Algorand BA

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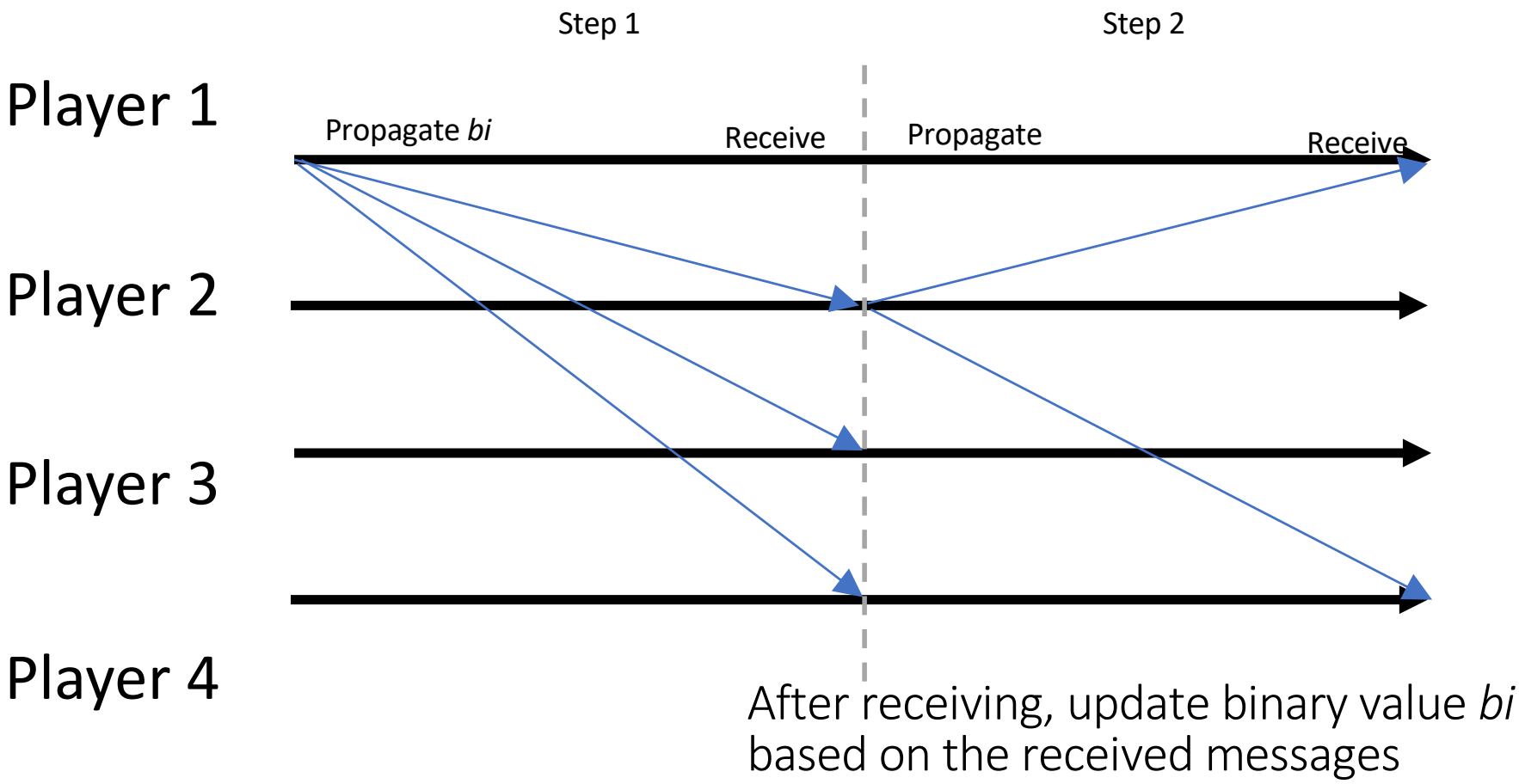
Warmup: The Binary BA Protocol BBA^*

- Although player-replaceability is a desired feature, let's start with a BA without player-replaceability.
- BBA^*: a probabilistic binary Byzantine agreement that targets $n = 3t + 1$ players and t-Byzantine fault tolerance.
- Each player i holds a binary value b_i on which they want to reach agreement.
- The protocol proceeds in synchronous steps, where messages are guaranteed to be delivered within a step.
- (Next slide shows an example of 2 steps)


Step 1
Step 2
Player 1
Player 2
Player 3
Player 4
Propagate *bi*
Receive
Propagate
Receive
After receiving, update binary value *bi* based on the received messages

A Straw-man Step

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If $\#_i(0) \geq 2t + 1$, then i sets $b_i = 0$.

Symmetrically, if $\#_i(1) \geq 2t + 1$, then i sets $b_i = 1$.

$\#_i(v)$ denotes the number of players from which i has received the value v.

A Straw-man Step: Analysis

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We need a cryptographic primitive called *common coin*: a new randomly and independently selected bit c for each step. (We will show how to implement it later.) Just let players set $b_i = c$.

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A Step: Player i propagates b_i.

1. If $\#_i(0) \geq 2t + 1$, then i sets $b_i = 0$.
2. Else, if $\#_i(1) \geq 2t + 1$, then i sets $b_i = 1$.
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(A) If, at the start of a step, the honest players (at least $2t+1$) are in agreement on a bit b, (i.e., if $b_i = b$ for all honest player i), then they remain in agreement on b by its end.

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A brief explanation for property (B) is that when honest players are not in agreement, they can be in either condition 1 and 3 (sets $b_i = 0$ and $b_i = c$) or condition 2 and 3 (sets $b_i = 1$ and $b_i = c$). In either case, coin c is equal to the bit with probability $1/2$.

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by running this straw-man step sufficiently many times, honest players will reach an agreement with overwhelming probability.

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- Coin-Fixed-To-0 Step. The common coin is replaced by a fixed bit 0.
- Coin-Fixed-To-1 Step. The common coin is replaced by a fixed bit 1.
- Coin-Genuinely-Flipped Step. The common coin is the genuinely random coin.

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A brief explanation is, taking Coin-Fixed-To-0 as an example, if an honest player i outputs, players can be in either condition 1 or 3, since there are at most t Byzantine players who vote twice so condition 2 is unreachable.

3 Steps of BBA^*: Analysis

These 3 properties make BBA^* a correct BA.

(A) If, at the start of a step, the honest players (at least $2t+1$) are in agreement on a bit b, (i.e., if $b_i = b$ for all honest player i), then they remain in agreement on b by its end.

(B) If the honest players are not in agreement (on any bit) at the start of Coin-Genuinely-Flipped step, then with probability $1/2$, they will be in agreement (on some bit) by its end.

(C) If, at Coin-Fixed-To-0 or Coin-Fixed-To-1 step, an honest player i outputs, then agreement will hold at the end of the step.

BBA^*: wrap up. Next: player-replaceability.

[Coin-Fixed-To-0 Step] Each player i propagates b_i.

1.1 If $\#_i(0) \geq 2t+1$, then i sets $b_i = 0$. Outputs 0 and do not change b_i. That is, for future steps, propagates 0.

1.2 If $\#_i(1) \geq 2t+1$, then i sets $b_i = 1$.

1.3 Else, i sets $b_i = 0$.

[Coin-Fixed-To-1 Step] Each player i propagates b_i.

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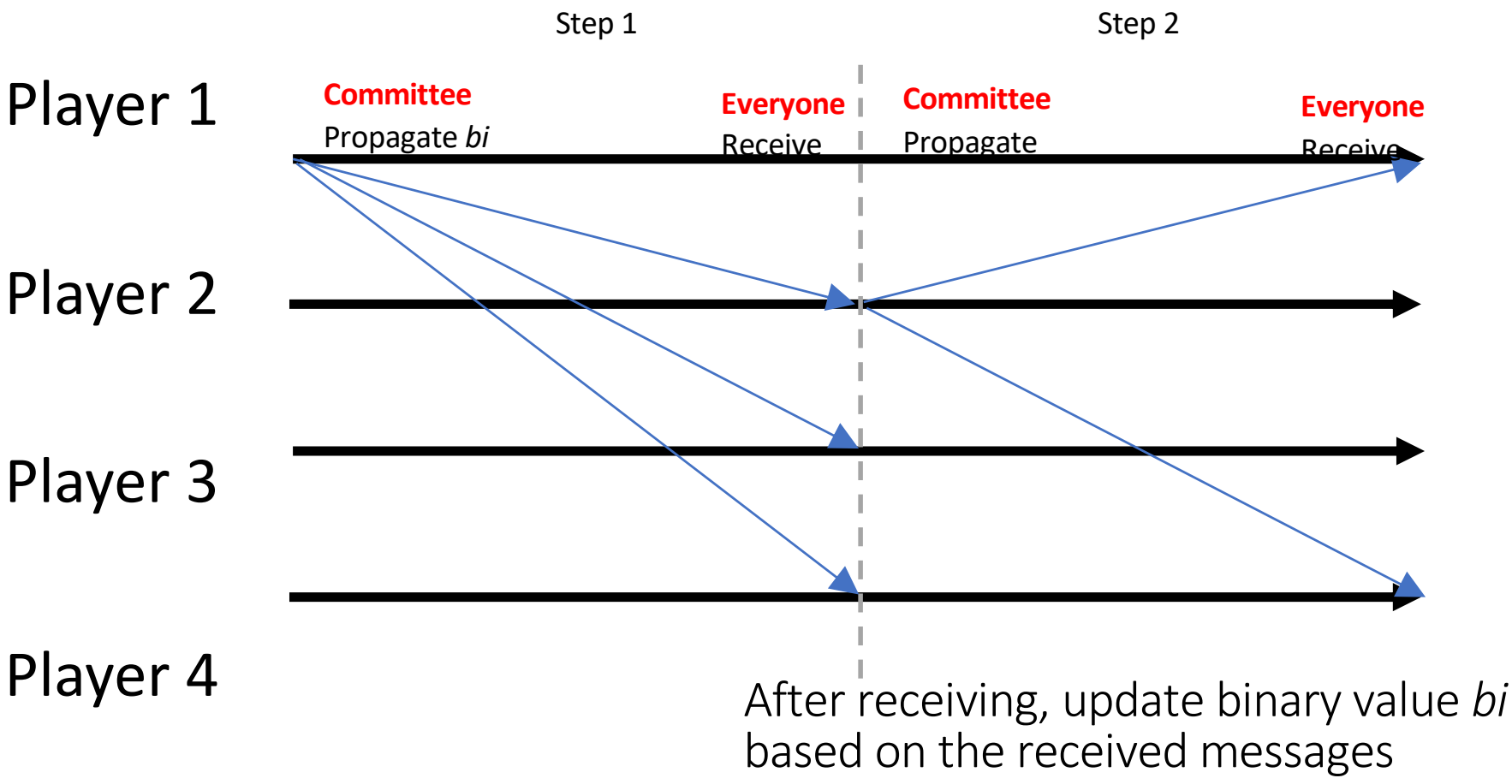
3.3 Else, i sets b_i to the common coin c.

Player-replaceability: selecting committees

Each step is assigned to a totally new committee, which is independently and randomly selected among all players by *sortition*.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- i propagates $\sigma_i^{r,s}$ with its message so that other players can verify it.

BBA^* player-replaceability. Just change $2t+1$ to $t_H \approx 2/3|\text{committee}|$

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To deal with this, players use ephemeral keys: public/secret key pairs that are single-use-only, and once used, are *destroyed*.

Implement the common coin

- In Coin-Genuinely-Flipped step, a player should receives messages from many players, denoted by SV. It picks the smallest credential hash from SV, hash the credential with the step counter s, and use the least significant bit as the coin c.
- Since the least significant bit of a hash is random, this coin implementation is almost a random common coin.

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  - if receive a valid block B^r from the leader and enough votes, $b_i = 0$.
  - otherwise, $b_i = 1$.
- Just run BBA^*!
- Players will agree on either $b_i = 0$ and a valid block, or $b_i = 1$ (in this case they use a default empty block).

Proof-of-Stake (PoS) Algorand

If we involve the stake held by a public key into the committee/leader selection, then we end up with a PoS blockchain.

Q and A