1. (a) (4 points) Ch3. Q12.
   Calculate the energy released by the alpha decay of ${}^{222}_{86}Rn$

   Solution: Alpha decay of ${}^{222}_{86}Rn$:

$$ {}^{222}_{86}Rn \longrightarrow {}^{218}_{84}Po + {}^{4}_{2}He $$

$$ \begin{aligned} Q &= \Delta_P - \Delta_D - \Delta_{He} \\ &= 16.274 - 8.36 - 2.42 \quad MeV \\ &= \boxed{5.594} \quad MeV \end{aligned} $$

   Alternative Method:
   Masses:

$$ \begin{aligned} {}^{222}_{86}Rn &: 222.0175777 \quad u \\ {}^{218}_{84}Po &: 218.0089730 \quad u \\ Alpha &: 4.001506179127 \quad u \\ Electron &: 0.5109989461 \quad MeV/c^2 \end{aligned} $$

   Energy Balance:

$$ \begin{aligned} m_{Rn-222}c^2 &= (m_{Po-218} + m_\alpha + 2m_{e^-})c^2 + Q \\ Q &= (m_{Rn-222} - m_{Po-218} - m_\alpha)c^2 - 2 \times 0.5109989461 \quad MeV \\ &= (0.007098521u)c^2 \times 931.4940954 \quad MeV/(u \cdot c^2) - 1.021997892 \quad MeV \\ &= 6.612230279 \quad MeV - 1.021997892 \quad MeV \\ &= 5.590232387 \quad MeV \approx \boxed{5.59 \quad MeV} \end{aligned} $$

   Note: The precision of the masses (in u) here is important as each u represents approximately 931.5 MeV.

   (b) (3 points) Calculate the energy of the alpha particle

   Solution:

$$ \begin{aligned} E_\alpha &= \frac{M}{m+M}Q \\ &= \frac{218}{4+218} \times 5.59 \quad MeV \\ &= \boxed{5.49} \quad MeV \end{aligned} $$

   Note: The precision of the masses here is not as important. (Do you know why? Try it yourself!)

   (c) (3 points) What is the energy of the recoil polonium atom?

   Solution:

$$ \begin{aligned} E_{Po} &= \frac{m}{m+M}Q \\ &= \frac{4}{4+218} \times 5.59 \quad MeV \\ &= \boxed{0.1} \quad MeV \end{aligned} $$

2. Ch3. Q14.
The $^{226}_{88}Ra$ nucleus emits a 4.60 MeV alpha particle 5.5% of the time when it decays to $^{222}_{86}Ra$
   (a) (5 points) Calculate the Q value of this decay.

Solution:
Alpha decay of $^{226}_{88}Ra$:

$$^{226}_{88}Rn \longrightarrow ^{222}_{86} Po + ^4_2 He$$

$$\begin{aligned} E_\alpha &= \frac{M}{m+M}Q \\ &= \frac{222}{4+222} \times Q \\ &= 4.60 \quad MeV \end{aligned}$$

$$\Longrightarrow \qquad Q = \boxed{4.68} \quad MeV$$

Note: You can't use the mass method as in 1(a) as you are given a specific energy value for the alpha particle.

   (b) (5 points) What is the recoil energy of the ^{222}Rn atom?

Solution:

$$\begin{aligned} E_{Rn} &= \frac{m}{m+M}Q \\ &= \frac{4}{4+222} \times 4.68 \quad MeV \\ &= \boxed{0.08} \quad MeV \end{aligned}$$

Alternative Method:

$$\begin{aligned} E_{Rn} &= Q - E_\alpha \\ &= 4.68 - 4.60 \quad MeV = \boxed{0.08} \quad MeV \end{aligned}$$

3. (a) (5 points) Ch3. Q17.
Calculate the energy released in the beta decay of $^{32}_{15}P$.

Solution: Beta decay of $^{32}_{15}P$:

$$^{32}_{15}P \longrightarrow ^{32}_{16} S + ^0_{-1}\beta + ^0_0 \overline{\nu}$$

$$\begin{aligned} Q &= \Delta_P - \Delta_D \\ &= -24.303 - (-26.013) \quad MeV \\ &= \boxed{1.71} \quad MeV \end{aligned}$$

   (b) (5 points) If a beta particle has 650 keV, how much energy does the antineutrino have?

Solution:

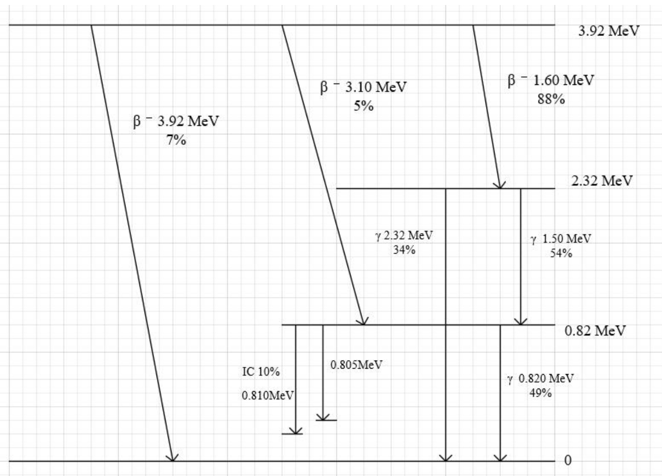
$$E_{\beta^-} + E_{\overline{\nu}=Q}$$

$$\begin{aligned} E_{\overline{\nu}} &= Q - E_{\beta^-} \\ &= 1.71 - 0.65 \quad MeV \\ &= \boxed{1.06} \quad MeV \end{aligned}$$

4. (20 points) Ch3. Q27.
   A parent nuclide decays by beta-particle emission into a stable daughter. The major radiations, energies (MeV), and frequencies are, in the notation of Appendix D:
   β^-: 3.92 max (7%), 3.10 max (5%), 1.60 max (88%)
   γ: 2.32 (34%), 1.50 (54%), 0.820 (88%)
   e^-: 0.818 0.805

   (a) (8 points) Draw the decay scheme

   Solution:



   (b) (3 points) What is the maximum energy that the antineutrino can receive in this decay?

   Solution:

$$\begin{aligned} E_{max,\overline{\nu}} &= E_{max,\beta^-} \\ &= \boxed{3.92} \quad MeV \end{aligned}$$

   (c) (3 points) What is the value of the internal-conversion coefficient?

   Solution:

$$\alpha = \frac{N_e}{N_\gamma} = \frac{10\%}{49\%} = \boxed{0.204} \qquad [Equation 3.31]$$

   Note: The reciprocal solution is also accepted.

   (d) (3 points) Estimate the L-shell electron binding energy of the daughter nuclide.

   Solution:

$$E_e = E^* - E_B$$

   where:

$$E^* = 0.82 \quad MeV$$

$$E_e = 0.818, 0.805 \quad MeV$$

   So:

$$E_B = 0.82 - 0.818 \quad MeV = 0.002 \quad MeV$$

$$E_B = 0.82 - 0.805 \quad MeV = 0.015 \quad MeV$$

   The larger E_B is for K-shell, so the binding energy for L-shell electron is $\boxed{0.002}$ MeV

(e) (3 points) Would daughter X rays be expected also? Why or why not?

Solution: Yes. The daughter atom would be missing a K-shell or L-shell electron after internal conversion. When an outer shell electron moves in to fill this hole, the energy difference could be released as X rays (or Auger electrons).

5. (10 points) Ch3. Q28.
Calculate the recoil energy of the technetium atom as a result of photon emission in the isomeric transition ${}^{99m}_{43}Tc \longrightarrow {}^{99}_{43}Tc + {}^{0}_{0}\gamma$.

Solution: From Appendix D, a 0.140 MeV gamma photon is emitted when Tc-99m undergoes gamma decay.
Momentum of gamma photon:

$$\begin{aligned} E_\gamma &= p_\gamma c \\ p_\gamma &= \frac{E_\gamma}{c} \\ &= \frac{0.140MeV}{299792458ms^{-1}} \times 1.60218 \times 10^{-13} J \cdot (MeV)^{-1} \\ &= 7.482016 \times 10^{-23} kg \cdot ms^{-1} \end{aligned}$$

Recoil energy of Tc-99:

$$\begin{aligned} E_{Tc-99} &= \frac{p^2}{2m_{Tc-99}} \\ &= (7.482016 \times 10^{-23} kg \cdot ms^{-1})^2 / (2\times \sim 99u \times 1.660539 \times 10^{-27} kg) \\ &= 1.70264 \times 10^{-20} \quad J \approx 1.70 \times 10^{-20} \quad J \quad (= 0.1063 \quad eV) \end{aligned}$$

6. (20 points) Ch3. Q33.
The nuclide ${}^{65}_{30}Zn$ decays by electron capture (98.5%) and by positron emission (1.5%).

(a) (6 points) Calculate the Q value for both modes of decay.

Solution: Electron capture:

$${}^{65}_{30}Zn + {}^{0}_{-1}e^- \longrightarrow {}^{65m}_{29}Cu + {}^{0}_{0}\gamma$$

$$\begin{aligned} Q &= \Delta_P - \Delta_D - E_B \\ &= (-65.911) - (-67.263) - 0.009 \quad MeV \\ &= \boxed{1.343} \quad MeV \end{aligned}$$

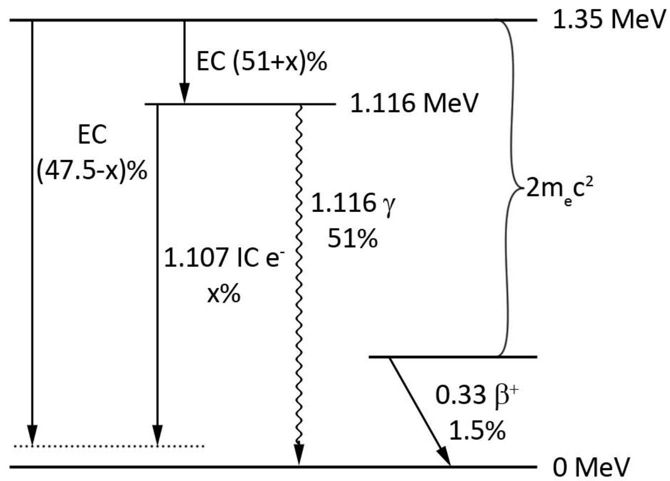
Positron emission:

$${}^{65}_{30}Zn \longrightarrow {}^{65}_{29}Cu + {}^{0}_{1}\beta^+ + {}^{0}_{0}\gamma$$

$$\begin{aligned} Q &= \Delta_P - \Delta_D - 2mc^2 \\ &= (-65.911) - (-67.263) - 1.022 \\ &= \boxed{0.33} \quad MeV \end{aligned}$$

(b) (8 points) Draw the decay scheme for ^{65}Zn.

Solution:



(c) (3 points) What are the physical processes responsible for each of the major radiations listed in Appendix D?

Solution: Positrons are produced by the positron decay of the parent atoms. When these positrons annihilate with electrons in the surrounding material, two annihilation photons of ∼511 keV are emitted per annihilation event.
At least 51% of the decay is via Electron Capture that populates the 1.116 MeV energy level. From this energy level, the excited daughter atoms either decays to the ground state via the 1.116 MeV gamma emissions (51%) or via Internal Conversion that ejects (most probably) the K-shell electron with an energy of 1.107 MeV.
The remaining Electron Capture events towards the nuclear ground-state of the daughter atoms as well as the above Internal Conversion events contribute to the population of excited daughter atoms (missing K-shell or L-shell electrons). These excited atoms de-excite when the outer shell electrons move in to fill the hole, producing daughter X-rays, or when the atoms eject auger electrons.

(d) (3 points) Estimate the binding energy of a K-shell electron in copper.

Solution:

$$\begin{aligned} E_B &= E^* - E_e \\ &= 1.116 - 1.107 \quad MeV \\ &= \boxed{0.009} \quad MeV \end{aligned}$$

7. (10 points) Ch3. Q35.
Show that $^{55}_{26}Fe$, which decays by electron capture, cannot decay by positron decay.

Solution: Q value of decay of $^{55}_{26}Fe$: $\quad ^{55}_{26}Fe + ^{0}_{-1}e^- \longrightarrow ^{55m}_{25}Mn + ^{0}_{0}\gamma$

$$\begin{aligned} Q &= \Delta_P - \Delta_D - E_B \\ &= -57.474 - (-57.705) - E_B \quad MeV \\ &= 0.231 \quad MeV - E_B \quad < (2 \times m_{e-}) \end{aligned}$$

Since the Q value of the decay of ${}^{55}_{26}Fe$ is less than two times of the mass of electrons required for positron decay, the decay cannot proceed via positron decay.

Note: If you compare the decay equations between Electron Capture and Positron Decay, you will find that there will be two extra leptons (an electron and an position) in the positron decay mode. The creation of these two leptons require energy and this energy is derived from the Q value of the reaction. If the Q value is less than two lepton masses, the decay cannot proceed via positron decay.

8. (30 points) Ch3. Q36. The isotope ${}^{126}_{53}I$ can decay by EC, β^-, and β^+ transitions.

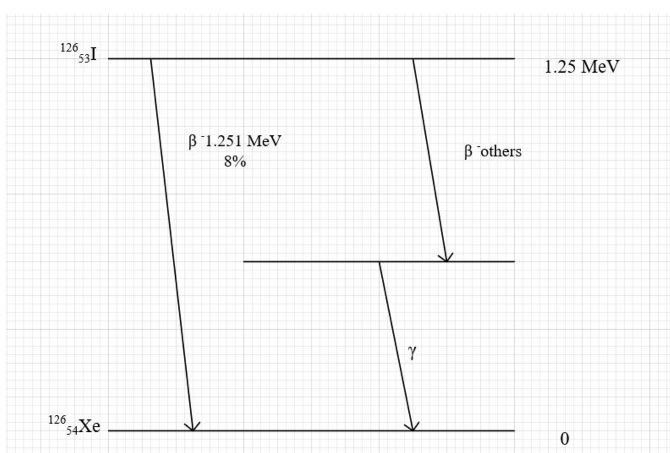
   (a) (12 points) Calculate the Q values for the three modes of decay to the ground states of the daughter nuclei.

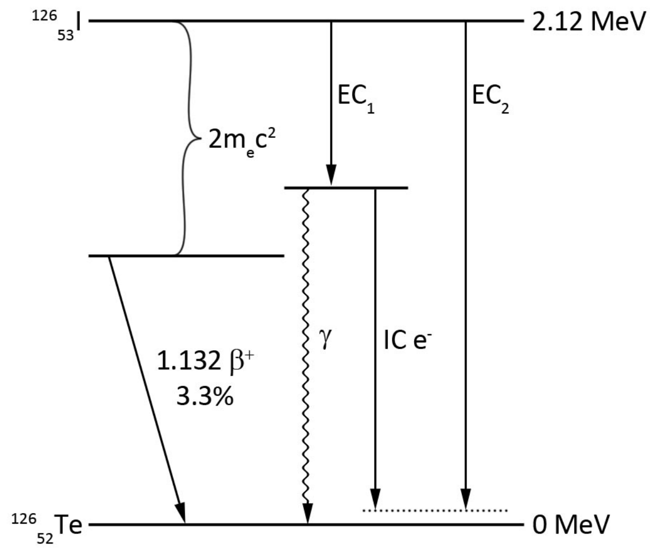
   Solution: EC:

$$
\begin{aligned}
&{}^{126}_{53}I + {}^{0}_{-1}e^- \longrightarrow {}^{126m}_{52}Te + {}^{0}_{0}\gamma \\
Q_{EC} &= \Delta_P + \Delta_D - E_B \\
&= -87.9 - (-90.05) - 0.033 \quad MeV \\
&= \boxed{2.12} \quad MeV \\
\beta^+ :&\quad {}^{126}_{53}I \longrightarrow {}^{126}_{52}Te + {}^{0}_{1}\beta^+ + {}^{0}_{0}\nu \\
Q_{\beta^+} &= \Delta_P + \Delta_D - 2mc^2 \\
&= -87.9 - (-90.05) - 1.022 \quad MeV \\
&= \boxed{1.13} \quad MeV \\
\beta^- :&\quad {}^{126}_{53}I \longrightarrow {}^{126}_{54}Xe + {}^{0}_{-1}\beta^- + {}^{0}_{0}\overline{\nu} \\
Q_{\beta^-} &= \Delta_P + \Delta_D \\
&= -87.9 - (-89.15) \quad MeV \\
&= \boxed{1.25} \quad MeV
\end{aligned}
$$

   (b) (13 points) Draw the decay scheme

   Solution:





(c) (5 points) What kinds of radiation can one expect from a ^{126}I source?

Solution: X-ray, Gamma-rays, β^-, β^+ as well as 511 keV annihilation photons can be expected.