Chapter 3: Radioactivity

Radioactivity

- Radioactivity is defined as the spontaneous nuclear transformation that results in the formation of new elements.

- Radioactivity and radioactive properties of nuclide are determined by nuclear considerations and independent of chemical and physical states of the radioisotope.

- The probability of radioactive transformation depends primarily on two factors:

  - Nuclear stability as related to the neutron-to-proton ratio.

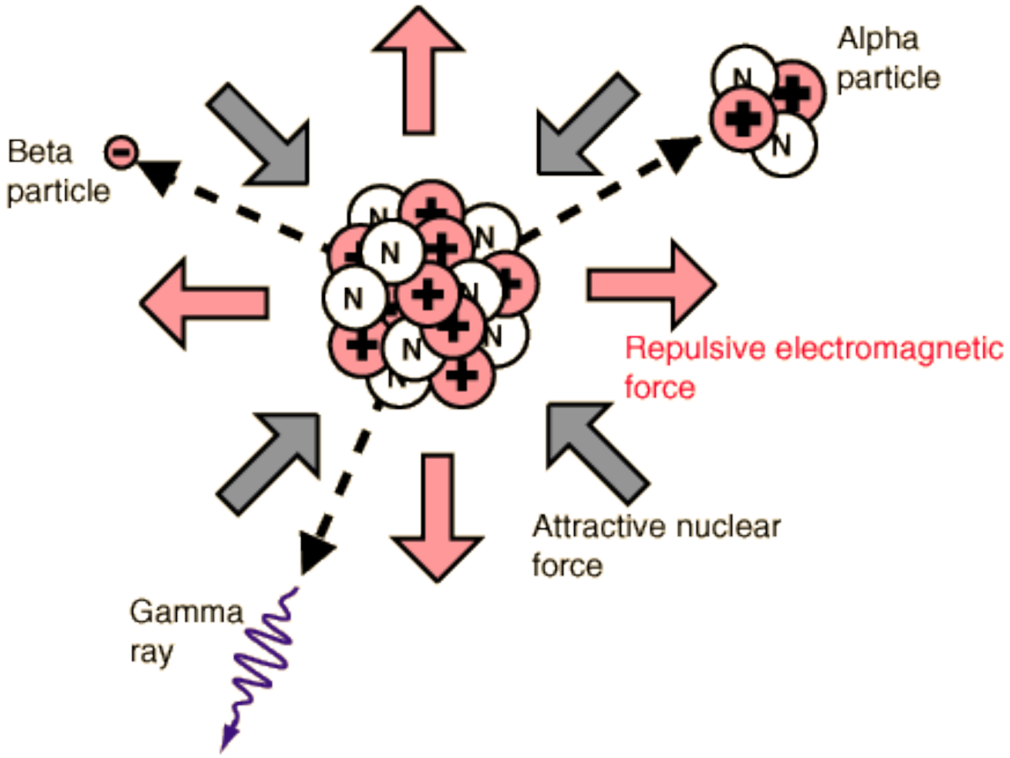
  - The mass-energy relationship among the parent nucleus, daughter nucleus and the emitted particles.

The Origin of Nuclear Radiation and a Few Related Concepts

- Nuclear force and Coulomb barrier.
- Nuclear binding energy and nuclear stability.
- Nuclear transformation as a way to achieve greater nuclear stability and associated energy release.

Nuclear Forces

Within the incredibly small nuclear size （$\sim 10^{-15}$m）, the two strongest forces in nature, Coulomb force and strong nuclear force, are pitted against each other. When the balance is broken, the resultant radioactivity yields particles of enormous energy.



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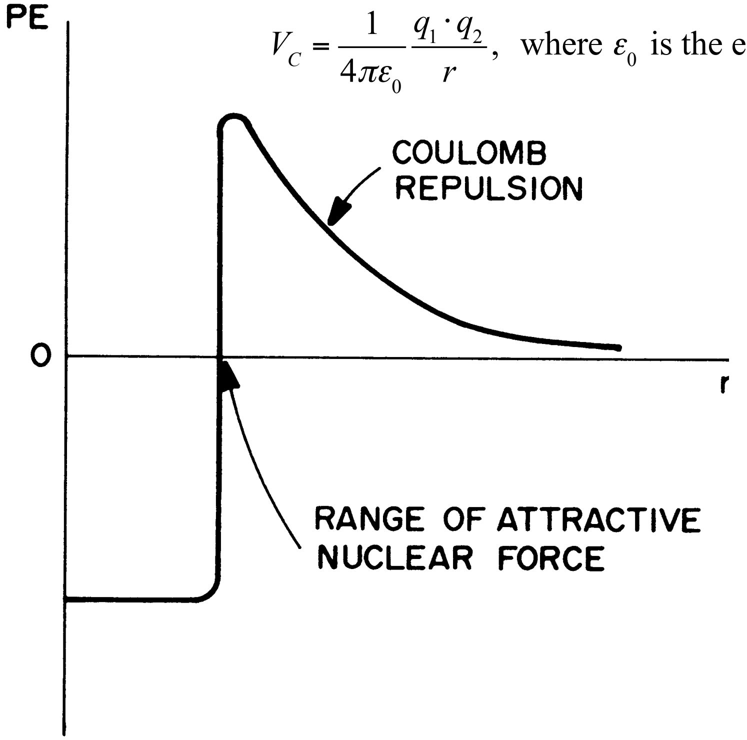
Potential Energy of Nucleus

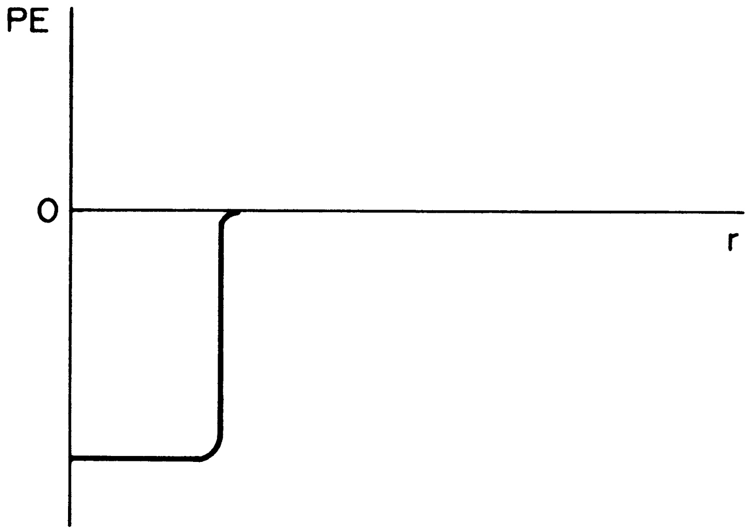
- Nucleons are bounded together in nucleus by the strong force, which has a short range of ~10^{-15}m.
- The strong force is powerful enough to overcome the Coulomb repulsion between the positively charged protons.

Coulomb potenital

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{q_1 \cdot q_2}{r}, \text{ where } \varepsilon_0 \text{ is the electrical permitivity}$$



(a) PROTON - NUCLEUS



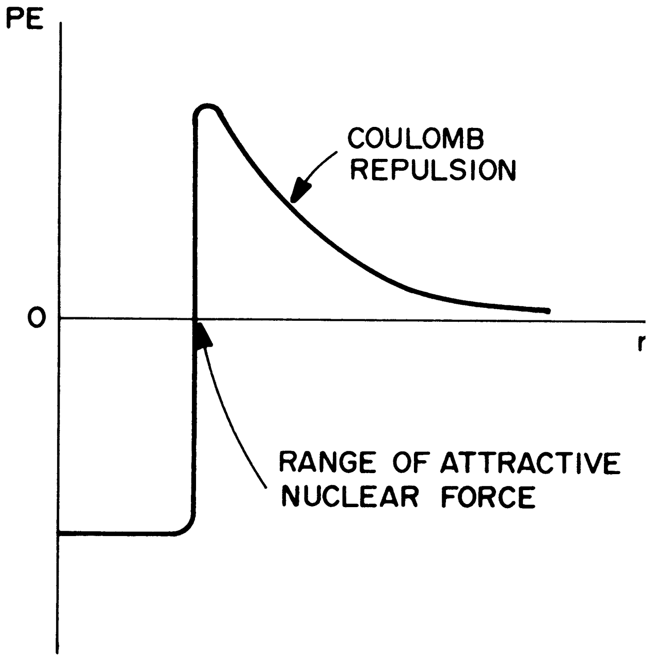
(b) NEUTRON - NUCLEUS

Coulomb Barrier

Example

Estimate the minimum energy that a proton would have to have in order to react with the nucleus of a stationary Cl atom.

In terms of Fig. 3.1(a), the proton would have to have enough energy to overcome the repulsive Coulomb barrier in a head-on collision. This would allow it to just reach the target nucleus.



(a) PROTON - NUCLEUS

Coulomb Barrier

We can use the following equation to estimate the radiuses of the Cl nucleus and the proton,

$$R \cong 1.3 A^{1/3} \times 10^{-15} m$$

With A=1 and A=35 for the proton and the Cl nucleus, we have

$$r_p = 1.3 \times 1^{1/3} \times 10^{-15} = 1.3 \times 10^{-15} \text{ m},$$

$$r_{Cl} = 1.3 \times 35^{1/3} \times 10^{-15} = 4.3 \times 10^{-15} \text{ m}.$$

The proton has unit positive charge, $e = 1.60 \times 10^{-19}$ C, and the chlorine ($Z = 17$) nucleus has a charge $17e$. The potential energy of the two charges separated by the distance $r_p + r_{Cl} = 5.6 \times 10^{-15}$ m is therefore

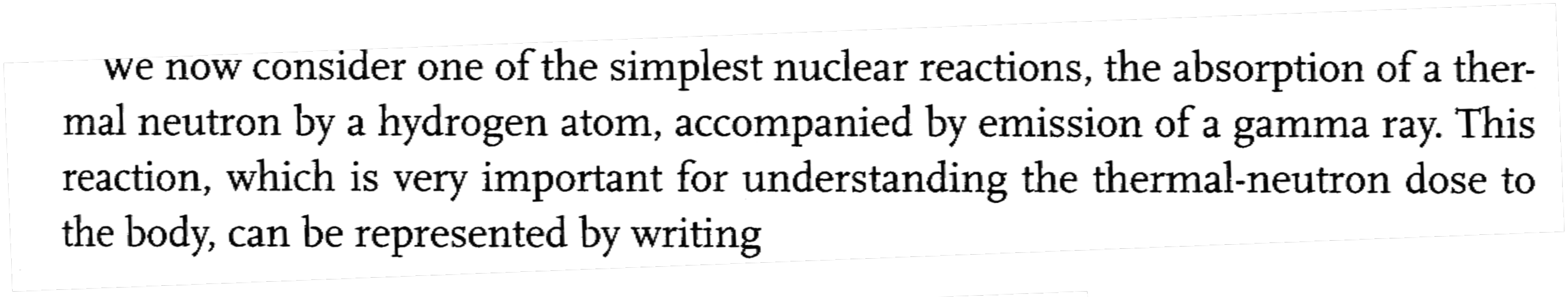
$$\text{PE} = \frac{8.99 \times 10^9 \times 17 \times (1.60 \times 10^{-19})^2}{5.6 \times 10^{-15}}$$

$$= 7.0 \times 10^{-13} \text{ J} = 4.4 \text{ MeV}.$$

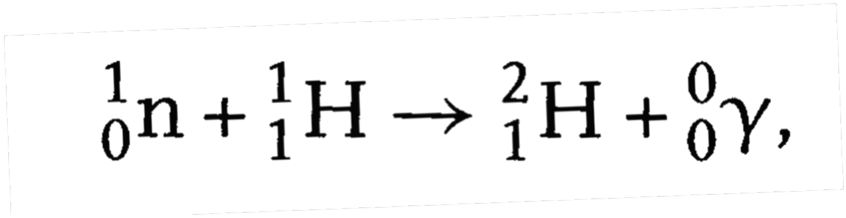
$$V_C = \frac{1}{4\pi\varepsilon_0} \frac{q_1 \cdot q_2}{r},$$

where ε_0 is the electrical permitivity

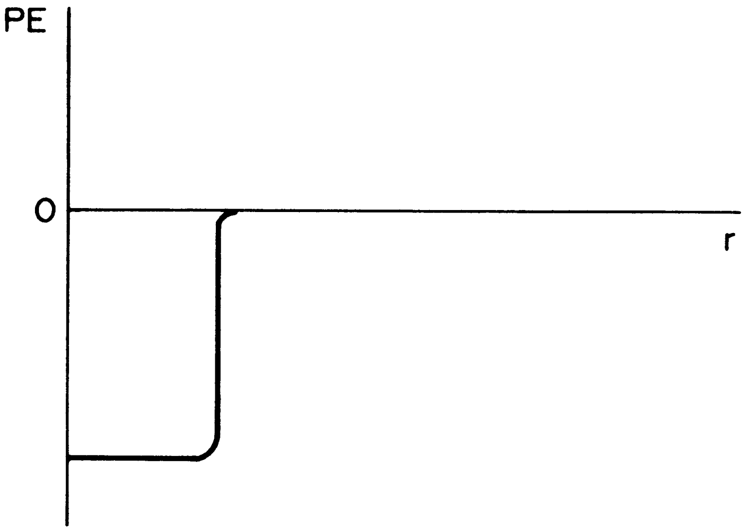
A Simple Nuclear Reaction



we now consider one of the simplest nuclear reactions, the absorption of a thermal neutron by a hydrogen atom, accompanied by emission of a gamma ray. This reaction, which is very important for understanding the thermal-neutron dose to the body, can be represented by writing



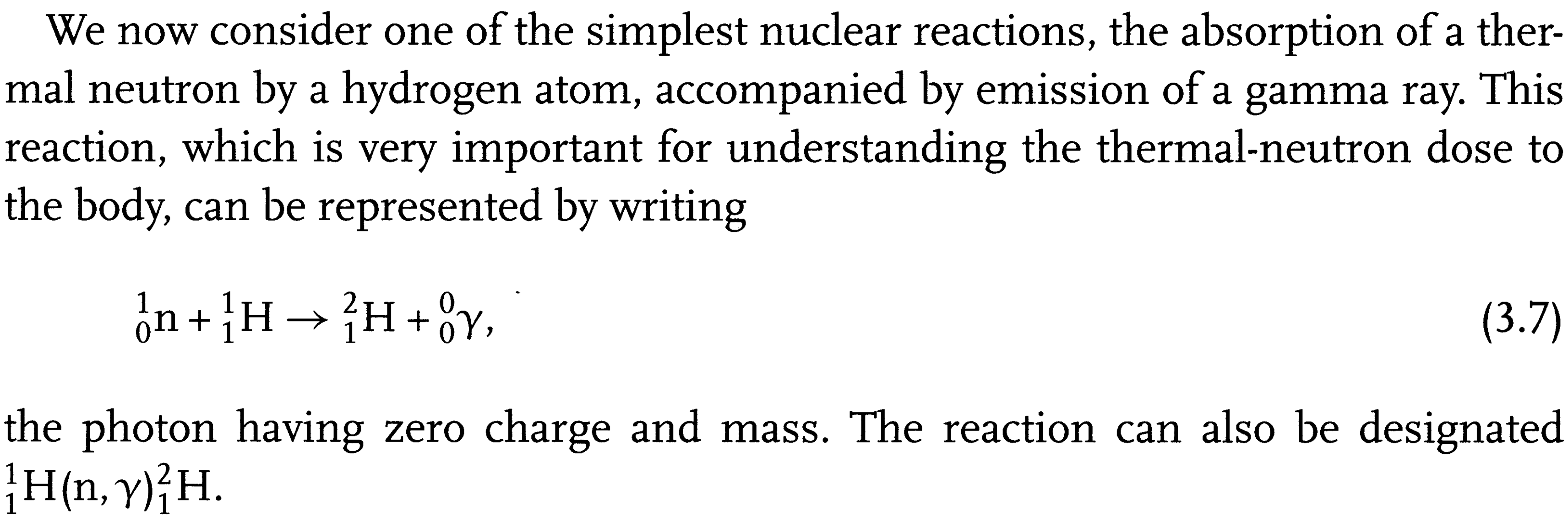
$$^{1}_{0}\mathrm{n} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + {}^{0}_{0}\gamma,$$



(b) NEUTRON - NUCLEUS

For example, thermal neutron capture by hydrogen nucleus.

Mass Defect and Nuclear Binding Energy

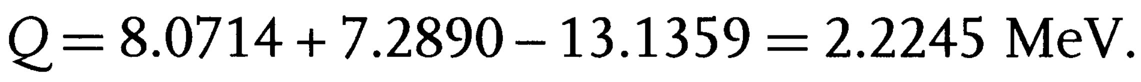


We now consider one of the simplest nuclear reactions, the absorption of a thermal neutron by a hydrogen atom, accompanied by emission of a gamma ray. This reaction, which is very important for understanding the thermal-neutron dose to the body, can be represented by writing

$$ {}_{0}^{1}\mathrm{n} + {}_{1}^{1}\mathrm{H} \rightarrow {}_{1}^{2}\mathrm{H} + {}_{0}^{0}\gamma, \tag{3.7} $$

the photon having zero charge and mass. The reaction can also be designated ${}_{1}^{1}\mathrm{H}(\mathrm{n},\gamma){}_{1}^{2}\mathrm{H}$.

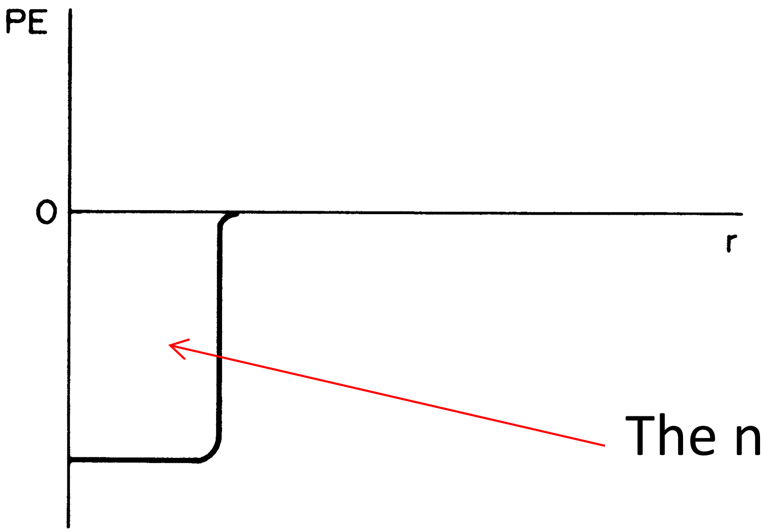
In this case, the energy transition due to the mass defect is



$$ Q = 8.0714 + 7.2890 - 13.1359 = 2.2245\ \mathrm{MeV}. $$

Nuclear Binding Energy

$${}_0^1\mathrm{n} + {}_1^1\mathrm{H} \rightarrow {}_1^2\mathrm{H} + {}_0^0\gamma,$$



The nuclear binding energy

(b) NEUTRON - NUCLEUS

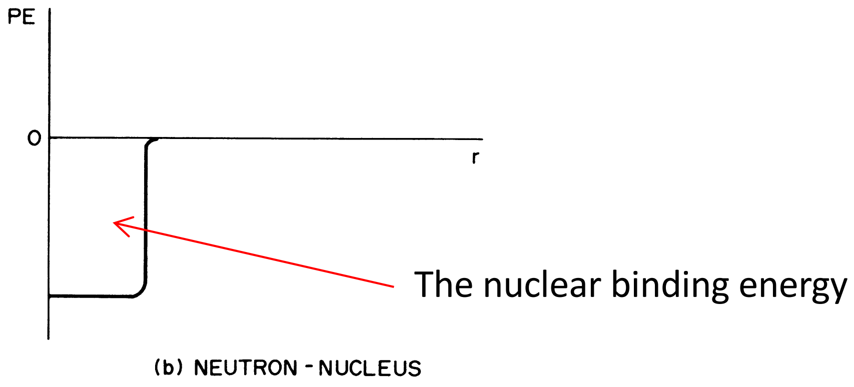
In this case, the binding energy for the deuterium nucleus is given by

$$Q = \overset{{}_0^1 n}{\underline{8.0714}} + \overset{{}_1^1 H}{\underline{7.2890}} - \overset{{}_1^2 H}{\underline{13.1359}} = 2.2245 \text{ MeV}.$$

Nuclear Binding Energy

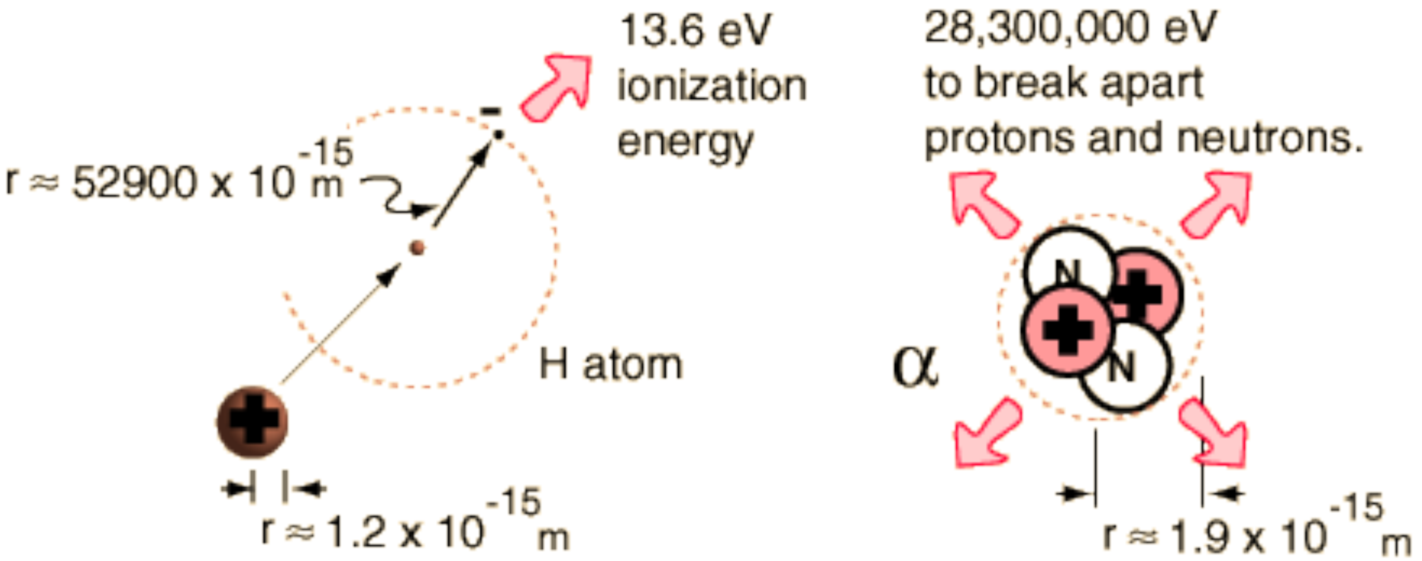
- Nuclei are made up of protons and neutron, but the mass of a nucleus is always less than the sum of the individual masses of the protons and neutrons which constitute it.
- This difference is a measure of the nuclear binding energy, which holds the nucleus together. The binding energy can be calculated from the Einstein relationship:

$$\text{Nuclear binding energy} = \Delta m \bullet c^2$$



Binding Energy of Atoms

The loss of mass that accompanies the binding of particles is not a specifically nuclear phenomenon. The mass of the hydrogen atom is smaller than the sum of the proton and electron masses by 1.46×10^{-8} AMU. This is equivalent to an energy $1.46 \times 10^{-8}\ \text{AMU} \times 931\ \text{MeV/AMU}^{-1} = 1.36 \times 10^{-5}\ \text{MeV} = 13.6\ \text{eV}$, the binding energy of the H atom.



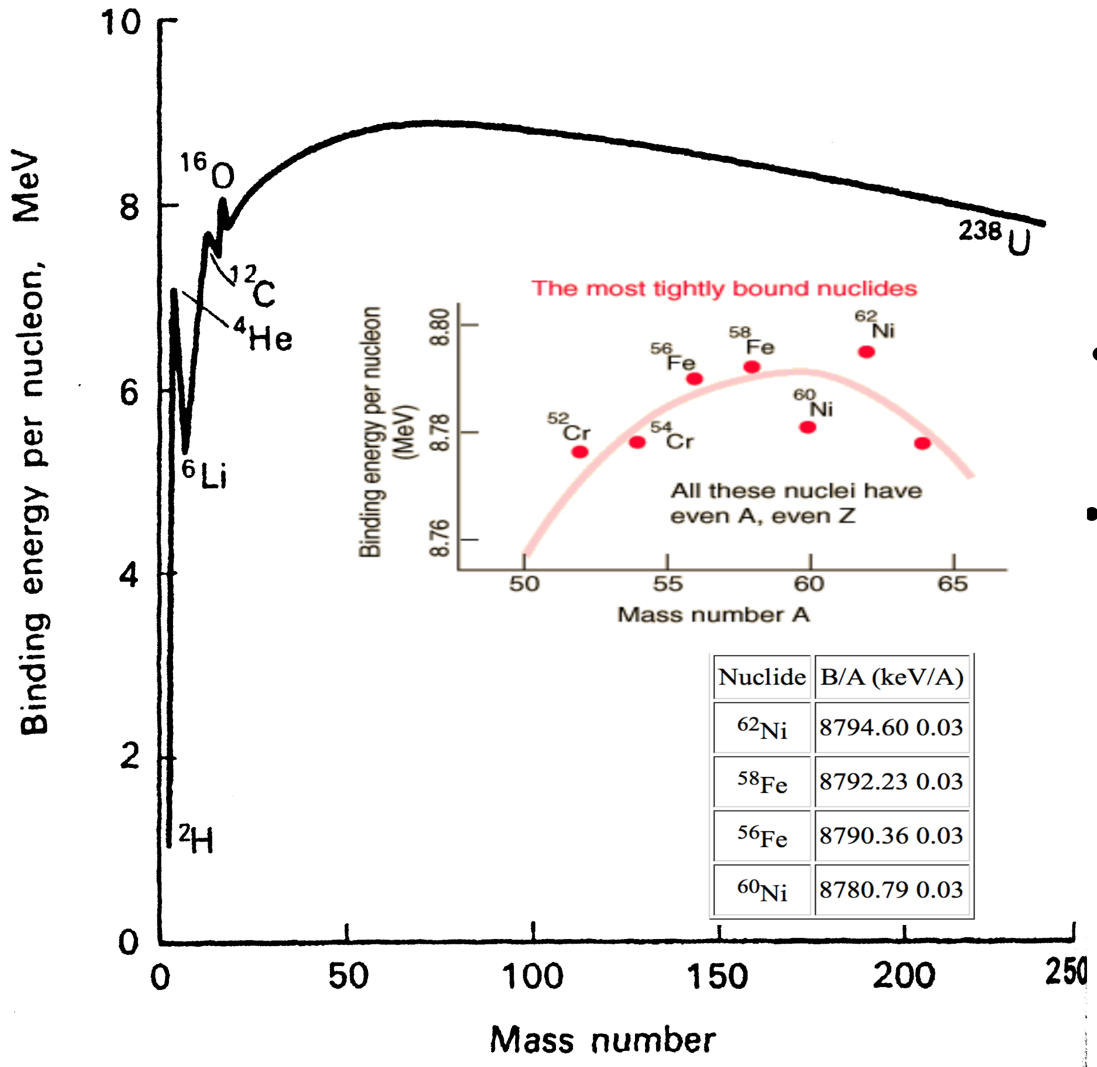
Comparison of atomic and nuclear scales and binding energy

http://230nsc1.phy-astr.gsu.edu/hbase/hframe.html

Nuclear Binding Energy

- Binding energy is always positive.
- The average binding energy per nucleon peaks for A = 40 to 120, with a maximum of ~8.5MeV.
- It then drops off for either higher or lower A.
- There are a few nuclei, ^{4}He, ^{12}C and ^{16}O at the lower mass number end that have binding energies (per nucleon) well above that for adjacent nuclei.
- In fact, these nuclei are all "multiples" of the alpha particle.
- And …

Average Binding Energy Per Nucleon



Nuclide	B/A (keV/A)
^{62}Ni	8794.60 0.03
^{58}Fe	8792.23 0.03
^{56}Fe	8790.36 0.03
^{60}Ni	8780.79 0.03

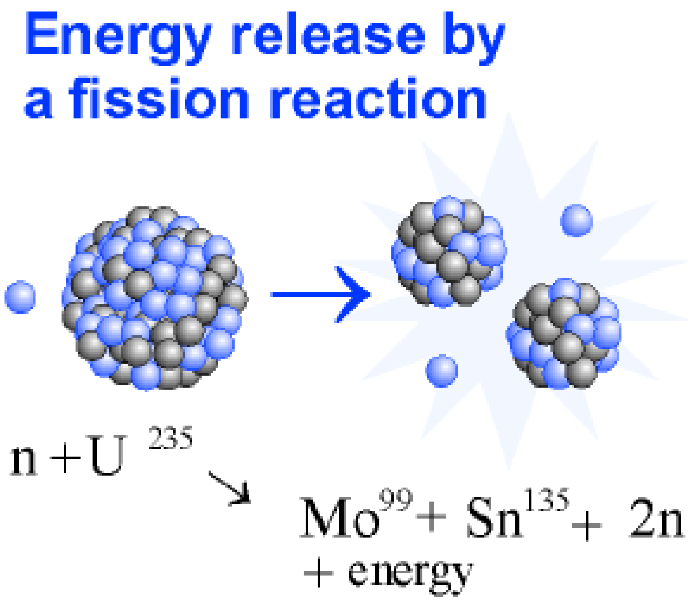
FIGURE 3.5. Variation of binding energy per nucleon with atomic mass num

What can we find from this graph?

- What are the more stable elements?
- What are the spikes on the left-hand side telling us?
- If we look at nuclear reactions from the viewpoint of energy production, can you give a few examples for producing energy through nuclear reactions?
- Which one(s) are more efficient in producing energy (say energy/kg fuel)?

Fission Reactions

- A fission reaction splits up a large nucleus into smaller pieces.
- A fission reaction typically happens when a neutron hits a nucleus with enough energy to make the nucleus unstable.



So Much Energy from Fission Reactions

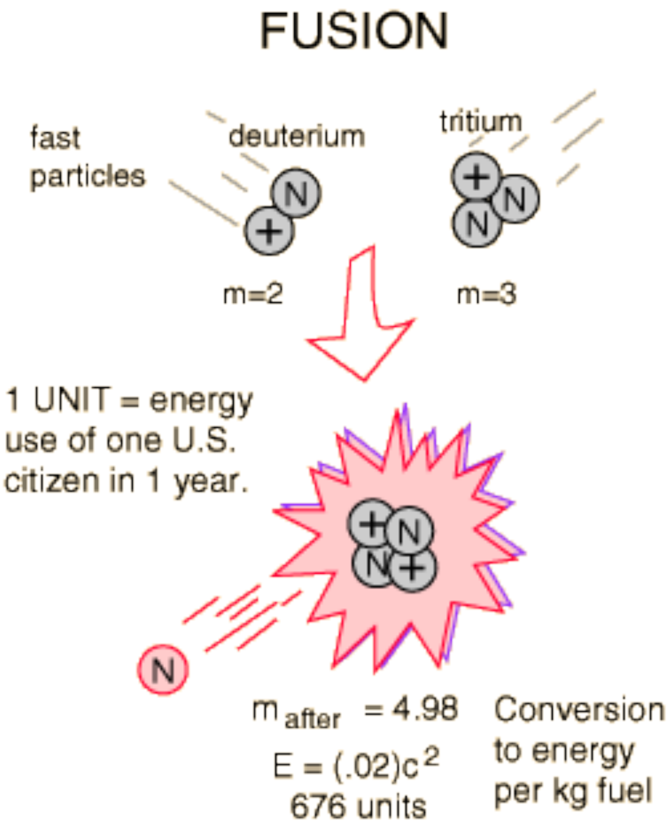
The fission of a single uranium-235 nucleus releases about 200 MeV of energy.

Verify the useful approximate fact that the fissioning of one gram of uranium releases about 1 megawatt(thermal)-day of energy:

Energy release = $(6\text{x}10^{23}/235)\text{x}(200\text{x}1.6\text{x}10^{-13}\text{J}) = 8\text{x}10^{10}$ J;
and
1 MW_{th}-day = $8.64\ \text{x}10^{10}$ J.

Compare with coal. 1 MW_{th}-day = 86.4 GJ is the heat of combustion of about 3 tons of coal. Mass ratio is 3 million.

Average Binding Energy Per Nucleon and Fusion Reaction



- The most promising of the hydrogen fusion reactions, which make up the deuterium cycle is the fusion of deuterium and tritium.
- The reaction yields 17.6 MeV of energy but requires a temperature of approximately 40 million Kelvins to overcome the coulomb barrier and ignite it.
- The deuterium fuel is abundant, but tritium must be either bred from lithium or gotten in the operation of the deuterium cycle.

*1 UNIT = energy use of one U.S. citizen in 1 year.

http://230nsc1.phy-astr.gsu.edu/hbase/hframe.html

Average Binding Energy Per Nucleon Comparing Fusion and Fission Reactions

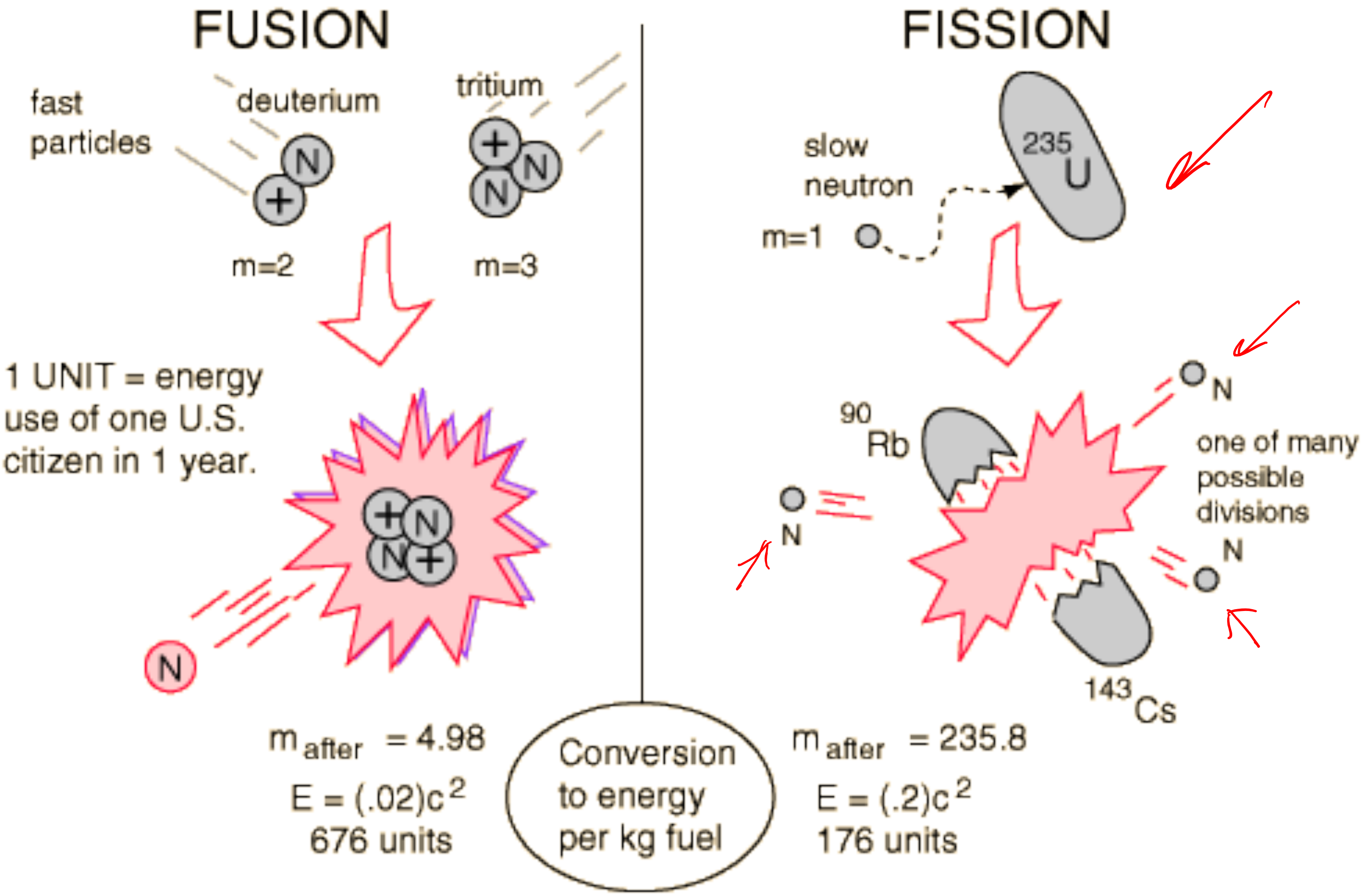


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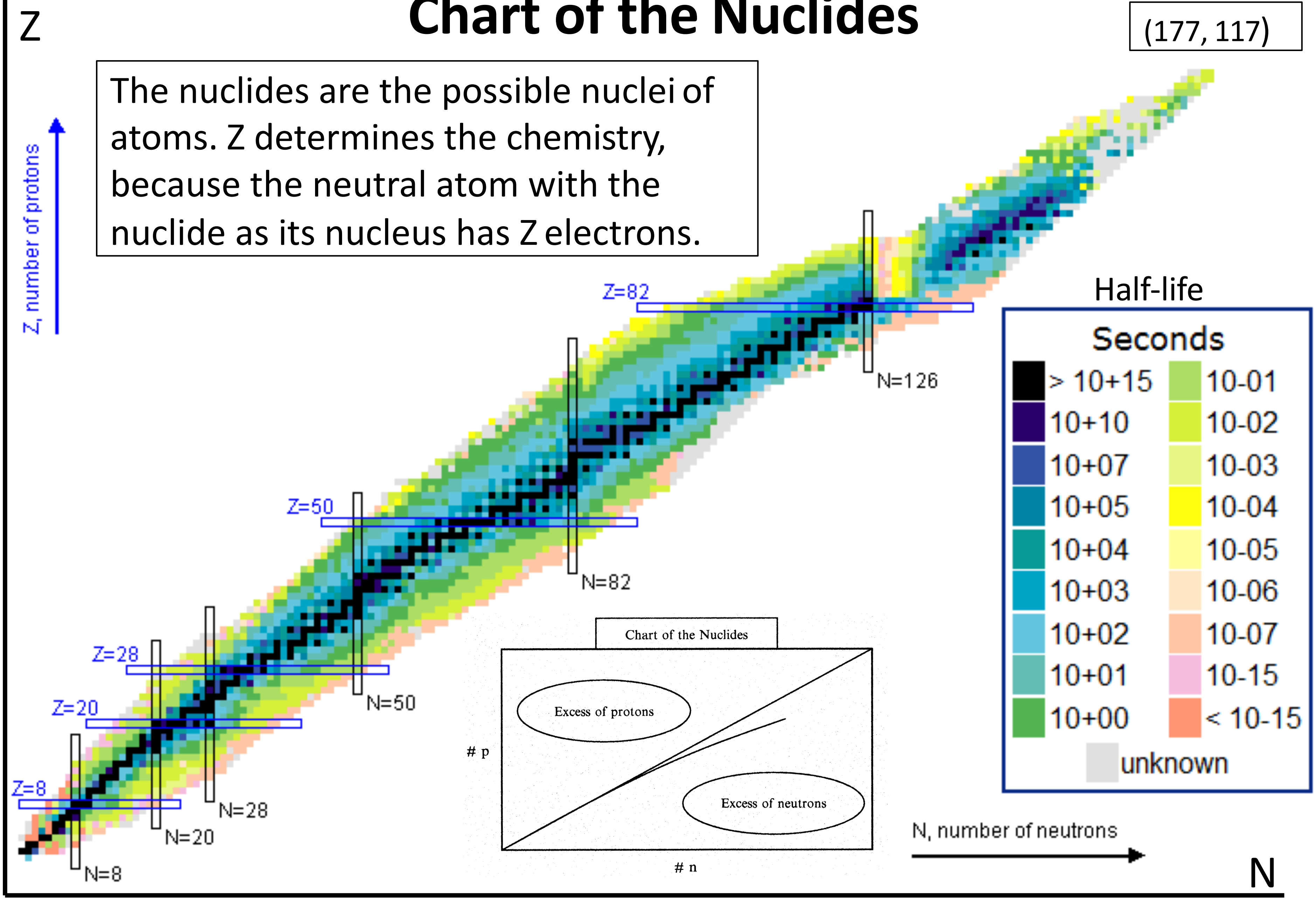
Chart of the Nuclides

(177, 117)

The nuclides are the possible nuclei of atoms. Z determines the chemistry, because the neutral atom with the nuclide as its nucleus has Z electrons.

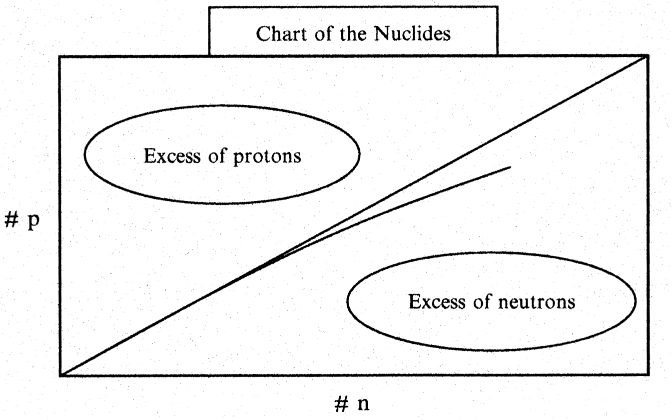


Source: http://www.nndc.bnl.gov/chart/reZoom.jsp?newZoom=5

The Vallley of Stability

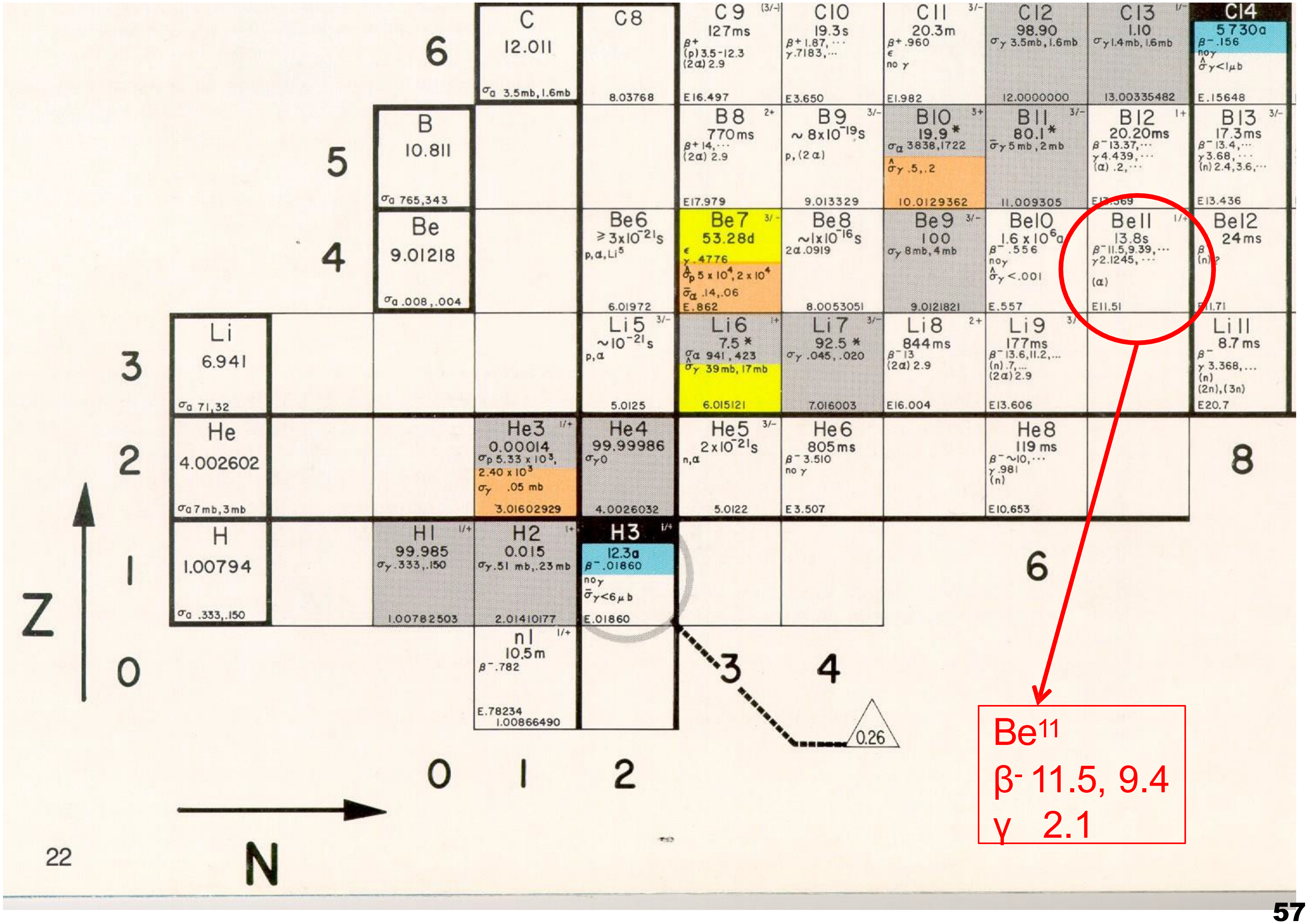
The stable isotopes lie along a curve on a (N, Z) plot that:

1. starts from the origin along the 45° line;
2. then falls below the 45° line, curving downward;
3. ends at N = 126, Z = 83 (bismuth-209).



- That there are stable nuclei at all is evidence of an attractive nuclear force between nucleons (P-P, P-N, and N-N).
- Starting from the origin along the 45° line is a consequence of the balance between P-P, P-N, and N-N interactions.
- Curving downward (toward more Ns than Ps) is a consequence of the electrostatic P-P repulsion, additive with the nuclear force.
- Ending (with Bi^{209}) is a sign that the nuclear force is short range, falling sharply in strength at distances of a few times the size of a nucleon (N or P).

Chart of the Nuclides



Be¹¹

β⁻ 11.5, 9.4

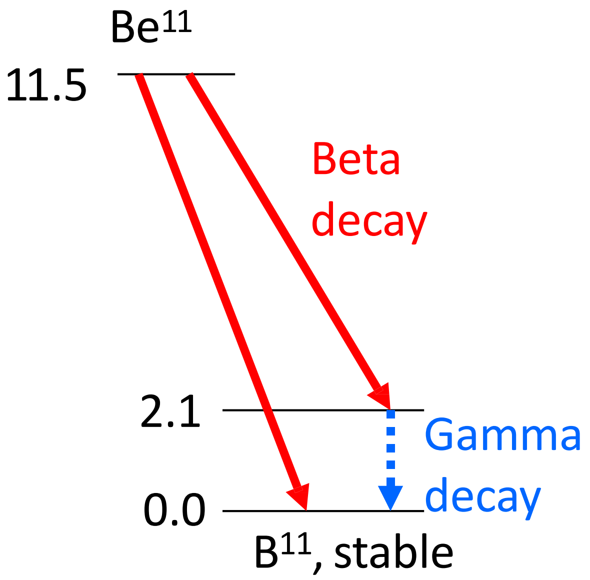
γ   2.1

Chart of the Nuclides

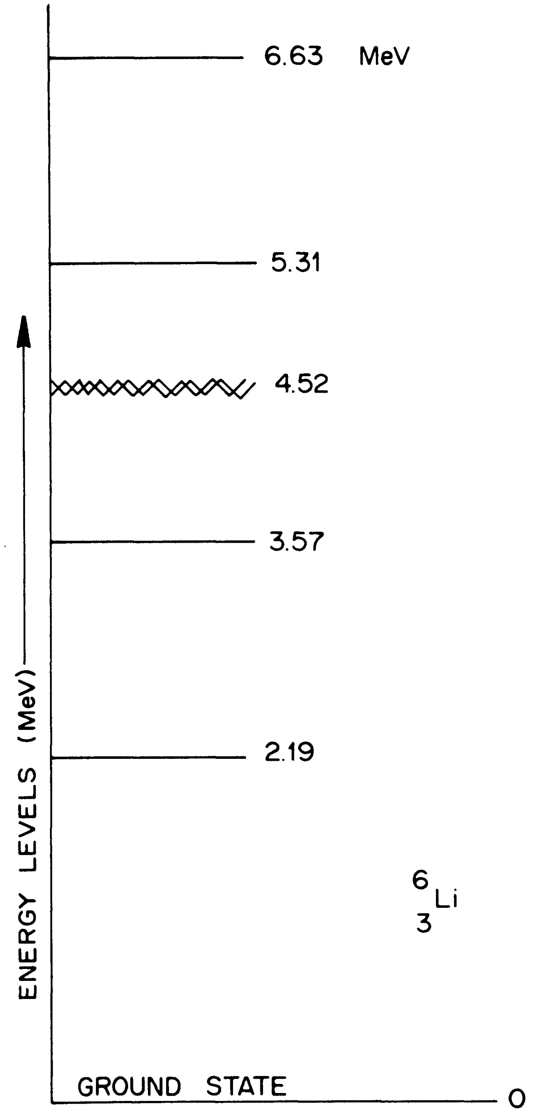
The boxes in the GE Chart of the Nuclides give information about energy release in the principal decay modes.

Example: Beryllium-11, half-life 13.8 sec.
Box shows beta energy releases of 11.5 and 9.4 Mev, gamma energy releases of 2.1 MeV.
Note that 11.5 – 9.4 = 2.1. What's up?

Answer: Beta-decay of Be^{11} produces both the ground state and an excited state of B^{11}; in the latter case, the excited state, through gamma decay, produces the ground state.



Nuclear Energy States and Subsequent Nuclear Transformation



- A nucleus is a quantum-mechanical bounded particles.
- A finite number of excited energy states exist in nuclei.
- ^{2}H and ^{4}He are examples of nuclei that have no bound excited states.

FIGURE 3.2. Energy levels of the $^{6}_{3}$Li nucleus, relative to the ground state of zero energy.

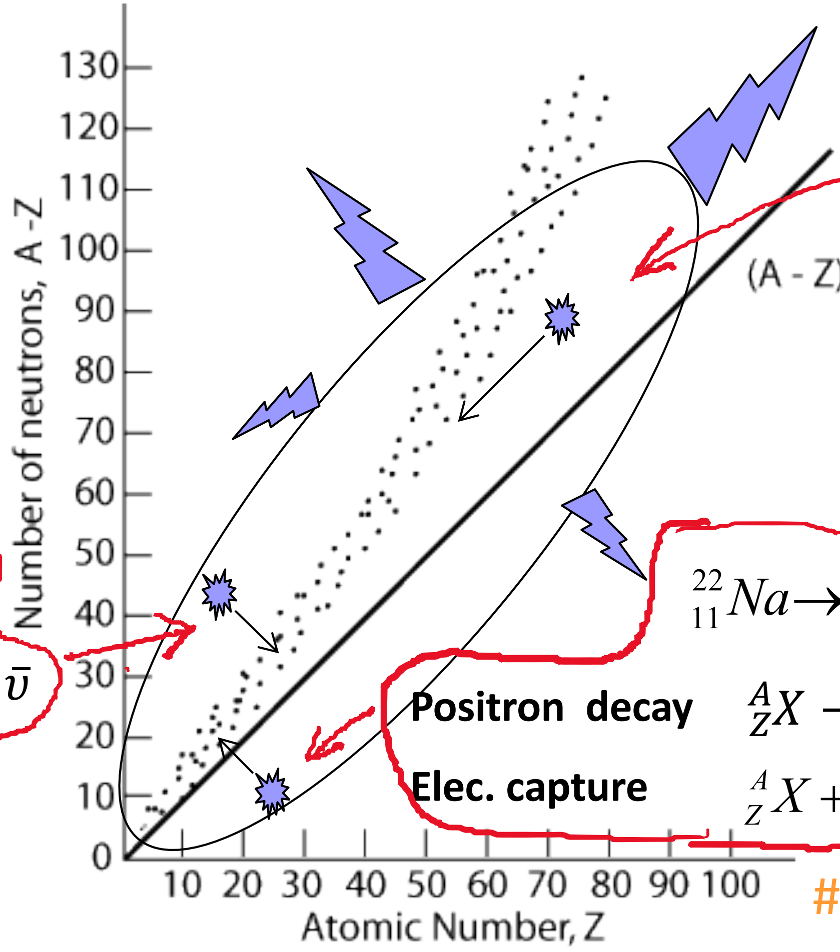
Nuclear Stability and the Origin of Radioactivity

N

\# of neutrons

Secondary radiations, e.g.

gamma rays, electrons



Alpha decay

Parent (Z, N)

→

Daughter (Z-2, N-2)

$${}^{210}_{84}Po \rightarrow {}^{206}_{82}Pb + {}^{4}_{2}He$$

Beta decay:

Parent (Z, N)

→

Daughter (Z+1, N-1)

$${}^{A}_{Z}X \rightarrow {}_{Z+1}^{\;\;\;A}X + e^{-} + \bar{\upsilon}$$

$${}^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + {}^{0}_{1}\beta + \upsilon$$

Positron decay ${}^{A}_{Z}X \rightarrow {}_{Z-1}^{\;\;\;A}X + e^{+} + \upsilon$

Elec. capture ${}^{A}_{Z}X + e^{-} \rightarrow {}^{\;\;\;A}_{Z-1}Y + \upsilon$

Parent (Z, N)

→

Daughter (Z-1, N+1)

\# of protons

Z