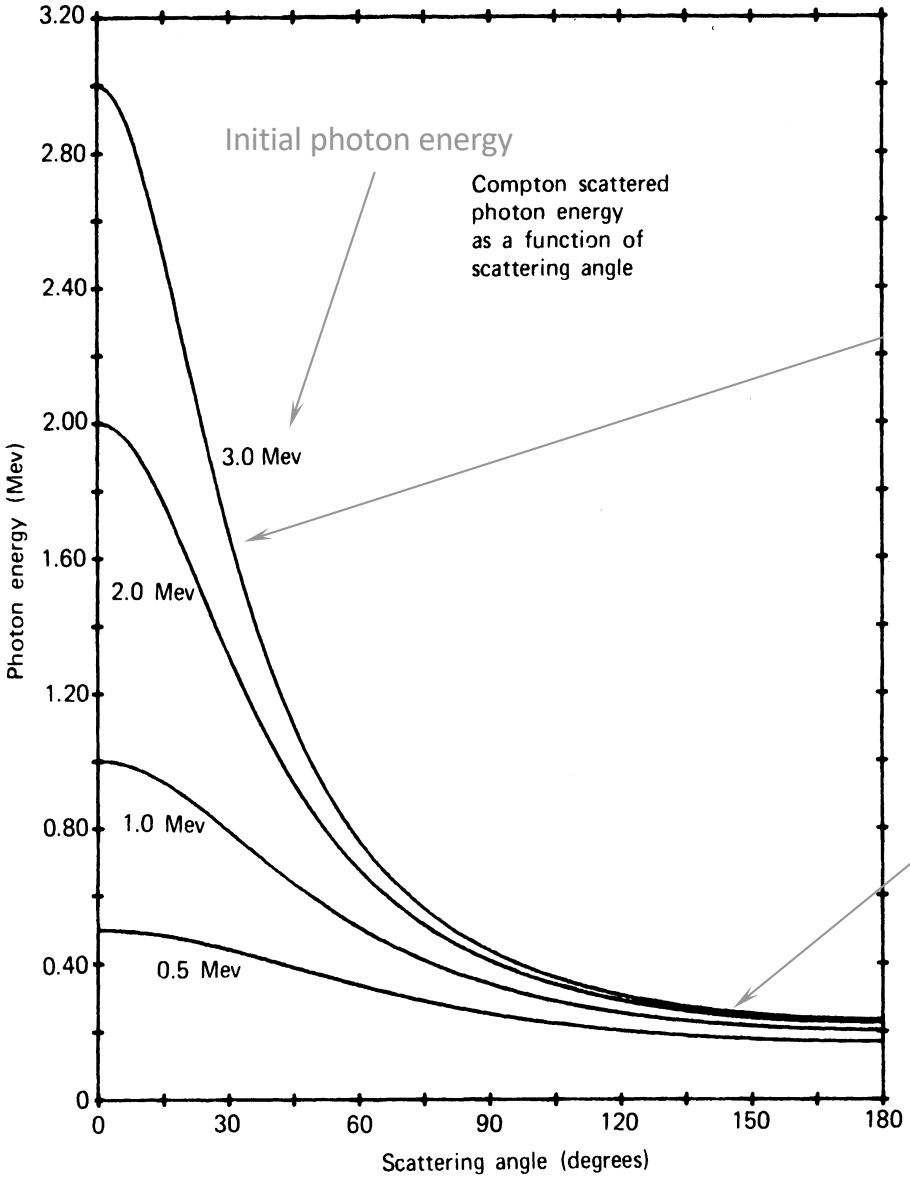
$$\tan\varphi = \frac{\cot(140°/2)}{1 + 1.332/0.511} = 0.101, \tag{8.26}$$

from which it follows that $\varphi = 5.76°$. This is a relatively hard collision in which the photon is backscattered, retaining only the fraction $0.238/1.332 = 0.179$ of its energy and knocking the electron in the forward direction. From Eq. (8.20),

$$T_{\max} = \frac{2 \times 1.332}{2 + 0.511/1.332} = 1.118 \text{ MeV}. \tag{8.27}$$

The maximum fractional energy loss is $T_{\max}/h\nu = 1.118/1.332 = 0.839$.

Energy Transfer in Compton Scattering



Small angle scattering:

Energy carried by the scattered gamma ray depends strongly on scattering angle

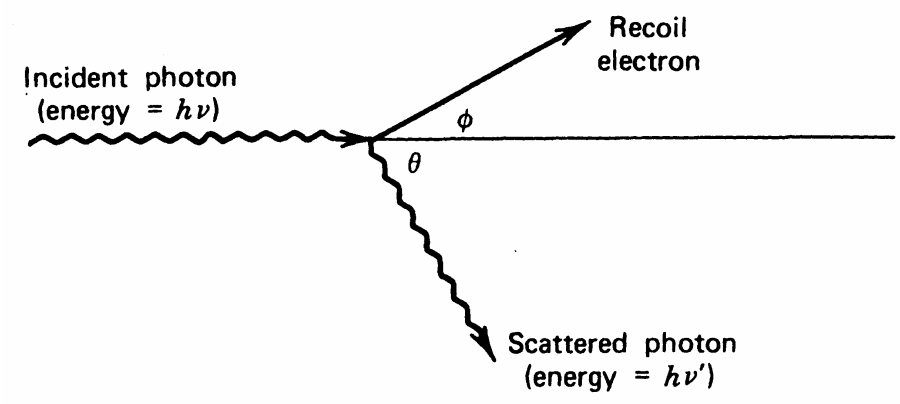
Large angle scattering:

Energy carried by the scattered gamma ray depends only weakly on scattering angle

Figure from Page 321, Radiation Detection and Measurements, Third Edition, G. F. Knoll.

Derivation of the Relationship Between Scattering Angle and Energy Loss

The relation between energy the scattering angle and energy transfer are derived based on the ***conservation of energy and momentum***:



$$\vec{p}_{h\nu} + \vec{p}_e = \vec{p}_{h\nu'} + \vec{p}_{e'}$$

$$E_{h\nu} + E_e = E_{h\nu'} + E_{e'}$$

Are those terms truly zero?

Compton Scattering with Non-stationary Electrons – Doppler Broadening

- It is so far assumed that (a) the ***electron is free and stationary*** and (b) the ***incident photon is unpolarized***.

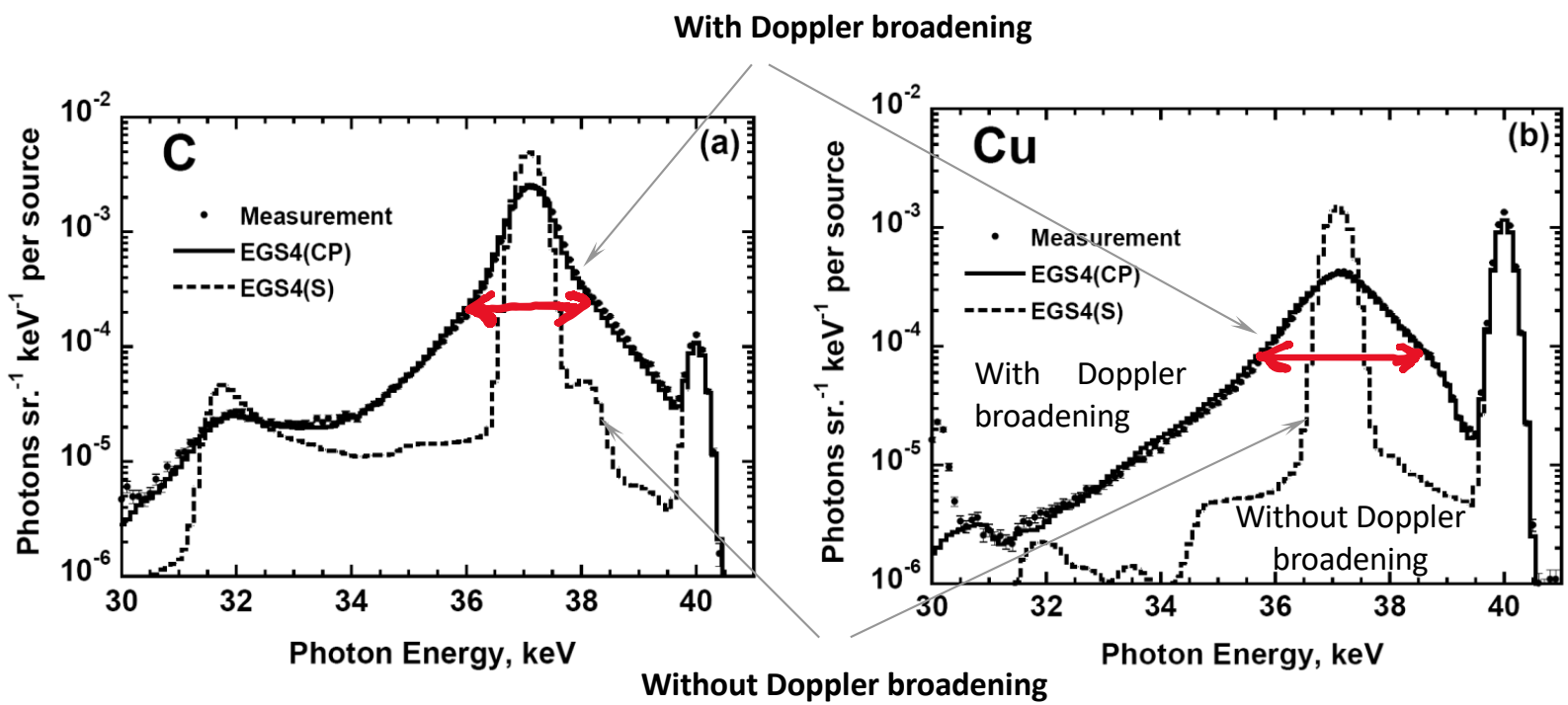
- When an incident photon is reflected by a ***non-stationary electron***, for example an bond electron, an extra uncertainty is added to the energy of the scattered photon. This extra uncertainty is called **Doppler broadening**.

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos(\theta))} \pm \boxed{\sigma(h\nu')}$$

The one-to-one relationship between scattering angle and energy loss holds only when incident photon energy is far greater than the bonding energy of the electron…

Compton Scattering with Non-stationary Electrons – Doppler Broadening

Comparison of the energy spectra for the photons scattered by C and Cu samples. $E_{h\nu}$=40keV, θ=90 degrees



The **Doppler broadening** is stronger in Cu than in C because of the Cu electrons have greater bonding energy.

Angular Distribution of the Scattered Gamma Rays

The **differential scattering cross section** of per electron is given by the **Klein-Nishina** formula:

$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2 \left(\frac{1}{1+\alpha(1-\cos\theta)} \right)^2 \left(\frac{1+\cos^2\theta}{2} \right) \left(1 + \frac{\alpha^2 (1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]} \right),$$

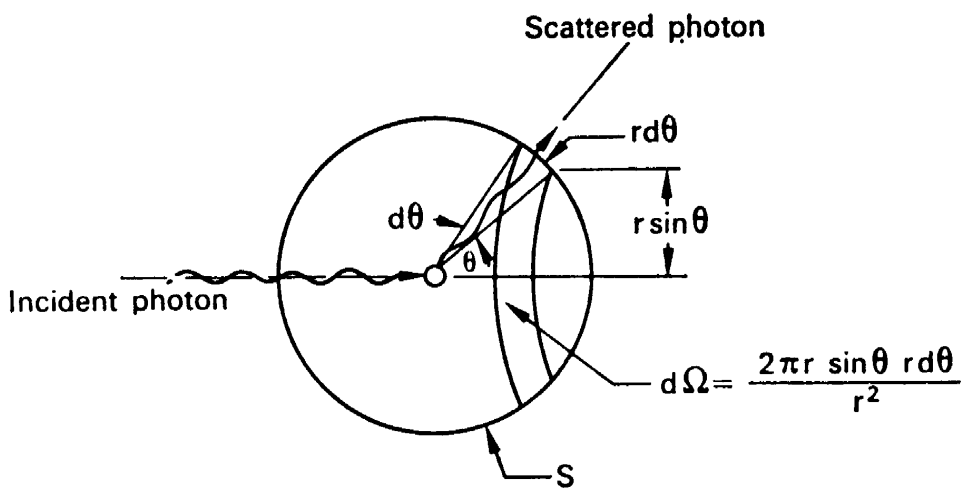
where

$$\alpha = \frac{h\nu}{m_0 c^2} \quad \text{and} \quad r_e = \frac{k_0 e^2}{m_0 c^2} \text{ is the classic electron radius } (2.818 \times 10^{-15} \text{ m})$$

Angular Distribution of the Scattered Gamma Rays

The differential scattering cross section per electron – **the probability of a photon scattered into a unit solid angle around the scattering angle θ, when the incident photon is passing normally through a thin layer of scattering material that contains one electron per unit area.**

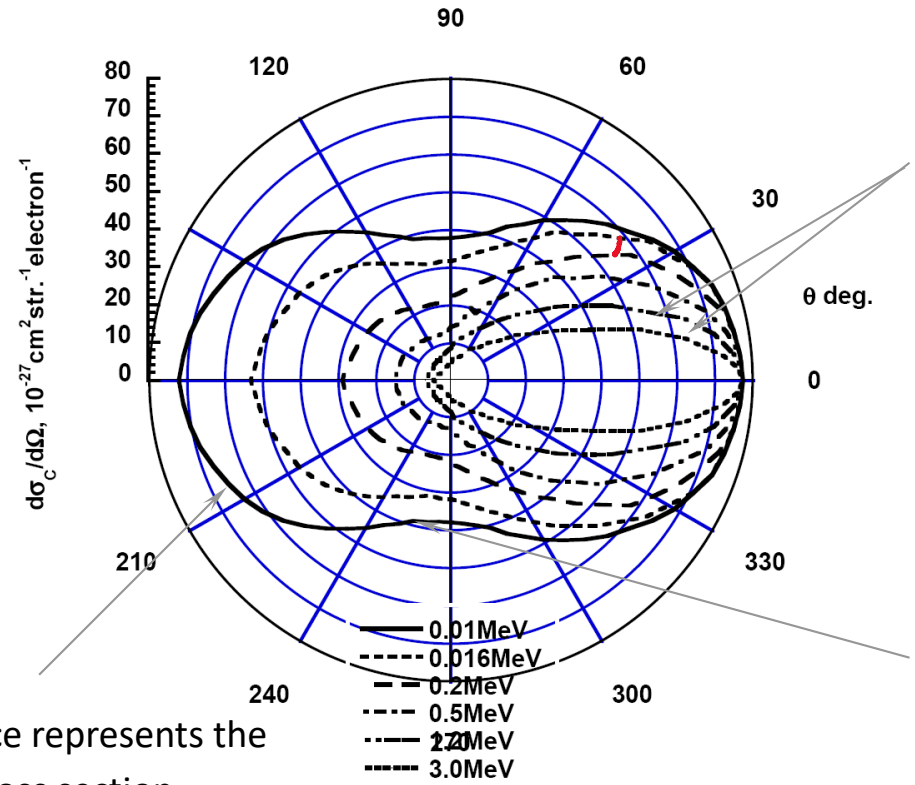
$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2 \left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2 \left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right)(m^2 sr^{-1})$$



$$d\Omega = 2\pi \sin\theta \, d\theta$$

FIG. 5.15. Compton scattering diagram to illustrate differential scattering cross section. S is a sphere of unit radius whose center is the scattering electron.

Angular Distribution of the Scattered Gamma Rays



Radial distance represents the differential cross section.

Incident photons with ***higher energies*** tend to scatter with smaller angles (forward scattering).

Incident photons with ***lower energy*** (a few hundred keV) have greater chance of undergoing large angle scattering (back scattering).

The higher the energy carried by an incident gamma ray, the more likely that the gamma ray undergoes forward scattering …

Total Compton Collision Cross Section for an Electron

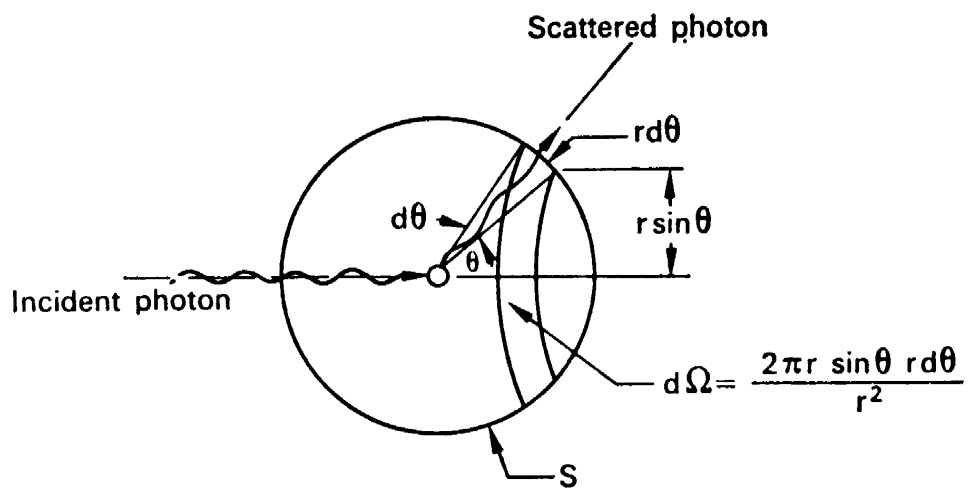
Compton Collision Cross Section is defined as the total cross section per electron for Compton scattering. It can be derived by integrating the differential cross section over the 4 π solid angle.

Since

$$d\Omega = 2\pi \sin\theta \, d\theta,$$

then the Compton scattering cross section per electron is given by

$$\sigma = 2\pi \int_{\Omega} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \cdot \sin\theta \cdot d\theta \;\; (m^2)$$



Note that the Compton scattering cross section per electron is given in unit of m^2.

Question

Given the Klein-Nishina formula, how do we derive the **energy spectrum of recoil electrons**? In other words, how do we derive the probability of a gamma ray that Compton scatted and transferred a given energy $\mathbf{E_{recoil}}$?

The differential scattering cross section per electron – **the probability of a photon scattered into a unit solid angle around the scattering angle θ, when passing normally through a very thing layer of scattering material that contains one electron per unit area.**

$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2\left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2\left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right)\left(m^2 sr^{-1}\right)$$

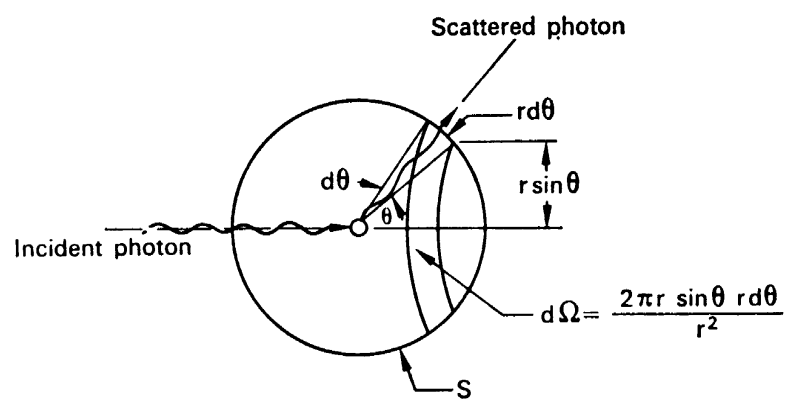


FIG. 5.15. Compton scattering diagram to illustrate differential scattering cross section. S is a sphere of unit radius whose center is the scattering electron.

Energy Distribution of Compton Recoil Electrons

Klein-Nishina formula can be used to calculate the expected **energy spectrum of recoil electrons** as the following:

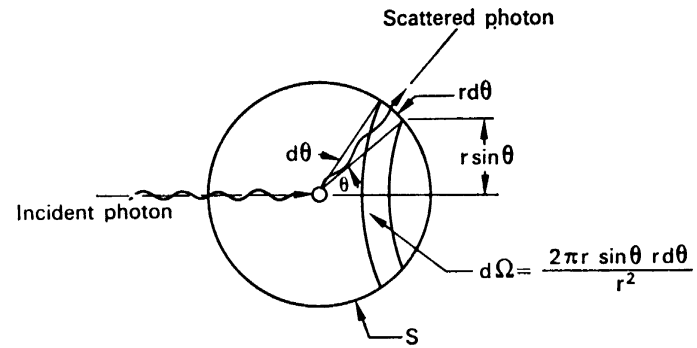
$$\frac{d\sigma}{dE_{recoil}} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{d\theta}\frac{d\theta}{dE_{recoil}}$$

and the probability that a recoil electron possesses an energy between E_{recoil}-$\Delta E/2$ and E_{recoil}+$\Delta E/2$ is given by

$$\propto \Delta E \cdot \frac{d\sigma}{dE_{recoil}}$$

Energy Distribution of Compton Recoil Electrons

$$\frac{d\sigma}{dE_{recoil}} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{d\theta}\frac{d\theta}{dE_{recoil}}$$



The three partial derivative terms on the right-hand side of the equation can be derived from the following relationships,

Klein-Nishina Formula

$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2\left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2\left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right)$$

$$E_{recoil} = h\nu - h\nu' = h\nu - \frac{h\nu}{1+\frac{h\nu}{m_0c^2}(1-\cos\theta)} \quad \Rightarrow$$

$$\frac{d\theta}{dE_{recoil}} = -\frac{m_0c^2}{E_{recoil}^2\sin\theta} = -\frac{m_0c^2}{\left[h\nu-\frac{h\nu}{1+\frac{h\nu}{m_0c^2}(1-\cos\theta)}\right]^2\sin\theta}$$

$$d\Omega = 2\pi\sin\theta\, d\theta \quad \Rightarrow \quad \frac{d\Omega}{d\theta} = 2\pi\sin\theta$$

Energy Distribution of Compton Recoil Electrons

Therefore the differential cross section becomes

$$\frac{d\sigma}{dE_{recoil}} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{d\theta}\frac{d\theta}{dE_{recoil}}$$

$$= \left[r_e^2\left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2\left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right)\right]\times\left[-\frac{m_0c^2}{E_{recoil}^2\sin\theta}\right]\times[2\pi\sin\theta]$$

Or the above equitation could be re-written as

$$\boxed{\frac{d\sigma}{dE_{recoil}} = F(\theta)\cdot\frac{1}{E_{recoil}^2}}$$

where

$$F(\theta) = \left[r_e^2\left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2\left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right)\right]\times\left[-\frac{m_0c^2}{\sin\theta}\right]\times[2\pi\sin\theta]$$

Application of the Klein-Nishina Formula (1)

Therefore the differential cross section becomes

$$\frac{d\sigma}{dE_{recoil}} = F(\theta) \cdot \frac{1}{E_{recoil}^2}$$

where

$$F(\theta) = \left[r_e^2 \left(\frac{1}{1+\alpha(1-\cos\theta)} \right)^2 \left(\frac{1+\cos^2\theta}{2} \right) \left(1 + \frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]} \right) \right] \times \left[-\frac{m_0 c^2}{\sin\theta} \right] \times [2\pi \sin\theta] \,.$$

Remember that

$$E_{recoil} = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1-\cos\theta)}.$$

$\Rightarrow \theta(E_{recoil})$

Finally, the distribution for energy carried by the recoil electron is given by

$$\boxed{\frac{d\sigma}{dE_{recoil}} = \frac{F[\theta(E_{recoil})]}{E_{recoil}^2}}$$

Application of the Klein-Nishina Formula (1)

Klein-Nishina formula can be used to calculate the expected **energy spectrum of recoil electrons** as the following:

$$\frac{d\sigma}{dE_{recoil}} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{d\theta}\frac{d\theta}{dE_{recoil}} \quad (m^2 \cdot \mathrm{keV}^{-1})$$

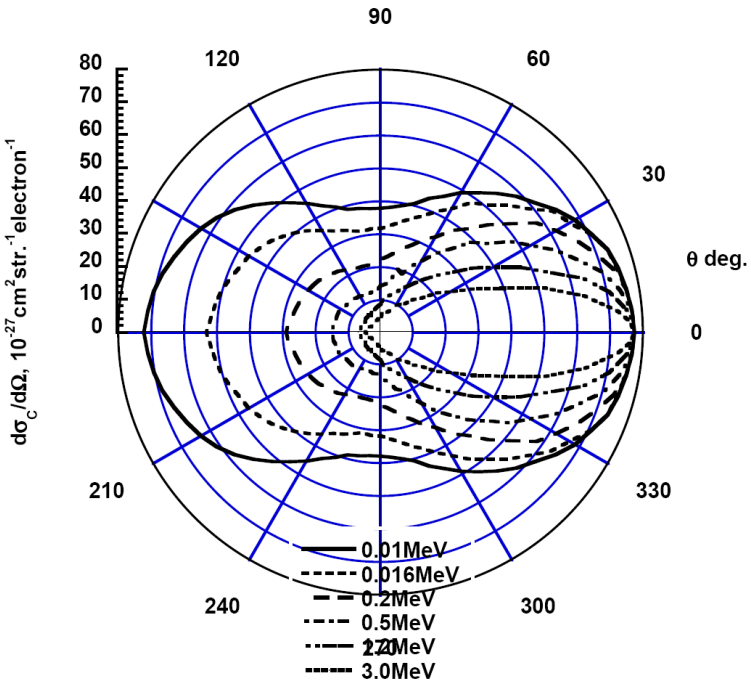
and the probability that a recoil electron possesses an energy between E_{recoil}-$\Delta E/2$ and E_{recoil}+$\Delta E/2$ is given by

$$P(E - {}^{\Delta E}/_{2} < E_{recoil} \leq E + {}^{\Delta E}/_{2}) \;=\; \Delta E \cdot \left[\left(\frac{d\sigma}{dE_{recoil}}\right)/\sigma\right]\Bigg|_{E_{recoil}=E} ,$$

where σ is the Compton scattering cross section per electron and is given by

$$\sigma = 2\pi \int_{\Omega} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \cdot \sin\theta \cdot d\theta \;\; (m^2) \,.$$

Application of the Klein-Nishina Formula (1)



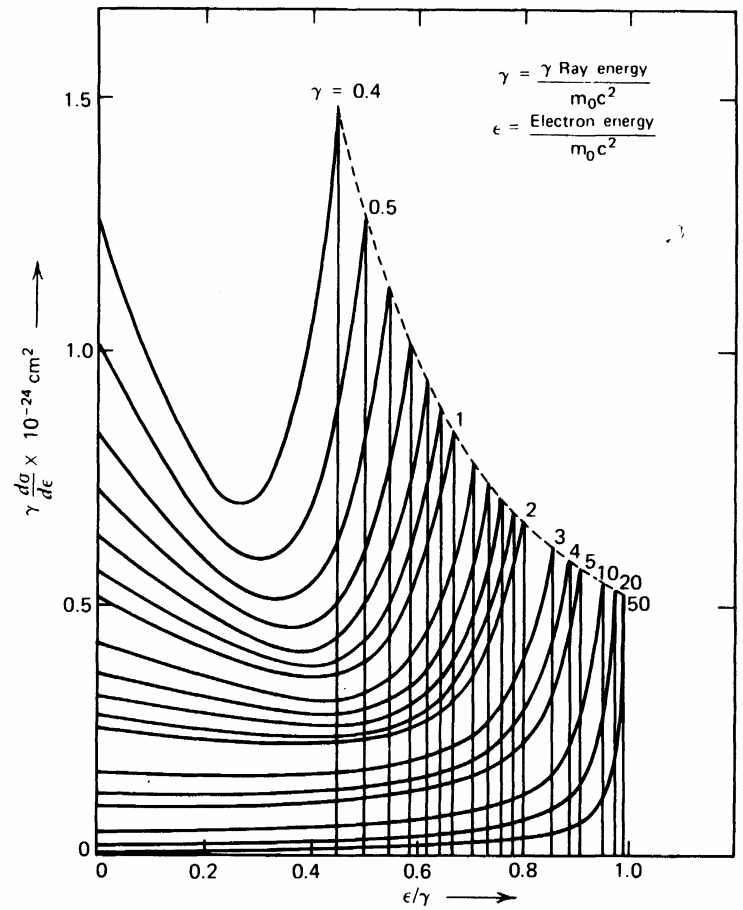


Figure 10.1 Shape of the Compton continuum for various gamma-ray energies. (From S. M. Shafroth (ed.), *Scintillation Spectroscopy of Gamma Radiation.* Copyright 1964 by Gordon & Breach, Inc. By permission of the publisher.)

☞ The energy distribution of the recoil electrons derived using the Klein-Nishina formula is closely related to **the energy spectrum measured with "small" detectors** (in particular, the so-called **Compton continuum**).

From Page 311, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Average fraction of energy transfer to the recoil electron through a single Compton Collision

Of special interest for dosimetry is the **average recoil electron energy** E_{avg_recoil}, since it is an approximation of the **radiation dose by each photon through Compton scattering**.

The **average fraction of energy transfer to the recoil electron** through a single Compton scattering is given by

$$\frac{E_{avg_recoil}}{h\nu} = \int_{E_{recoil}} \frac{E_{recoil}}{h\nu} \cdot \left[\left(\frac{d\sigma}{dE_{recoil}}\right)/\sigma\right] \cdot dE_{recoil} \ ,$$

where σ is the Compton scattering cross section per electron and is given by

$$\sigma = 2\pi \int_{\Omega} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \cdot \sin\theta \cdot d\theta \ \ (m^2) \ .$$

Average fraction of energy transfer to the recoil electron through a single Compton Collision

TABLE 8.1. Average Kinetic Energy, T_{avg}, of Compton Recoil Electrons and Fraction of Incident Photon Energy, $h\nu$

Photon Energy $h\nu$ (MeV)	Average Recoil Electron Energy T_{avg} (MeV)	Average Fraction of Incident Energy $T_{avg}/h\nu$
0.01	0.0002	0.0187
0.02	0.0007	0.0361
0.04	0.0027	0.0667
0.06	0.0056	0.0938
0.08	0.0094	0.117
0.10	0.0138	0.138
0.20	0.0432	0.216
0.40	0.124	0.310
0.60	0.221	0.368
0.80	0.327	0.409
1.00	0.440	0.440
2.00	1.06	0.531
4.00	2.43	0.607
6.00	3.86	0.644
8.00	5.34	0.667
10.0	6.84	0.684
20.0	14.5	0.727
40.0	30.4	0.760
60.0	46.6	0.776
80.0	62.9	0.787
100.0	79.4	0.794

Linear Attenuation Coefficient through Compton Scattering

The **differential Compton cross section** given by the Klein-Nishina Formula can also be related to another important parameter for gamma ray dosimetry – **the linear attenuation coefficient**.

$$\sigma_{linear} = NZ\sigma \quad (m^{-1}),$$

which is the **probability of a photon interacting with the absorber through Compton scattering while traversing a unit distance**. NZ is the electron density of the absorber materials (number of electrons per m^3)

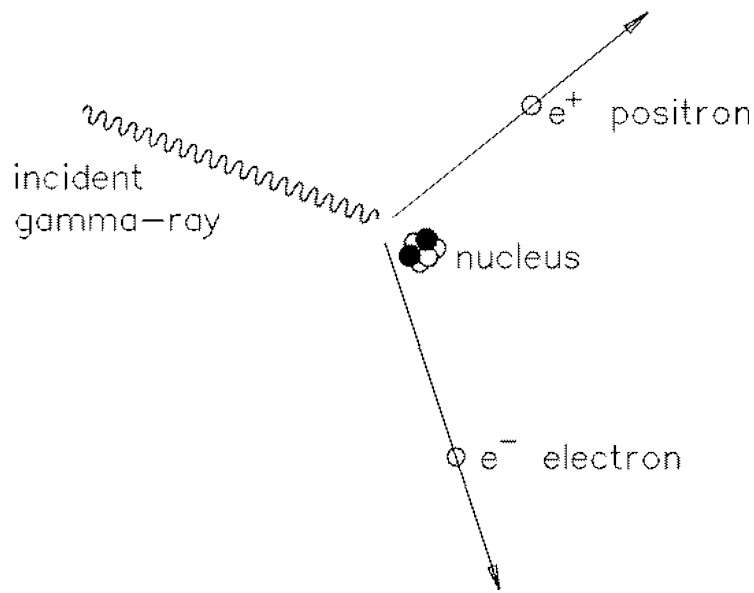
Note that σ is the Compton scattering cross section per electron and is given by

$$\sigma = 2\pi \int_{\Omega} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \cdot \sin\theta \cdot d\theta \ \ (m^2) \ .$$

Pair Production

Definition:

Pair production refers to the creation of an electron-positron pair by an incident gamma ray in the vicinity of a nucleus.



Characteristics

☞ The minimum energy required is

$$E_\gamma \geq 2m_e c^2 + \frac{2m_e^2 c^2}{m_{nucleus}} \approx 2m_e c^2 = 1.022 MeV$$

☞ The process is more probable with a ***heavy nucleus*** and incident ***gamma rays with higher energies***.

☞ The positrons emitted will soon **annihilate** with ordinary electrons near by and produces two 511keV gamma rays.

Photonuclear Reaction

- A photon can be absorbed by an atomic nucleus and knock out a nucleon. This process is called **photonuclear reaction**. For example,

$$ {}^{9}_{4}Be + h\nu \Rightarrow {}^{8}_{4}Be + {}^{1}_{0}n, \quad Q-value: -1.666MeV $$

$$ {}^{2}_{1}H + h\nu \Rightarrow {}^{1}_{1}H + {}^{1}_{0}n, \quad Q-value: -2.226MeV $$

- The photon **must possess enough energy to overcome the nuclear binding energy**, which is generally several MeV.
- The threshold, or the minimum photon energy required, for (γ,p) reaction is generally higher than that for (γ,n) reactions. Since the repulsive Coulomb barrier that a proton must overcome to escape from the nucleus.
- Other nuclear reaction s are also possible, such as (γ, 2n), (γ, np), (γ, α) and photon induced fission reaction.

Interaction of Photons in Matter

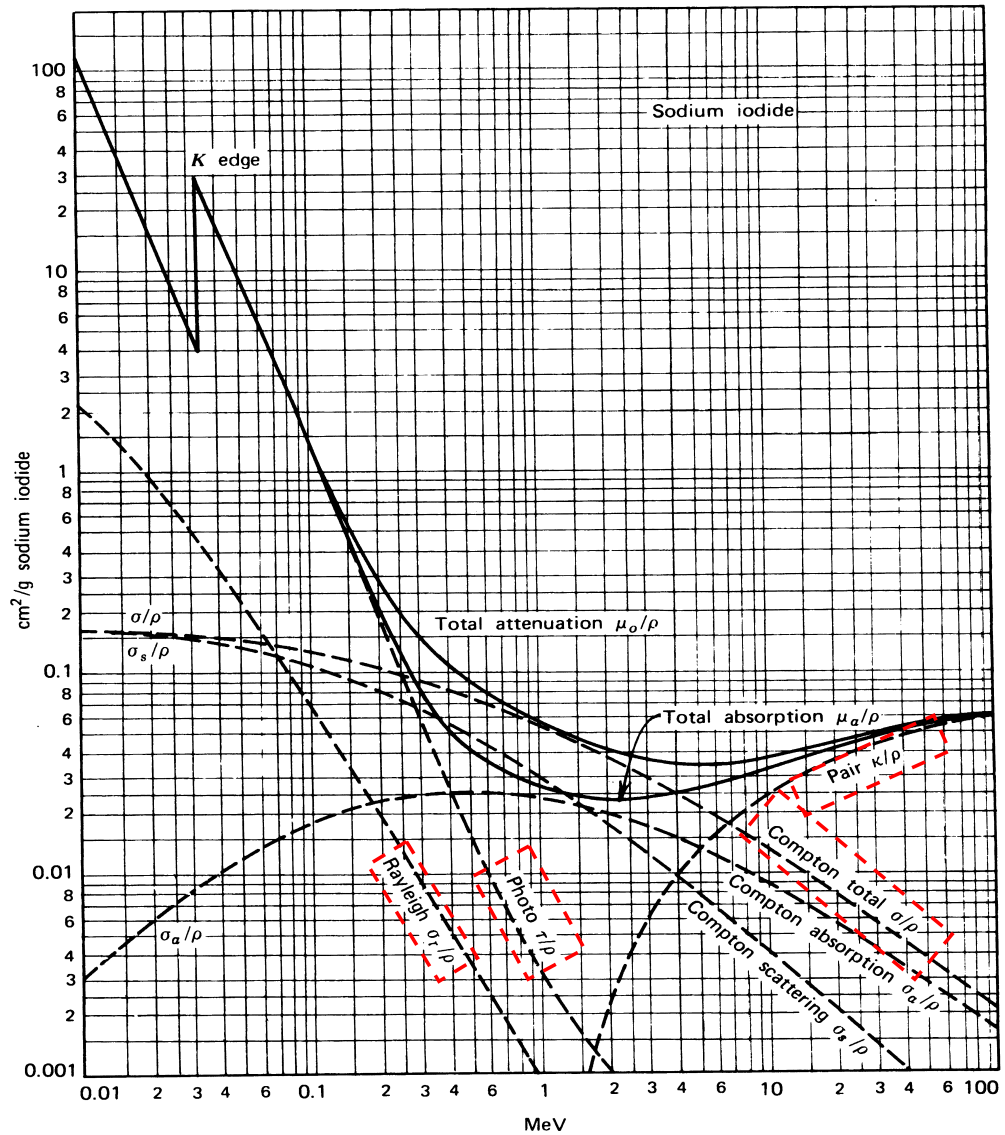
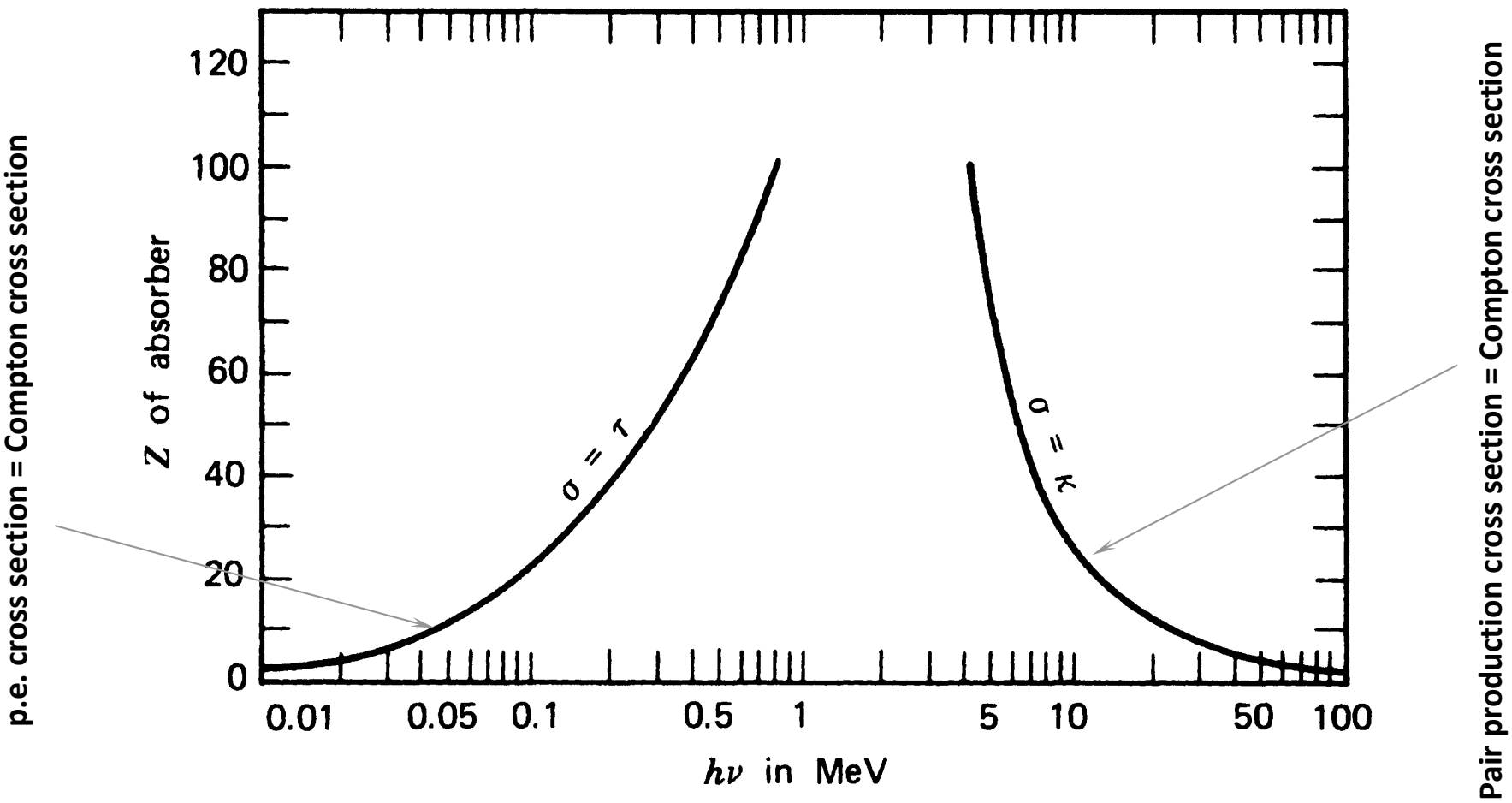


Figure 2.18 Energy dependence of the various gamma–ray interaction processes in sodium iodide. (From *The Atomic Nucleus* by R. D. Evans. Copyright 1955 by the McGraw-Hill Book Company. Used with permission.)

From Page 50, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

The Relative Importance of the Three Major Type of X and Gamma Ray Interactions



Page 52, Chapter 2 in Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Interactions of Photons with Matter (Revisited)

Interaction of Photons in Matter

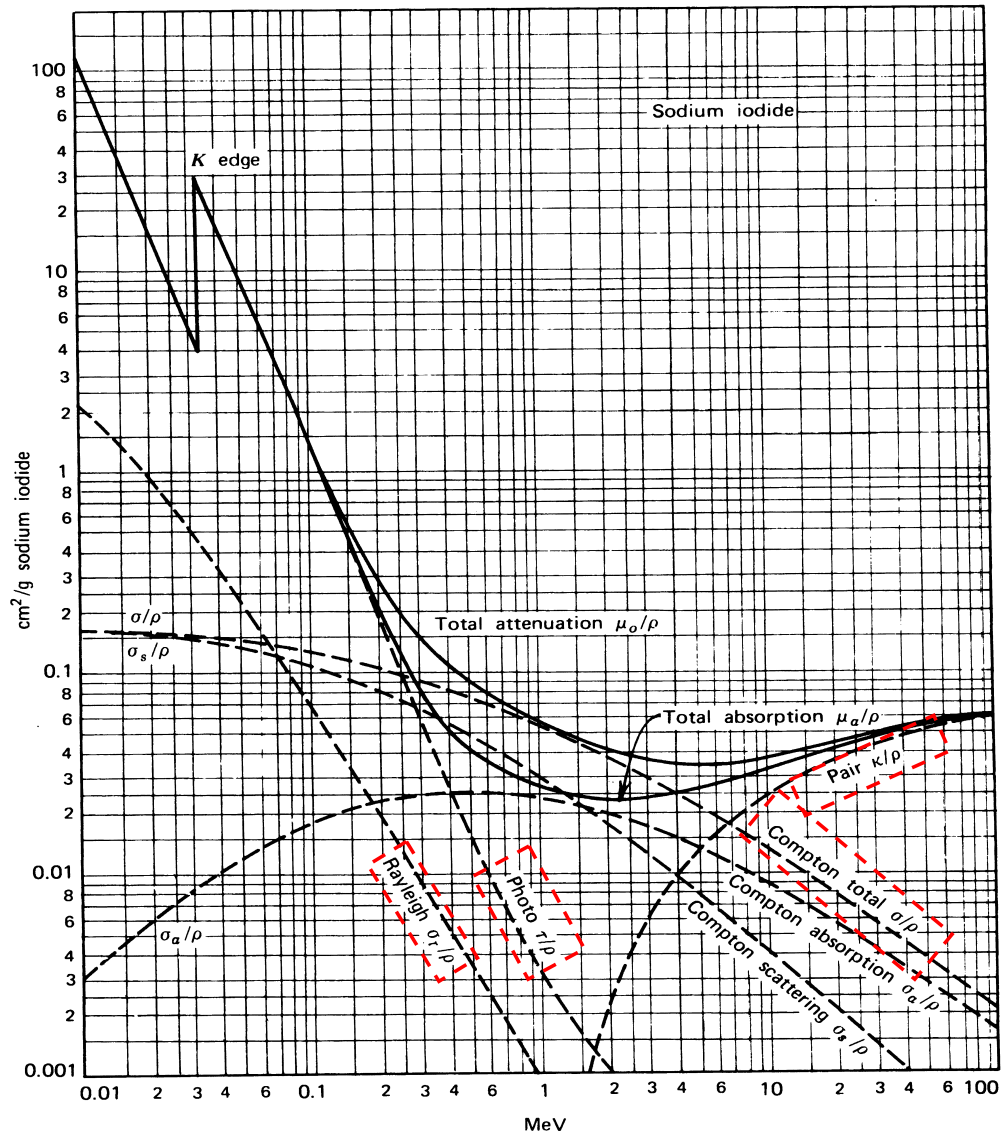
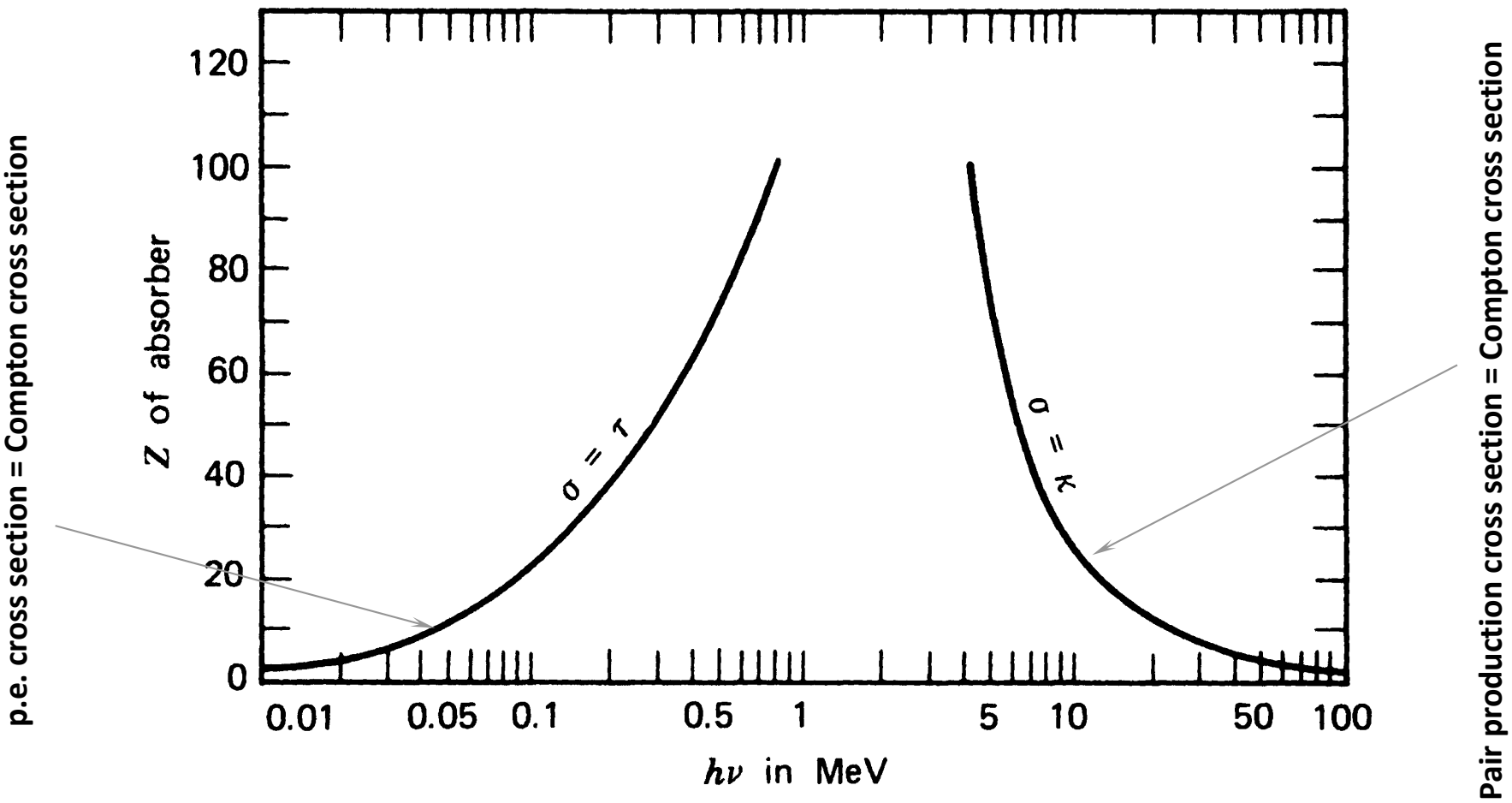


Figure 2.18 Energy dependence of the various gamma–ray interaction processes in sodium iodide. (From *The Atomic Nucleus* by R. D. Evans. Copyright 1955 by the McGraw-Hill Book Company. Used with permission.)

From Page 50, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

The Relative Importance of the Three Major Type of X and Gamma Ray Interactions

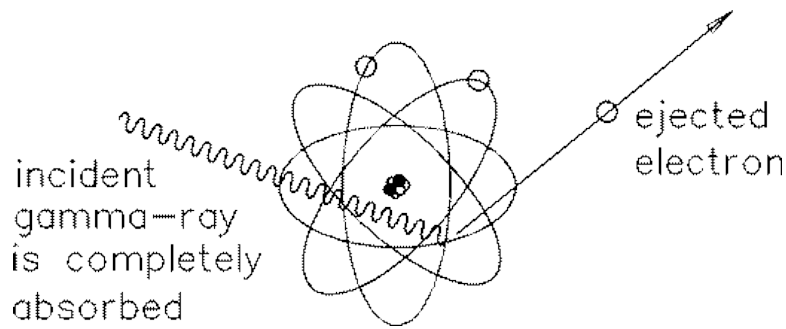


Page 52, Chapter 2 in Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Photoelectric Effect

Photoelectric Effect

In **photoelectric** process, an incident photon transfer its energy to an orbital electron, causing it to be ejected from the atom.



$$E_{e^-} = h\nu - E_b$$

h is the Planck's constant

ν is the frenquency of the photon

- ☞ Photoelectric interaction is **with the atom in a whole** and can not take place with free electrons.
- ☞ Photoelectric effect **creates a vacancy in one of the electron shells**, which leaves the atom at an excited state.

Photoelectric Effect – Absorption Edges

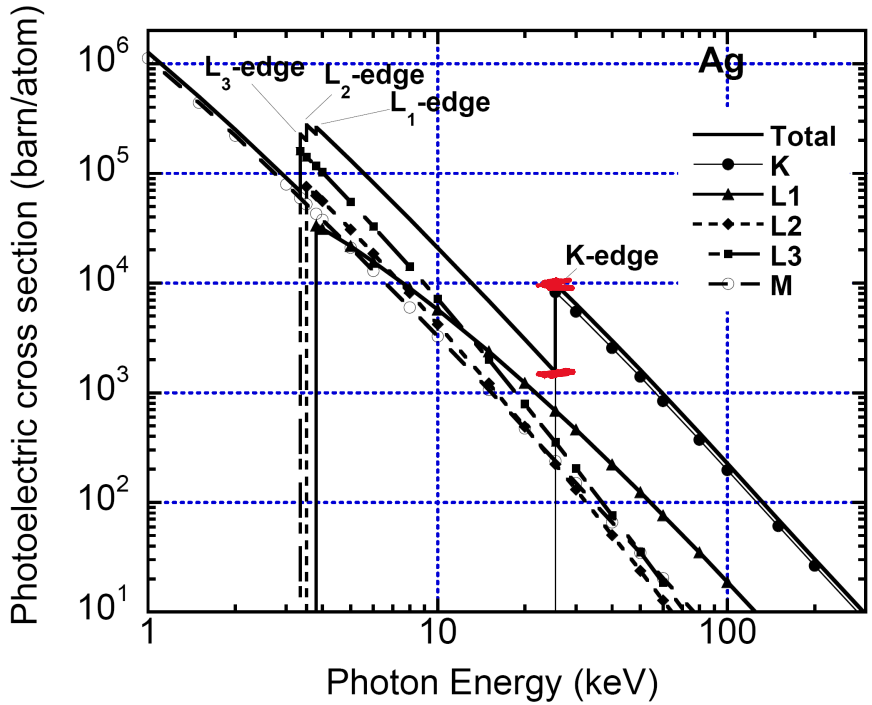


Figure 2: Total and partial atomic photoeffect of Ag.

- Requires **sufficient photon energy** for P.E. interaction.
- Interaction probability decreases dramatically with increasing energy.
- P.E. interaction is significant only for low energy photons, when the photon energy is close to the binding energies of the target atoms.

Photoelectric Effect Cross Section

Probability of photoelectric absorption per atom is

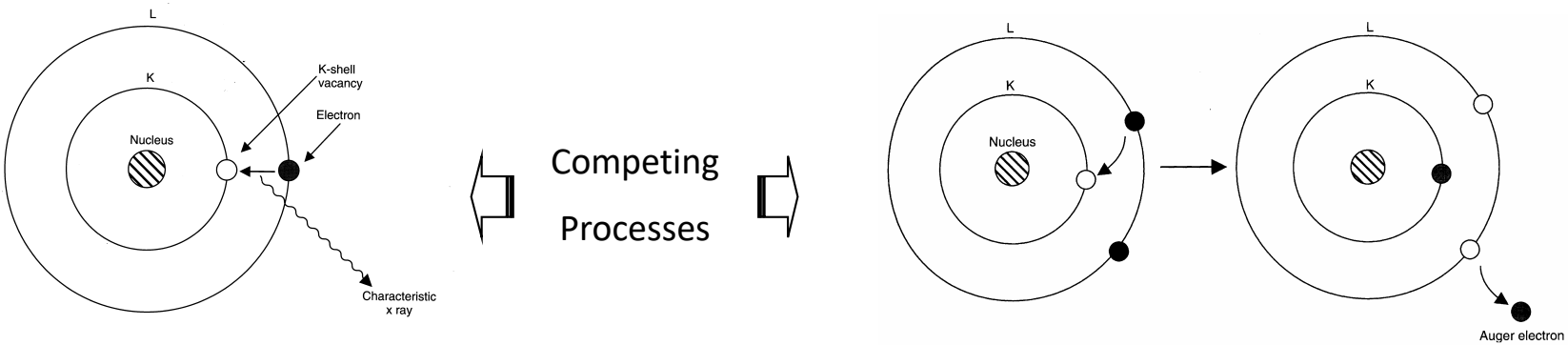
$$\sigma \propto \begin{cases} \dfrac{Z^4}{(h\nu)^{3.5}} & low\ energy \\ \dfrac{Z^5}{(h\nu)^{3.5}} & high\ energy \end{cases}$$

- The interaction cross section for photoelectric effect **depends strongly on Z**.
- Photoelectric effect is **favored at lower photon energies**. It is the major interaction process for photons at low hundred keV energy range.

Page 49, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Relaxation Process after Photoelectric Effect

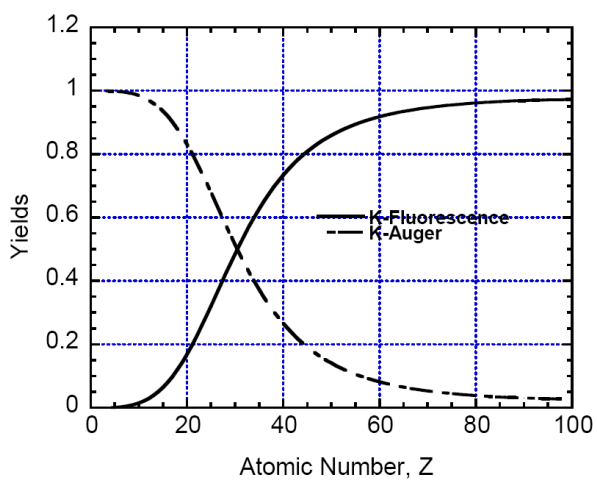
☞ The excited atoms will **de-excite** through one of the following processes:



Generation of characteristic X-rays

Generation of Auger electrons

☞ **Auger electron** emission dominates in **low-Z** elements. **Characteristic X-ray** emission dominates in **higher-Z** elements.



Auger Electrons

The relative probability of the emission of characteristic radiation to the emission of an Auger electron is called the fluorescent yield, ω:

$$\omega_K = \frac{\text{Number K x ray photons emitted}}{\text{Number K shell vacancies}} \qquad (3\text{-}12)$$

Values for ω_K are given in Table 3-1. We see that for large Z values fluorescent radiation is favored, while for low values of Z Auger electrons tend to be produced.

From this table we see that if a nucleus with Z = 40 had a K shell hole, then on the average 0.74 fluorescent photons and 0.26 Auger electrons would be emitted.

TABLE 3-1
Fluorescent Yield

Z	ω_K	Z	ω_K	Z	ω_K
10	0	40	.74	70	.92
15	.05	45	.80	75	.93
20	.19	50	.84	80	.95
25	.30	55	.88	85	.95
30	.50	60	.89	90	.97
35	.63	65	.90		

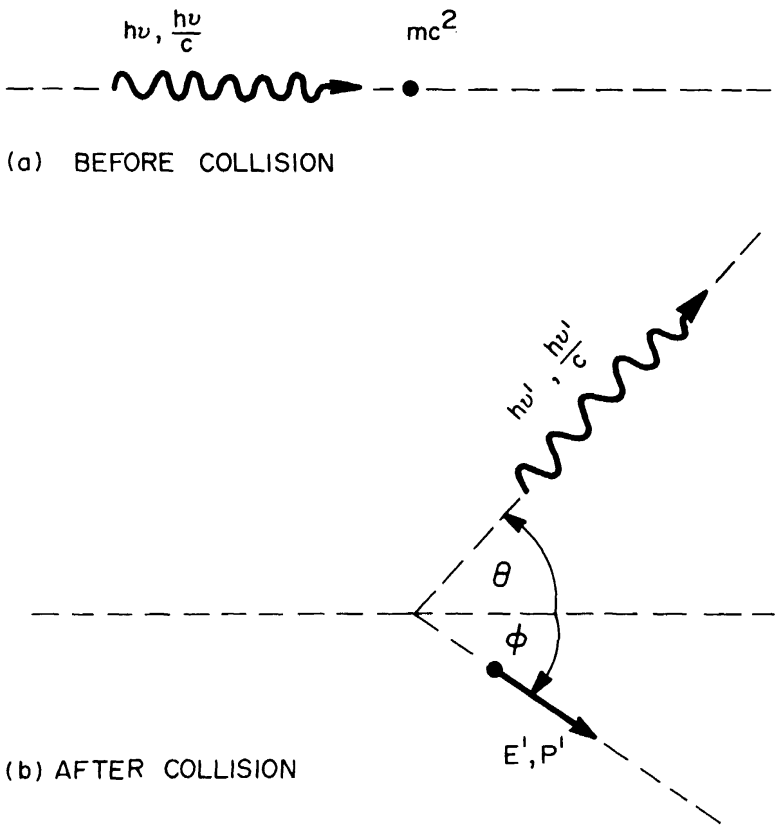
From Evans (E1)

Compton Scattereing

Basic Kinematics in Compton Scattering

The **energy transfer** in Compton scattering may be derived as the following:

- Assuming that the **electron binding energy is small** compared with the energy of the incident photon – **elastic scattering**.
- Write out the **conservation of energy and momentum**:



Conservation of energy

$$h\nu + mc^2 = h\nu' + E'$$

Conservation of momentum

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + P'\cos\varphi$$

$$\frac{h\nu'}{c}\sin\theta = P'\sin\varphi$$

Energy Transfer in Compton Scattering

If we assume that the electron is free and at rest, the scattered gamma ray has an energy

Initial photon energy, *v*: photon frequency

$$hv' = \frac{hv}{1 + \frac{hv}{m_0 c^2}(1 - \cos\theta)},$$

mass of electron

Scattering angle

and the photon transfers part of its energy to the electron (assumed to be at rest before the collision), which is known as the ***recoil electron***. Its energy is simply

$$E_{recoil} = hv - hv' = hv - \frac{hv}{1 + \frac{hv}{m_0 c^2}(1 - \cos(\theta))}$$

assuming the binding energy of the electron is negligible.

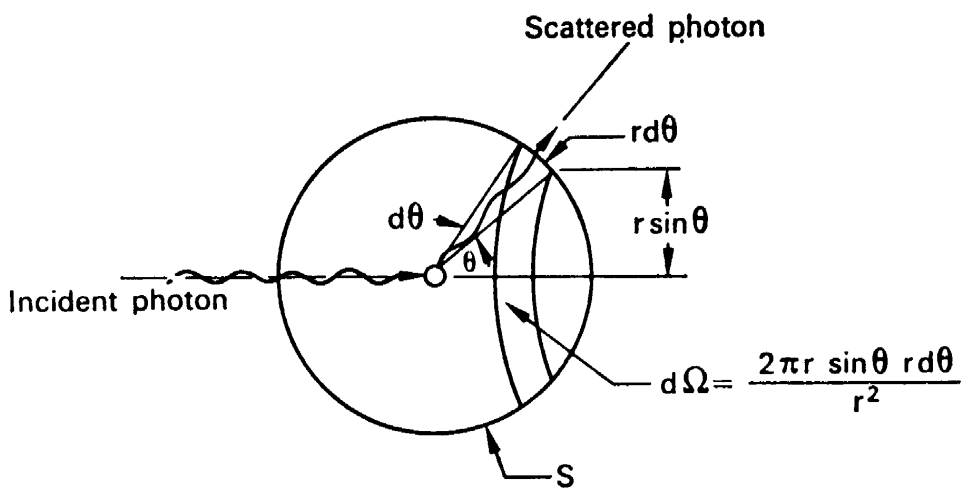
In the simplified elastic scattering case, there is an one-to-one relationship between scattering angle and energy loss!!

Reading: Page 51, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Angular Distribution of the Scattered Gamma Rays

The differential scattering cross section per electron – **the probability of a photon scattered into a unit solid angle around the scattering angle θ, when the incident photon is passing normally through a thin layer of scattering material that contains one electron per unit area.**

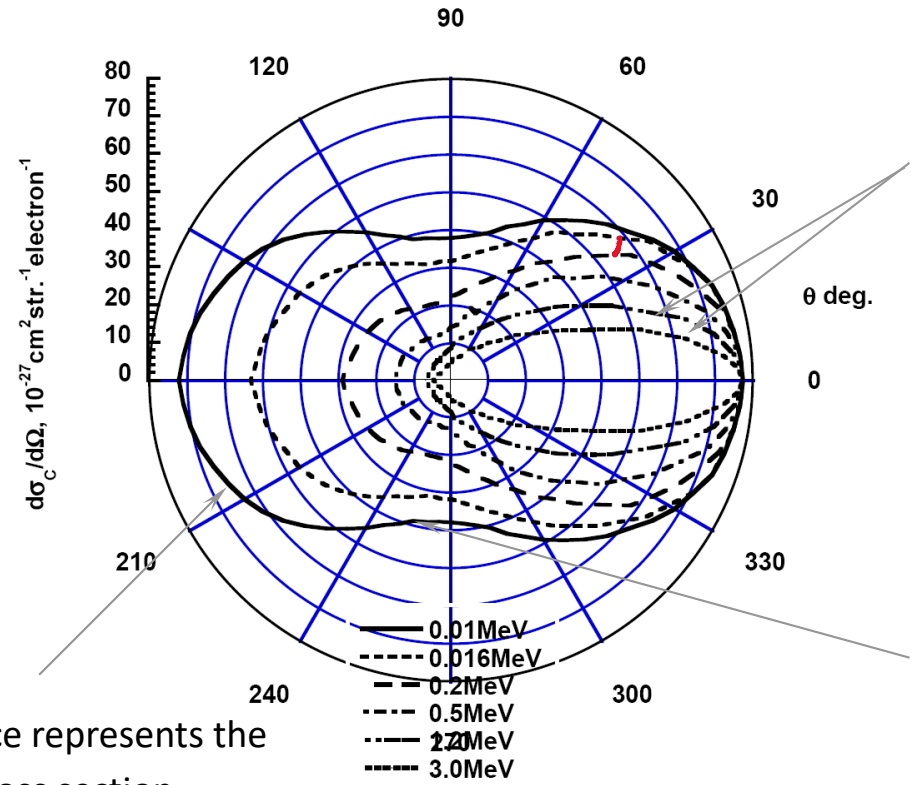
$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2 \left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2 \left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right)(m^2 sr^{-1})$$



$$d\Omega = 2\pi \sin\theta \, d\theta$$

FIG. 5.15. Compton scattering diagram to illustrate differential scattering cross section. S is a sphere of unit radius whose center is the scattering electron.

Angular Distribution of the Scattered Gamma Rays



Radial distance represents the differential cross section.

Incident photons with ***higher energies*** tend to scatter with smaller angles (forward scattering).

Incident photons with ***lower energy*** (a few hundred keV) have greater chance of undergoing large angle scattering (back scattering).

The higher the energy carried by an incident gamma ray, the more likely that the gamma ray undergoes forward scattering …

Energy Distribution of Compton Recoil Electrons

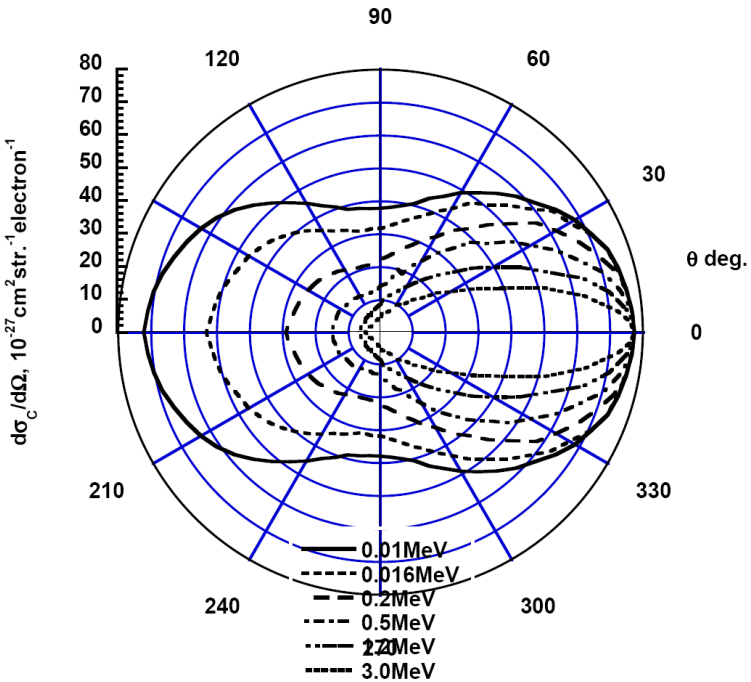
Klein-Nishina formula can be used to calculate the expected **energy spectrum of recoil electrons** as the following:

$$\frac{d\sigma}{dE_{recoil}} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{d\theta}\frac{d\theta}{dE_{recoil}}$$

and the probability that a recoil electron possesses an energy between E_{recoil}-$\Delta E/2$ and E_{recoil}+$\Delta E/2$ is given by

$$\propto \Delta E \cdot \frac{d\sigma}{dE_{recoil}}$$

Application of the Klein-Nishina Formula (1)



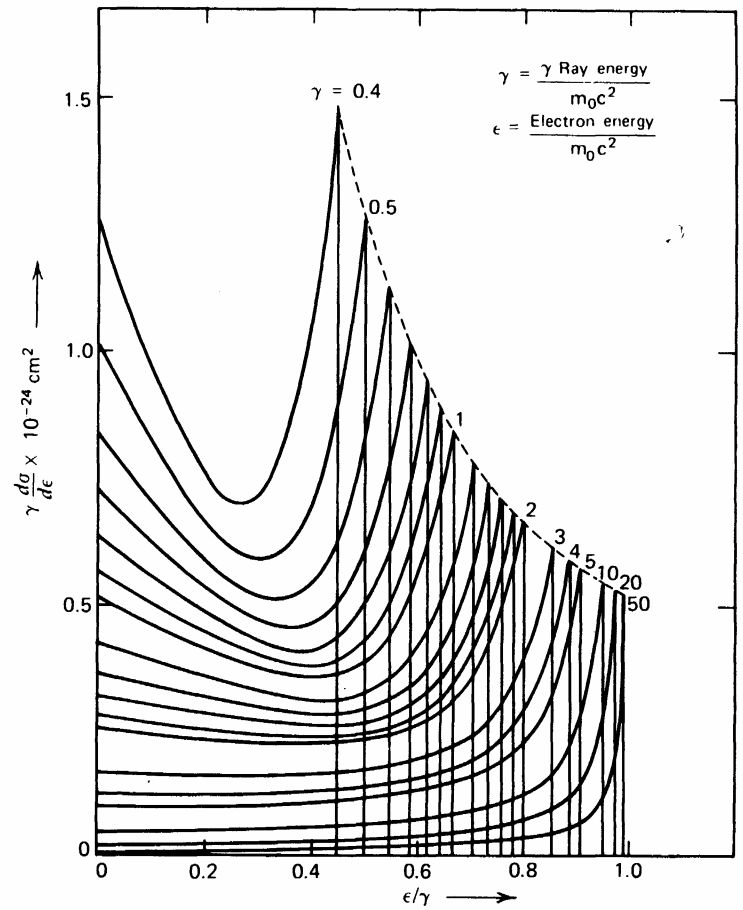


Figure 10.1 Shape of the Compton continuum for various gamma-ray energies. (From S. M. Shafroth (ed.), *Scintillation Spectroscopy of Gamma Radiation.* Copyright 1964 by Gordon & Breach, Inc. By permission of the publisher.)

☞ The energy distribution of the recoil electrons derived using the Klein-Nishina formula is closely related to **the energy spectrum measured with "small" detectors** (in particular, the so-called **Compton continuum**).

From Page 311, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Average fraction of energy transfer to the recoil electron through a single Compton Collision

Of special interest for dosimetry is the **average recoil electron energy** E_{avg_recoil}, since it is an approximation of the **radiation dose by each photon through Compton scattering**.

The **average fraction of energy transfer to the recoil electron** through a single Compton scattering is given by

$$\frac{E_{avg_recoil}}{h\nu} = \int_{E_{recoil}} \frac{E_{recoil}}{h\nu} \cdot \left[\left(\frac{d\sigma}{dE_{recoil}}\right)/\sigma\right] \cdot dE_{recoil} \ ,$$

where σ is the Compton scattering cross section per electron and is given by

$$\sigma = 2\pi \int_{\Omega} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \cdot \sin\theta \cdot d\theta \ \ (m^2) \ .$$

Linear Attenuation Coefficient through Compton Scattering

The **differential Compton cross section** given by the Klein-Nishina Formula can also be related to another important parameter for gamma ray dosimetry – **the linear attenuation coefficient**.

$$\sigma_{linear} = NZ\sigma \quad (m^{-1}),$$

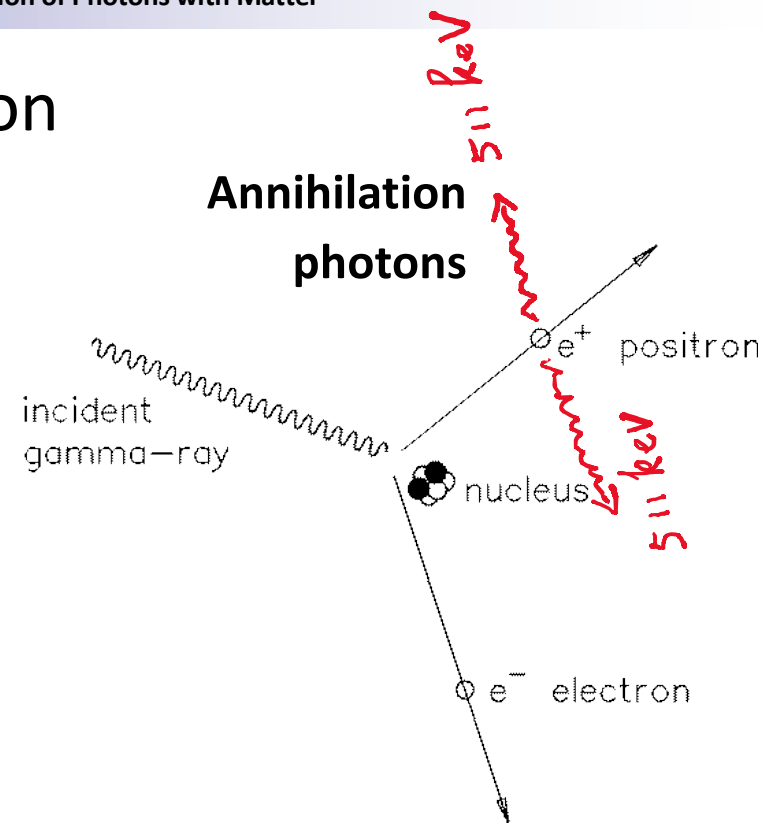
which is the **probability of a photon interacting with the absorber through Compton scattering while traversing a unit distance**. NZ is the electron density of the absorber materials (number of electrons per m^3)

Note that σ is the Compton scattering cross section per electron and is given by

$$\sigma = 2\pi \int_{\Omega} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \cdot \sin\theta \cdot d\theta \ \ (m^2) \ .$$

Pair Production

Pair Production



Definition:

Pair production refers to the creation of an electron-positron pair by an incident gamma ray in the vicinity of a nucleus.

Characteristics

☞ The minimum energy required is

$$E_\gamma \geq 2m_e c^2 + \frac{2m_e^2 c^2}{m_{nucleus}} \approx 2m_e c^2 = 1.022 MeV$$

☞ The process is more probable with a ***heavy nucleus*** and incident ***gamma rays with higher energies***.

☞ The positrons emitted will soon **annihilate** with ordinary electrons near by and produces two 511keV gamma rays.