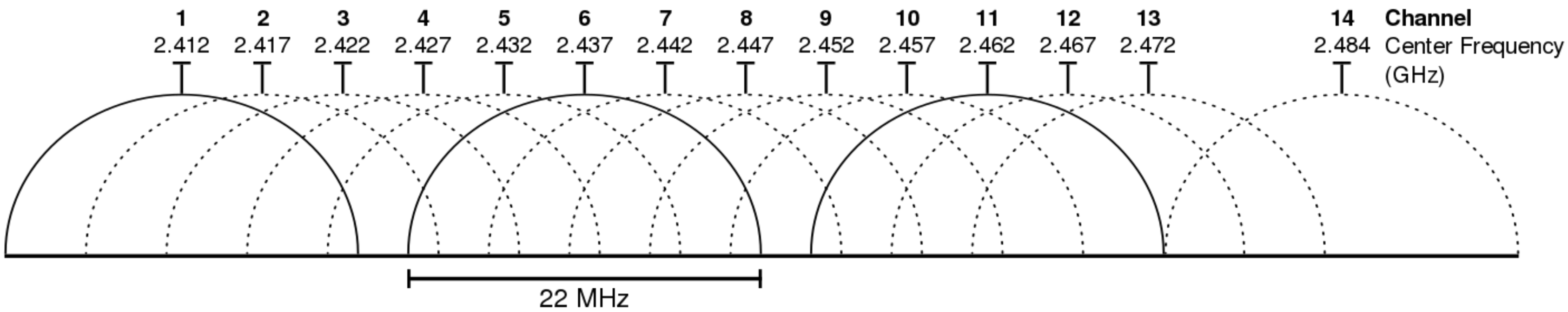
# ECE 598HH: Advanced Wireless Networks and Sensing Systems

## Lecture 6: MIMO Part 1

Haitham Hassanieh

# WiFi Standards from WiFi 1 to WiFi 6

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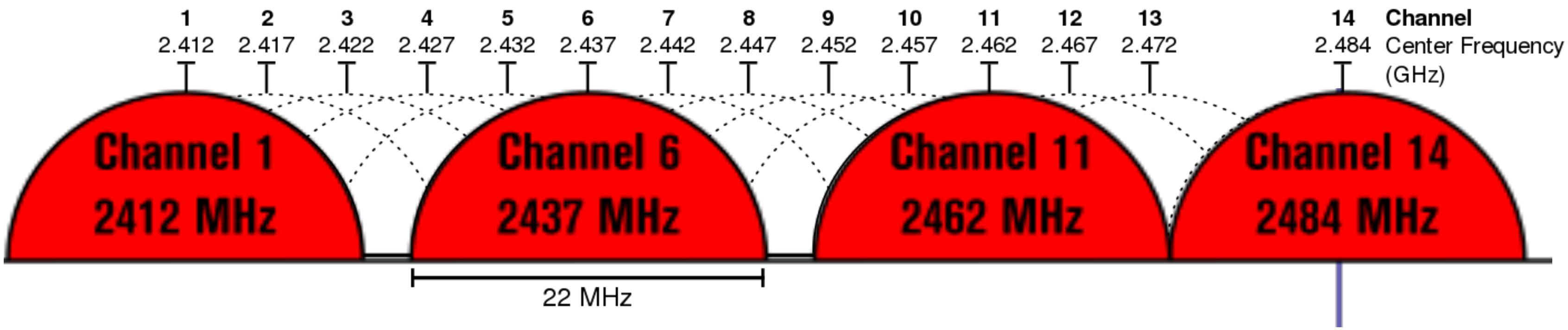


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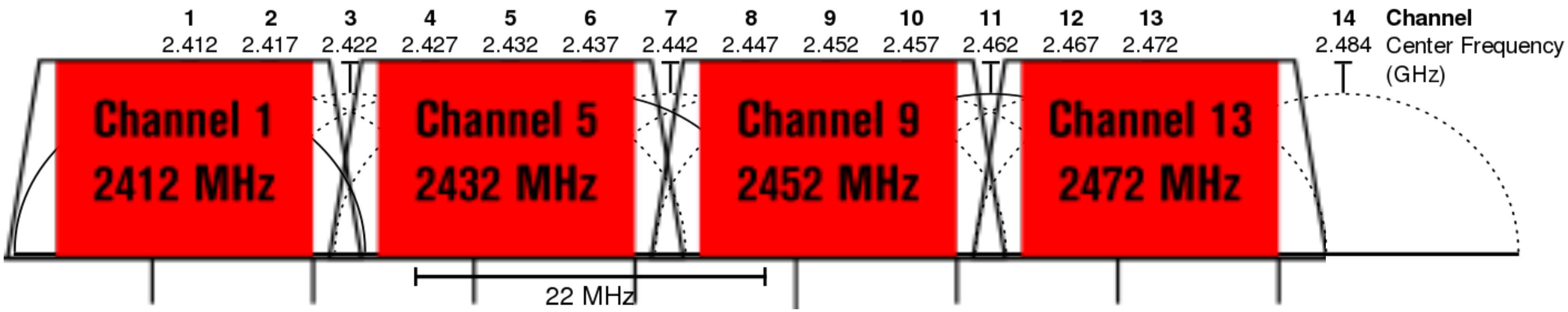


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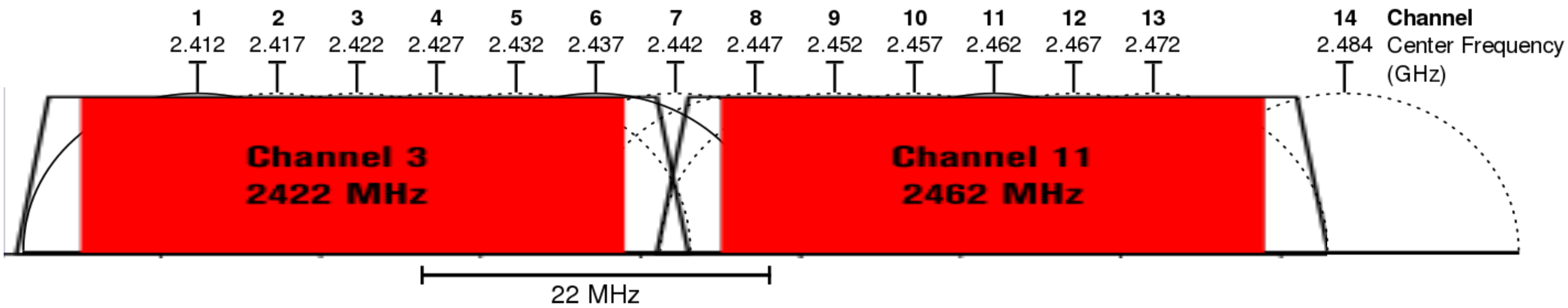


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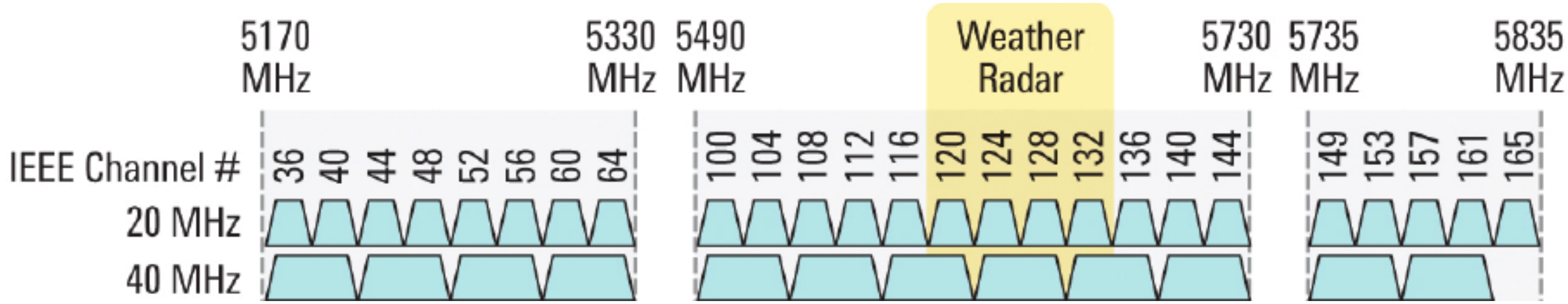


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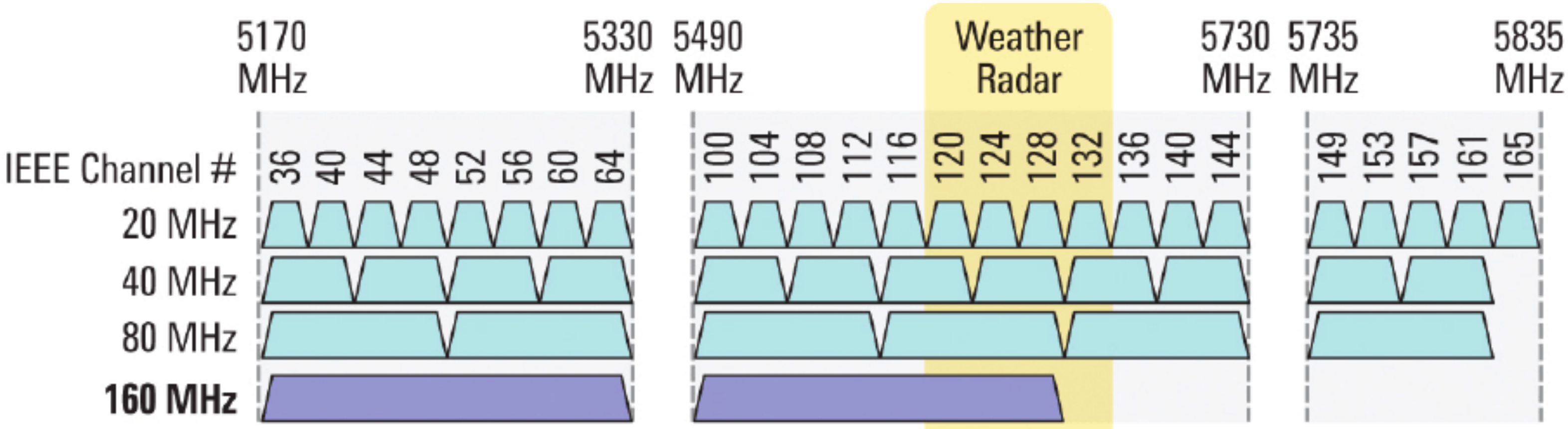


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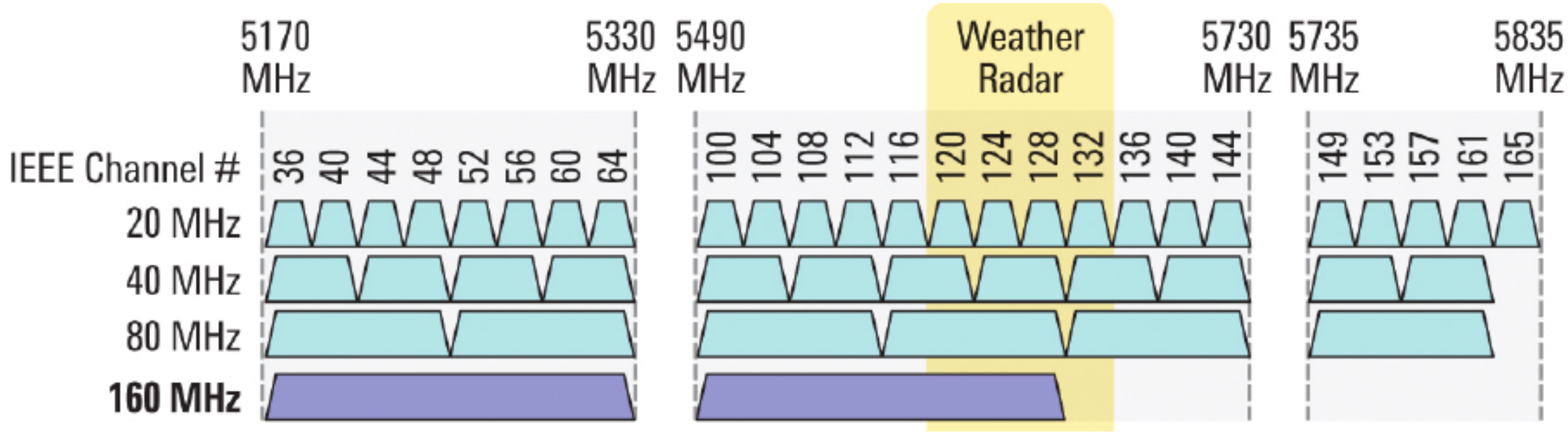


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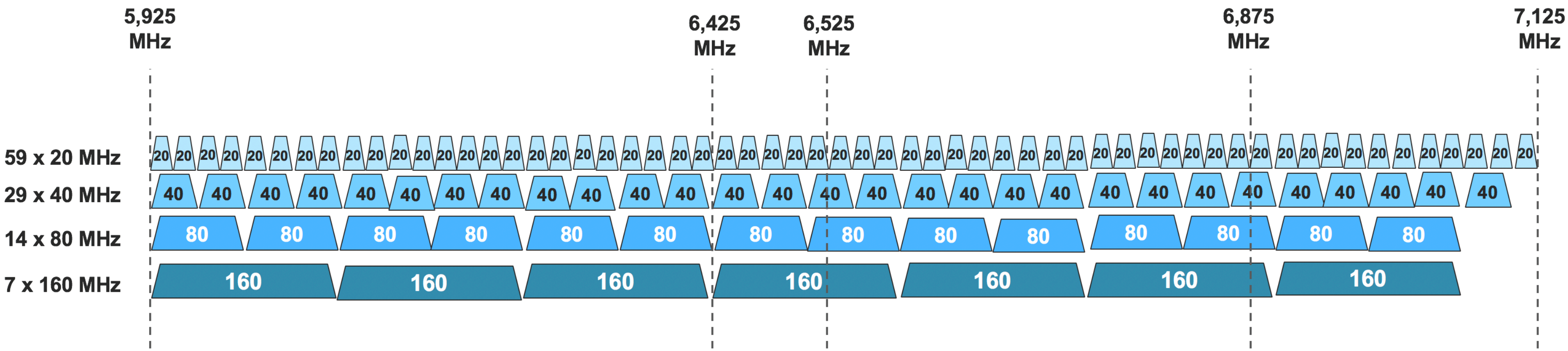
| WiFi 6 (802.11ax) | 2019 | OFDM (N=256, 512, 1024, 2048) MIMO (8x8) MU-MIMO (Up & Down) OFDMA | BPSK, QPSK, 16-QAM, 64-QAM 256-QAM, 1024-QAM | 2.4 GHz 5 GHz | 20 MHz 40 MHz 80 MHz 160 MHz | 9.608 Gb/s |

# WiFi Standards from WiFi 1 to WiFi 6

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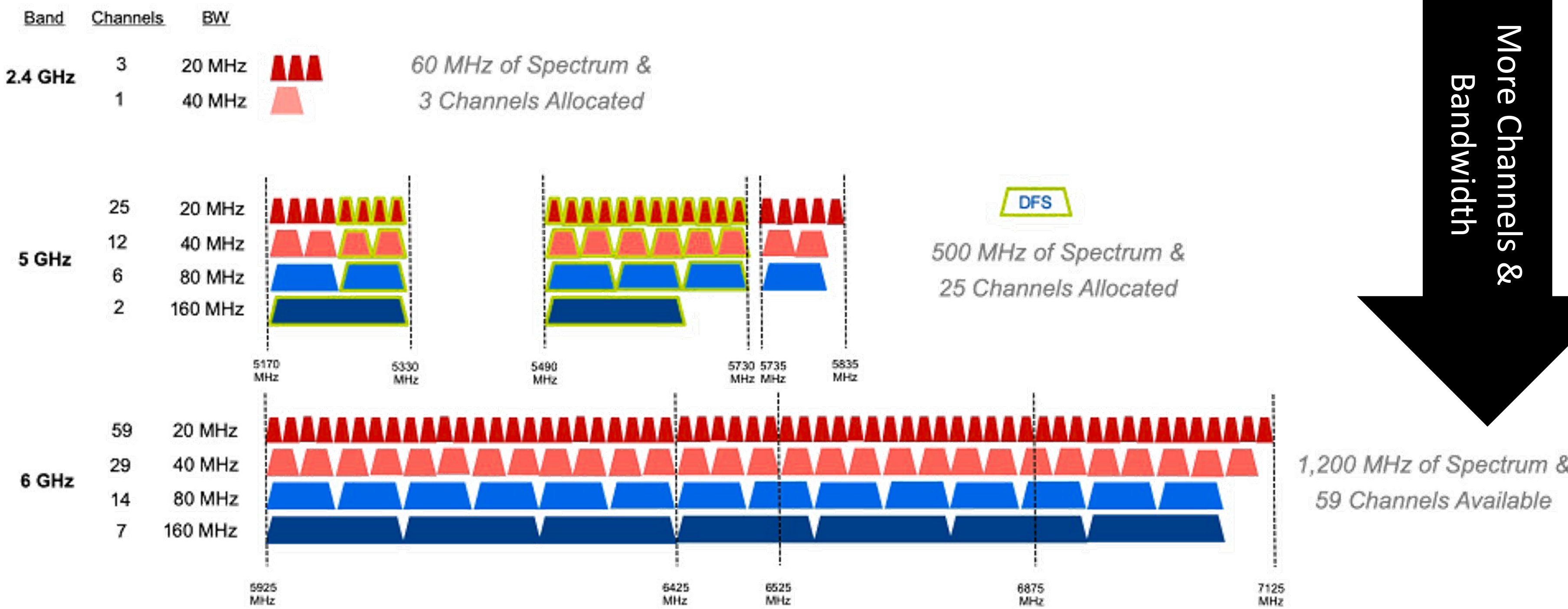
| WiFi 6 (802.11ax)<br>**WiFi 6E** | **2020** | OFDM (N=256, 512, 1024, 2048)<br>MIMO (8x8)<br>MU-MIMO (Up & Down)<br>OFDMA | BPSK, QPSK,<br>16-QAM, 64-QAM<br>256-QAM,<br>1024-QAM | 2.4 GHz<br>5 GHz<br>**6 GHz** | 20 MHz<br>40 MHz<br>80 MHz<br>160 MHz | 9.608 Gb/s |

# WiFi Standards from WiFi 1 to WiFi 6

More Channels & Bandwidth

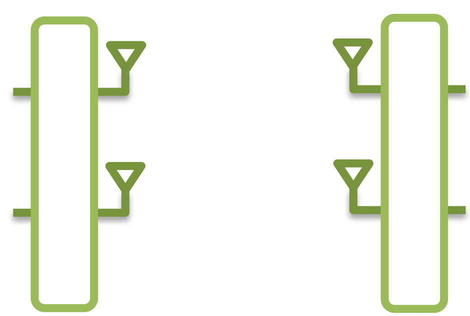



WiFi 1 More MIMO Antennas WiFi 5/6

# MIMO: Multiple Input Multiple Output

So far: single input single output

MIMO: multiple input multiple output



Increase capacity of channel using multiple transmit and receive antennas.

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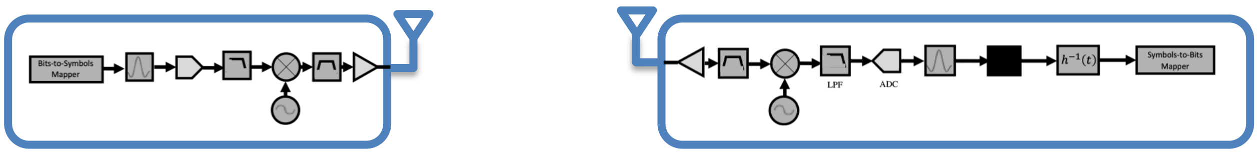
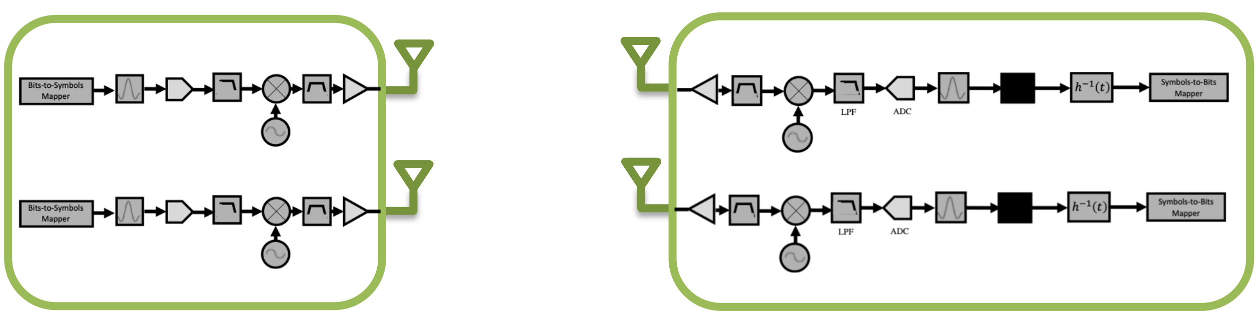
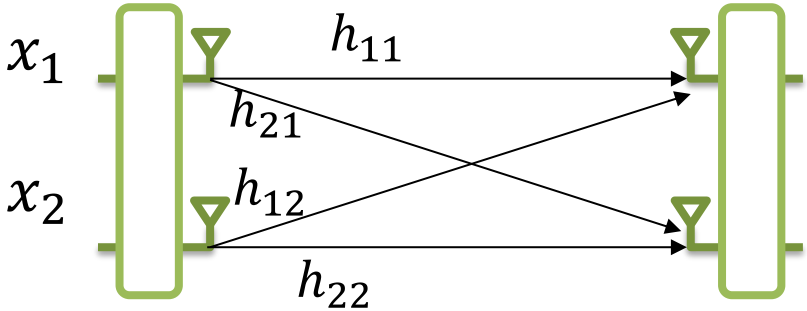



MIMO: multiple input multiple output




Increase capacity of channel using multiple transmit and receive antennas.

# MIMO: Multiple TX-RX streams




$$y_1 = h_{11}x_1 + h_{12}x_2$$

$$y_2 = h_{21}x_1 + h_{22}x_2$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \Rightarrow \mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

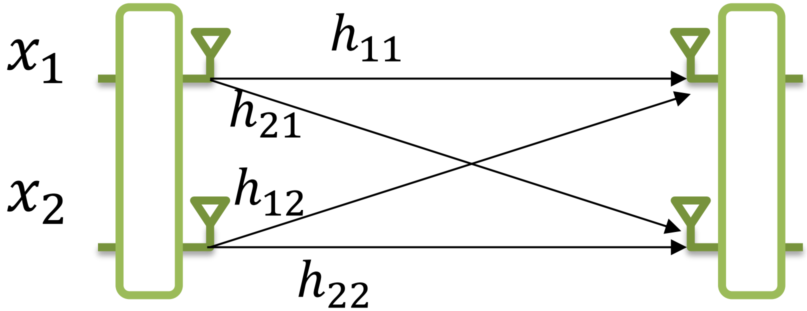
How to recover $x_1$ and $x_2$?

Estimate $\mathbf{H}$, compute $\mathbf{H}^{-1}$ and invert the channel!

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{Hx} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

**Transmit 2 packets at the same time!**

# MIMO: Multiple TX-RX streams

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**For $N$ antennas, $\mathbf{H}$ is $N \times N$ matrix → $O(N^3)$**

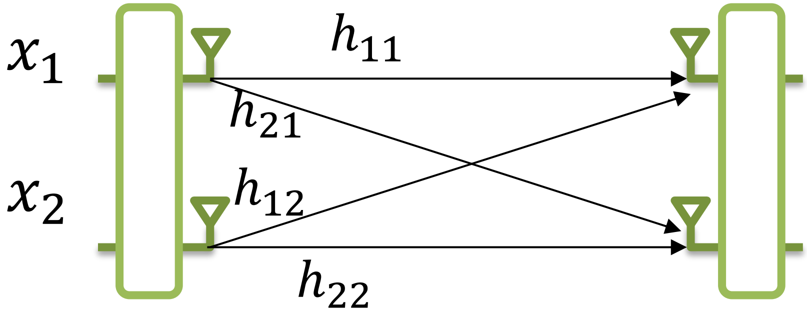
How to recover $x_1$ and $x_2$?

Estimate $\mathbf{H}$, compute $\mathbf{H}^{-1}$ and invert the channel!

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{Hx} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

**Noise amplification**

# MIMO: Vector Representation

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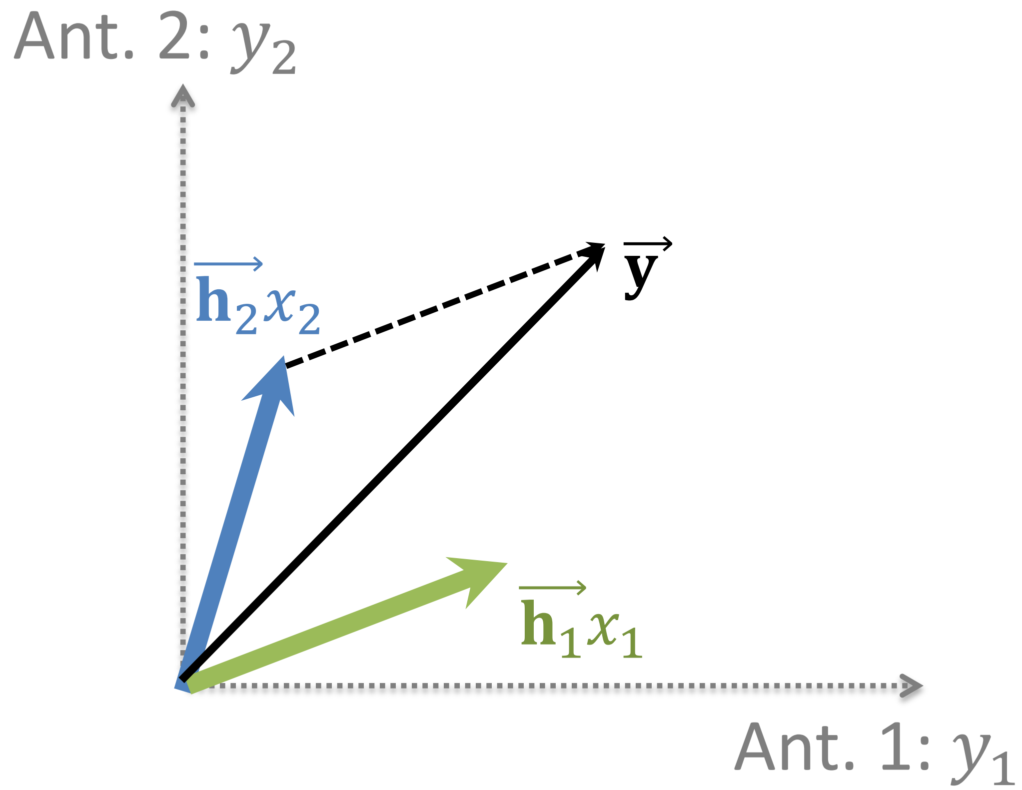
How to recover $x_1$ and $x_2$?

$$\mathbf{y} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} x_1 + \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2 = \overrightarrow{\mathbf{h}_1} x_1 + \overrightarrow{\mathbf{h}_2} x_2$$

# MIMO: Antenna Space

$$\overrightarrow{\mathbf{y}} = \overrightarrow{\mathbf{h}_1} x_1 + \overrightarrow{\mathbf{h}_2} x_2$$



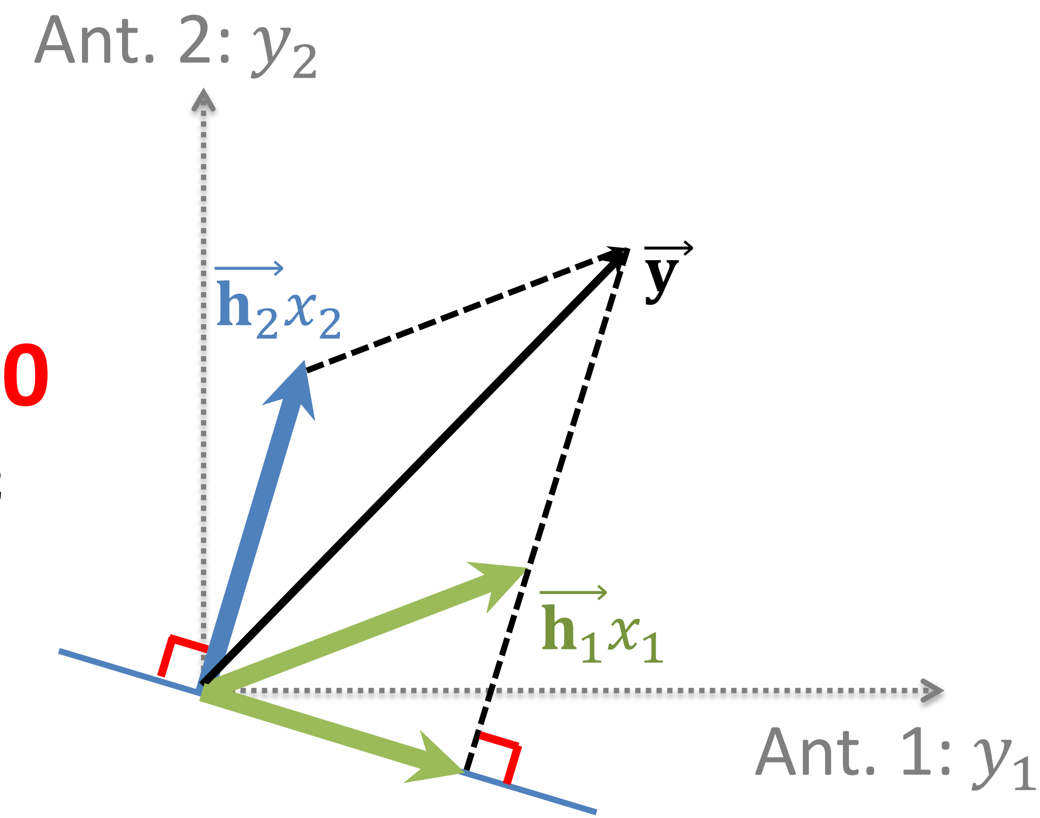
# MIMO: Antenna Space

$$\overrightarrow{\mathbf{y}} = \overrightarrow{\mathbf{h}_1}x_1 + \overrightarrow{\mathbf{h}_2}x_2$$

To decode $x_1$, project on a vector $\overrightarrow{\mathbf{h}_2}^{\perp}$ orthogonal to $\overrightarrow{\mathbf{h}_2}$

$$\overrightarrow{\mathbf{h}_2}^{\perp}\overrightarrow{\mathbf{y}} = \overrightarrow{\mathbf{h}_2}^{\perp}\overrightarrow{\mathbf{h}_1}x_1 + \cancelto{0}{\overrightarrow{\mathbf{h}_2}^{\perp}\overrightarrow{\mathbf{h}_2}x_2}$$

$$= \overrightarrow{\mathbf{h}_2}^{\perp}\overrightarrow{\mathbf{h}_1}x_1$$




$$\mathbf{y} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} x_1 + \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2$$

$$\overrightarrow{\mathbf{h}_2}^{\perp} = \boxed{\begin{matrix} h_{22} & -h_{12} \end{matrix}}$$

$$\overrightarrow{\mathbf{h}_2}^{\perp}\overrightarrow{\mathbf{y}} = h_{22}h_{11}x_1 - h_{12}h_{21}x_1$$

$$\cancelto{0}{+ h_{22}h_{12}x_2 - h_{12}h_{22}x_2}$$

$$= (h_{22}h_{11} - h_{12}h_{21})\, x_1$$

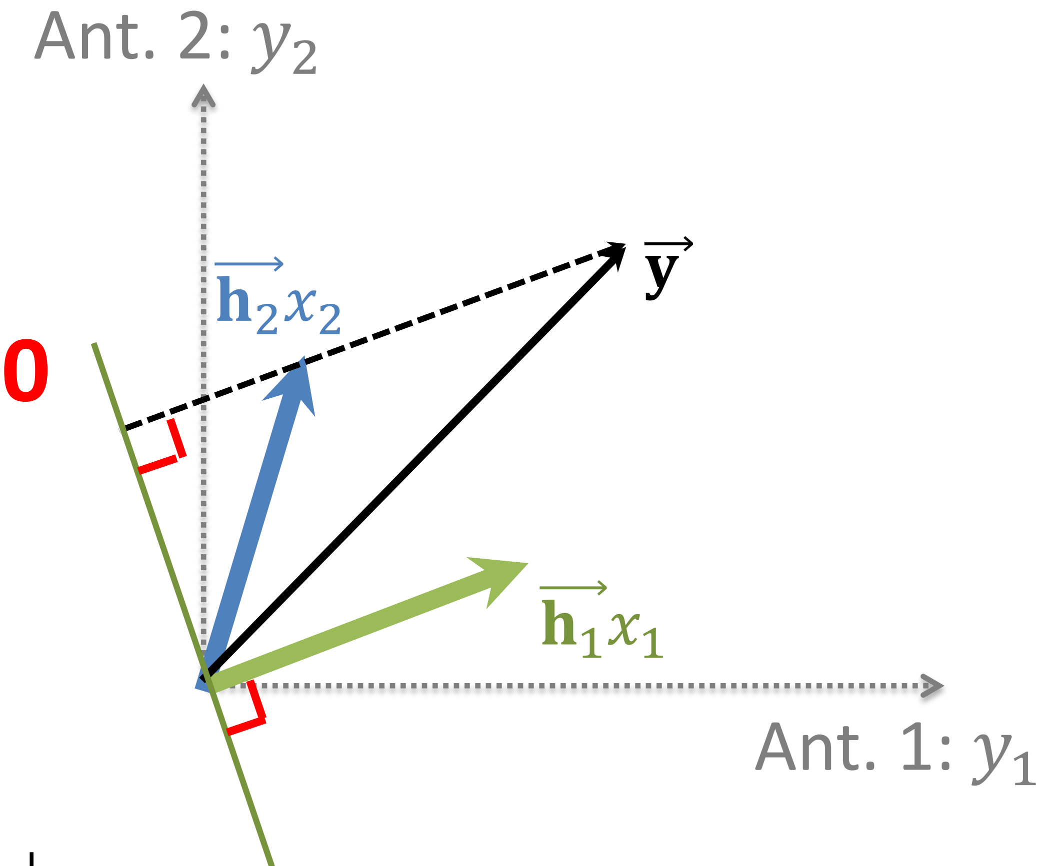
# MIMO: Antenna Space

$$\vec{\mathbf{y}} = \vec{\mathbf{h}_1} x_1 + \vec{\mathbf{h}_2} x_2$$

To decode $x_2$, project on a vector $\vec{\mathbf{h}_1}^{\perp}$ orthogonal to $\vec{\mathbf{h}_1}$

$$\vec{\mathbf{h}_1}^{\perp} \vec{\mathbf{y}} = \vec{\mathbf{h}_1}^{\perp} \vec{\mathbf{h}_1} x_1 + \cancelto{0}{\vec{\mathbf{h}_1}^{\perp} \vec{\mathbf{h}_2}} x_2$$

$$= \vec{\mathbf{h}_1}^{\perp} \vec{\mathbf{h}_2} x_2$$

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$$\vec{\mathbf{h}_1}^{\perp} = \boxed{h_{21} \quad -h_{11}}$$

$$\vec{\mathbf{h}_1}^{\perp} \vec{\mathbf{y}} = h_{21} h_{11} x_1 - h_{21} h_{11} x_1$$

$$\cancelto{0}{+ h_{21} h_{12} x_2 - h_{11} h_{22} x_2}$$

$$= (h_{21} h_{12} - h_{11} h_{22})\, x_2$$

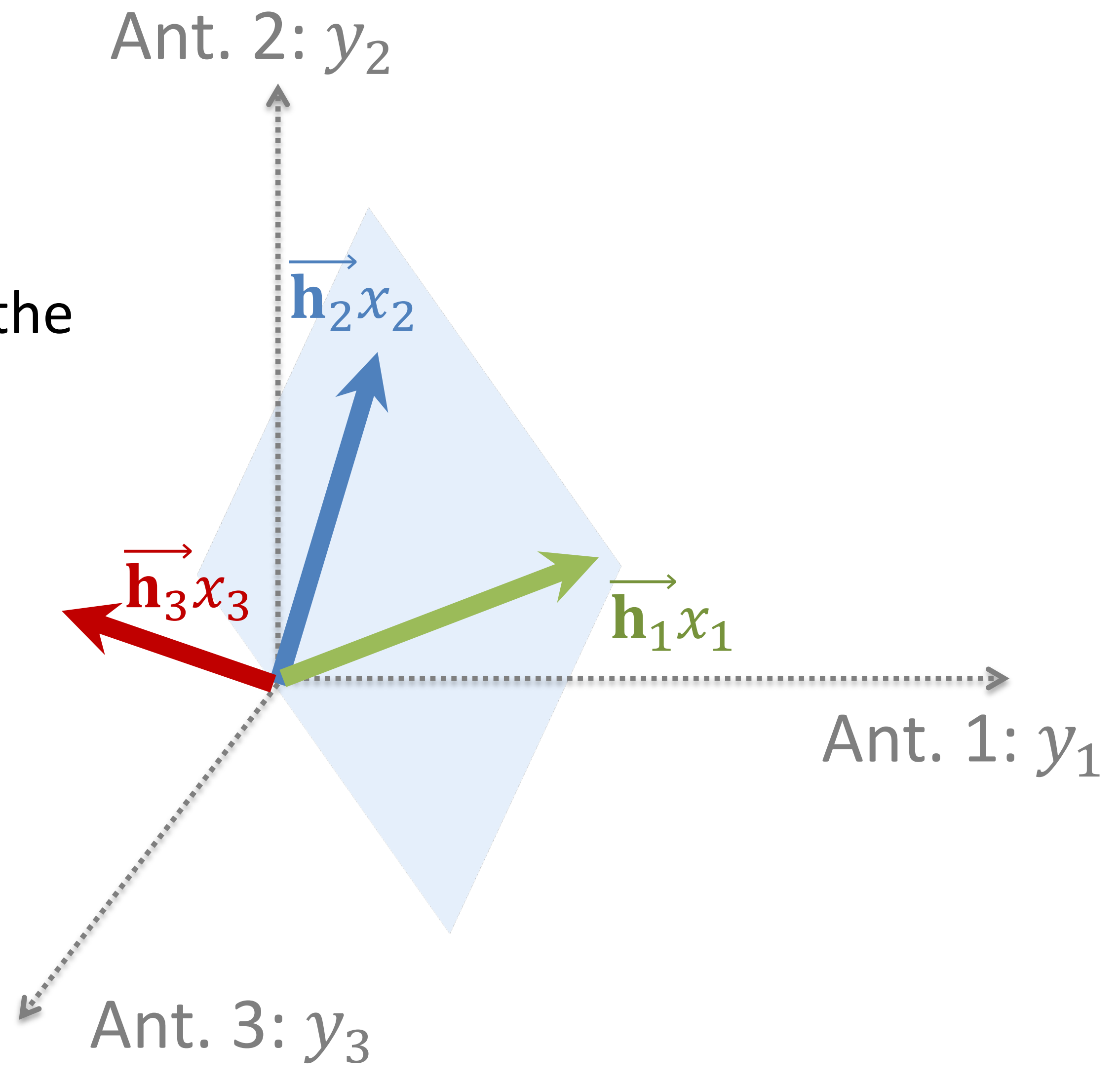
# MIMO: Antenna Space

**What about MIMO with more antennas?**

$$\vec{\mathbf{y}} = \overrightarrow{\mathbf{h}_1} x_1 + \overrightarrow{\mathbf{h}_2} x_2 + \overrightarrow{\mathbf{h}_3} x_3$$

To decode $x_1$, project on a vector $\overrightarrow{\mathbf{h}_{23}}^{\perp}$ orthogonal to the plane formed by $\overrightarrow{\mathbf{h}_2}$ and $\overrightarrow{\mathbf{h}_3}$

$$\overrightarrow{\mathbf{h}_{23}}^{\perp} = \overrightarrow{\mathbf{h}_2} \times \overrightarrow{\mathbf{h}_3}$$



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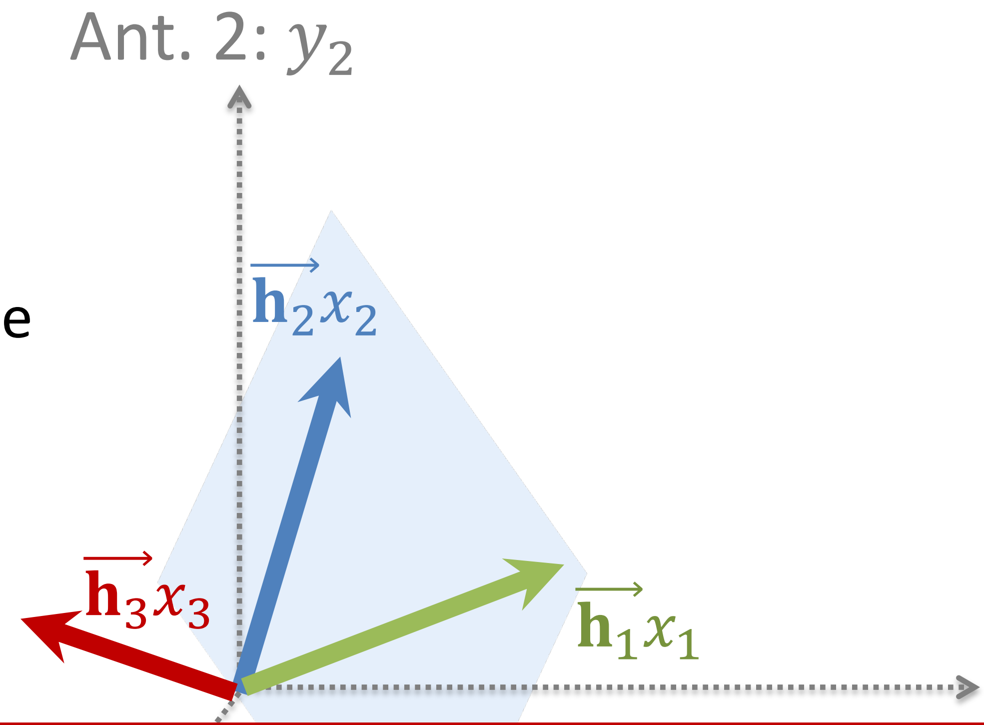
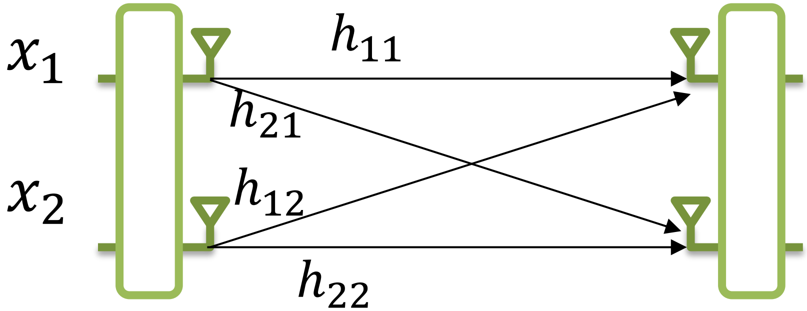



$N$ antenna MIMO, receives signals in $N$ dimensional space

→ Can decode $N$ parallel signals

Decoding complexity scales $O(N^2)$ operations with the number of antennas

# MIMO Channel

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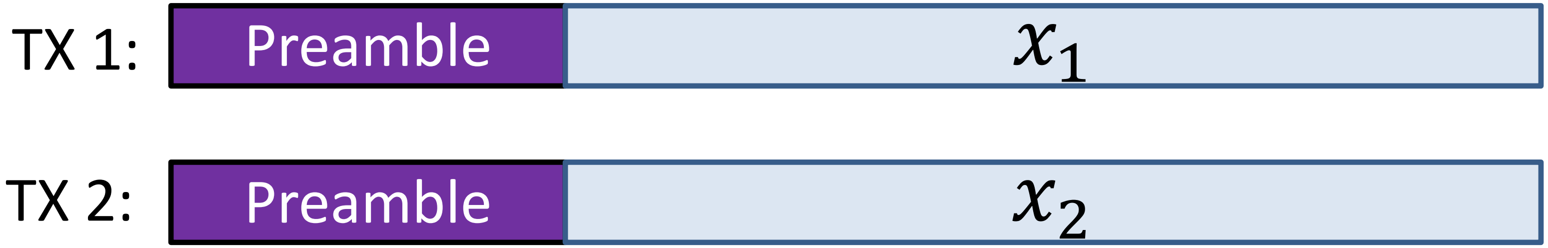
How to estimate the channels: $h_{11}, h_{12}, h_{21}, h_{22}$?



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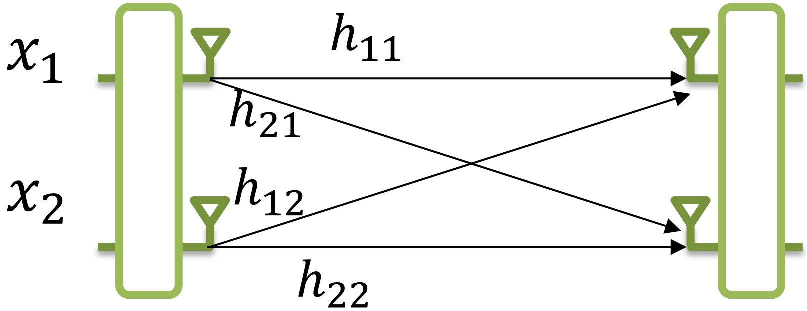


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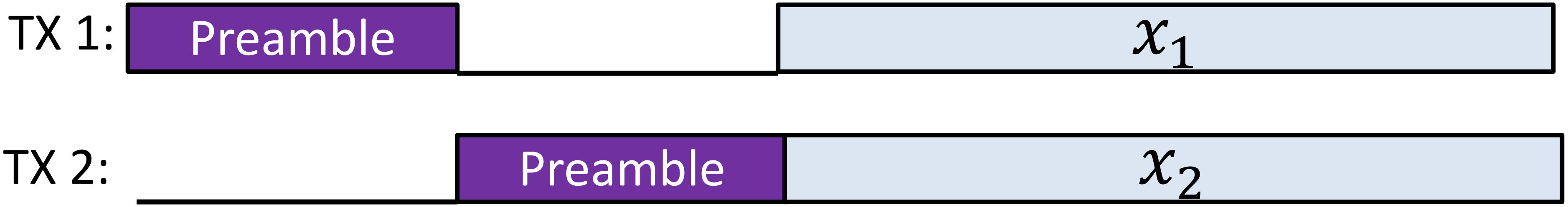


# MIMO Gains

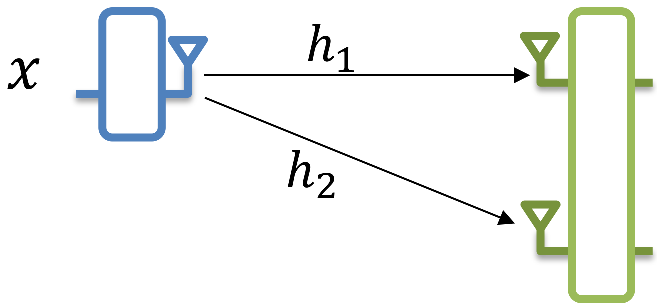
**Multiplexing Gain:**

- Send multiple packets at the same time
- $N \times N$ MIMO → $N\times$ more packets

**Diversity Gain:**

- Send/Receive the same packet on multiple antennas
- Increase SNR of the received packets → transmit at higher data rates

# Receiver Diversity




$y_1 = h_1 x + n_1$

$y_2 = h_2 x + n_2$

How to best decode $x$?

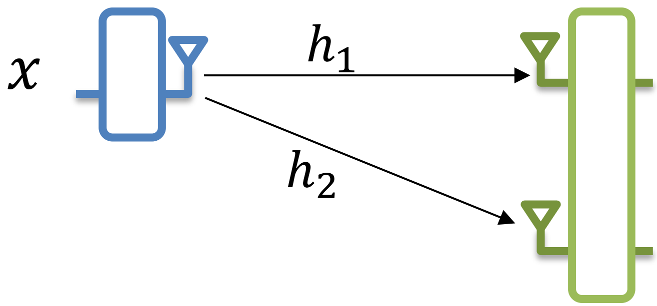
Option 1: Add the received signals

$$y_1 + y_2 = h_1 x + n_1 + h_2 x + n_2$$

$$= (h_1 + h_2)x + n_1 + n_2$$

Channels can sum up destructively! $h_1 + h_2 \approx 0$

# Receiver Diversity




$y_1 = h_1 x + n_1$

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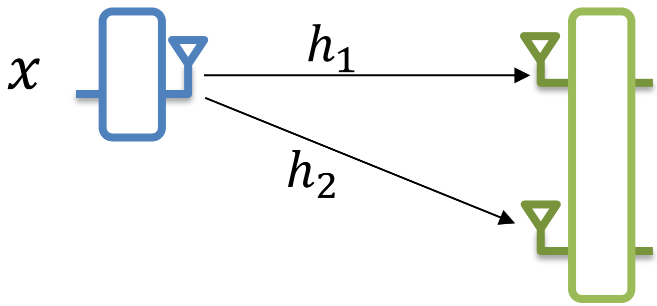
How to best decode $x$?

Option 1: Add the received signals

Option 2: Decode independently

Sub-optimal!

# Receiver Diversity

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Option 1: Add the received signals

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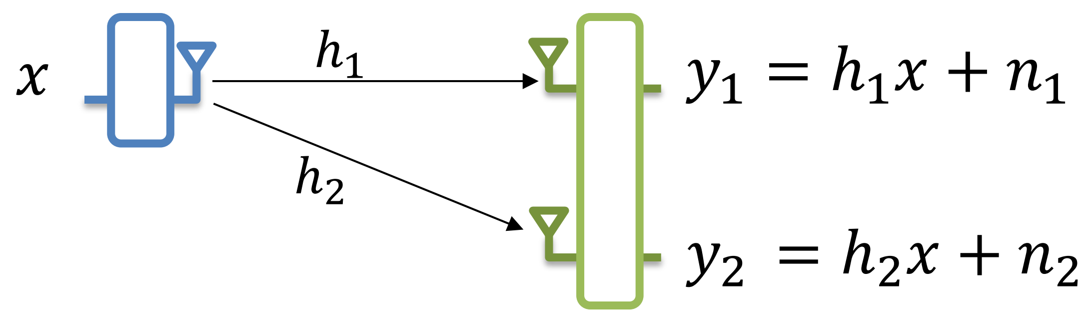
Optimal Solution: Maximum Ratio Combining (MRC)

# Maximum Ratio Combining




$$\alpha_1 y_1 + \alpha_2 y_2 = h_1^* y_1 + h_2^* y_2 = h_1^* h_1 x + h_1^* n_1 + h_2^* h_2 x + h_2^* n_2$$

$$= (|h_1|^2 + |h_2|^2)x + h_1^* n_1 + h_2^* n_2$$

---

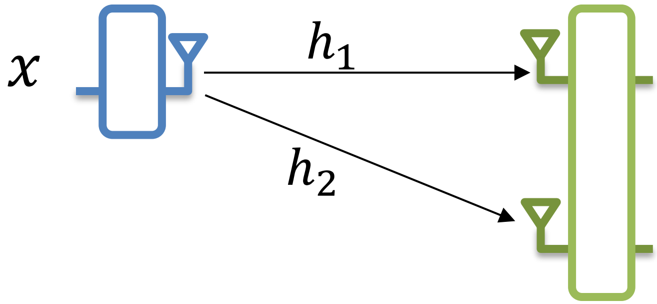
Let $P = \mathbb{E}[|x|^2]$ and $\sigma^2 = \mathbb{E}[|n_1|^2] = \mathbb{E}[|n_2|^2]$

$$Signal\ Power = \mathbb{E}[|(|h_1|^2 + |h_2|^2)x|^2]$$

$$= (|h_1|^2 + |h_2|^2)^2\ \mathbb{E}[|x|^2]$$

$$= (|h_1|^2 + |h_2|^2)^2\ P$$

# Maximum Ratio Combining

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$$= (|h_1|^2 + |h_2|^2)x + h_1^* n_1 + h_2^* n_2$$

---

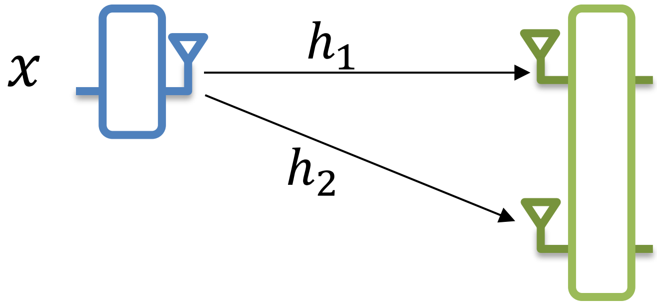
Let $P = \mathbb{E}[|x|^2]$ and $\sigma^2 = \mathbb{E}[|n_1|^2] = \mathbb{E}[|n_2|^2]$

$$Signal\ Power = (|h_1|^2 + |h_2|^2)^2\ P$$

$$Noise\ Power = \mathbb{E}[|h_1^* n_1 + h_2^* n_2|^2] = \mathbb{E}[|h_1^* n_1|^2] + \mathbb{E}[|h_2^* n_2|^2]$$

$$= |h_1|^2 \mathbb{E}[|n_1|^2] + |h_2|^2 \mathbb{E}[|n_2|^2] = (|h_1|^2 + |h_2|^2)\sigma^2$$

# Maximum Ratio Combining




$$y_1 = h_1 x + n_1$$

$$y_2 = h_2 x + n_2$$

$$\alpha_1 y_1 + \alpha_2 y_2 = h_1^* y_1 + h_2^* y_2 = h_1^* h_1 x + h_1^* n_1 + h_2^* h_2 x + h_2^* n_2$$

$$= (|h_1|^2 + |h_2|^2) x + h_1^* n_1 + h_2^* n_2$$

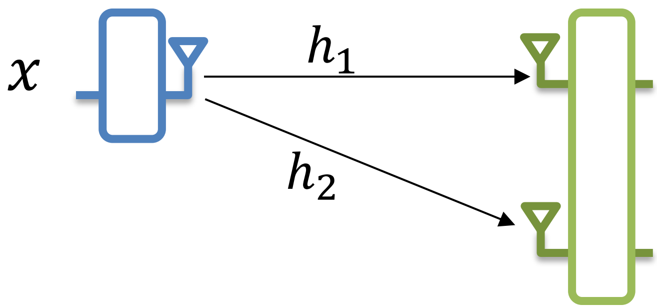
Let $P = \mathbb{E}[|x|^2]$ and $\sigma^2 = \mathbb{E}[|n_1|^2] = \mathbb{E}[|n_2|^2]$

$$Signal\ Power = (|h_1|^2 + |h_2|^2)^2\, P$$

$$Noise\ Power = (|h_1|^2 + |h_2|^2)\sigma^2$$

$$SNR = \frac{(|h_1|^2 + |h_2|^2)^2\, P}{(|h_1|^2 + |h_2|^2)\sigma^2} = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$$

# Maximum Ratio Combining




$$y_1 = h_1 x + n_1$$

$$y_2 = h_2 x + n_2$$

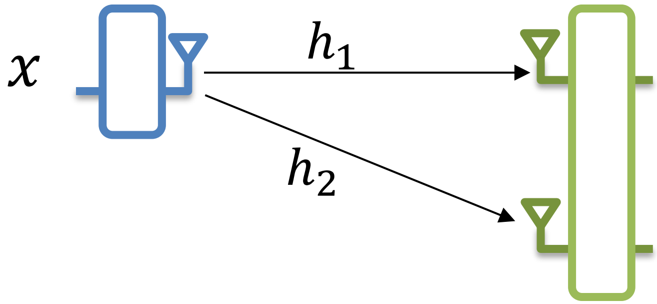
$$\alpha_1 y_1 + \alpha_2 y_2 = h_1^* y_1 + h_2^* y_2 = h_1^* h_1 x + h_1^* n_1 + h_2^* h_2 x + h_2^* n_2$$

$$= (|h_1|^2 + |h_2|^2)x + h_1^* n_1 + h_2^* n_2$$

With Receiver Diversity: $SNR = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$

Single Receiver: $SNR = |h_1|^2 \frac{P}{\sigma^2}$

# Receiver Diversity Gain




$$y_1 = h_1 x + n_1$$

$$y_2 = h_2 x + n_2$$

With Receiver Diversity: $SNR = (|h_1|^2 + |h_2|^2)\dfrac{P}{\sigma^2}$

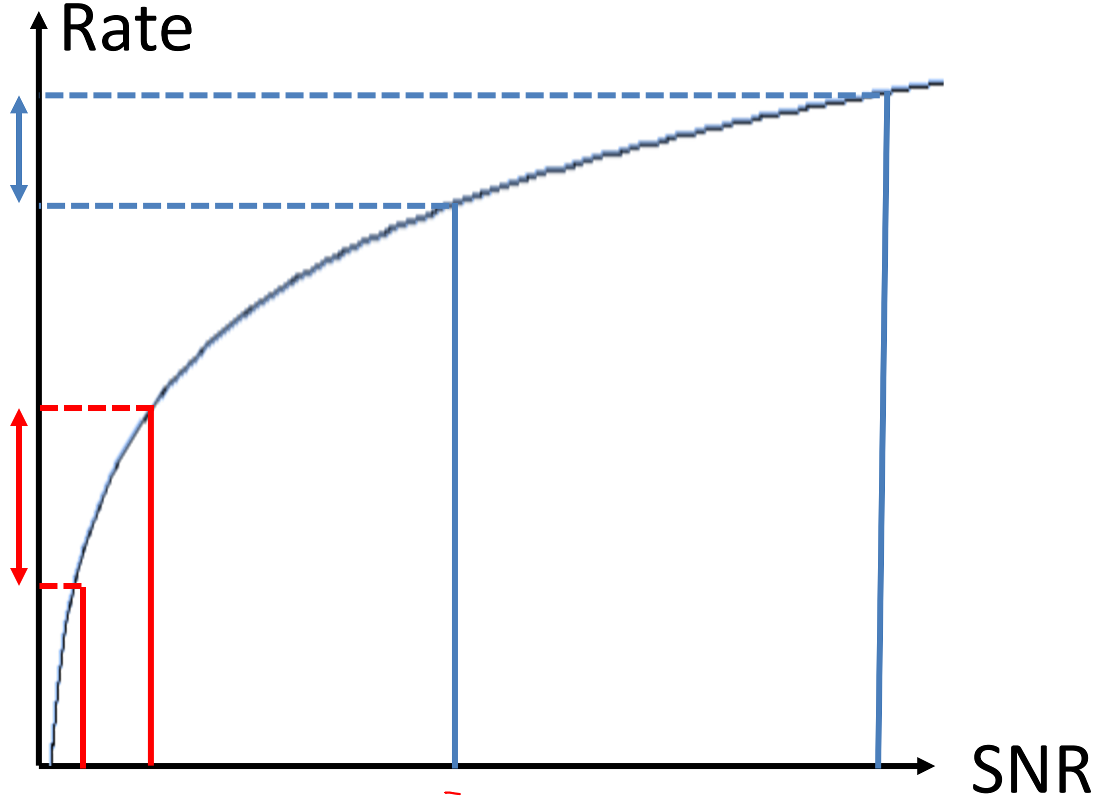
Single Receiver: $SNR = |h_1|^2 \dfrac{P}{\sigma^2}$

- $|h_1|^2 \approx |h_2|^2$ → Can double SNR!

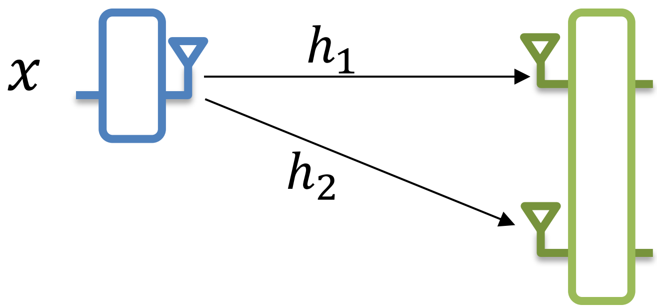
# Receiver Diversity Gain

## Do we care about doubling the SNR?

$$Capacity \propto \log(SNR)$$



# Receiver Diversity Gain




$y_1 = h_1 x + n_1$

$y_2 = h_2 x + n_2$

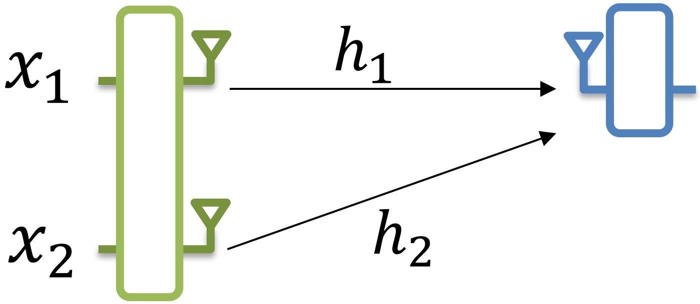
With Receiver Diversity: $SNR_{2RX} = (|h_1|^2 + |h_2|^2)\dfrac{P}{\sigma^2}$

Single Receiver: $SNR_{1RX} = |h_1|^2 \dfrac{P}{\sigma^2}$

- $|h_1|^2 \approx |h_2|^2$ → Can double SNR!
- $|h_1|^2 \ll |h_2|^2$ → Huge Gain in SNR
- $|h_1|^2 \gg |h_2|^2$ → Little Gain in SNR

It is unlikely that both antennas experience channel fading.

# Transmitter Diversity




$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Option 1: transmit the same thing $x$ on both antennas

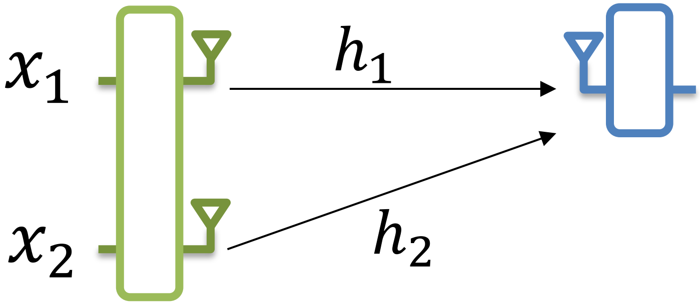
$$y = (h_1 + h_2)x + n$$

Channels can sum up destructively! $h_1 + h_2 \approx 0$

Total transmit power $= 2P$ → Doubled TX power

→ Why not use 1 TX with $2P$

# Transmitter Diversity



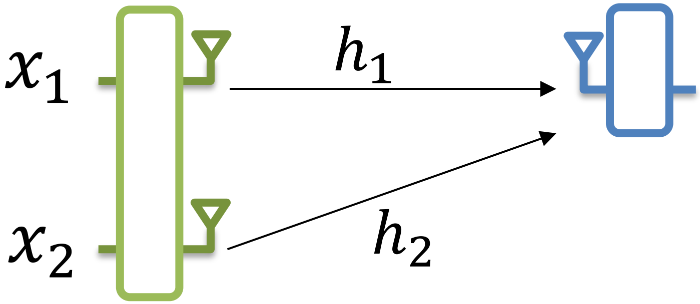

$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Option 1: transmit the same thing $x$ on both antennas

- Must ensure signals sum up constructively.
- Must ensure total TX power $= \mathbb{E}[|x_1|^2] + \mathbb{E}[|x_2|^2]$
  $= \mathbb{E}[|x|^2] = P$

# Transmitter Diversity




$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

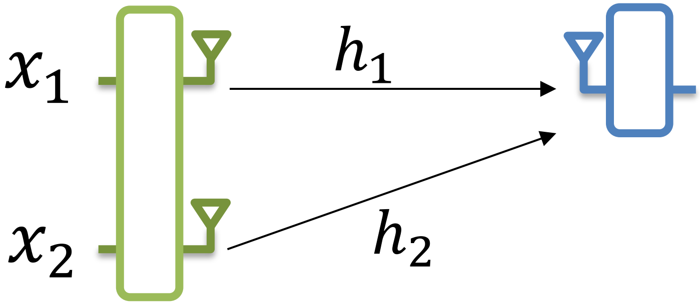
Option 2: Maximum Ratio Combining (MRC)

$$\left.\begin{aligned} x_1 &= \alpha_1 x \\ x_2 &= \alpha_2\, x \end{aligned}\right\} y = (\alpha_1 h_1 + \alpha_2 h_2)x + n$$

Set: $\alpha_1 = h_1^*, \alpha_2 = h_2^*$ $\rightarrow y = (h_1^* h_1 + h_2^* h_2)x + n$

$$= (|h_1|^2 + |h_2|^2)x + n$$

# Transmitter Diversity




$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

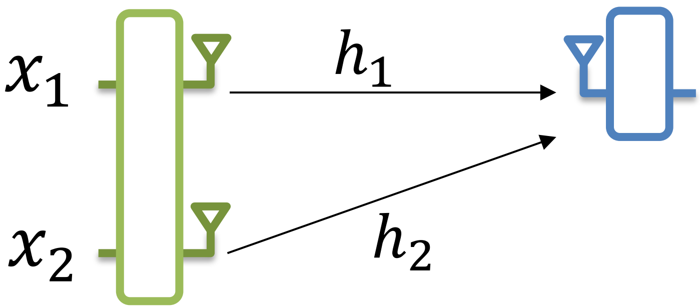
Option 2: Maximum Ratio Combining (MRC)

$$x_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x \qquad\qquad x_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x$$

$$Total\ TX\ Power = \mathbb{E}[|x_1|^2] + \mathbb{E}[|x_1|^2]$$

$$= \frac{|h_1|^2}{|h_1|^2 + |h_2|^2} P + \frac{|h_2|^2}{|h_1|^2 + |h_2|^2} P = P$$

# Transmitter Diversity




$y = h_1 x_1 + h_2 x_2 + n$

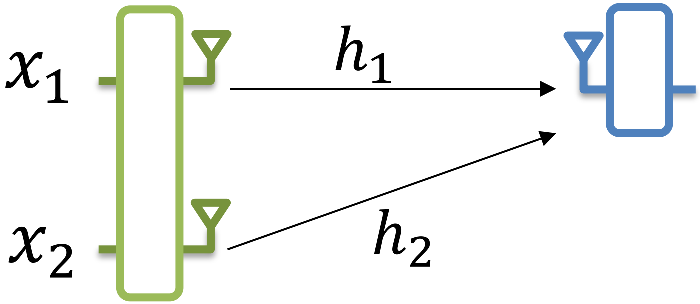
What should we transmit on each antenna?

Option 2: Maximum Ratio Combining (MRC)

$$x_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x \qquad x_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x$$

$$y = \frac{h_1 h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x + \frac{h_2 h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x + n$$

# Transmitter Diversity




$$y = h_1 x_1 + h_2 x_2 + n$$

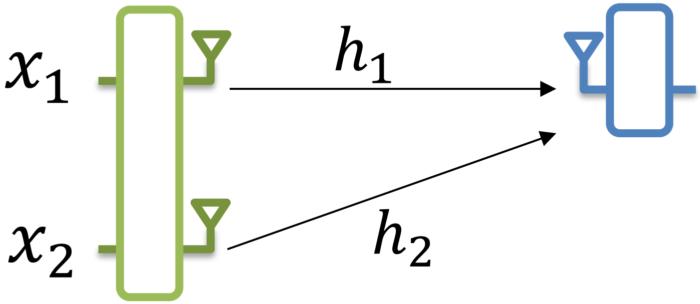
What should we transmit on each antenna?

Option 2: Maximum Ratio Combining (MRC)

$$x_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x \qquad x_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x$$

$$y = \frac{|h_1|^2 + |h_2|^2}{\sqrt{|h_1|^2 + |h_2|^2}}\, x + n = \left(\sqrt{|h_1|^2 + |h_2|^2}\right) x + n$$

# Transmitter Diversity




$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

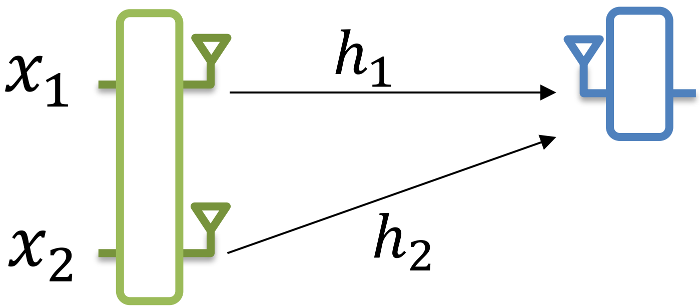
Option 2: Maximum Ratio Combining (MRC)

$$y = \left(\sqrt{|h_1|^2 + |h_2|^2}\right) x + n$$

$$SNR = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$$ → Similar SNR Gain to RX Diversity

**Caveat: MRC at TX Requires Feedback from the Receiver!**

# Transmitter Diversity




$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

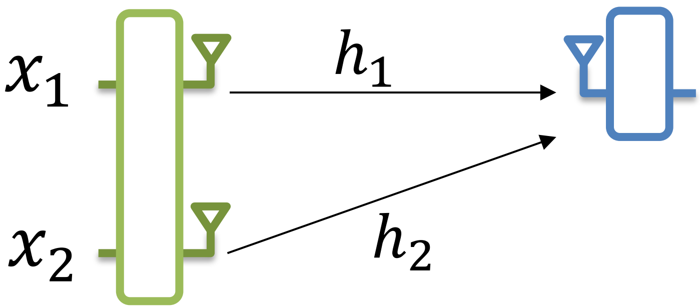
MRC codes across space only (Requires Channel Feedback):

| | | |
|---|---|---|
| TX1: $x_1$ | $\alpha_1 x[1]$ | $\alpha_1 x[2]$ |
| TX2: $x_2$ | $\alpha_2 x[1]$ | $\alpha_2 x[2]$ |
| | $t = 1$ | $t = 2$ |

$$\alpha_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}$$

$$\alpha_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}$$

# Transmitter Diversity




$$y = h_1x_1 + h_2x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

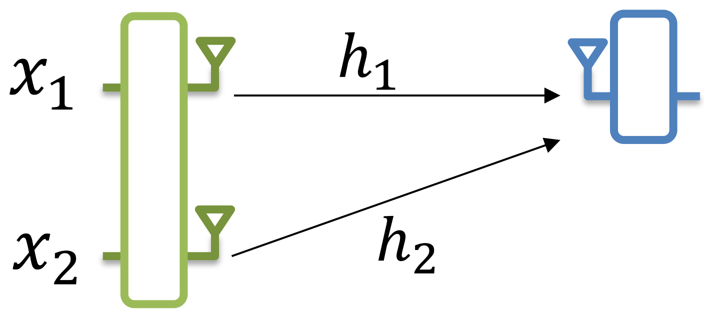
Alamouti Codes:

| | $t = 1$ | $t = 2$ |
|---|---|---|
| TX1: $x_1$ | $x[1]$ | $-x^*[2]$ |
| TX2: $x_2$ | $x[2]$ | $x^*[1]$ |

$$y[1] = h_1x[1] + h_2x[2]$$

$$y[2] = -h_1x^*[2] + h_2x^*[1]$$

# Transmitter Diversity




$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

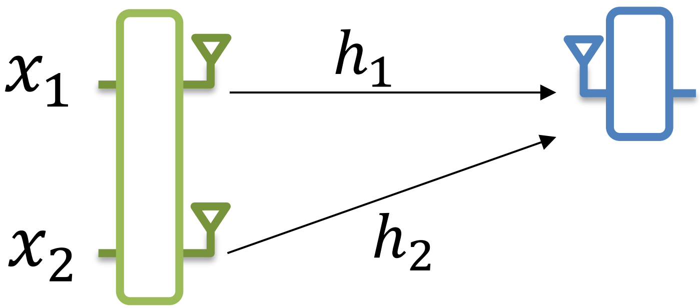
Solution: Use Space-Time Codes

Alamouti Codes:

$$y[1] = h_1 x[1] + h_2 x[2] \qquad y[2] = -h_1 x^*[2] + h_2 x^*[1]$$

$$\begin{aligned} h_1^* y[1] + h_2 y^*[2] &= h_1^* h_1 x[1] + \cancel{h_1^* h_2 x[2]} \\ &\quad \cancel{-h_2 h_1^* x[2]} + h_2 h_2^* x[1] \\ &= (|h_1|^2 + |h_2|^2) x[1] \end{aligned}$$

# Transmitter Diversity




$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

Alamouti Codes:

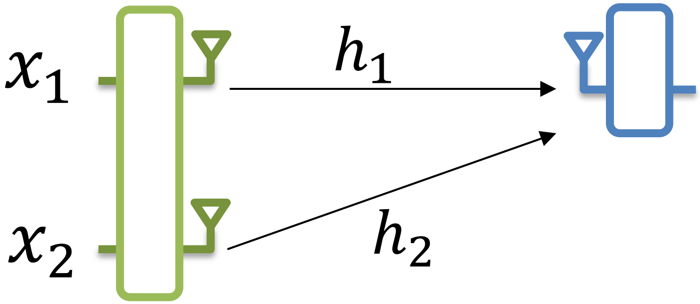
$$y[1] = h_1 x[1] + h_2 x[2] \qquad y[2] = -h_1 x^*[2] + h_2 x^*[1]$$

$$h_2^* y[1] - h_1 y^*[2] = h_2^* h_1 x[1] + \cancel{h_2^* h_2 x[2]}$$

$$+ \cancel{h_1 h_1^* x[2]} - h_1 h_2^* x[1]$$

$$= (|h_1|^2 + |h_2|^2) x[2]$$

# Transmitter Diversity




$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

Alamouti Codes:

| | $t = 1$ | $t = 2$ |
|---|---|---|
| TX1: $x_1$ | $\frac{x[1]}{\sqrt{2}}$ | $\frac{-x^*[2]}{\sqrt{2}}$ |
| TX2: $x_2$ | $\frac{x[2]}{\sqrt{2}}$ | $\frac{x^*[1]}{\sqrt{2}}$ |

$$h_1^* y[1] + h_2 y^*[2]$$

$$= (|h_1|^2 + |h_2|^2)\frac{x[1]}{\sqrt{2}}$$

$$h_2^* y[1] - h_1 y^*[2]$$

$$= (|h_1|^2 + |h_2|^2)\frac{x[2]}{\sqrt{2}}$$

# MIMO Gains

**Multiplexing Gain:**

- Send multiple packets at the same time
- $N \times N$ MIMO → $\times N$ more packets
- Data Rate: $\propto N \log(SNR/N)$

**Diversity Gain:**

- Increase SNR of the received packets
- $N \times N$ MIMO → $\times \log N^2$ data rate
- Data Rate $\propto \log(SNR \times N^2)$

# WiFi 802.11ac MCS (Modulations and Coding Schemes)

| MCS Index | | | Spatial Stream | Modulation | Coding | 20MHz | | 40MHz | | 80MHz | | 160MHz | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| HT | VHT | HE | | | | 0.8μs GI | 0.4μs GI | 0.8μs GI | 0.4μs GI | 0.8μs GI | 0.4μs GI | 0.8μs GI | 0.4μs GI |
| 0 | 0 | 0 | 1 | BPSK | 1/2 | 6.5 | 7.2 | 13.5 | 15 | 29.3 | 32.5 | 58.5 | 65 |
| 1 | 1 | 1 | 1 | QPSK | 1/2 | 13 | 14.4 | 27 | 30 | 58.5 | 65 | 117 | 130 |
| 2 | 2 | 2 | 1 | QPSK | 3/4 | 19.5 | 21.7 | 40.5 | 45 | 87.8 | 97.5 | 175.5 | 195 |
| 3 | 3 | 3 | 1 | 16-QAM | 1/2 | 26 | 28.9 | 54 | 60 | 117 | 130 | 234 | 260 |
| 4 | 4 | 4 | 1 | 16-QAM | 3/4 | 39 | 43.3 | 81 | 90 | 175.5 | 195 | 351 | 390 |
| 5 | 5 | 5 | 1 | 64-QAM | 2/3 | 52 | 57.8 | 108 | 120 | 234 | 260 | 468 | 520 |
| 6 | 6 | 6 | 1 | 64-QAM | 3/4 | 58.5 | 65 | 121.5 | 135 | 263.3 | 292.5 | 526.5 | 585 |
| 7 | 7 | 7 | 1 | 64-QAM | 5/6 | 65 | 72.2 | 135 | 150 | 292.5 | 325 | 585 | 650 |
| | 8 | 8 | 1 | 256-QAM | 3/4 | 78 | 86.7 | 162 | 180 | 351 | 390 | 702 | 780 |

# WiFi 802.11ac MCS (Modulations and Coding Schemes)

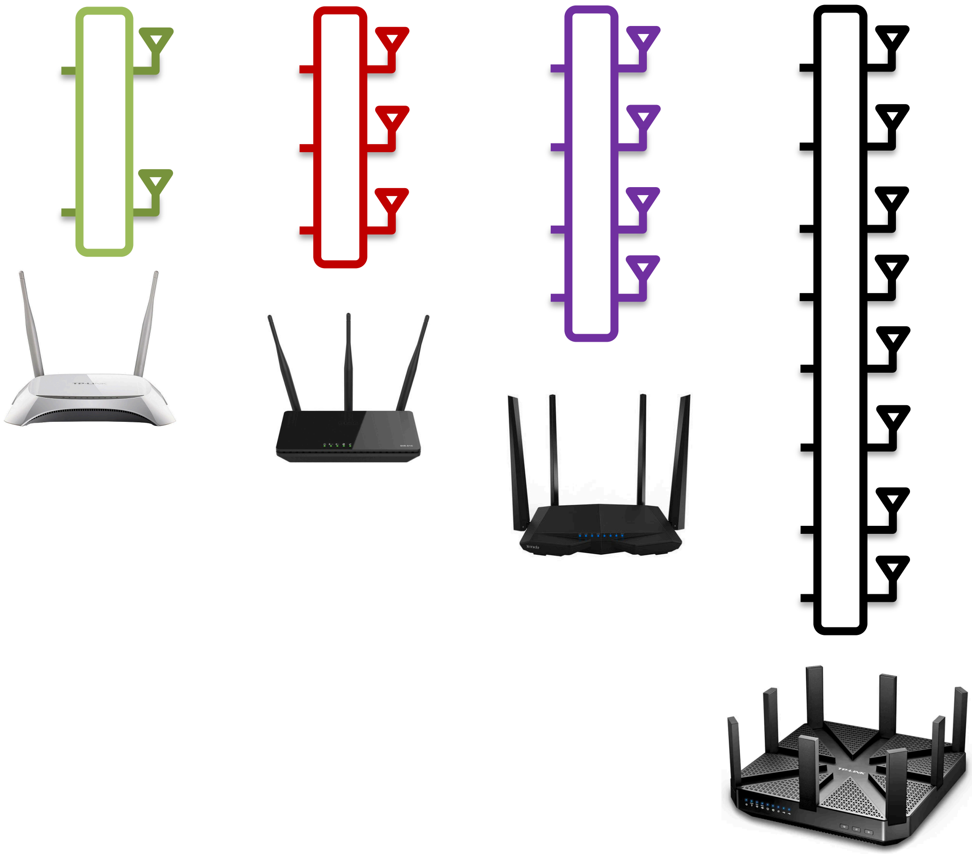
| MCS Index | | | Spatial Stream | Modulation | Coding | 20MHz | | 40MHz | | 80MHz | | 160MHz | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| HT | VHT | HE | | | | 0.8μs GI | 0.4μs GI | 0.8μs GI | 0.4μs GI | 0.8μs GI | 0.4μs GI | 0.8μs GI | 0.4μs GI |
| 0 | 0 | 0 | 1 | BPSK | 1/2 | 6.5 | 7.2 | 13.5 | 15 | 29.3 | 32.5 | 58.5 | 65 |
| 1 | 1 | 1 | 1 | QPSK | 1/2 | 13 | 14.4 | 27 | 30 | 58.5 | 65 | 117 | 130 |
| 2 | 2 | 2 | 1 | QPSK | 3/4 | 19.5 | 21.7 | 40.5 | 45 | 87.8 | 97.5 | 175.5 | 195 |
| 3 | 3 | 3 | 1 | 16-QAM | 1/2 | 26 | 28.9 | 54 | 60 | 117 | 130 | 234 | 260 |
| 4 | 4 | 4 | 1 | 16-QAM | 3/4 | 39 | 43.3 | 81 | 90 | 175.5 | 195 | 351 | 390 |
| 5 | 5 | 5 | 1 | 64-QAM | 2/3 | 52 | 57.8 | 108 | 120 | 234 | 260 | 468 | 520 |
| 6 | 6 | 6 | 1 | 64-QAM | 3/4 | 58.5 | 65 | 121.5 | 135 | 263.3 | 292.5 | 526.5 | 585 |
| 7 | 7 | 7 | 1 | 64-QAM | 5/6 | 65 | 72.2 | 135 | 150 | 292.5 | 325 | 585 | 650 |
| | 8 | 8 | 1 | 256-QAM | 3/4 | 78 | 86.7 | 162 | 180 | 351 | 390 | 702 | 780 |
| | 9 | 9 | 1 | 256-QAM | 5/6 | N/A | N/A | 180 | 200 | 390 | 433.3 | 780 | 866.7 |
| | | 10 | 1 | 1024-QAM | 3/4 | | | | | | | | |
| | | 11 | 1 | 1024-QAM | 5/6 | | | | | | | | |
| 8 | 0 | 0 | 2 | BPSK | 1/2 | 13 | 14.4 | 27 | 30 | 58.5 | 65 | 117 | 130 |
| 9 | 1 | 1 | 2 | QPSK | 1/2 | 26 | 28.9 | 54 | 60 | 117 | 130 | 234 | 260 |
| 10 | 2 | 2 | 2 | QPSK | 3/4 | 39 | 43.3 | 81 | 90 | 175.5 | 195 | 351 | 390 |
| 11 | 3 | 3 | 2 | 16-QAM | 1/2 | 52 | 57.8 | 108 | 120 | 234 | 260 | 468 | 520 |
| 12 | 4 | 4 | 2 | 16-QAM | 3/4 | 78 | 86.7 | 162 | 180 | 351 | 390 | 702 | 780 |
| 13 | 5 | 5 | 2 | 64-QAM | 2/3 | 104 | 115.6 | 216 | 240 | 468 | 520 | 936 | 1040 |
| 14 | 6 | 6 | 2 | 64-QAM | 3/4 | 117 | 130 | 243 | 270 | 526.5 | 585 | 1053 | 1170 |
| 15 | 7 | 7 | 2 | 64-QAM | 5/6 | 130 | 144.4 | 270 | 300 | 585 | 650 | 1170 | 1300 |
| | 8 | 8 | 2 | 256-QAM | 3/4 | 156 | 173.3 | 324 | 360 | 702 | 780 | 1404 | 1560 |

# WiFi 802.11ac MCS (Modulations and Coding Schemes)

| MCS Index | | | Spatial Stream | Modulation | Coding | 20MHz | | 40MHz | | 80MHz | | 160MHz | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| HT | VHT | HE | | | | 0.8µs GI | 0.4µs GI | 0.8µs GI | 0.4µs GI | 0.8µs GI | 0.4µs GI | 0.8µs GI | 0.4µs GI |
| 0 | 0 | 0 | 1 | BPSK | 1/2 | 6.5 | 7.2 | 13.5 | 15 | 29.3 | 32.5 | 58.5 | 65 |
| 1 | 1 | 1 | 1 | QPSK | 1/2 | 13 | 14.4 | 27 | 30 | 58.5 | 65 | 117 | 130 |
| 2 | 2 | 2 | 1 | QPSK | 3/4 | 19.5 | 21.7 | 40.5 | 45 | 87.8 | 97.5 | 175.5 | 195 |
| 3 | 3 | 3 | 1 | 16-QAM | 1/2 | 26 | 28.9 | 54 | 60 | 117 | 130 | 234 | 260 |
| 4 | 4 | 4 | 1 | 16-QAM | 3/4 | 39 | 43.3 | 81 | 90 | 175.5 | 195 | 351 | 390 |
| 5 | 5 | 5 | 1 | 64-QAM | 2/3 | 52 | 57.8 | 108 | 120 | 234 | 260 | 468 | 520 |
| 6 | 6 | 6 | 1 | 64-QAM | 3/4 | 58.5 | 65 | 121.5 | 135 | 263.3 | 292.5 | 526.5 | 585 |
| 7 | 7 | 7 | 1 | 64-QAM | 5/6 | 65 | 72.2 | 135 | 150 | 292.5 | 325 | 585 | 650 |
| | 8 | 8 | 1 | 256-QAM | 3/4 | 78 | 86.7 | 162 | 180 | 351 | 390 | 702 | 780 |
| | 9 | 9 | 1 | 256-QAM | 5/6 | N/A | N/A | 180 | 200 | 390 | 433.3 | 780 | 866.7 |
| | | 10 | 1 | 1024-QAM | 3/4 | | | | | | | | |
| | | 11 | 1 | 1024-QAM | 5/6 | | | | | | | | |
| 8 | 0 | 0 | 2 | BPSK | 1/2 | 13 | 14.4 | 27 | 30 | 58.5 | 65 | 117 | 130 |
| 9 | 1 | 1 | 2 | QPSK | 1/2 | 26 | 28.9 | 54 | 60 | 117 | 130 | 234 | 260 |
| 10 | 2 | 2 | 2 | QPSK | 3/4 | 39 | 43.3 | 81 | 90 | 175.5 | 195 | 351 | 390 |
| 11 | 3 | 3 | 2 | 16-QAM | 1/2 | 52 | 57.8 | 108 | 120 | 234 | 260 | 468 | 520 |
| 12 | 4 | 4 | 2 | 16-QAM | 3/4 | 78 | 86.7 | 162 | 180 | 351 | 390 | 702 | 780 |
| 13 | 5 | 5 | 2 | 64-QAM | 2/3 | 104 | 115.6 | 216 | 240 | 468 | 520 | 936 | 1040 |
| 14 | 6 | 6 | 2 | 64-QAM | 3/4 | 117 | 130 | 243 | 270 | 526.5 | 585 | 1053 | 1170 |
| 15 | 7 | 7 | 2 | 64-QAM | 5/6 | 130 | 144.4 | 270 | 300 | 585 | 650 | 1170 | 1300 |
| | 8 | 8 | 2 | 256-QAM | 3/4 | 156 | 173.3 | 324 | 360 | 702 | 780 | 1404 | 1560 |
| | 9 | 9 | 2 | 256-QAM | 5/6 | N/A | N/A | 360 | 400 | 780 | 866.7 | 1560 | 1733.3 |
| | | 10 | 2 | 1024-QAM | 3/4 | | | | | | | | |
| | | 11 | 2 | 1024-QAM | 5/6 | | | | | | | | |
| 16 | 0 | 0 | 3 | BPSK | 1/2 | 19.5 | 21.7 | 40.5 | 45 | 87.8 | 97.5 | 175.5 | 195 |
| 17 | 1 | 1 | 3 | QPSK | 1/2 | 39 | 43.3 | 81 | 90 | 175.5 | 195 | 351 | 390 |
| 18 | 2 | 2 | 3 | QPSK | 3/4 | 58.5 | 65 | 121.5 | 135 | 263.3 | 292.5 | 526.5 | 585 |
| 19 | 3 | 3 | 3 | 16-QAM | 1/2 | 78 | 86.7 | 162 | 180 | 351 | 390 | 702 | 780 |
| 20 | 4 | 4 | 3 | 16-QAM | 3/4 | 117 | 130 | 243 | 270 | 526.5 | 585 | 1053 | 1170 |
| 21 | 5 | 5 | 3 | 64-QAM | 2/3 | 156 | 173.3 | 324 | 360 | 702 | 780 | 1404 | 1560 |
| 22 | 6 | 6 | 3 | 64-QAM | 3/4 | 175.5 | 195 | 364.5 | 405 | N/A | N/A | 1579.5 | 1755 |
| 23 | 7 | 7 | 3 | 64-QAM | 5/6 | 195 | 216.7 | 405 | 450 | 877.5 | 975 | 1755 | 1950 |
| | 8 | 8 | 3 | 256-QAM | 3/4 | 234 | 260 | 486 | 540 | 1053 | 1170 | 2106 | 2340 |
| | 9 | 9 | 3 | 256-QAM | 5/6 | 260 | 288.9 | 540 | 600 | 1170 | 1300 | N/A | N/A |
| | | 10 | 3 | 1024-QAM | 3/4 | | | | | | | | |
| | | 11 | 3 | 1024-QAM | 5/6 | | | | | | | | |

# MIMO: Multiple TX-RX streams

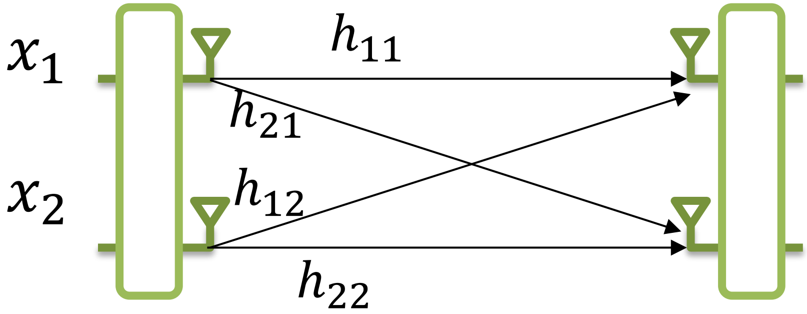
## WiFi Access Points 2 to 8 MIMO



## WiFi Devices 1 to 2 MIMO

- Power
- Form Factor & Antenna Separation
- Most mobile phones support 2 MIMO

# MIMO



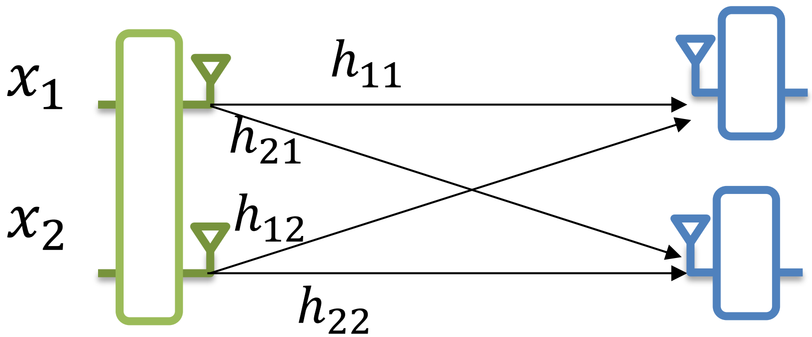

$$y_1 = h_{11}x_1 + h_{12}x_2$$

$$y_2 = h_{21}x_1 + h_{22}x_2$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{H}\mathbf{x} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

# Multi-User MIMO




$$y_1 = h_{11}x_1 + h_{12}x_2$$

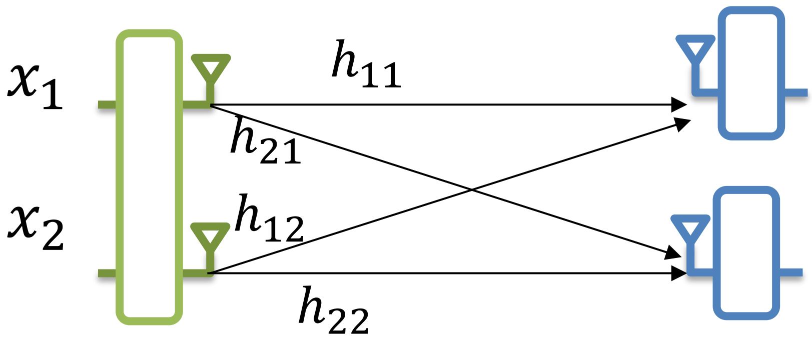
$$y_2 = h_{21}x_1 + h_{22}x_2$$

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{Hx} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

Does not work. Receivers do not have access to each other's signals

# Precoding




$y_1 = h_{11}x_1 + h_{12}x_2$

$y_2 = h_{21}x_1 + h_{22}x_2$

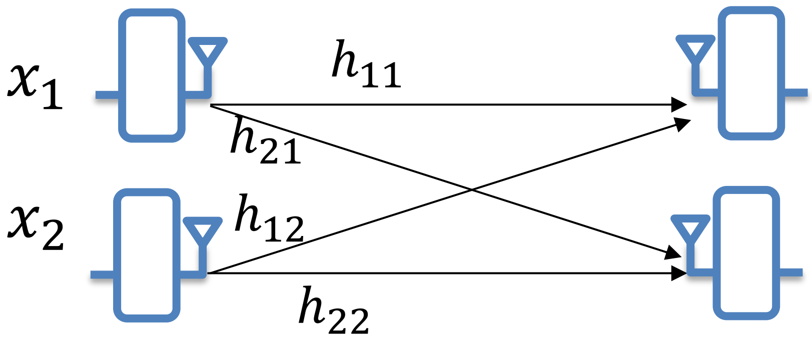
Send: $\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{x}$

Receive: $\mathbf{y} = \mathbf{H}\tilde{\mathbf{x}} + \mathbf{n} = \mathbf{H}\mathbf{H}^{-1}\mathbf{x} + \mathbf{n} = \mathbf{x} + \mathbf{n}$

**Also known as Beamforming or Zero-Forcing**

Requires Channel Feedback!

# What about distributed transmitters?




$$y_1 = h_{11}x_1 + h_{12}x_2$$

$$y_2 = h_{21}x_1 + h_{22}x_2$$

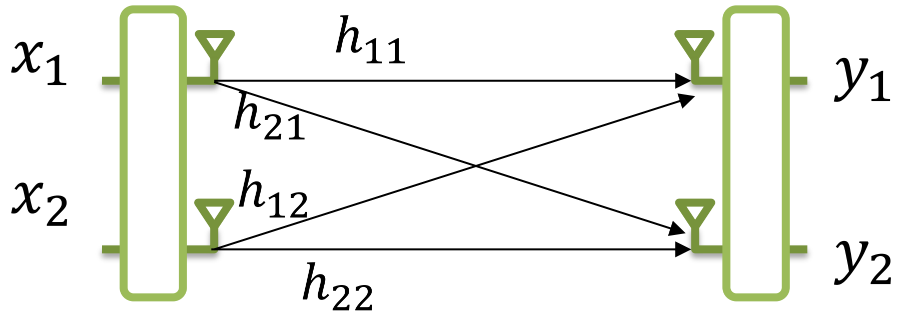
Send: $\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{x}$

Receive: $\mathbf{y} = \mathbf{H}\tilde{\mathbf{x}} + \mathbf{n} = \mathbf{H}\mathbf{H}^{-1}\mathbf{x} + \mathbf{n} = \mathbf{x} + \mathbf{n}$

Does not work. Transmitters do not have access to each other's signals

False: what if transmitters are APs connected over Ethernet?

# Clock Synchronization in MIMO

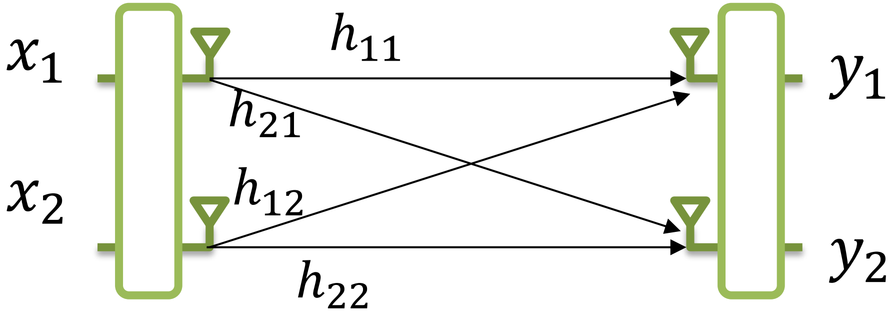



$$y_1 = h_{11}x_1e^{-j2\pi\Delta f_{11}t} + h_{12}x_2e^{-j2\pi\Delta f_{12}t}$$

$$y_2 = h_{21}x_1e^{-j2\pi\Delta f_{21}t} + h_{22}x_2e^{-j2\pi\Delta f_{22}t}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

# Clock Synchronization in MIMO

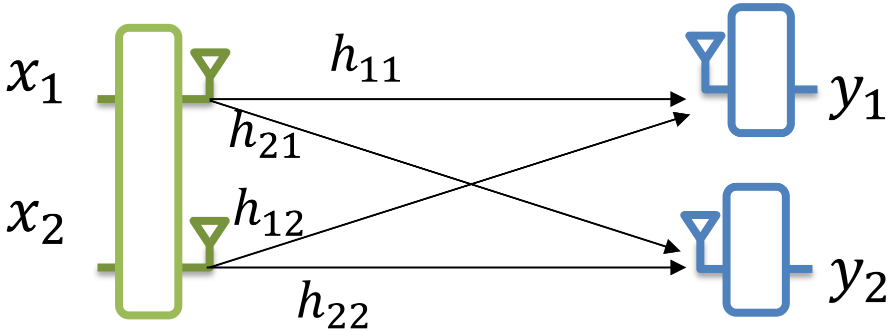



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{11}t} & h_{12}e^{-j2\pi\Delta f_{12}t} \\ h_{21}e^{-j2\pi\Delta f_{21}t} & h_{22}e^{-j2\pi\Delta f_{22}t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

**On chip MIMO: $\Delta f_{11} = \Delta f_{21} = \Delta f_{12} = \Delta f_{22} = \Delta f$**

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} e^{-j2\pi\Delta f t} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

# Clock Synchronization in MIMO

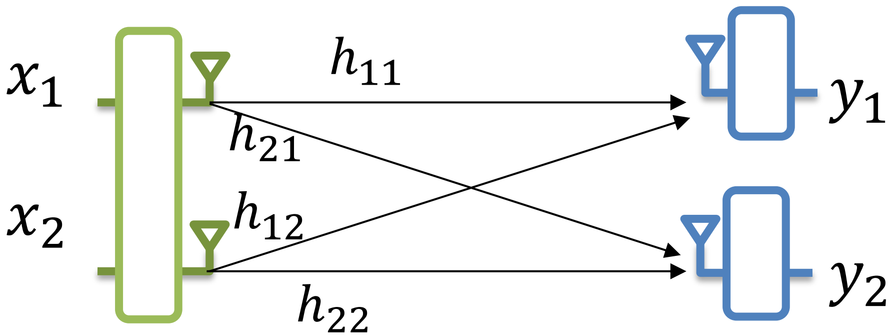



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{11} t} & h_{12} e^{-j2\pi\Delta f_{12} t} \\ h_{21} e^{-j2\pi\Delta f_{21} t} & h_{22} e^{-j2\pi\Delta f_{22} t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

**MU-MIMO: $\Delta f_{11} = \Delta f_{12}$ and $\Delta f_{21} = \Delta f_{22}$**

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_1 t} & h_{12} e^{-j2\pi\Delta f_1 t} \\ h_{21} e^{-j2\pi\Delta f_2 t} & h_{22} e^{-j2\pi\Delta f_2 t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

# Clock Synchronization in MIMO

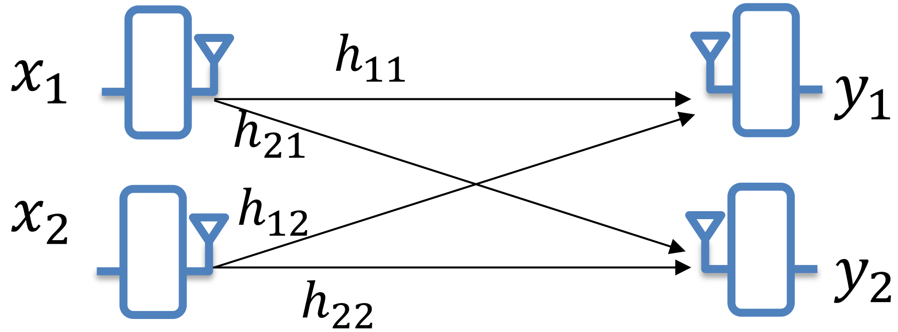



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{11} t} & h_{12} e^{-j2\pi\Delta f_{12} t} \\ h_{21} e^{-j2\pi\Delta f_{21} t} & h_{22} e^{-j2\pi\Delta f_{22} t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

**MU-MIMO: $\Delta f_{11} = \Delta f_{12}$ and $\Delta f_{21} = \Delta f_{22}$**

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} e^{-j2\pi\Delta f_1 t} & 0 \\ 0 & e^{-j2\pi\Delta f_2 t} \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

# Clock Synchronization in MIMO




$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{11}t} & h_{12}e^{-j2\pi\Delta f_{12}t} \\ h_{21}e^{-j2\pi\Delta f_{21}t} & h_{22}e^{-j2\pi\Delta f_{22}t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Distributed MIMO: $\Delta f_{11} \neq \Delta f_{12} \neq \Delta f_{21} \neq \Delta f_{22}$

Next Lecture!