



Key Elements of X-ray CT Physics

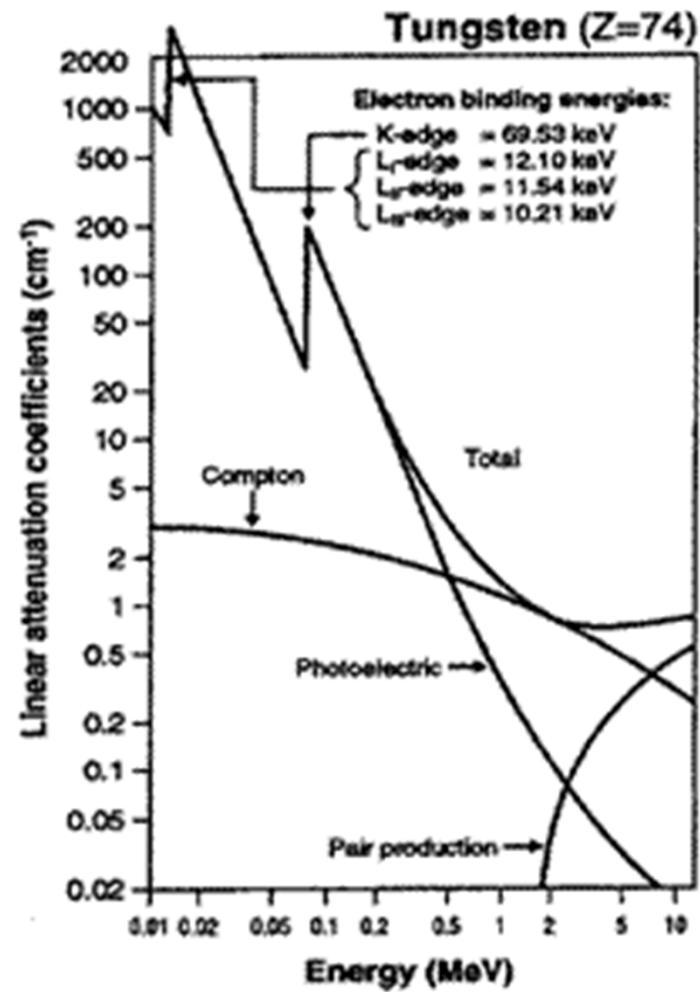
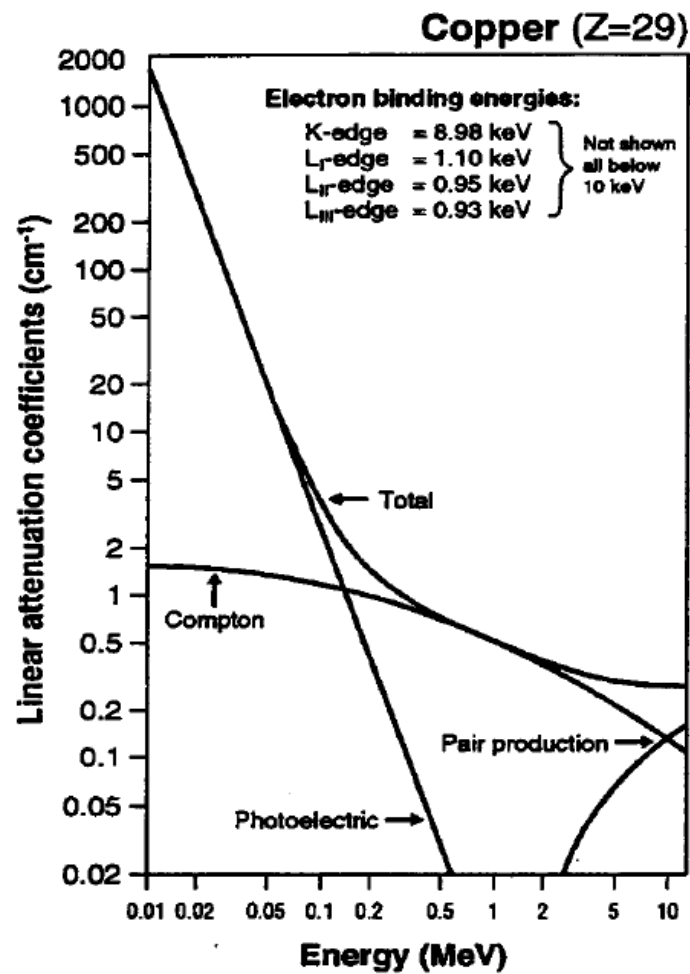
Part 2: X-ray Interactions



Photoelectric Effect

- Photoe⁻ absorption is the preferred interaction for X-ray imaging.
- Rem.: $E_b \propto Z^2$; characteristic x-rays and/or Auger e⁻ → **preferred in high Z material.**
- Probability of photoe⁻ absorption $\propto Z^3/E^3$ (Z = atomic no.) → **provide contrast according to different Z.**
- Due to the absorption of the incident x-ray without scatter, maximum subject contrast arises with a photoe⁻ effect interaction → **No scattering contamination** → better contrast
- Explains why contrast ↓ as higher energy x-rays are used in the imaging process
- Increased probability of photoe⁻ absorption just above the E_b of the inner shells cause discontinuities in the **attenuation profiles** (e.g., K-edge)

Photoelectric Effect





X-ray Cross Section and Linear Attenuation Coefficient

- **Cross section** is a measure of the probability ('apparent area') of interaction: $\sigma(E)$ measured in barns (10^{-24} cm²)
- Interaction probability can also be expressed in terms of the thickness of the material – **linear attenuation coefficient**: $\mu(E)$ = **fractional number of photons removed (attenuated) from the beam after traveling through a unit length** in media by absorption or scattering
- $\mu(E)$ [cm⁻¹] = Z [e⁻ /atom] · N_{avg} [atoms/mole] · $1/A$ [moles/gm] · ρ [gm/cm³] · $\sigma(E)$ [cm²/e⁻]
- Multiply by 100% to get % removed from the beam/cm
- $\mu(E) \downarrow$ as $E \uparrow$, e.g., for soft tissue $\mu(30 \text{ keV}) = 0.35 \text{ cm}^{-1}$ and $\mu(100 \text{ keV}) = 0.16 \text{ cm}^{-1}$

Calculation of the Linear Attenuation Coefficient

To the extent that Compton scattered photons are completely removed from the beam, the attenuation coefficient can be approximated as

$$\mu = \rho N_g \left[f(E) + C_p \frac{Z^m}{E^n} \right]$$

The (effective) Z value of a material is particularly important for determining μ .

$$N_g = N_A \frac{Z}{A}$$

$Z = \text{atomic number}$

$A = \text{atomic mass}$

$N_A = \text{Avogadro's number}$

$\rho = \text{density}$

$$f(E) = 0.597 \times 10^{-24} e^{-0.0028(E-30)}$$

$= \text{Compton scattering part for low } E$

$$C_p = 9.8 \times 10^{-24}$$

$$m = 3.8, n = 3.2$$

Calculation of Effective Z

- This is quite approximate but does permit simple computation provided that (a) the photon energy is high enough (b) scattered photons are removed from the beam and (c) $E < 200 \text{keV}$.
- The effective Z (Z_{eff}) is given as

$$Z_{\text{eff}} = \left(\sum_i \alpha_i Z_i^m \right)^{1/m}$$

where α_i is the electron fraction of the i^{th} element.

$$\alpha_i = \frac{N_{g_i}}{\sum_j N_{g_j}}$$

$$N_{g_i} = N_A w_i \left(\frac{Z_i}{A_i} \right)$$

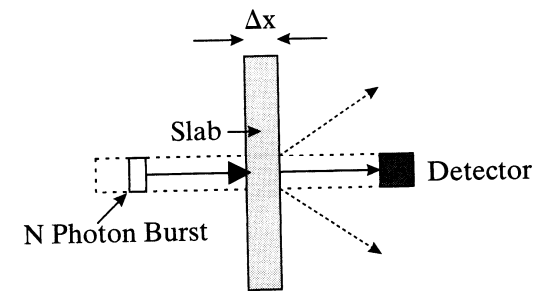
→ fraction by weight of the element

Measure of X-ray Beam Strength

- An exponential relationship between the incident radiation intensity (I_0) and the transmitted intensity (I) with respect to thickness:

$$I(E) = I_0(E) e^{-\mu(E) \cdot x}$$

$$\mu_{\text{total}}(E) = \mu_{\text{PE}}(E) + \mu_{\text{CS}}(E) + \mu_{\text{RS}}(E) + \mu_{\text{PP}}(E)$$



- At low x-ray E: $\mu_{\text{PE}}(E)$ dominates and $\mu(E) \propto Z^3/E^3$
- At high x-ray E: $\mu_{\text{CS}}(E)$ dominates and $\mu(E) \propto \rho$
- Only at very-high E ($> 1\text{MeV}$) does $\mu_{\text{PP}}(E)$ contribute
- The value of $\mu(E)$ is dependent on the phase state: $\mu_{\text{water vapor}} \ll \mu_{\text{ice}} < \mu_{\text{water}}$

Effective Z – An Example

- The effective Z (Z_{eff}) of H_2O is

$$N_{g_o} = N_A \left(\frac{16}{18} \right) \left(\frac{8}{16} \right) \quad ; \quad N_{g_H} = N_A \left(\frac{2}{18} \right) \left(\frac{1}{1.008} \right)$$

$$= 0.444 N_A \quad ; \quad = 0.111 N_A$$

$$\alpha_o = \frac{0.444}{0.555} = 0.8 \quad ; \quad \alpha_H = 0.2$$

$$Z_{\text{eff}} = \left[0.8(8)^{3.8} + 0.2(1)^{3.8} \right]^{1/3.8}$$
$$= 7.54$$

A_{eff} is just a normal weighted sum.

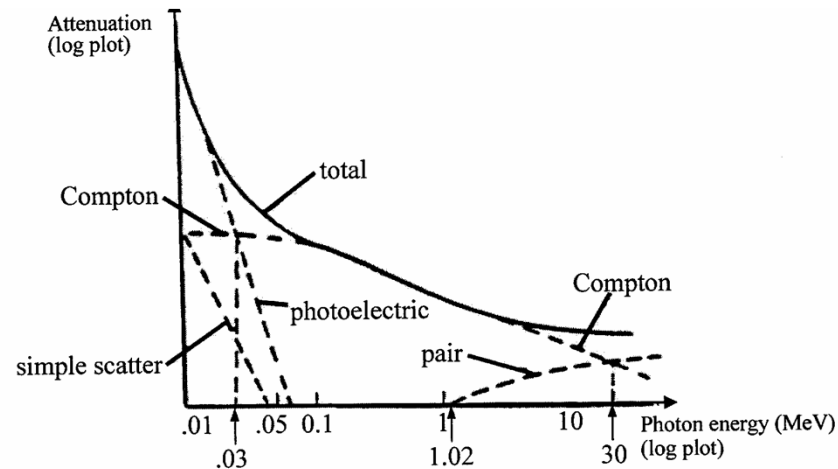
Measure of X-ray Beam Strength

- Attenuation mechanisms as a function of energy

<u>Mechanism</u>	μ dependence		<u>Energy Range in Soft Tissue</u>
	<u>E</u>	<u>Z</u>	
simple scatter	$1/E$	Z^2	1-20 keV
photoelectric	$1/E^3$	Z^3	1-30 keV
Compton	falls slowly with E	independent	30 keV-20 MeV
pair production	rises slowly with E	Z^2	above 20 MeV

Attenuation Mechanism

- Attenuation mechanisms as a function of energy



Attenuation mechanisms in water

The optimum photon energy is about 30 keV (tube voltage 80-100 kV) where the photoelectric effect dominates. The Z^3 dependence leads to good contrast:

Z_{fat}	5.9
Z_{muscles}	7.4
Z_{bone}	13.9

⇒ Photoelectric attenuation from bone is about 11x that due to soft tissue, which is dominated by Compton scattering.

Attenuation Mechanism

- Attenuation mechanisms as a function of energy

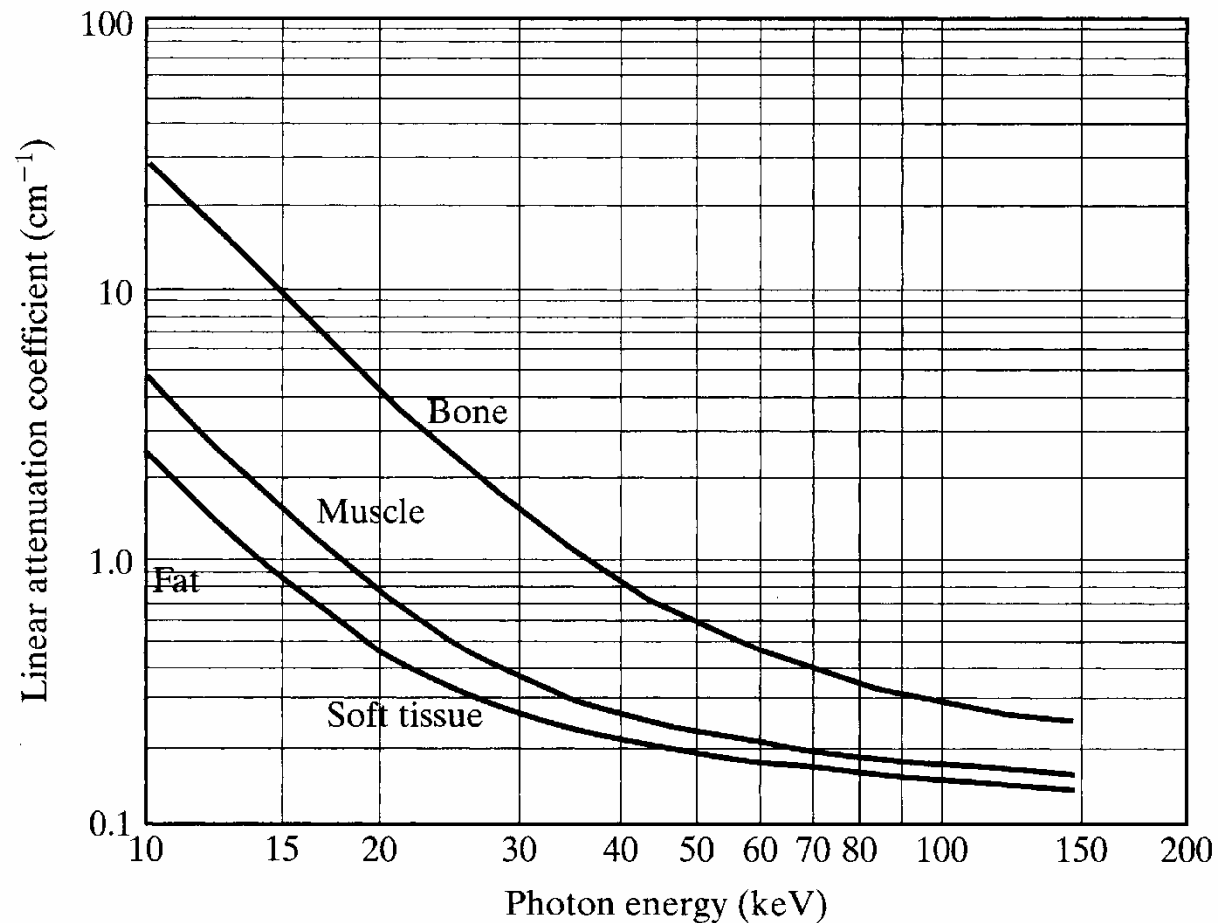


Figure 4.8
Linear attenuation coefficient for bone, muscle, and fat as a function of incident x-ray photon energy.



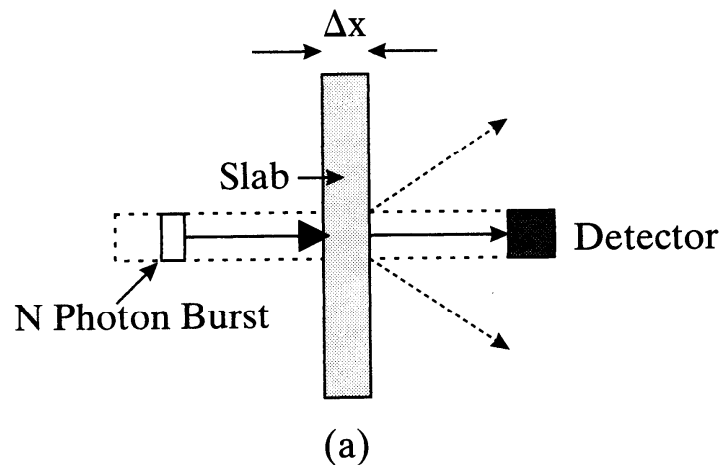
Half-Value Layer

- Thickness of material required to reduce the intensity of the incident beam by $\frac{1}{2}$
- $\frac{1}{2} = e^{-\mu(E) \cdot \text{HVL}}$ or $\text{HVL} = 0.693/\mu(E)$
- Units of HVL expressed in mm Al for a Dx x-ray beam
- For a monoenergetic incident photon beam (i.e., that from a synchrotron), the HVL is easily calculated
- For each HVL, $I \downarrow$ by $\frac{1}{2}$: $5 \text{ HVL} \rightarrow I/I_0 = 100\%/32 = 3.1\%$

X-ray Linear Attenuation Coefficient

Monoenergetic and Narrow Beam Case


- Suppose the slab is not homogeneous $\rightarrow \mu$ depends on x (along the beam direction)



$$\mu = \frac{n/N}{\Delta x},$$

- Assuming monoenergetic

$$N = N_0 e^{-\mu \Delta x}, \quad \Rightarrow \quad I = I_0 e^{-\mu \Delta x},$$



X-ray Linear Attenuation Coefficient Polyenergetic and Narrow Beam Case

- The attenuation of a poly energetic X-ray beam by a thin slab of material can be decomposed as x-ray flux at different energies.

$$S(E) = S_0(E)e^{-\mu(E)\Delta x} .$$

- The attenuation of a poly energetic X-ray beam by a heterogeneous slab is then

$$S(x; E) = S_0(E) \exp \left\{ - \int_0^x \mu(x'; E) dx' \right\} .$$



X-ray Linear Attenuation Coefficient Polyenergetic and Narrow Beam Case

- The energy dependent X-ray flux can be converted to the intensity as

$$I = \int_0^{\infty} S_0(E') E' \exp \{ -\mu(E') \Delta x \} dE'$$

- Taking into account the spatial variation of the energy-dependent attenuation coefficient, the remaining beam intensity at a given position x is

$$I(x) = \int_0^{\infty} S_0(E') E' \exp \left\{ - \int_0^x \mu(x'; E') dx' \right\} dE' .$$



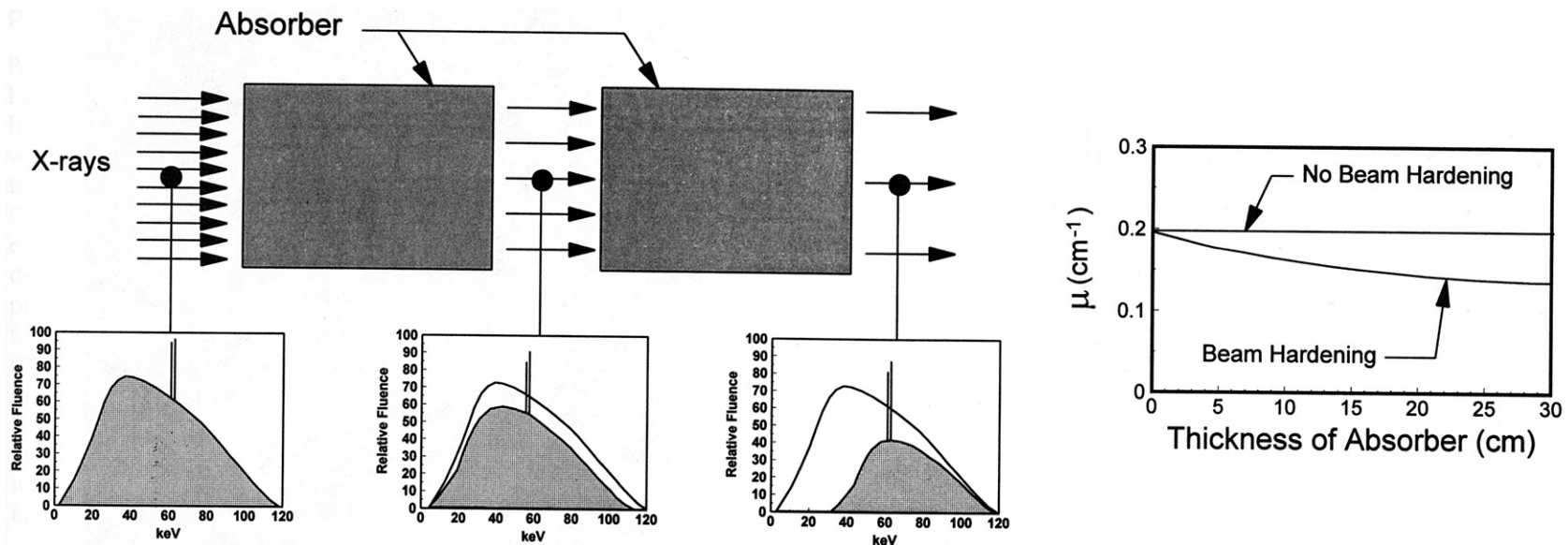
Beam Hardening Effect

Mean Free Path and Beam Hardening

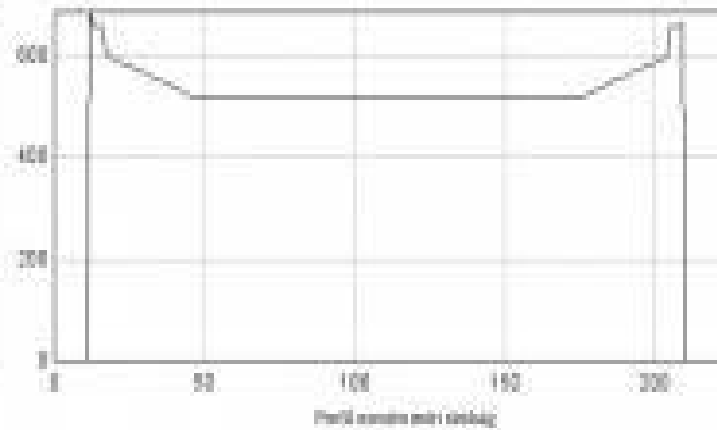
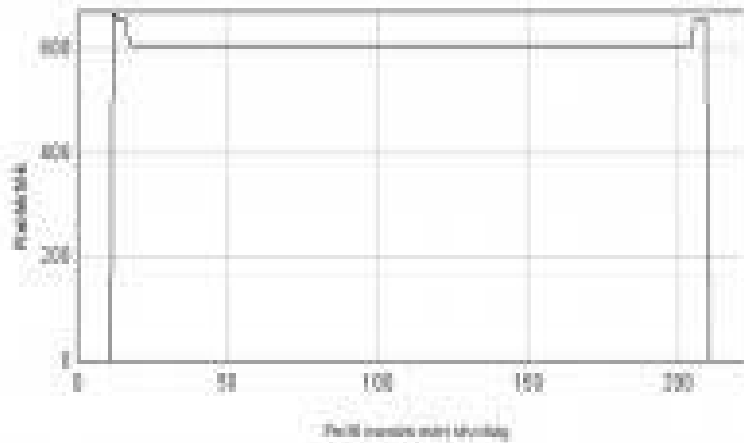
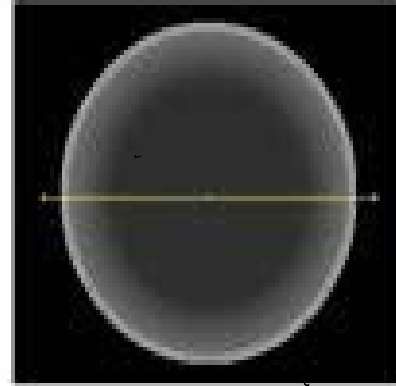
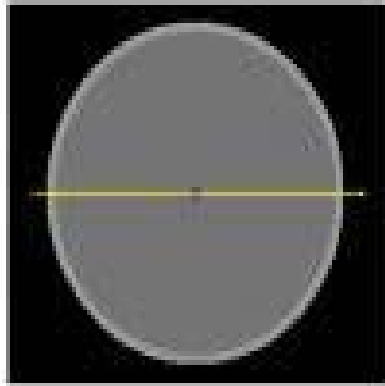
Mean free path (avg. path length of x-ray): $= 1/\mu = \text{HVL}/0.693$

Beam hardening:

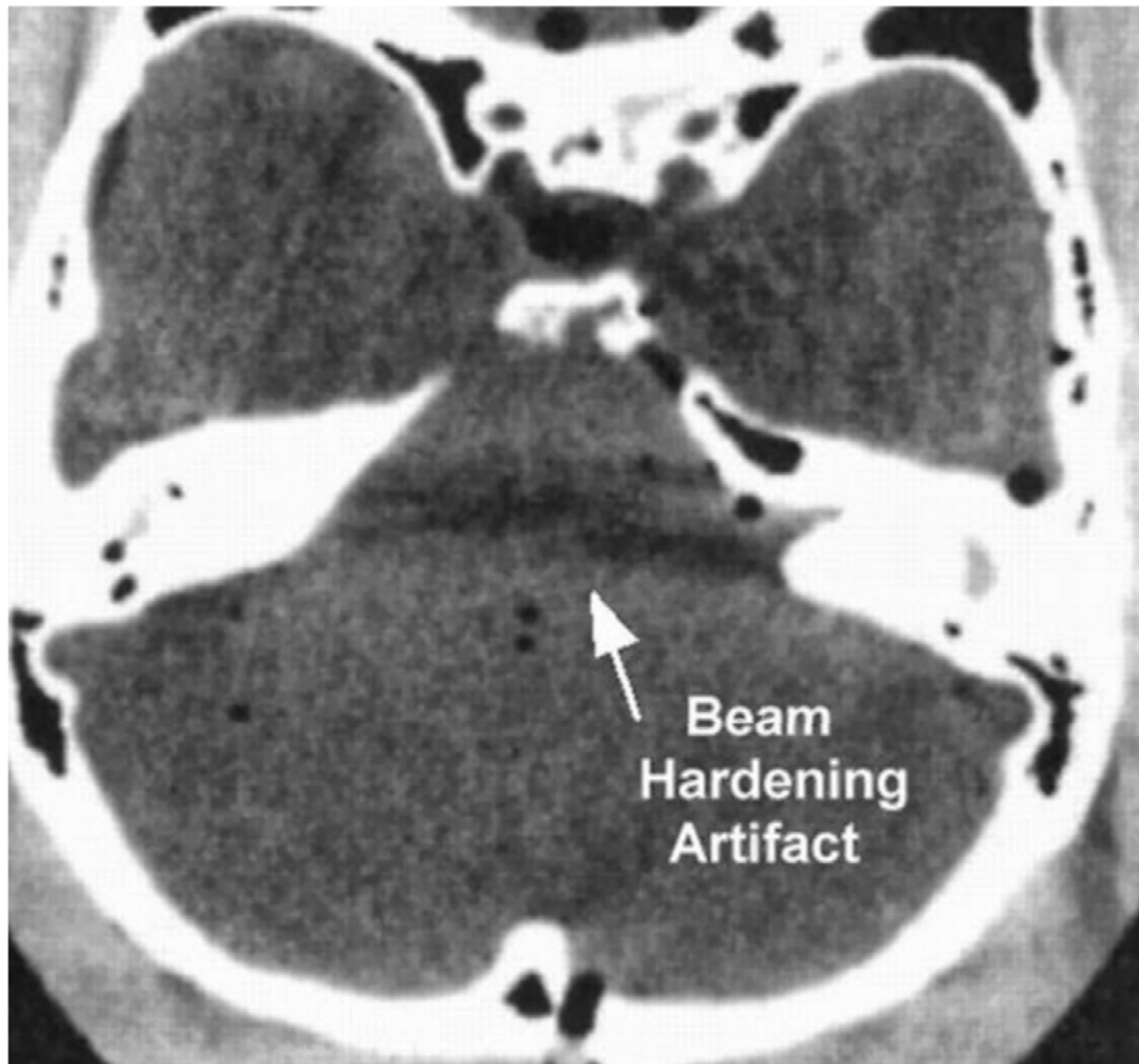
- The Bremsstrahlung process produces a wide spectrum of energies, resulting in a polyenergetic (polychromatic) x-ray beam
- As lower E photons have a greater attenuation coefficient, they are preferentially removed from the beam
- Thus the mean energy of the resulting beam is shifted to higher E



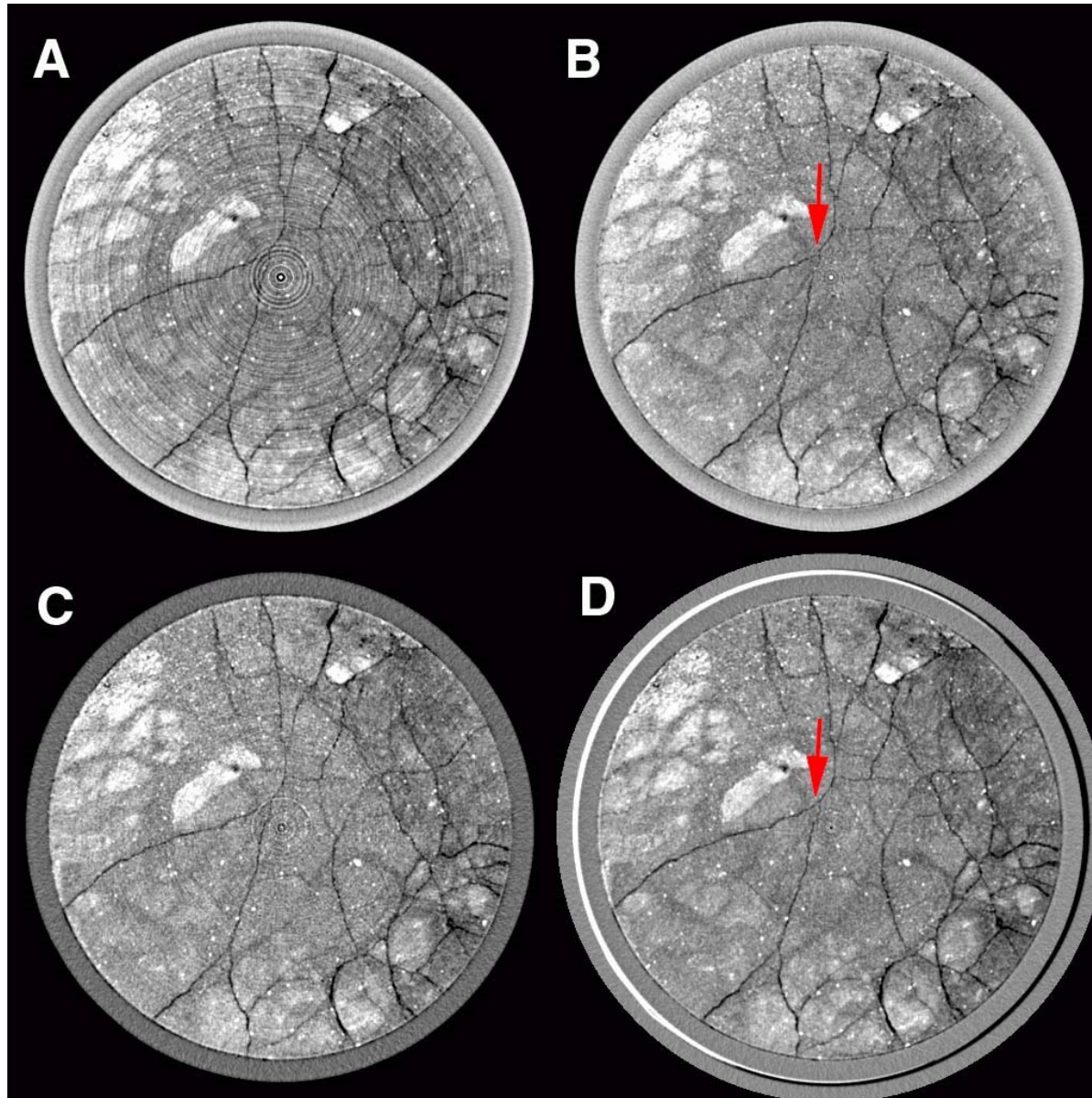
Mean Free Path and Beam Hardening



Beam-hardening effect and correction. The upper left picture shows a water-filled cylinder beam-hardening corrected homogeneous cross-sectional image, below the yellow line corresponding profile curve. The upper right is the image of the same cylinder without beam-hardening correction and the corresponding profile curve is shown below.



<http://www.ctlab.geo.utexas.edu/about-ct/artifacts-and-partial-volume-effects/>

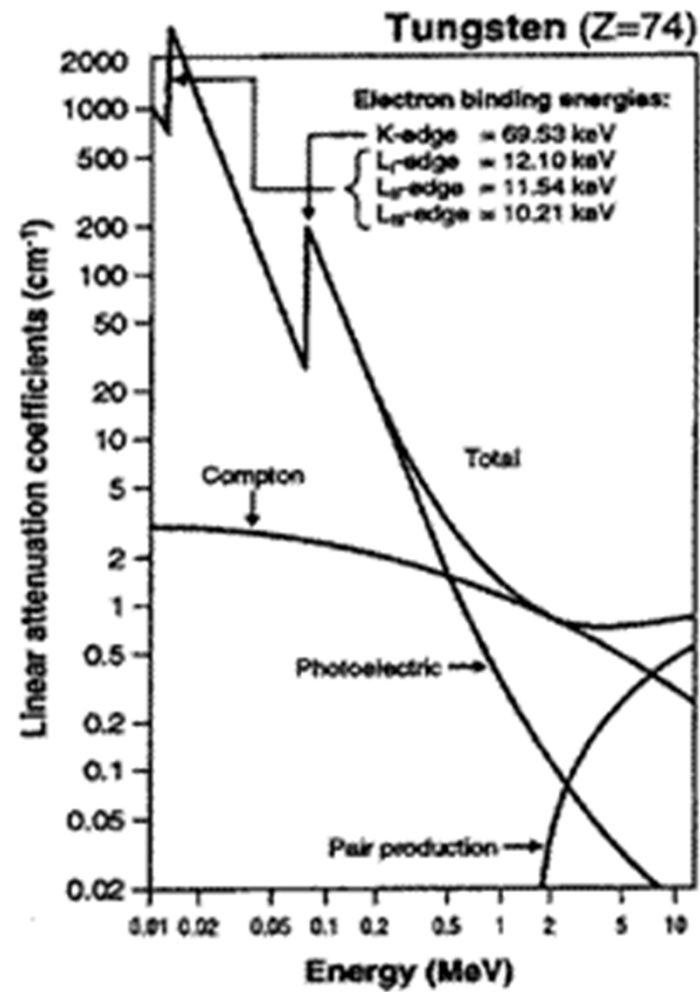
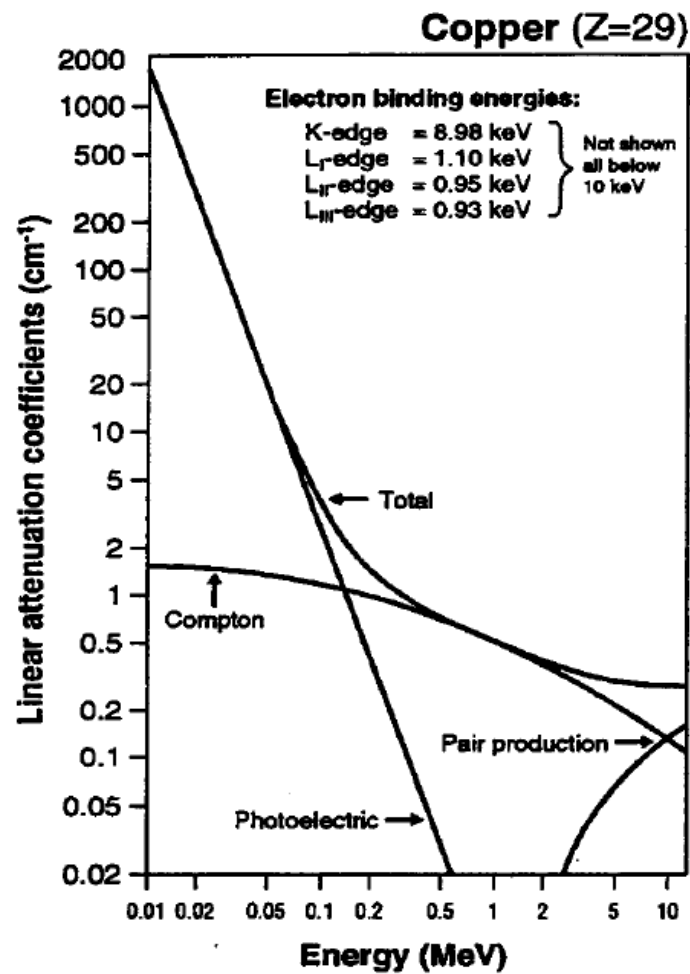


<http://www.ctlab.geo.utexas.edu/about-ct/artifacts-and-partial-volume-effects/>



Filtration of X-ray Generators

Photoelectric Effect



X-ray Generation – Filtration

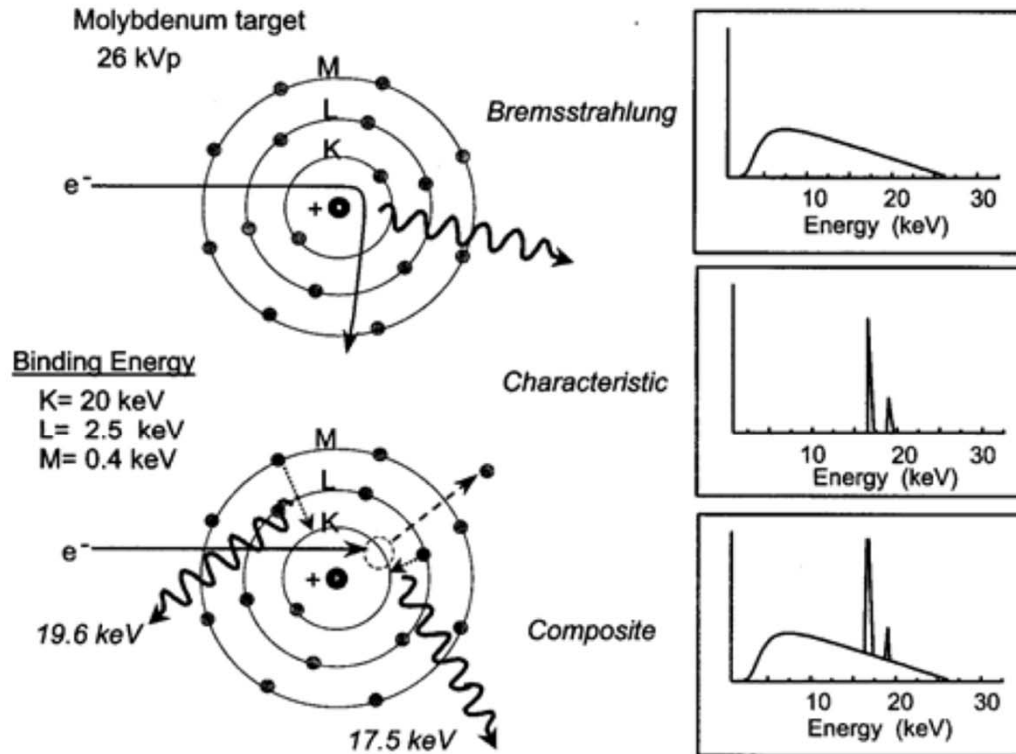


FIGURE 8-7. The output of a mammography x-ray system is composed of bremsstrahlung and characteristic radiation. The characteristic radiation energies of molybdenum (17.5 and 19.6 keV) are nearly optimal for detection of low-contrast lesions in breasts of 3- to 6-cm thickness.

Options:

Molybdenum (Mo)

Ruthenium (Ru)

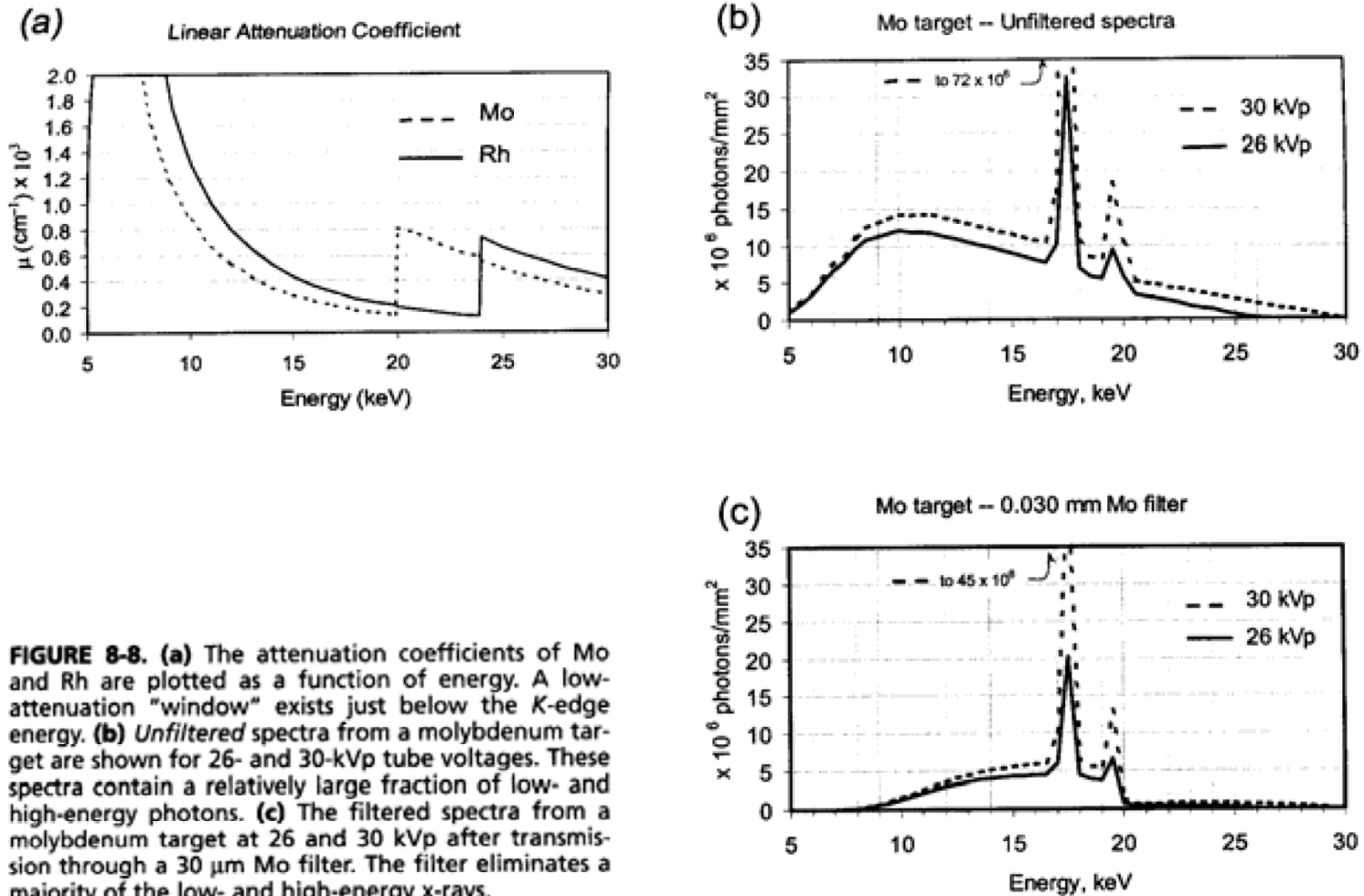
Rhodium (Rh)

Palladium (Pd)

Silver (Ag)

Cadmium (Cd)

X-ray Generation – Filtration



X-ray Generation – Filtration

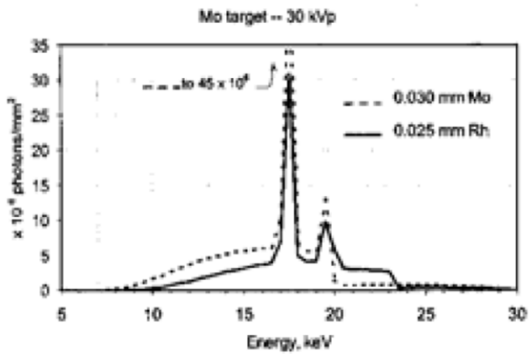


FIGURE 8-9. Output spectra from a Mo target for a 30-kVp tube voltage with a 0.030-mm Mo filter and 0.025-mm Rh filter show the relative bremsstrahlung photon transmission “windows” just below their respective K-edges.

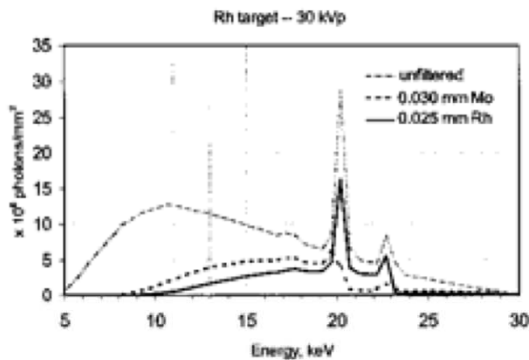


FIGURE 8-10. The rhodium target provides characteristic x-ray energies 2 to 3 keV higher than the corresponding Mo target. The unfiltered spectrum (light dashed line) is modified by 0.030-mm Mo (heavy dashed line) and 0.025-mm Rh (solid line) filters. A Mo filter with an Rh target inappropriately attenuates Rh characteristic x-rays.

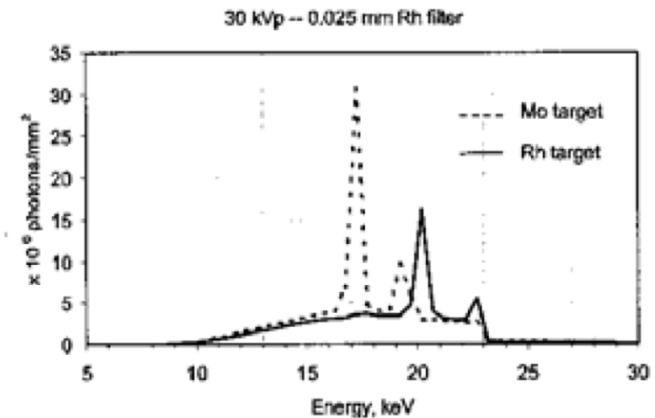


FIGURE 8-11. Spectra from beams filtered by 0.025 mm Rh are depicted for a Mo and an Rh target. The major difference is the higher energies of the characteristic x-rays generated by the Rh target, compared to the Mo target.

- filtering reduces the x-ray energy photons below the K-shell edge providing a transmission window for characteristic x-rays.
- typical values – Mo target with 0.03 mm Mo filter (Mo/Mo)
 - Rh target with 0.025 mm Rh filter (Rh/Rh)
 - Mo target with Rh filter
 - note: cannot use Rh target with Mo filter!

Ref: Bushberg ¹⁰



Application of Contrast Agents



Photoelectric Effect and Absorption Edge

- Edges become significant factors for higher Z materials as the E_b are in the diagnostic energy range:
- **Contrast agents** – barium (Ba, Z=56) and iodine (I, Z=53)
- Rare earth materials used for intensifying screens – lanthanum (La, Z=57) and gadolinium (Gd, Z=64)
- **Computed radiography (CR)** and **digital radiography (DR)** acquisition – europium (Eu, Z=63) and cesium (Cs, Z=55)
- Increased absorption probabilities improve subject contrast and quantum detective efficiency
- At photon $E \ll 50 \text{ keV}$, the photoelectric effect plays an important role in **imaging soft tissue**, amplifying small differences in tissues of slightly different Z, thus improving subject contrast (e.g., in mammography)

Example of Contrast Agents

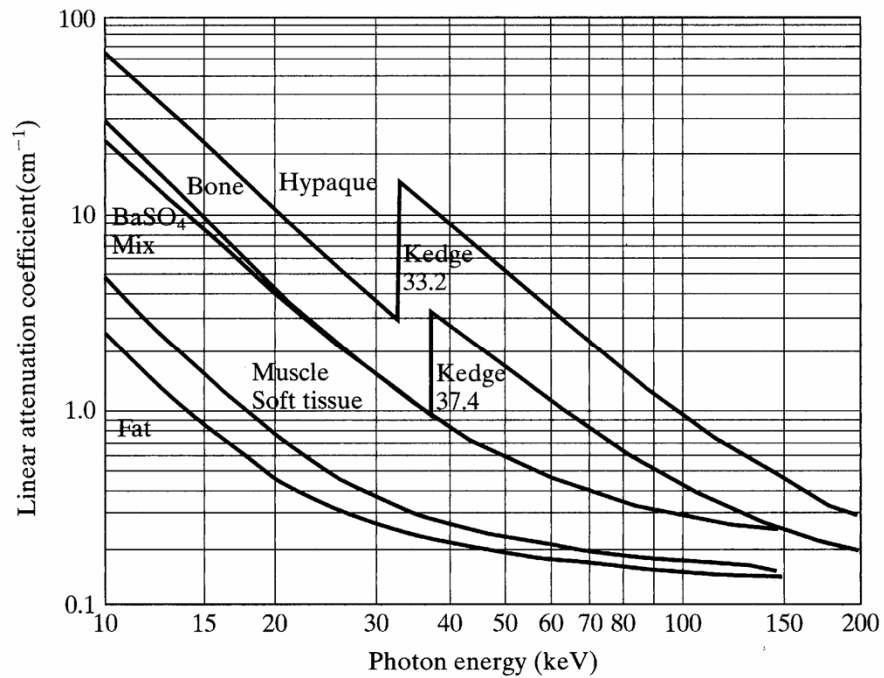


Figure 5.8
Linear attenuation coefficients of bone, muscle, fat, and two contrast agents. (From Johns and Cunningham, 1983.)

Bring out the difference between fat and soft tissues

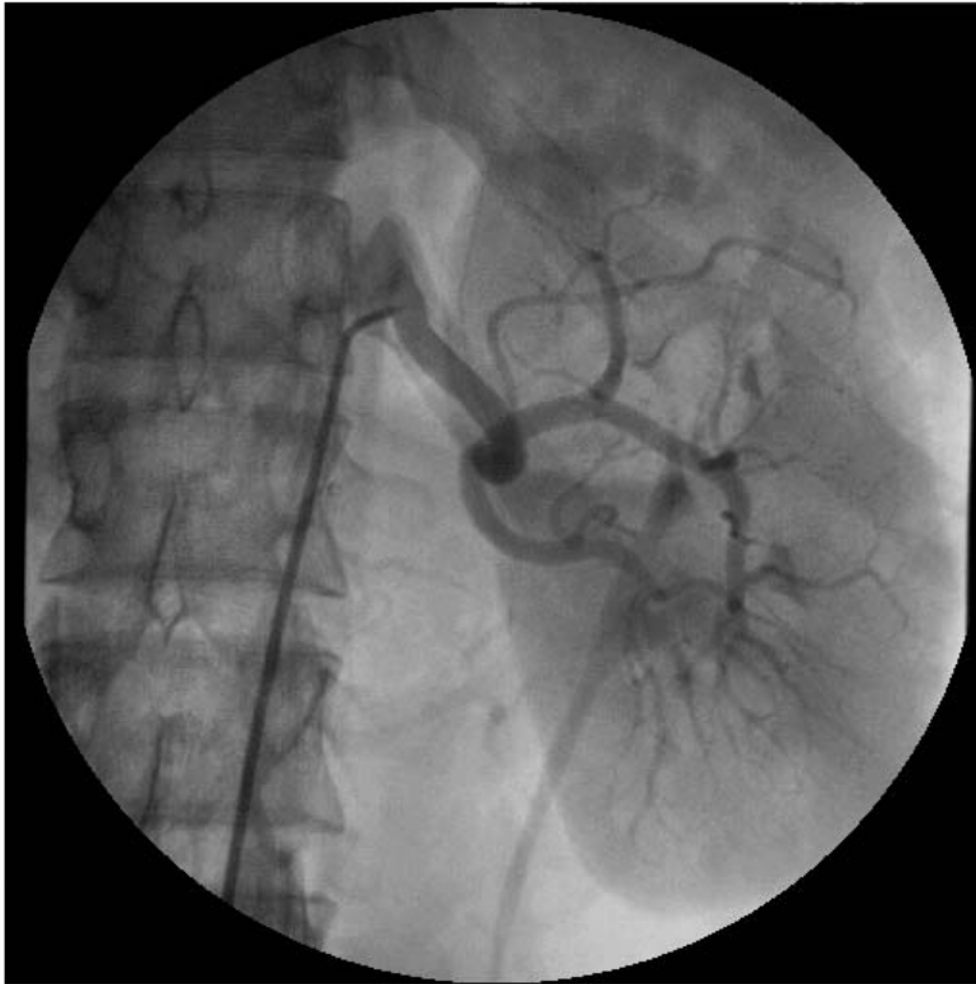


Example of Contrast Agents

- Blood vessels are not normally seen in an x-ray image, because they do not contrast sufficiently with the surrounding tissues.
- To increase image contrast, contrast agents, which are dense fluids with elements of high atomic numbers, such as iodine, are injected into a blood vessel during angiography. Because of its higher density and high atomic number, iodine absorbs photons more than blood and tissue.
- This creates detailed images of the blood vessels in real time.
- The first contrast media used for intravascular injection were called highosmolar contrast media (HOCM). (Osmolality is the measure of the particle concentration in a solution.)

Example of Contrast Agents

Angiographic Imaging:



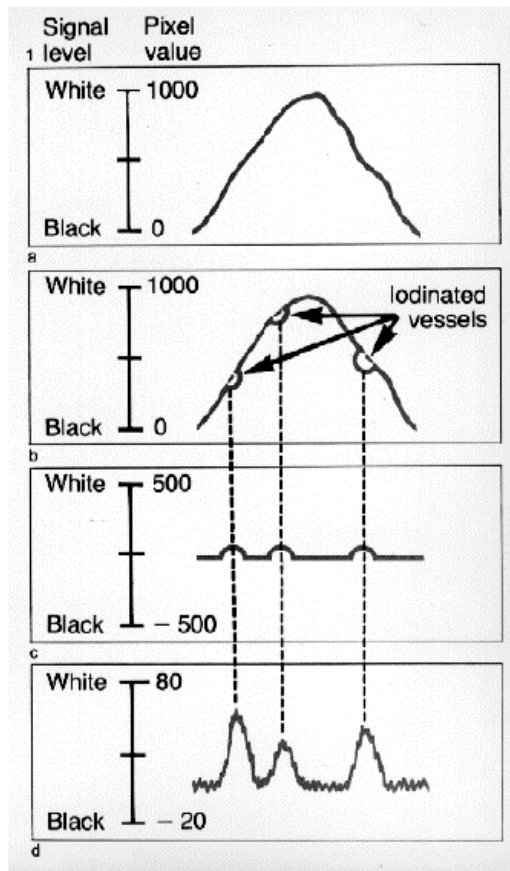
Blood vessels are visualized by injecting a “contrast medium”.

Example:
Blood vessels in kidney.

Example of Contrast Agents

Principles of DSA (Digital Subtraction Angiography)

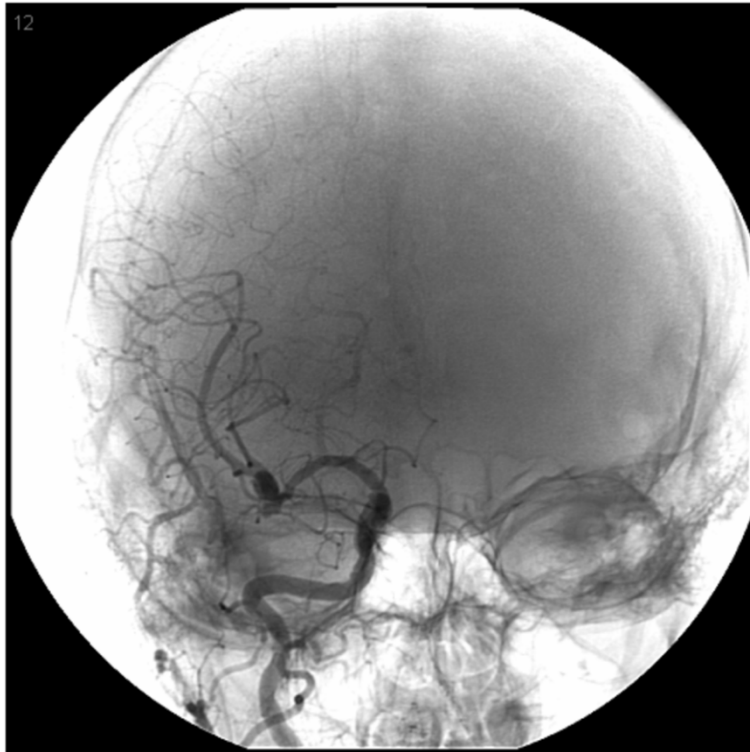
1. Recording of image I_1 without contrast medium ("mask image").
2. Recording of image I_2 after contrast injection.
3. Subtraction $I_1 - I_2$.



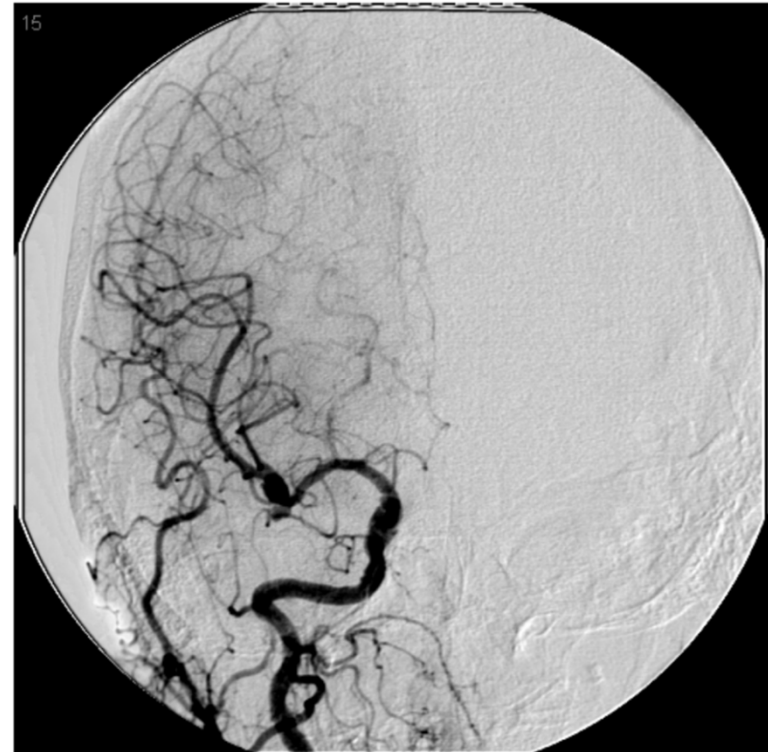
DAS of aorta.

Medical Image Processing (Med Img 1)

X-ray Generation – Characteristic X-rays



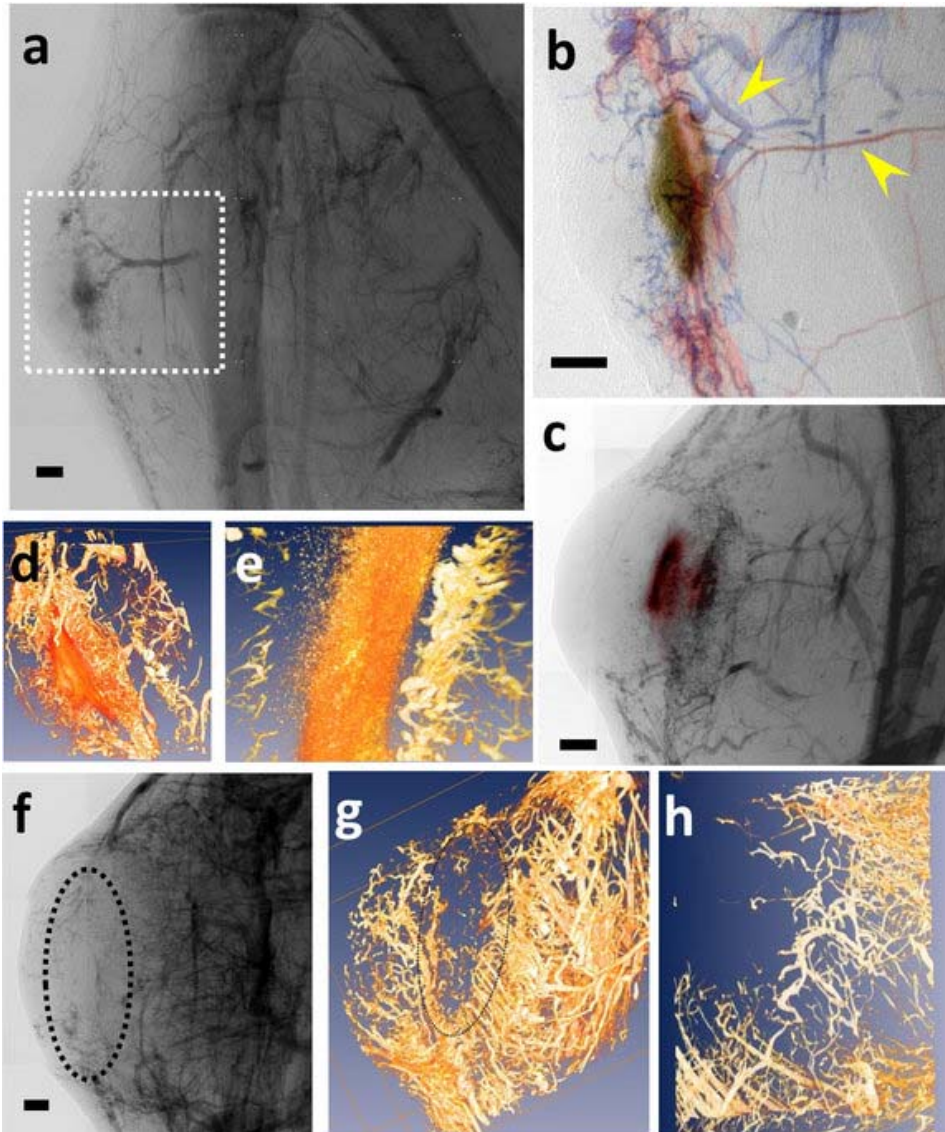
This is a mask image showing the background bone which obscures many of the smaller vessels.



Subtracted image with the background details removed.

Both images from Bushberg et al. 2003

Contrast Enhanced X-ray Imaging



(a) and (b): in vivo projection x-ray tumor images at day 3. (b): the magnified, color-coded view of the rectangular portion of (a), showing that the tumor already developed a vessel network with feeding arterioles (arrow, red color vessels) and venules (arrow head, blue color vessels). The brown area is the inoculated site. (c): projection image similar to (a), at day 7. (d) and (e): tomographic reconstructed images corresponding to (c) with two levels of magnification ([supplementary data S2 d and e](#)). Tumor angiogenesis at day 7 is denser than at day 3. Most of inoculated primary tumor cells stayed at the inoculation site and angiogenesis developed around but not inside the central area. (f)–(h): tomography results at day 7 without Au-NPs also show angiogenesis around but not inside the inoculation site (black dotted area). (f): projection image (the dotted circle marks the tumor whereas (g) and (h) are tomographic reconstructed image with low and high magnification ([supplementary data S2 g and h](#)). All scale bars are 500 μm .

Fig. from “X-ray imaging of tumor growth in live mice by detecting gold-nanoparticle-loaded cells”, [Chia-Chi Chien et al., Scientific Report, 2012](#).

Contrast Enhanced X-ray Imaging

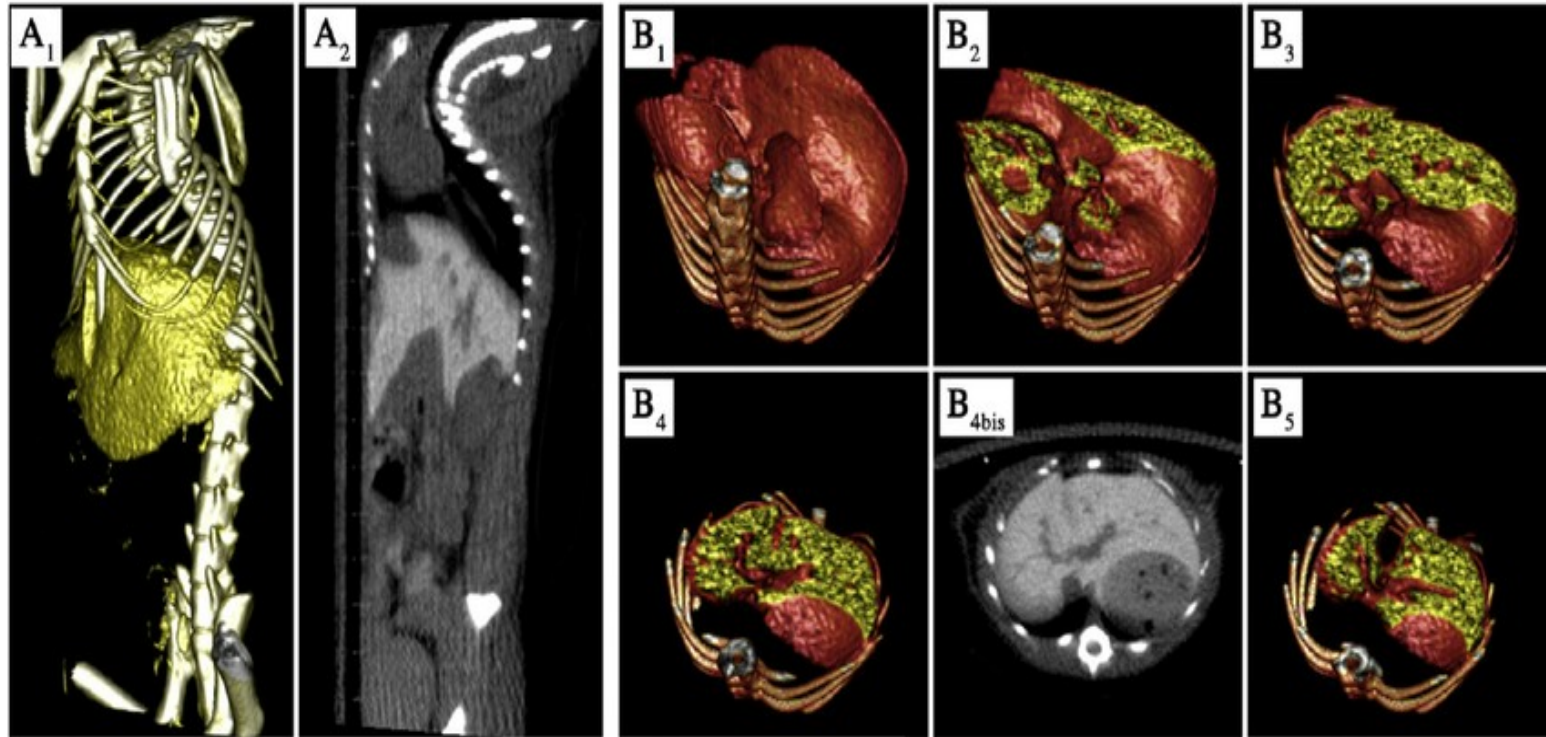


Fig. 5. Micro-CT liver imaging with hepatocyte-specific contrast agent: vitamin E nano-emulsions, from [41]. (A 1) Left lateral view, 3D rendering, (A 2) sagittal view, maximal intensity projection. (B 1) to (B 5) show 3D rendering of liver sections, emphasizing the clear contrast different between the liver tissues and its vascularization. (B 4 bis) is the transverse view of maximal intensity projection corresponding to (B 4).

From “contrast agents for preclinical targeted X-ray imaging”, 2014