4.2 Interaction of Heavy Charged Particles

Energy Loss Mechanisms

- Heavy charged particles loss energy primarily though the ionization and excitation of atoms.

- Heavy charged particles can transfer only a small fraction of its energy in a single collision. Its deflection in collision is almost negligible. Therefore heavy charged particles travel in a almost straight paths in matter, losing energy continuously through a large number of collisions with atomic electrons.

- At low velocity, a heavy charged particle may losses a negligible amount of energy in nuclear collisions. It may also pick up free electrons along its path, which reduces it net charge.

Energy Loss Mechanisms

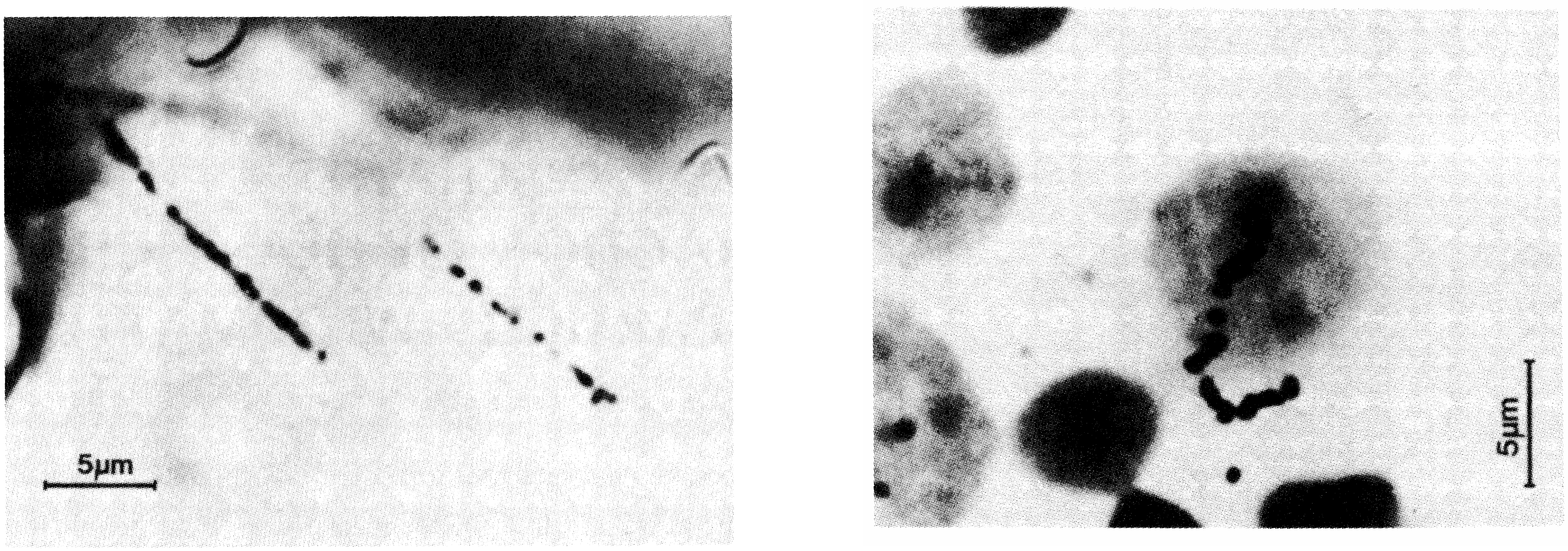


FIGURE 5.1. (Top) Alpha-particle autoradiograph of rat bone after inhalation of ^{241}Am. Biological preparation by R. Masse and N. Parmentier. (Bottom) Beta-particle autoradiograph of isolated rat-brain nucleus. The ^{14}C-thymidine incorporated in the nucleolus is located at the track origin of the electron emitted by the tracer element. Biological preparation by M. Wintzerith and P. Mandel. (Courtesy R. Rechenmann and E. Wittendorp-Rechenmann, Laboratoire de Biophysique des Rayonnements et de Methodologie INSERM U.220, Strasbourg, France.)

Energy Loss Mechanisms

For heavy charged particles, the maximum energy that can be transferred in a single collision is given by the conservation of energy and momentum:

$$\tfrac{1}{2}MV^2 = \tfrac{1}{2}MV_1^2 + \tfrac{1}{2}mv_1^2$$

$$MV = MV_1 + mv_1.$$

where M and m are the mass of the heavy charged particle and the electron. V is the initial velocity of the charged particle. V_1 and v_1 are the velocities of both particles after the collision.

The maximum energy transfer is therefore given by

$$Q_{\max} = \frac{1}{2}MV^2 - \frac{1}{2}MV_1^2 = \frac{4mME}{(M+m)^2}$$

Maximum Energy Loss by a Single Collision

For a more general case, which includes the relativistic effect, the maximum energy transferred by a single collision is

$$Q_{\max} = \frac{2\gamma^2 m V^2}{1 + 2\gamma m/M + m^2/M^2}$$

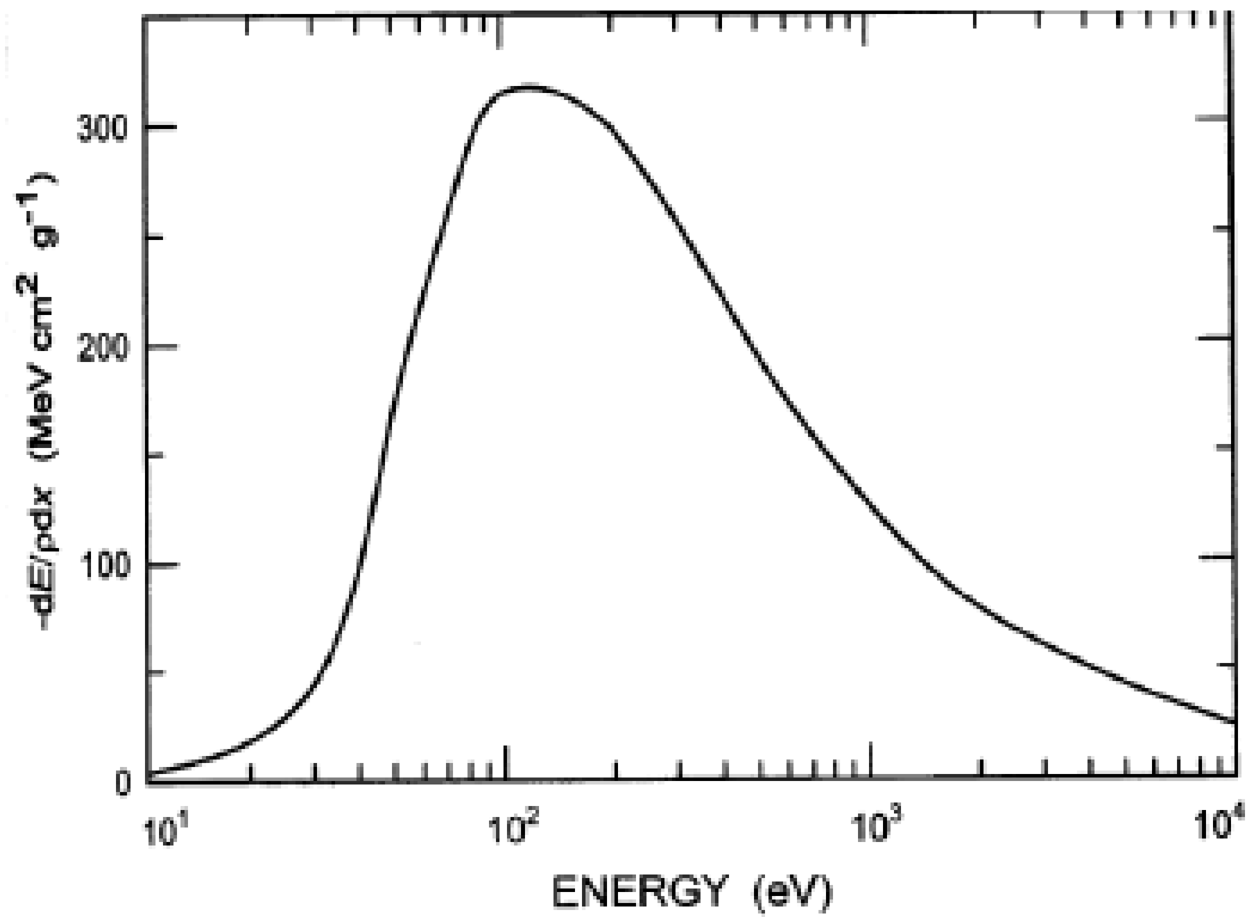
where $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = V/c$, and c is the speed of light

Maximum Energy Loss by a Single Collision

TABLE 5.1. Maximum Possible Energy Transfer, Q_{max}, in Proton Collision with Electron

Proton Kinetic Energy E (MeV)	Q_{max} (MeV)	Maximum Percentage Energy Transfer $100Q_{max}/E$
0.1	0.00022	0.22
1	0.0022	0.22
10	0.0219	0.22
100	0.229	0.23
10^3	3.33	0.33
10^4	136.	1.4
10^5	1.06×10^4	10.6
10^6	5.38×10^5	53.8
10^7	9.21×10^6	92.1

Mass Stopping Power



Mass stopping power of low energy electrons in water

Single Collision Energy-Loss Spectrum

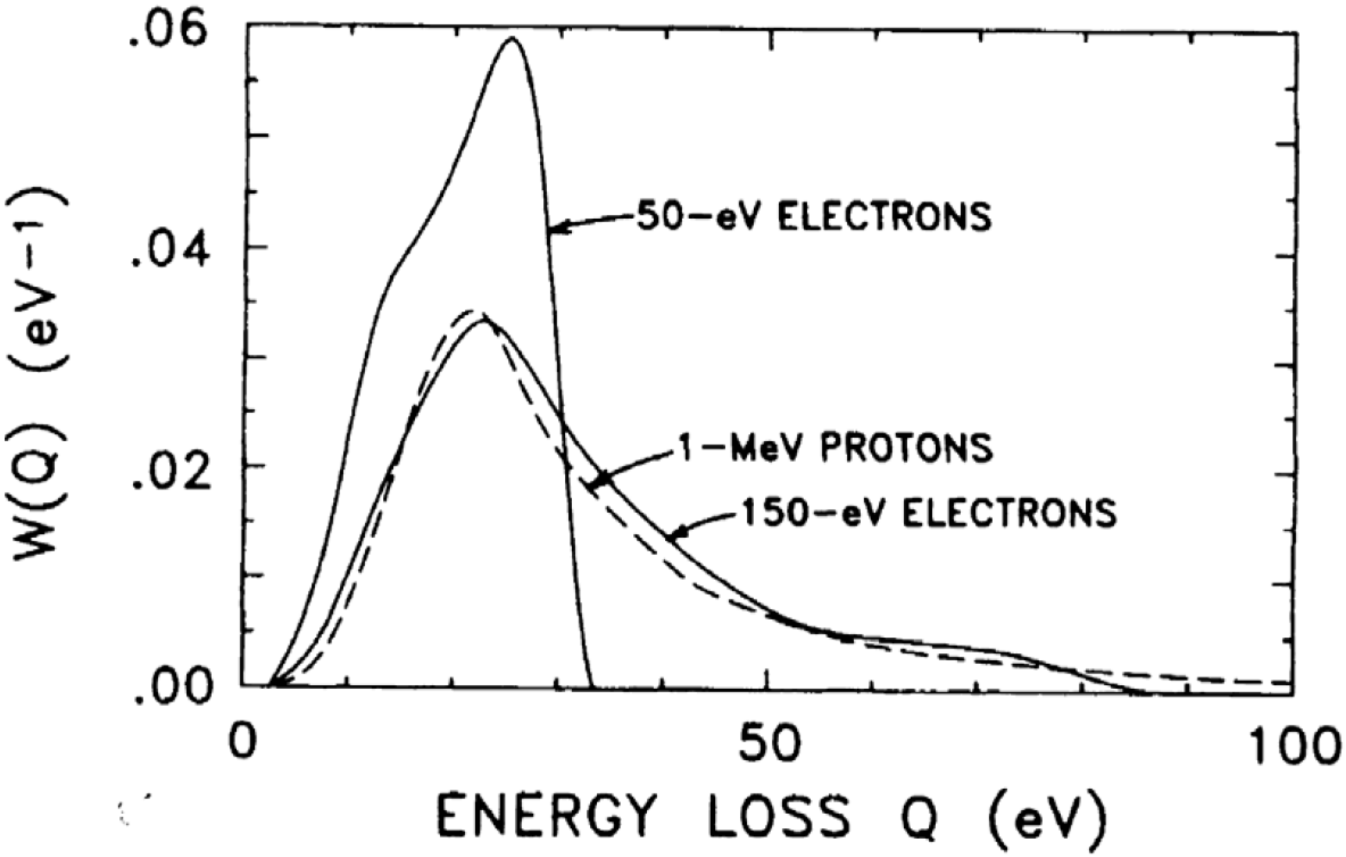


Fig. 5.3 Single-collision energy-loss spectra for 50-eV and 150-eV electrons and 1-MeV protons in liquid water. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)

Single Collision Energy-Loss Spectrum

- Single collision energy-loss spectra for fast charged particles (v>0.1c) are remarkably similar.
- Energy loss spectra for particles at slower speed differ from each other.

- At lower speed, charged particles are more likely to excite atoms rather than to ionize them.

- Energy loss can not be infinitely small – a certain amount of energy is needed for excitation or ionization.

Single Collision Energy-Loss Spectrum

Example

Estimate the probability that a 50-MeV proton will lose between 30 eV and 40 eV in a collision with an atomic electron in penetrating the soft tissue of the body.

Solution:

Soft tissue is similar in atomic composition to liquid water (Table 12.3), and so we use Fig. 5.3 to make the estimate.

As implied in the text, the energy-loss spectrum for 50-MeV protons is close to that for 1-MeV protons, except that it extends out to a different value of Q_{max}.

Therefore,

$$W(Q)\Delta Q = (0.019\ \text{eV}^{-1})(40 - 30)\ \text{eV} = 0.19.$$

Thus, a 50-MeV proton has about a 20% chance of losing between 30 eV and 40 eV in a single electronic collision in soft tissue.

Linear Stopping Power for Heavy Charged Particles

The **linear stopping power** for heavy charged particles may be estimated using the single collision energy-loss spectra discussed previously.

For a given type of charged particle at a given energy, the stopping power is given by the product of (1) the probability μ per unit distance of travel that an electronic collision occurs and (2) the average energy loss per collision, Q_{avg}. The former is called the macroscopic cross section, or attenuation coefficient, and has the dimensions of inverse length. The latter is given by

$$Q_{avg} = \int_{Q_{min}}^{Q_{max}} QW(Q)\,dQ,$$

where Q_{min} was introduced at the end of the last section.

Therefore, the **linear stopping power** is given by

$$-\frac{dE}{dx} = \mu Q_{avg} = \mu \int_{Q_{min}}^{Q_{max}} QW(Q)\,dQ.$$

Linear Stopping Power for Heavy Charged Particles

To understand what "short distances" mean in the present context, we recall that the macroscopic cross section μ is the probability per unit distance of travel that an electronic collision takes place. Its role in charged-particle penetration is analogous to that of the decay constant λ, which is the probability of disintegration per unit time in radioactive decay. Equation (4.22) showed that the reciprocal of the decay constant is equal to the mean life. In the same way, the reciprocal of μ is the mean distance of travel, or mean free path, of a charged particle between collisions. In the last example, the mean free path of the 1-MeV proton is $1/\mu = 1/(410\ \mu\text{m}^{-1}) = 0.0024\ \mu\text{m} = 24$ Å. Atomic diameters are of the order of 1 Å to 2 Å.

Linear Stopping Power – A Semiclassic Treatment

Consider the following diagram

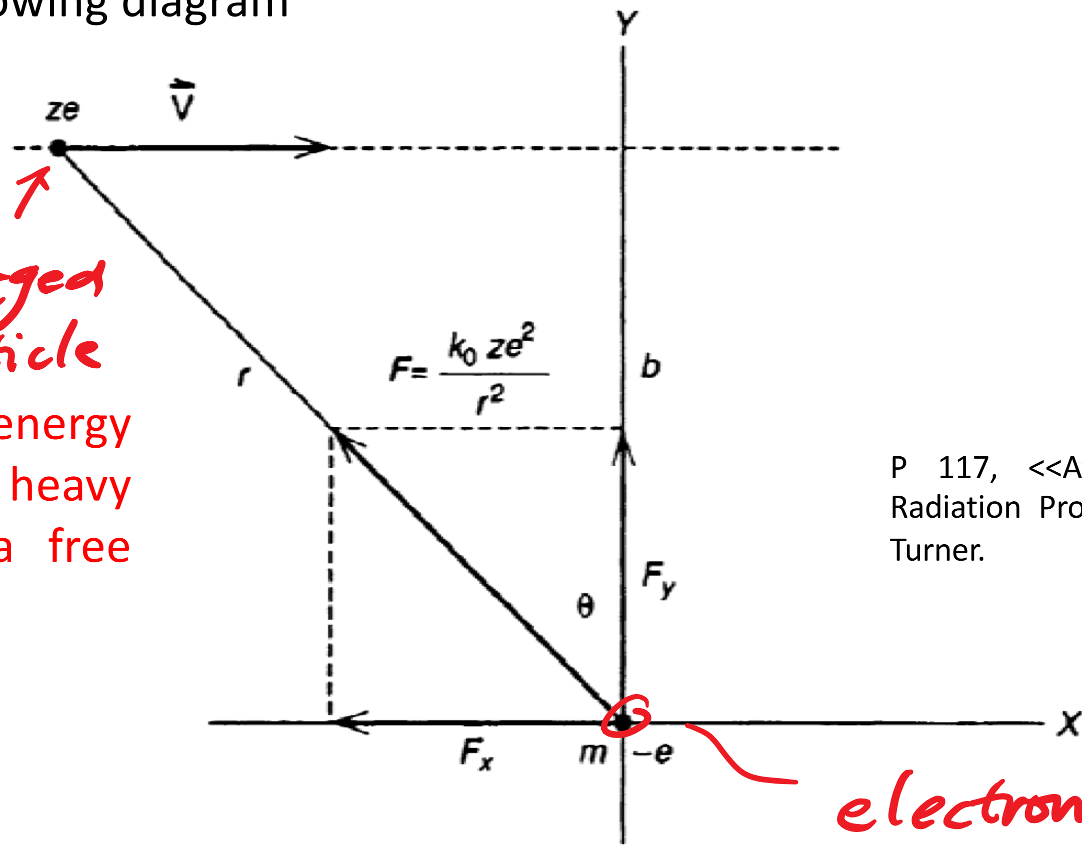


Fig. 5.4 Representation of the sudden collision of a heavy charged particle with an electron, located at the origin of XY coordinate axes shown. See text.

Step 1: Deriving the energy transfer from the heavy charged particle to a free electron nearby.

P 117, <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.

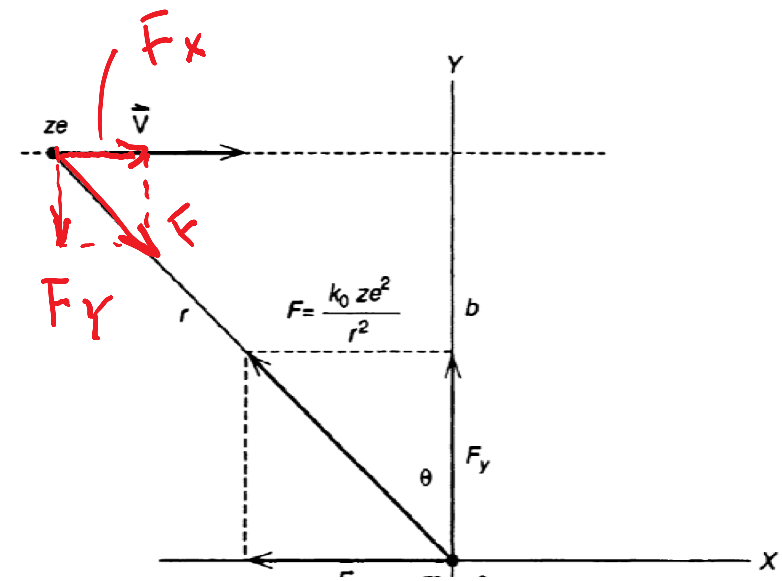
and assuming the electron is stationary during the collision…

Deriving the Linear Stopping Power for Heavy Charged Particles – A Semiclassic Treatment

The total momentum imparted to the electron is given by

$$p = \int_{-\infty}^{\infty} F_Y \, dt = \int_{-\infty}^{\infty} F \cos\theta \, dt = k_0 z e^2 \int_{-\infty}^{\infty} \frac{\cos\theta}{r^2} \, dt. \qquad (5.11)$$

Coulumb force $F = \frac{k_0 z e^2}{r^2}$



To carry out the integration, we let $t = 0$ represent the time at which the heavy charged particle crosses the Y-axis in Fig. 5.4. Since $\cos\theta = b/r$ and the integral is symmetric in time, we write

$r = \sqrt{b^2 + V^2 t^2}$

$$\int_{-\infty}^{\infty} \frac{\cos\theta}{r^2} \, dt = 2\int_0^{\infty} \frac{b}{r^3} \, dt = 2b \int_0^{\infty} \frac{dt}{(b^2 + V^2 t^2)^{3/2}}$$

$$= 2b \left[\frac{t}{b^2 (b^2 + V^2 t^2)^{1/2}} \right]_0^{\infty} = \frac{2}{Vb}. \qquad (5.12)$$

P 118, <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.

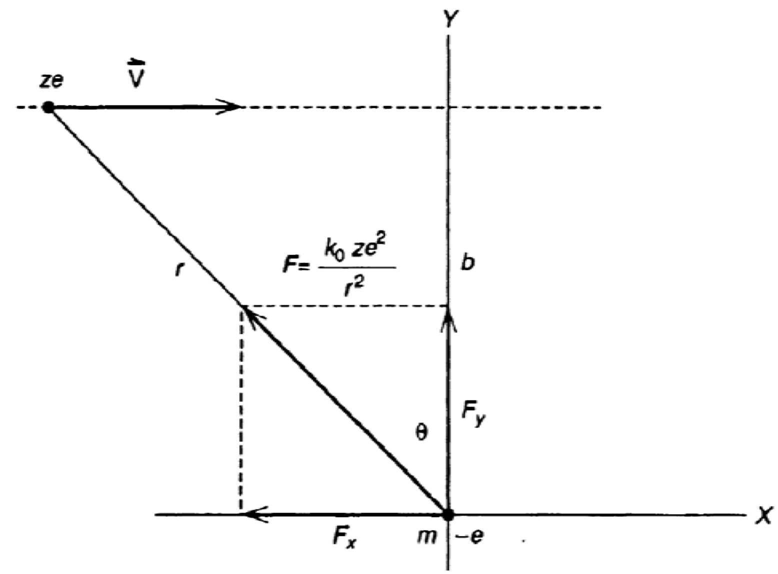
Linear Stopping Power – A Semiclassic Treatment

Combining this result with (5.11) gives, for the momentum transferred to the electron in the collision,[2)]

$$p = \frac{2k_0 z e^2}{Vb}. \qquad (5.13)$$

The energy transferred is

$$Q = \frac{p^2}{2m} = \frac{2k_0^2 z^2 e^4}{mV^2 b^2}.$$



P 118, <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.

Linear Stopping Power – A Semiclassic Treatment

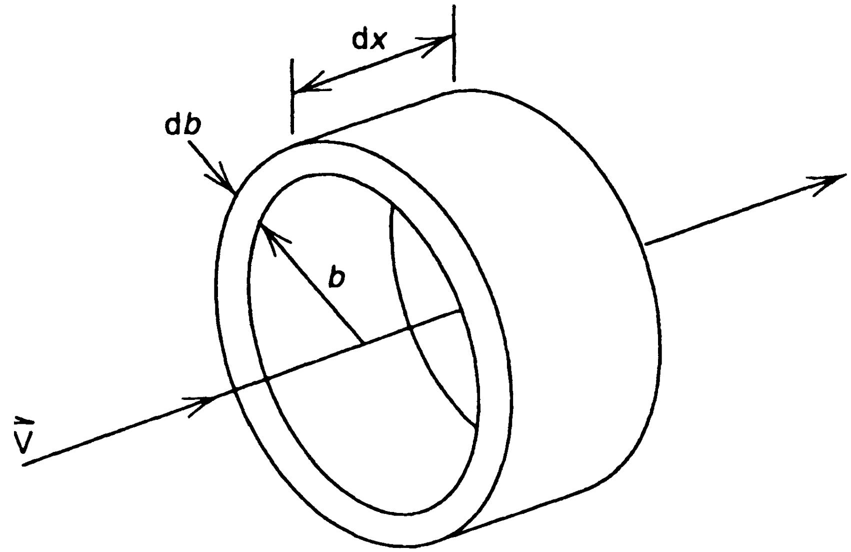


Fig. 5.5 Annular cylinder of length dx centered about path of heavy charged particle. See text.

Step 2: Integrate the energy transfer to all the electrons surrounding the path of the heave charged particle.

In traversing a distance dx in a medium having a uniform density of n electrons per unit volume, the heavy particle encounters $2\pi nb\,\mathrm{d}b\,\mathrm{d}x$ electrons at impact parameters between b and $b + \mathrm{d}b$, as indicated in Fig. 5.5. The energy lost to these electrons per unit distance traveled is therefore $2\pi nQb\,\mathrm{d}b$. The total linear rate of

Therefore, the total linear rate of energy-loss is given by

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 2\pi n \int_{Q_{\min}}^{Q_{\max}} Qb\,\mathrm{d}b = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2} \int_{b_{\min}}^{b_{\max}} \frac{\mathrm{d}b}{b} = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2} \ln \frac{b_{\max}}{b_{\min}}. \quad (5.15)$$

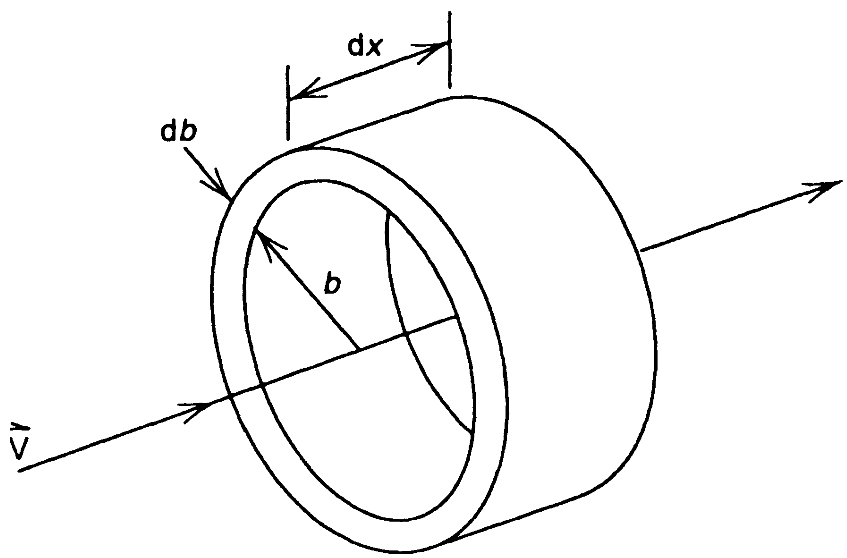
Linear Stopping Power – A Semiclassic Treatment

The maximum value of the impact parameter can be estimated from the physical principle that a quantum transition is likely only when the passage of the charged particle is rapid compared with the period of motion of the atomic electron. We denote the latter time by $1/f$, where f is the orbital frequency. The duration of the collision is of the order of b/V. Thus, the important impact parameters are restricted to values approximately given by

the duration of the collision

$$\frac{b}{V} < \frac{1}{f} \quad \text{or} \quad b_{\max} \sim \frac{V}{f}.$$

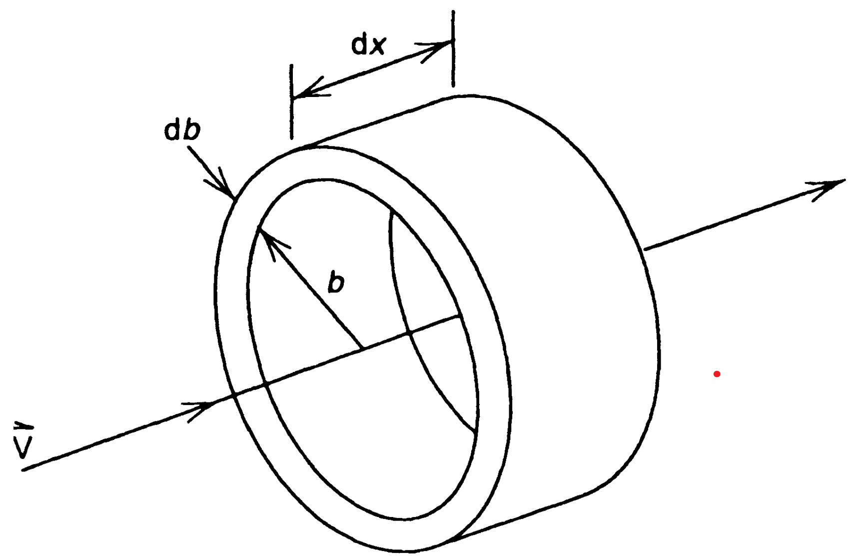
period of the orbital electron



Linear Stopping Power – A Semiclassic Treatment

For the minimum impact parameter, the analysis implies that the particles' positions remain separated by a distance b_{min} at least as large as their de Broglie wavelengths during the collision. This condition is more restrictive for the less massive electron than for the heavy particle. In the rest frame of the latter, the electron has a de Broglie wavelength $\lambda = h/mV$, since it moves approximately with speed V relative to the heavy particle. Accordingly, we choose

$$b_{\text{min}} \sim \frac{h}{mV}. \tag{5.17}$$



Linear Stopping Power – A Semiclassic Treatment

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 2\pi n \int_{Q_{\min}}^{Q_{\max}} Qb\,\mathrm{d}b = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2} \int_{b_{\min}}^{b_{\max}} \frac{\mathrm{d}b}{b} = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2} \ln \frac{b_{\max}}{b_{\min}}. \qquad (5.15)$$

Combining the relations (5.15), (5.16), and (5.17) gives the semiclassical formula for stopping power,

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2} \ln \frac{mV^2}{hf}. \qquad (5.18)$$

Linear Stopping Power – A Semiclassic Treatment

Example

Calculate the maximum and minimum impact parameters for electronic collisions for an 8-MeV proton. To estimate the orbital frequency f, assume that it is about the same as that of the electron in the ground state of the He^+ ion.

$$b_{\mathrm{min}} \sim \frac{h}{mV}.$$

$$\frac{b}{V} < \frac{1}{f} \quad \text{or} \quad b_{\mathrm{max}} \sim \frac{V}{f}.$$

P 119, <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.

Linear Stopping Power – A Semiclassic Treatment

Solution

$$b_{\max} \sim \frac{V}{f}.$$

The proton velocity is given by $V = (2T/M)^{1/2}$, where T is the kinetic energy and M is the mass:

$$V = \left[\frac{2 \times 8\ \text{MeV} \times 1.60 \times 10^{-13}\,\text{J MeV}^{-1}}{1.67 \times 10^{-27}\ \text{kg}}\right]^{1/2} = 3.92 \times 10^{7}\ \text{m s}^{-1}. \quad (5.19)$$

The orbital frequency of the electron in the ground state of He^{+} can be found by using Eqs. (2.8) and (2.9) with $Z = 2$ and $n = 1$:

$$f = \frac{v_1}{2\pi r_1} = \frac{4.38 \times 10^{6}\ \text{m s}^{-1}}{2\pi \times 2.65 \times 10^{-11}\ \text{m}} = 2.63 \times 10^{16}\ \text{s}^{-1}. \quad (5.20)$$

Equation (5.16) gives for the maximum impact parameter

$$b_{\max} \sim \frac{V}{f} = \frac{3.92 \times 10^{7}\ \text{m s}^{-1}}{2.63 \times 10^{16}\ \text{s}^{-1}} = 1.49 \times 10^{-9}\ \text{m} = 15\ \text{Å}. \quad (5.21)$$

Linear Stopping Power – A Semiclassic Treatment

$$b_{\min} \sim \frac{h}{mV}.$$

The minimum impact parameter is, from Eq. (5.17),

$$b_{\min} \sim \frac{h}{mV} = \frac{6.63 \times 10^{-34}\ \mathrm{J\,s}}{9.11 \times 10^{-31}\ \mathrm{kg} \times 3.92 \times 10^{7}\ \mathrm{m\,s^{-1}}}$$

$$= 1.86 \times 10^{-11}\ \mathrm{m} = 0.19\ \text{Å}. \qquad (5.22)$$

Linear Stopping Power for Heavy Charged Particles

The linear stopping power of a medium is given by the Bethe formula,

Combining the relations (5.15), (5.16), and (5.17) gives the semiclassical formula for stopping power,

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2} \ln \frac{mV^2}{hf}. \quad (5.18)$$

$k_0 = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

z = atomic number of the heavy particle,

e = magnitude of the electron charge,

n = number of electrons per unit volume in the medium,

m = electron rest mass,

c = speed of light in vacuum,

$\beta = V/c$ = speed of the particle relative to c,

I = mean excitation energy of the medium.

Linear Stopping Power of a Medium for Heavy Charged Particles (revisited)

The linear stopping power of a medium is given by the Bethe formula,

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2\beta^2}\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2\right].$$

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mV^2}\ln\frac{mV^2}{hf}.$$

k_0 = 8.99×10^9 N m^2 C^{-2} (Appendix C),
z = atomic number of the heavy particle,
e = magnitude of the electron charge,
n = number of electrons per unit volume in the medium,
m = electron rest mass,
c = speed of light in vacuum,
β = V/c = speed of the particle relative to c,
I = mean excitation energy of the medium.

Mean Excitation Energies

The main excitation energy (I) for an element having atomic number Z, can be approximately given by

$$I \cong \begin{cases} 19.0 \text{ eV}, \; Z = 1 \text{ (hydrogen)} \\ 11.2 + 11.7\, Z \text{ eV}, \; 2 \leq Z \leq 13 \\ 52.8 + 8.71\, Z \text{ eV}, \; Z > 13. \end{cases}$$

For compound or mixture,

If there are N_i atoms cm^{-3} of an element with atomic number Z_i and mean excitation energy I_i, then in formula (5.23) one makes the replacement

mean excitation energy for compound

$$n \ln I = \sum_i N_i Z_i \ln I_i,$$

Mean Excitation Energies

Example

Calculate the mean excitation energy of H_2O.

Solution

We obtain the I values for H and O from Eqs. (5.24) and (5.25), and then apply (5.27). For H, $I_H = 19.0$ eV, and for O, $I_O = 11.2 + 11.7 \times 8 = 105$ eV. The electronic densities $N_i Z_i$ and n can be computed in a straightforward way. However, only the ratios $N_i Z_i / n$ are needed to find I, and these are much simpler to use. Since the H_2O molecule has 10 electrons, 2 of which belong to H ($Z = 1$) and 8 to O ($Z = 8$), we may write from Eq. (5.27)

$$\ln I = \frac{2 \times 1}{10} \ln 19.0 + \frac{1 \times 8}{10} \ln 105 = 4.312, \quad (5.28)$$

giving $I = 74.6$ eV.

$$n \ln I = \sum_i N_i Z_i \ln I_i$$

For the equations used in this derivation, please see p. 123, <<Atoms, Radiation, and Radiation Protection>>, by James E Turner.

Linear Stopping Power of a Medium for Heavy Charged Particles

The Bethe formula can be further simplified by substituting known constants, which gives

$$-\frac{dE}{dx} = \frac{5.08 \times 10^{-31} z^2 n}{\beta^2}\left[\ln \frac{1.02 \times 10^6 \beta^2}{I\ (1-\beta^2)} - \beta^2\right] \text{MeV cm}^{-1}$$

It may be further simplified to emphasize some important quantities related to the stopping power, the "speed" of the particle β, atomic mass of the charged particle z, the number of electron per cm^3 n and the mean excitation-ionization potential I:

$$-\frac{dE}{dx} = \frac{5.08 \times 10^{-31} z^2 n}{\beta^2}\left[F(\beta) - \ln I\right] \text{MeV cm}^{-1}$$

where

$$F(\beta) = \ln \frac{1.02 \times 10^6 \beta^2}{1-\beta^2} - \beta^2$$

Table for Computation of Stopping Power

Table 5.2 Data for Computation of Stopping Power for Heavy Charged Particles

Proton Kinetic Energy (MeV)	β^2	$F(\beta)$ Eq. (5.34)
0.01	0.000021	2.179
0.02	0.000043	3.775
0.04	0.000085	4.468
0.06	0.000128	4.873
0.08	0.000171	5.161
0.10	0.000213	5.384
0.20	0.000426	6.077
0.40	0.000852	6.771
0.60	0.001278	7.175
0.80	0.001703	7.462
1.00	0.002129	7.685
2.00	0.004252	8.376
4.00	0.008476	9.066
6.00	0.01267	9.469
8.00	0.01685	9.753
10.00	0.02099	9.972
20.00	0.04133	10.65
40.00	0.08014	11.32
60.00	0.1166	11.70
80.00	0.1510	11.96
100.0	0.1834	12.16
200.0	0.3205	12.77
400.0	0.5086	13.36
600.0	0.6281	13.73
800.0	0.7088	14.02
1000.	0.7658	14.26

Stopping Power of Water for Protons

For protons, $z = 1$ in Eq. (5.33). The gram molecular weight of water is 18.0 g, and the number of electrons per molecule is 10. Since 1 m^3 of water has a mass of 10^6 g, the density of electrons is

$$n = 6.02 \times 10^{23} \times \frac{10^6 \text{ g m}^{-3}}{18.0 \text{ g}} \times 10 = 3.34 \times 10^{29} \text{ m}^{-3}. \tag{5.36}$$

Also, as found at the end of Section 5.7, $\ln I_{eV} = 4.312$. From Eq. (5.33) it follows that the stopping power of water for a proton of speed β is given by

$$-\frac{dE}{dx} = \frac{0.170}{\beta^2}[F(\beta) - 4.31] \text{ MeV cm}^{-1}. \tag{5.37}$$

At 1 MeV, for example, we find in Table 5.2 that $\beta^2 = 0.00213$ and $F(\beta) = 7.69$; therefore Eq. (5.37) gives

$$-\frac{dE}{dx} = \frac{0.170}{0.00213}(7.69 - 4.31) = 270 \text{ MeV cm}^{-1}. \tag{5.38}$$

From Atoms, Radiation, and Radiation Protection, James E Turner

Table for Computation of Stopping Power

Example

Compute $F(\beta)$ for a proton with kinetic energy $T = 10$ MeV.

Solution

In the second example in Section 5.2 we found that $\beta^2 = 0.02099$. Substitution of this value into Eq. (5.34) gives $F(\beta) = 9.972$.

Can we use the results for proton to predict the linear stopping power for other heavy charged particles?

$$-\frac{dE}{dx} = \frac{5.08 \times 10^{-31} z^2 n}{\beta^2} [F(\beta) - \ln I \quad] \text{ MeV cm}^{-1}$$

Table for Computation of Stopping Power

$$-\frac{dE}{dx} = \frac{5.08 \times 10^{-31} z^2 n}{\beta^2} [F(\beta) - \ln I \quad] \text{ MeV cm}^{-1}$$

The quantities β^2 and $F(\beta)$ are given for protons of various energies in Table 5.2. Since, for a given value of β, the kinetic energy of a particle is proportional to its rest mass, the table can also be used for other heavy particles as well. For example, the ratio of the kinetic energies T_d and T_p of a deuteron and a proton traveling at the same speed is

$$\frac{T_d}{T_p} = \frac{M_d}{M_p} = 2. \tag{5.35}$$

The value of $F(\beta) = 9.973$ that we just computed for a 10-MeV proton applies, therefore, to a 20-MeV deuteron. Linear interpolation can be used where needed in the table.

Phenomena Associated with for Charged Particles

Key Things to Remember

- Interaction mechanisms.
- Bethe formula for linear stopping power
- First collision energy transfer
- Restricted stopping power and linear energy transfer
- Stopping time and range of heavy charged particles

Restricted Stopping Power

- Energy lost versus energy absorbed…
- Restricted Stopping Power is introduced to better associate the energy lost in a target with the energy actually absorbed there.

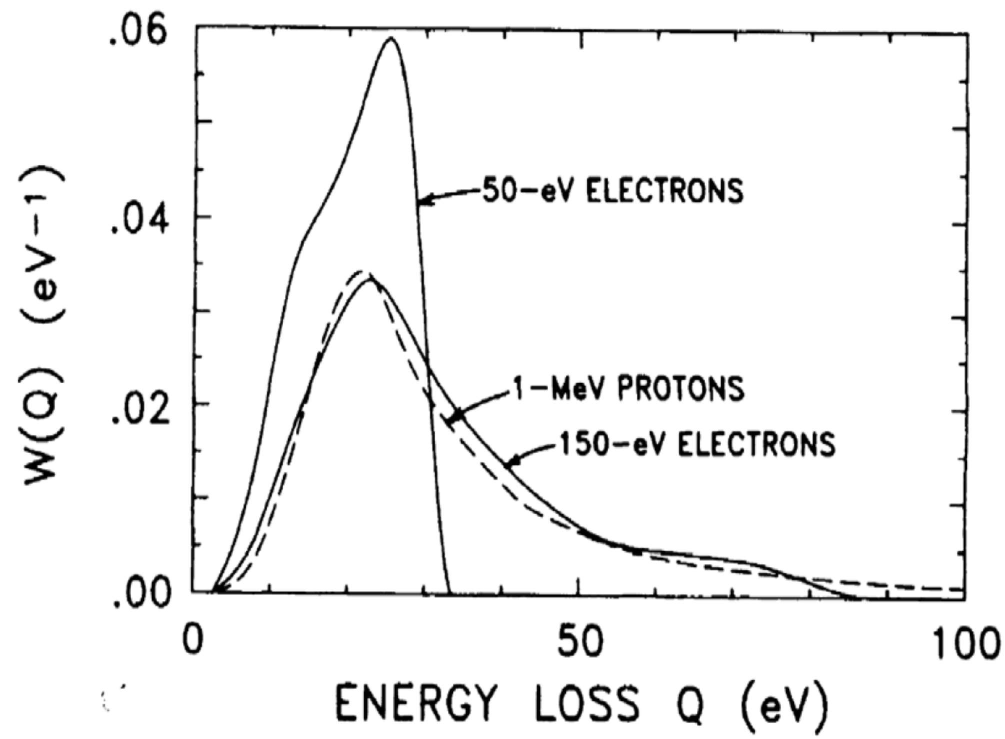


Fig. 5.3 Single-collision energy-loss spectra for 50-eV and 150-eV electrons and 1-MeV protons in liquid water. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)

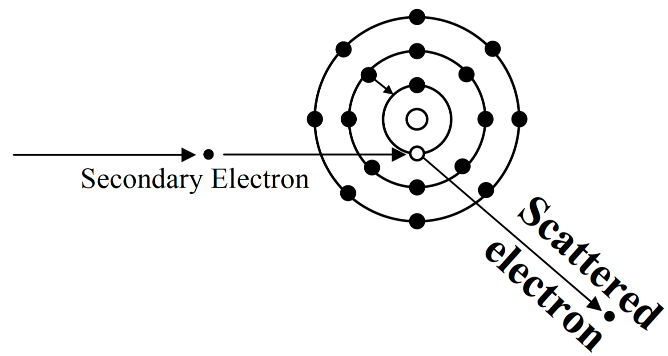
$$\left(-\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\Delta} = \mu \int_{Q_{\min}}^{\Delta} Q W(Q)\, \mathrm{d}Q.$$

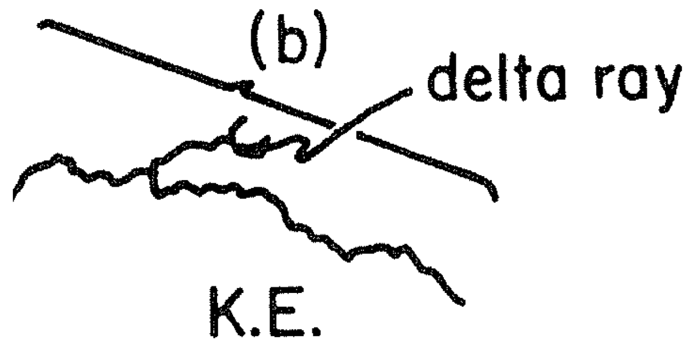
Restricted Stopping Power

In hard collisions, the scattered electron – or delta ray - can receive significant amounts of energy.

The delta ray can carry this energy a significant distance from the initial interaction site.

Need to separate this from the energy loss that is deposited locally.





The Rationale behind Restricted Stopping Power

- If we are interested in microscopic events, in which incident particles deposit energy in local regions with finite sizes …
- Since the predominate way for heavy charged particles to loss energy is to transferring its energies to energetic delta-rays …
- If the range of the delta-ray is large compare to the dimension of the region-of-interest (ROI), it is likely that the energy carried by these delta-rays will not be fully deposited in the ROI.
- To account for this effect, we will consider those delta-rays that carry energy less than a threshold, this give rise to the Restrict Stopping Power.
- The value for the threshold is typically determined by the dimension of the ROI associated with the given application.

Restricted Stopping Power

Restricted stopping power L_{Δ} is the fraction of the mass collision stopping power that includes all soft collisions and only those hard collisions which result in δ-rays with energy less than Δ.

This is sometimes known as linear energy transfer

Restricted Stopping Power

Table 7.1 Restricted Mass Stopping Power of Water, $(-dE/\rho dx)_\Delta$ in MeV cm^2 g^{-1}, for Protons

Energy (MeV)	$\left(-\frac{dE}{\rho dx}\right)_{100\ \text{eV}}$	$\left(-\frac{dE}{\rho dx}\right)_{1\ \text{keV}}$	$\left(-\frac{dE}{\rho dx}\right)_{10\ \text{keV}}$	$\left(-\frac{dE}{\rho dx}\right)_{\infty}$
0.05	910.	910.	910.	910.
0.10	711.	910.	910.	910.
0.50	249.	424.	428.	428.
1.00	146.	238.	270.	270.
10.0	24.8	33.5	42.2	45.9
100.	3.92	4.94	5.97	7.28

Restricted Stopping Power

Example

A sample of viruses, assumed to be in the shape of spheres of diameter 300 Å, is to be irradiated by a charged-particle beam in an experiment. Estimate a cutoff value that would be appropriate for determining a restricted stopping power that would be indicative of the actual energy *absorbed* in the individual virus particles.

Solution

As an approximation, one can specify that the range of the most energetic delta ray should not exceed 300 Å $= 3 \times 10^{-6}$ cm. We assume that the virus sample has unit density. Table 6.1 shows that this distance is approximately the range of a 700-eV secondary electron. Therefore, we choose $\Delta = 700$ eV and use the restricted stopping power $(-\mathrm{d}E/\mathrm{d}x)_{700\ \mathrm{eV}}$ as a measure of the average energy absorbed in an individual virus particle from a charged particle traversing it.

Linear Energy Transfer (LET)

used with exactly the same meaning. In 1980, the ICRU defined LET_Δ as the restricted stopping power for energy losses not exceeding Δ:

$$\mathrm{LET}_\Delta = \left(-\frac{\mathrm{d}E}{\mathrm{d}x}\right)_\Delta, \tag{7.3}$$

with the symbol LET_∞ denoting the usual (unrestricted) stopping power.

In 1998, the ICRU introduced the following new definition, also called "linear energy transfer, or restricted linear electronic stopping power, L_Δ":

$$L_\Delta = -\frac{\mathrm{d}E_\Delta}{\mathrm{d}x}.$$

$$L_\Delta = -\frac{\mathrm{d}E}{\mathrm{d}x} - \frac{\mathrm{d}E_{\mathrm{ke},\Delta}}{\mathrm{d}x},$$

Restricted Stopping Power

Example

Use Table 7.1 to determine $LET_{1\ keV}$ and $LET_{5\ keV}$ for 1-MeV protons in water.

Solution

Note that Eq. (7.3) for LET involves stopping power rather than mass stopping power. Since $\rho = 1$ for water, the numbers in Table 7.1 also give $(-dE/dx)_\Delta$ in MeV cm^{-1}. We find $LET_{1\ keV} = 238$ MeV cm^{-1} given directly in the table. Linear interpolation gives $LET_{5\ keV} = 252$ MeV cm^{-1}.

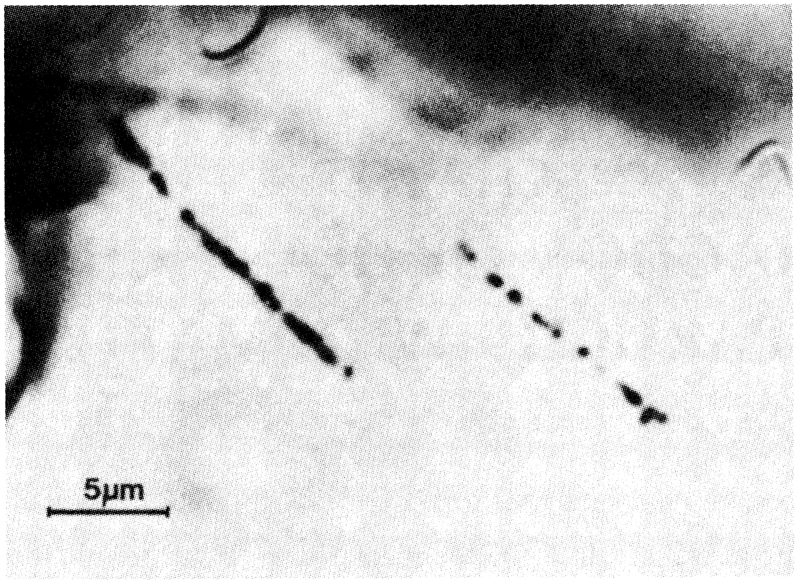
Table 7.1 Restricted Mass Stopping Power of Water, $(-dE/\rho dx)_\Delta$ in MeV cm^2 g^{-1}, for Protons

Energy (MeV)	$\left(-\frac{dE}{\rho dx}\right)_{100\ eV}$	$\left(-\frac{dE}{\rho dx}\right)_{1\ keV}$	$\left(-\frac{dE}{\rho dx}\right)_{10\ keV}$	$\left(-\frac{dE}{\rho dx}\right)_{\infty}$
0.05	910.	910.	910.	910.
0.10	711.	910.	910.	910.
0.50	249.	424.	428.	428.
1.00	146.	238.	270.	270.
10.0	24.8	33.5	42.2	45.9
100.	3.92	4.94	5.97	7.28

Key Things to Remember

- Interaction mechanisms.
- Bethe formula for linear stopping power
- First collision energy transfer
- Restricted stopping power and linear energy transfer
- Stopping time and range of heavy charged particles

Energy Loss Mechanisms



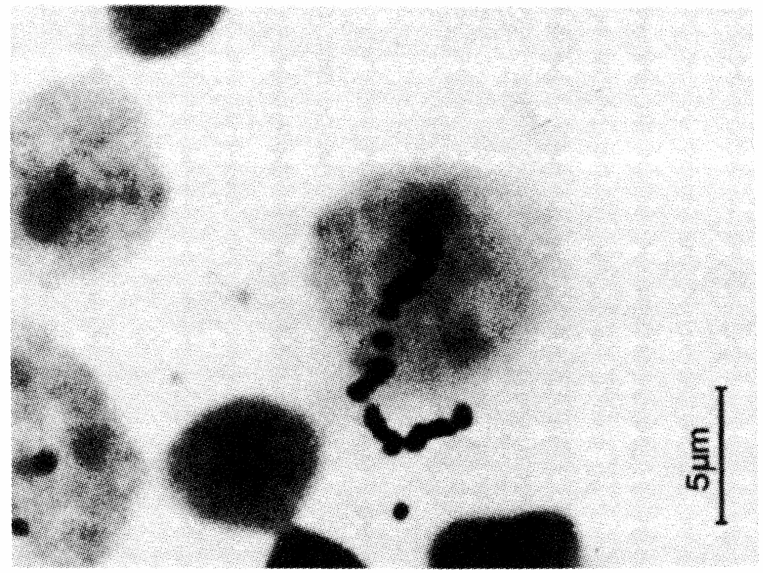


FIGURE 5.1. (Top) Alpha-particle autoradiograph of rat bone after inhalation of ^{241}Am. Biological preparation by R. Masse and N. Parmentier. (Bottom) Beta-particle autoradiograph of isolated rat-brain nucleus. The ^{14}C-thymidine incorporated in the nucleolus is located at the track origin of the electron emitted by the tracer element. Biological preparation by M. Wintzerith and P. Mandel. (Courtesy R. Rechenmann and E. Wittendorp-Rechenmann, Laboratoire de Biophysique des Rayonnements et de Methodologie INSERM U.220, Strasbourg, France.)

Range for Heavy Charged Particles

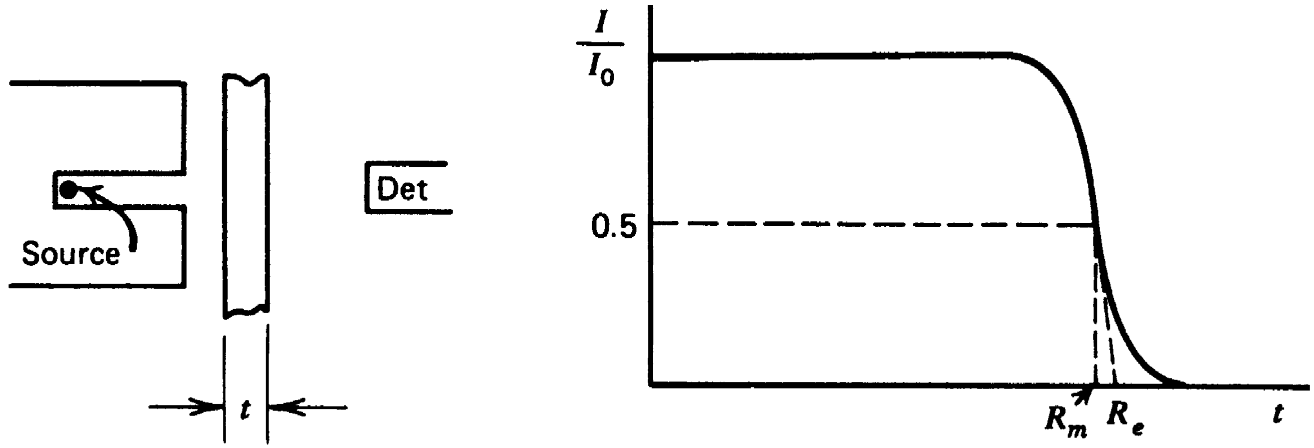


Figure 2.5 An alpha particle transmission experiment. I is the detected number of alpha particles through an absorber thickness t, whereas I_0 is the number detected without the absorber. The mean range R_m and extrapolated range R_e are indicated.

There are two related definitions of the range of heavy charged particles:

1. Mean range: the absorber thickness that reduces the alpha particle count to exactly one-half of its value in the absence of the absorber.
2. Extrapolated range: extrapolating the linear portion of the end of the transmission curve to zero.

Range for Alpha Particles

☞ The range of alpha particles in air (15°C, 1atm) can be approximately given by

$$R = 0.56E, \qquad E < 4;$$
$$R = 1.24E - 2.62, \qquad 4 < E < 8.$$

where E is given in MeV and R is given in cm.

☞ The range of alpha particles in any other medium with a similar atomic composition can be computed from the following relationship:

$$R_m,\ \mathrm{mg/cm^2} = 0.56A^{1/3}\,R,$$

where A = atomic mass number of the medium,
R = range of the alpha particle in air, cm.

☞ Because the effective atomic composition of tissue is not very much different from that of air, the following relationship may be used to calculate the range of alpha particles in tissue:

$$R_a \times \rho_a = R_t \times \rho_t,$$

Range for Heavy Charged Particles

- The mean range of any heavy charged particle can be related to the linear stopping power as the following:

$$R(T) = \int_0^T \left(-\frac{dE}{dx}\right)^{-1} dE \quad \text{where} \quad -\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2\beta^2}\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2\right]$$

where reciprocal of the linear stopping power gives the distance traveled per unit energy loss.

- Substitute the expression for the linear stopping power into the above relationship, we have

$$R(T) = \frac{1}{z^2}\int_0^T \frac{dE}{G(\beta)} \quad \textit{where} \quad G(\beta) = \frac{4\pi k_0^2 e^4 n}{mc^2\beta^2}\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2\right]$$

- Since the energy of the moving particle is $T=Mc^2/(1-\beta^2)^{1/2}$, the range can be further written as

$$R(\beta) = \frac{M}{z^2}\int_0^\beta \frac{g(\beta)}{G(\beta)}\, d\beta = \frac{M}{z^2} f(\beta) \quad \textit{where} \quad g(\beta) = \frac{d}{d\beta}\left(\frac{c^2}{\sqrt{1-\beta^2}}\right)$$

where M is the mass of the charged particle.

Range for Heavy Charged Particles

- It easily follows that the range of two different types of particles with the same speed satisfy the following relationship:

$$\frac{R_1(\beta)}{R_2(\beta)} = \frac{z_2^2 M_1}{z_1^2 M_2}$$

- Therefore, one can get the range of other heavy charged particles as

$$R(\beta) = \frac{M}{z^2} R_{proton}(\beta)$$

where M and z are the rest mass and the charge of the heavy charged particle.

Range for Heavy Charged Particles

TABLE 5.3. Mass Stopping Power $-dE/\rho dx$ and Range R_p for Protons in Water

Kinetic Energy (MeV)	β^2	$-dE/\rho dx$ (MeV cm^2 g^{-1})	R_p (g cm^{-2})
0.01	.000021	500.	3×10^{-5}
0.04	.000085	860.	6×10^{-5}
0.05	.000107	910.	7×10^{-5}
0.08	.000171	920.	9×10^{-5}
0.10	.000213	910.	1×10^{-4}
0.50	.001065	428.	8×10^{-4}
1.00	.002129	270.	0.002
2.00	.004252	162.	0.007
4.00	.008476	95.4	0.023
6.00	.01267	69.3	0.047
8.00	.01685	55.0	0.079
10.0	.02099	45.9	0.118
12.0	.02511	39.5	0.168
14.0	.02920	34.9	0.217
16.0	.03327	31.3	0.280
18.0	.03731	28.5	0.342
20.0	.04133	26.1	0.418
25.0	.05126	21.8	0.623
30.0	.06104	18.7	0.864
35.0	.07066	16.5	1.14
40.0	.08014	14.9	1.46
45.0	.08948	13.5	1.80
50.0	.09867	12.4	2.18
60.0	.1166	10.8	3.03
70.0	.1341	9.55	4.00
80.0	.1510	8.62	5.08
90.0	.1675	7.88	6.27
100.	.1834	7.28	7.57
150.	.2568	5.44	15.5
200.	.3207	4.49	25.5
300.	.4260	3.52	50.6
400.	.5086	3.02	80.9
500.	.5746	2.74	115.
600.	.6281	2.55	152.
700.	.6721	2.42	192.
800.	.7088	2.33	234.
900.	.7396	2.26	277.
1000.	.7658	2.21	321.
2000.	.8981	2.05	795.
4000.	.9639	2.09	1780.

Range for Charged Particles

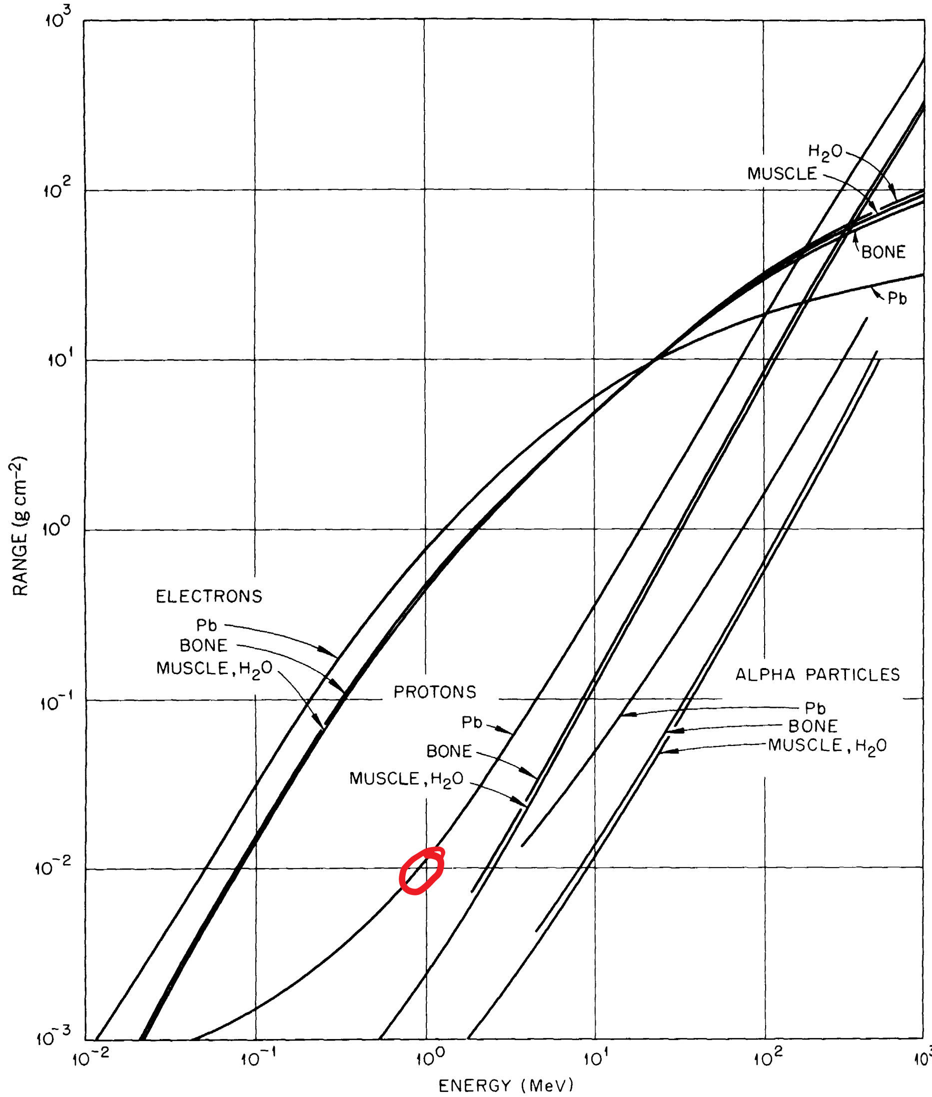
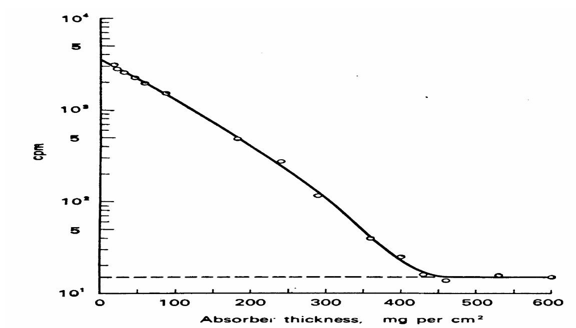
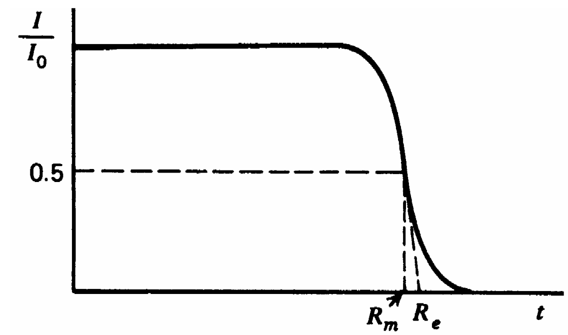


FIGURE 5.7. Ranges of protons, alpha particles, and electrons in water, muscle, bone, and lead, expressed in g cm^{-2}. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)

A Brief Summary

	Beta Particles	Heavy Charged Particles
Types of Interactions	Ionization and excitation Bremsstrahlung	Ionization and excitation
Linear Energy Exchange (by Ionization and Excitation)	$\frac{dE}{dx} = \frac{2\pi q^4 NZ \times (3 \times 10^9)^4}{E_m \beta^2 (1.6 \times 10^{-6})^2} \left\{ \ln \left[\frac{E_m E_k \beta^2}{I^2(1-\beta^2)} \right] - \beta^2 \right\}$	$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2 \beta^2} \left[\ln \frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2 \right]$
Range in Media		

Calculation of the Range

Beta Particles — Using the following relationship

$$R = 412\, E^{1.265 - 0.0954 \ln E}$$

for $0.01 \le E \le 2.5$ MeV,

$$\ln E = 6.63 - 3.2376(10.2146 - \ln R)^{\frac{1}{2}}$$

for $R \le 1200$,

$$R = 530\, E - 106$$

for $E > 2.5$ MeV, $R > 1200$,

where R = range, mg/cm^2
E = maximum beta-ray energy, MeV.

Heavy Charged Particles:

- Calculate the linear range of alpha in air

$$R = 0.56E, \quad E < 4;$$
$$R = 1.24E - 2.62, \quad 4 < E < 8.$$

- Convert to alpha range in terms of mass thickness

$$R_m, \text{mg/cm}^2 = 0.56A^{1/3}\, R$$

- Derive the alpha range in other media:

$$R_a \times \rho_a = R_t \times \rho_t,$$

Range and Energy Straggling of Charged Particles

- As charged particle penetrates matter, statistical fluctuation occur in the number of collisions along its path and in the amount of energy lose in each collision.
- As a result, a number of identical particles starting out under identical conditions will show (1) a distribution of energies as they pass a given depth – the energy straggling and (2) a distribution of path-lengths traversed before they stop – the range straggling.

Range Straggling of Charged Particles

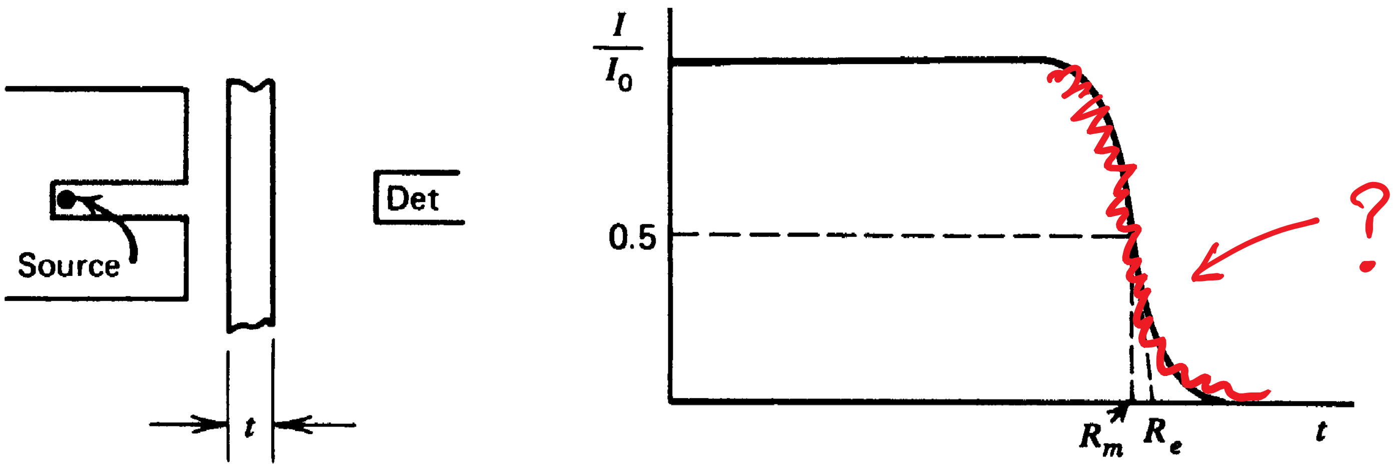


Figure 2.5 An alpha particle transmission experiment. I is the detected number of alpha particles through an absorber thickness t, whereas I_0 is the number detected without the absorber. The mean range R_m and extrapolated range R_e are indicated.

- For example, for 100MeV protons in tissue, the root-mean-square fluctuation in path-length is about 0.09cm,which is about 1.2% of the average path-length.

Energy Straggling of Charged Particles

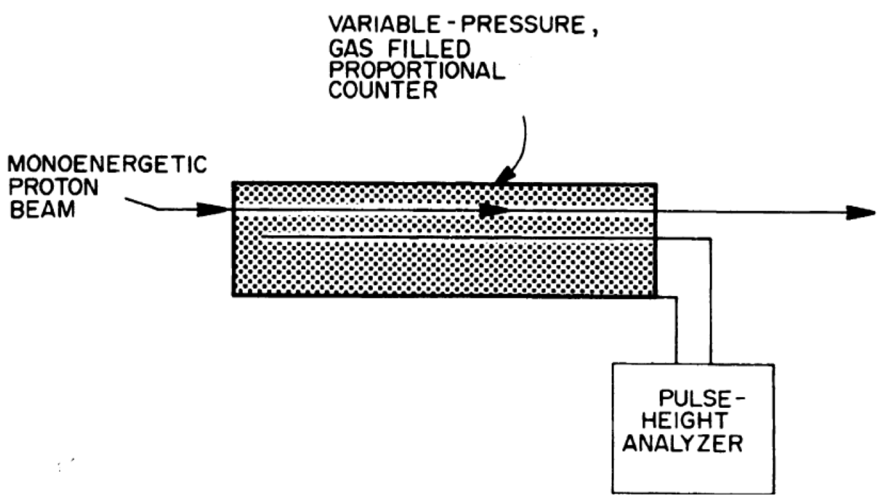


Fig. 7.2 Schematic arrangement for studying energy straggling experimentally.

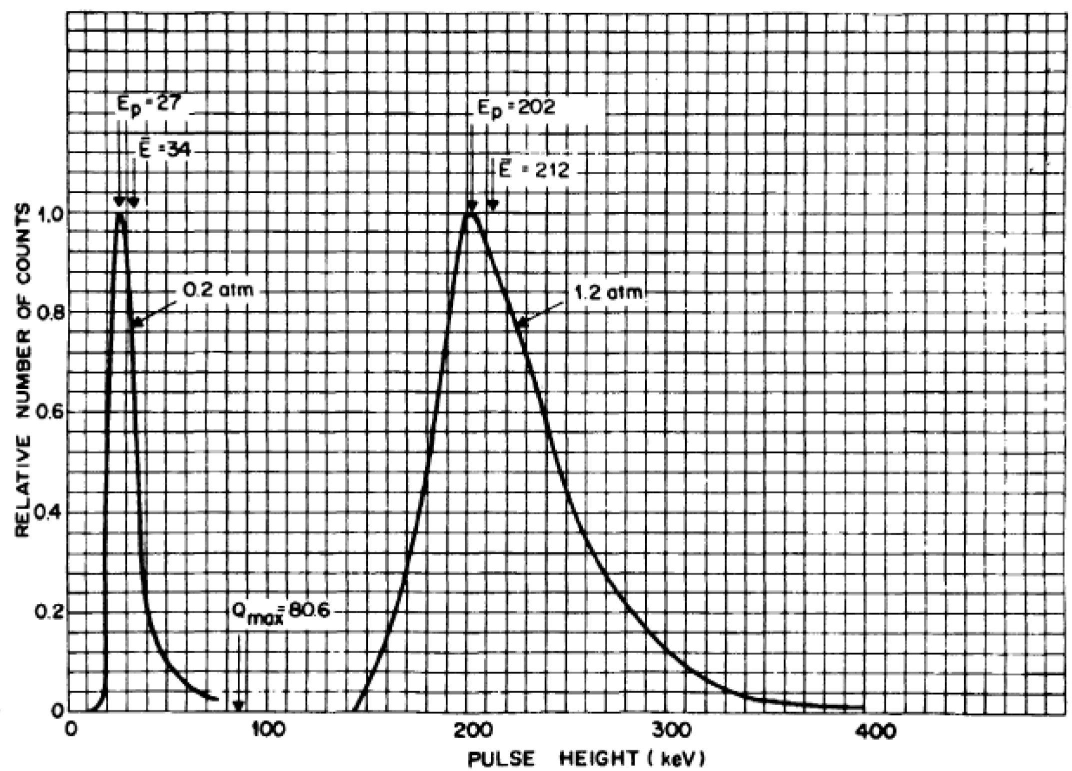
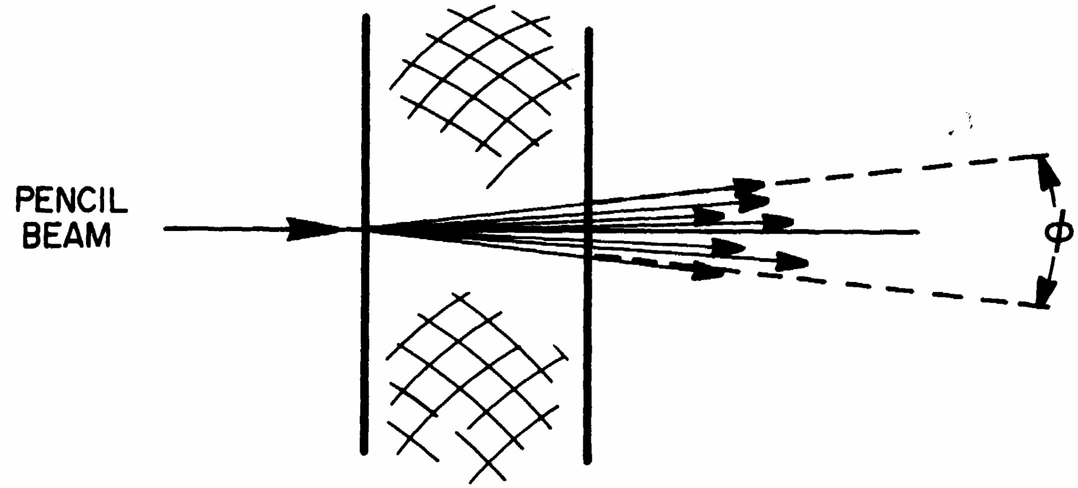


Fig. 7.3 Pulse-height spectra for 37-MeV protons traversing proportional counter with gas at 0.2-atm and 1.2-atm pressure. See text. [Based on T. J. Gooding and R. M. Eisberg, "Statistical Fluctuations in Energy Losses of 37-MeV Protons," Phys. Rev. **105**, 357–360 (1957).]

Multiple Coulomb Scattering

The path of a heavy charged particle in matter deviates from a straight line because it undergoes frequent small-angle scattering events.



URE 7.7. Multiple Coulomb scattering causes spread in a pencil beam of charged icles as they penetrate matter.

In radiotherapy with charged particle beams, multiple scattering often significantly diminishes the dose delivery to a specific target area.

Energy Loss Mechanisms

For heavy charged particles, the maximum energy that can be transferred in a single collision is given by the conservation of energy and momentum:

$$\tfrac{1}{2}MV^2 = \tfrac{1}{2}MV_1^2 + \tfrac{1}{2}mv_1^2$$

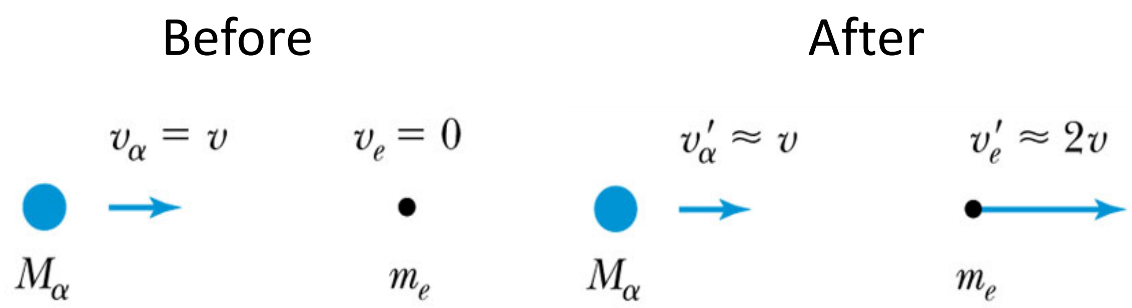
$$MV = MV_1 + mv_1.$$

where M and m are the mass of the heavy charged particle and the electron. V is the initial velocity of the charged particle. V_1 and v_1 are the velocities of both particles after the collision.

The maximum energy transfer is therefore given by

$$Q_{\max} = \frac{1}{2}MV^2 - \frac{1}{2}MV_1^2 = \frac{4mME}{(M+m)^2}$$

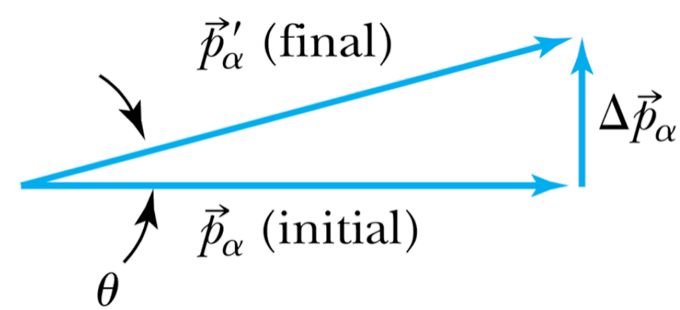
Geiger and Marsden Experiment (revisited)



It can be shown that the maximum momentum transfer to the a particle is:

$$\Delta p_{\max} = 2 m_e \upsilon_\alpha$$

Determine q_{max} by letting Δp_{max} be perpendicular to the direction of motion:



$$\theta_{\max} = \frac{\Delta p_\alpha}{p_\alpha} = \frac{2 m_e \mathrm{v}_\alpha}{M_\alpha \mathrm{v}_\alpha} = 2.7\times10^{-4}\ \mathrm{rad} = 0.016^\circ$$

Key Things to Remember

- Interaction mechanisms.
- Bethe formula for linear stopping power
- First collision energy transfer
- Restricted stopping power and linear energy transfer
- Stopping time and range of heavy charged particles

Stopping Time for Heavy Charged Particles

☞ The formula for the stopping power can be used to calculate the rate at which a heavy charged particle slows down,

$$-dE/dt = (-dE/dx)/(dt/dx) = V(-dE/dx)$$

where

$V = dx/dt$, is the velocity of the particle,

and

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2\beta^2}\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2\right]$$

☞ A rough estimate of the stopping time is given by

$$\tau \approx \frac{E_{initial}}{-dE/dt} = \frac{E_{initial}}{V\cdot(-dE/dx)}$$

☞ Note that since the stopping power is higher at lower particle energies, the actual slowing down time is shorter than the one estimated with the above equation.

Stopping Time for Heavy Charged Particles

Example of estimated stopping time for protons in water, which is normally in ns to ps range.

TABLE 5.4. Calculated Slowing-Down Rates, $-dE/dt$, and Estimated Stopping Times τ for Protons in Water

Proton Energy T (MeV)	Slowing-down Rate $-dE/dt$ (MeV s^{-1})	Estimated Stopping Time τ (s)
0.5	4.19×10^{11}	1.2×10^{-12}
1.0	3.74×10^{11}	2.7×10^{-12}
10.0	2.00×10^{11}	5.0×10^{-11}
100.0	9.35×10^{10}	1.1×10^{-9}
1000.0	5.81×10^{10}	1.7×10^{-8}

Limitation of the Bethe Formula

Since almost all analytical descriptions of the behavior of heavy charged particles are based on the Bathe formula, it is important to realize the limitation of this formula.

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2\beta^2}\left[\ln \frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2\right]$$

- Bethe formula is valid for high energies as long as the inequality $\gamma m/M \ll 1$ holds.
- At low energy, it fails because the term $\ln[2mc^2\beta^2/I(1-\beta^2)]-\beta^2$ eventually becomes negative giving a negative value for the stopping power.
- It does not account for the fact that at low energies, a charged particle may capture electrons as it moves, this will reduce its net charge and reduce the stopping power of the medium.
- The dependence of the Bethe formula on z^2 implies that a pair of particles, with the same amount of mass but opposite charge, have the same stopping power and range. Departures from this predication has been measured and theoretically predicted.