Although player-replaceability is a desired feature, let’s start with a BA without player-replaceability.

\( BBA^* \): a probabilistic binary Byzantine agreement that targets \( n = 3t + 1 \) players and \( t \)-Byzantine fault tolerance.

Each player \( i \) holds a binary value \( b_i \) on which they want to reach agreement.

The protocol proceeds in synchronous steps, where messages are guaranteed to be delivered within a step.

(Next slide shows an example of 2 steps)
After receiving, update binary value $bi$ based on the received messages.
A Straw-man Step

Intuition about how player $i$ updates $b_i$. 
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Intuition about how player $i$ updates $b_i$. Since the assumption of Byzantine players is $n/3$, honest $2n/3$, $(n = 3t + 1)$:

\[ \text{If } \#_i(0) \geq 2t + 1, \text{ then } i \text{ sets } b_i = 0. \]
\[ \text{Symmetrically, if } \#_i(1) \geq 2t + 1, \text{ then } i \text{ sets } b_i = 1. \]

$\#_i(v)$ denotes the number of players from which $i$ has received the value $v$. 
A Straw-man Step: Analysis

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An obvious question with the above step is what if $\#_i(0) < 2t + 1$ and $\#_i(1) < 2t + 1$?
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If there is a \( 2/3 \) majority: reach agreement.

An obvious question with the above step is what if \( \#_i(0) < 2t + 1 \) and \( \#_i(1) < 2t + 1 \)?
We need a cryptographic primitive called \textit{common coin}: a new randomly and independently selected bit \( c \) for each step. (We will show how to implement it later.) Just let players set \( b_i = c \).
A Straw-man Step: Analysis

A Step: Player $i$ propagates $b_i$.

1. If $\#_i(0) \geq 2t + 1$, then $i$ sets $b_i = 0$.
2. Else, if $\#_i(1) \geq 2t + 1$, then $i$ sets $b_i = 1$.
3. Else, $i$ sets $b_i = c$.

Easy to see the following properties,

(A) If, at the start of a step, the honest players (at least $2t + 1$)
are in agreement on a bit $b$ (i.e., if $b_i = b$ for all honest
player $i$), then they remain in agreement on $b$ by its end.

(B) If the honest players are not in agreement (on any bit) at the
start of a step, then with probability $1/2$, they will be in
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A brief explanation for property (B) is that when honest players are not in agreement, they can be in either condition 1 and 3 (sets \( b_i = 0 \) and \( b_i = c \)) or condition 2 and 3 (sets \( b_i = 1 \) and \( b_i = c \)). In either case, coin \( c \) is equal to the bit with probability 1/2.
A Straw-man Step: Analysis

A Step: Player $i$ propagates $b_i$.

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by running this straw-man step sufficiently many times, honest players will reach an agreement with overwhelming probability.
3 Steps of \textit{BBA}\

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- **Coin-Fixed-To-0 Step.** The common coin is replaced by a fixed bit 0.
- **Coin-Fixed-To-1 Step.** The common coin is replaced by a fixed bit 1.
- **Coin-Genuinely-Flipped Step.** The common coin is the genuinely random coin.
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(C) If, at Coin-Fixed-To-0 or Coin-Fixed-To-1 step, an honest player $i$ outputs, then agreement will hold at the end of the step.
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  (C) If, at Coin-Fixed-To-0 or Coin-Fixed-To-1 step, an honest player $i$ outputs, then agreement will hold at the end of the step.

A brief explanation is, taking Coin-Fixed-To-0 as an example, if an honest player $i$ outputs, players can be in either condition 1 or 3, since there are at most $t$ Byzantine players who vote twice so condition 2 is unreachable.
3 Steps of \textit{BBA}*: Analysis

These 3 properties make \textit{BBA}* a correct BA.

(A) If, at the start of a step, the honest players (at least $2t + 1$) are in agreement on a bit $b$, (i.e., if $b_i = b$ for all honest player $i$), then they remain in agreement on $b$ by its end.

(B) If the honest players are not in agreement (on any bit) at the start of Coin-Genuinely-Flipped step, then with probability 1/2, they will be in agreement (on some bit) by its end.

(C) If, at Coin-Fixed-To-0 or Coin-Fixed-To-1 step, an honest player $i$ outputs, then agreement will hold at the end of the step.
BBA*: wrap up. Next: player-replaceability.

[Coin-Fixed-To-0 Step] Each player $i$ propagates $b_i$.

1.1 If $\#_i(0) \geq 2t + 1$, then $i$ sets $b_i = 0$. Outputs 0 and do not change $b_i$. That is, for future steps, propagates 0.

1.2 If $\#_i(1) \geq 2t + 1$, then $i$ sets $b_i = 1$.

1.3 Else, $i$ sets $b_i = 0$.

[Coin-Fixed-To-1 Step] Each player $i$ propagates $b_i$.

2.1 If $\#_i(1) \geq 2t + 1$, then $i$ sets $b_i = 1$. Outputs 1 and do not change $b_i$. That is, for future steps, propagates 1.

2.2 If $\#_i(0) \geq 2t + 1$, then $i$ sets $b_i = 0$.

2.3 Else, $i$ sets $b_i = 1$.

[Coin-Genuinely-Flipped Step] Each player $i$ propagates $b_i$.

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3.2 If $\#_i(1) \geq 2t + 1$, then $i$ sets $b_i = 1$.

3.3 Else, $i$ sets $b_i$ to the common coin $c$. 
Player-replaceability: selecting committees

Each step is assigned to a totally new committee, which is independently and randomly selected among all players by sortition.
After receiving, update binary value $b_i$ based on the received messages.
Player-replaceability: selecting committees

- Player $i$ uses a quantity $\textit{credential} \ \sigma$ to secretly determine whether it is selected.
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- $i$ is committee in round $r$ and step $s$ if $H(\sigma_{i}^{r,s}) < p$ ($H$ is a hash function, $p$ is a threshold)
- $i$ propagates $\sigma_{i}^{r,s}$ with its message so that other players can verify it.
BBA* player-replaceability. Just change $2t + 1$ to $t_H \approx 2/3|\text{committee}|

[Coin-Fixed-To-0 Step] Each player $i$ propagates $b_i$.
1.1 If $\#_i(0) \geq 2t + 1$, then $i$ sets $b_i = 0$. Outputs 0 and do not change $b_i$. That is, for future steps, propagates 0.
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Ephemeral keys

Although the adversary cannot predict beforehand which users will be the committee, it would know their identities after seeing their messages, and could then corrupt all of them.
Ephemeral keys

Although the adversary cannot predict beforehand which users will be the committee, it would know their identities after seeing their messages, and could then corrupt all of them. To deal with this, players use ephemeral keys: public/secret key pairs that are single-use-only, and once used, are destroyed.
Implement the common coin

- In Coin-Genuinely-Flipped step, a player should receive messages from many players, denoted by $SV$. It picks the smallest credential hash from $SV$, hash the credential with the step counter $s$, and use the least significant bit as the coin $c$.
- Since the least significant bit of a hash is random, this coin implementation is almost a random common coin.
Extending $BBA^*$ to multi-valued (block) Byzantine agreement

- Elect a leader (just like electing a committee), let the leader propagate a valid block for round $r$, $B^r$. 
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- Elect a leader (just like electing a committee), let the leader propagate a valid block for round $r$, $B^r$.
- two-round voting to ensure that if two honest players receive a block and enough votes, they have the same block $B^r$. 
Extending $BBA^*$ to multi-valued (block) Byzantine agreement

- Elect a leader (just like electing a committee), let the leader propagate a valid block for round $r$, $B^r$.
- two-round voting to ensure that if two honest players receive a block and enough votes, they have the same block $B^r$.
- Decide the input $b_i$ to $BBA^*$:
  - if receive a valid block $B^r$ from the leader and enough votes, $b_i = 0$.
  - otherwise, $b_i = 1$. 

Players will agree on either $b_i = 0$ a valid block, or $b_i = 1$ (in this case they use a default empty block).
Extending $BBA^*$ to multi-valued (block) Byzantine agreement

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- Decide the input $b_i$ to $BBA^*$:
  - if receive a valid block $B^r$ from the leader and enough votes, $b_i = 0$.
  - otherwise, $b_i = 1$.
- Just run $BBA^*$!
- Players will agree on either $b_i = 0$ and a valid block, or $b_i = 1$ (in this case they use a default empty block).
If we involve the stake held by a public key into the committee/leader selection, then we end up with a PoS blockchain.
Q and A